Structural Breaks, Orders of Integration, and the Neutrality Hypothesis: Further Evidence

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Abstract

In this paper, we analyze the issue of the impact of multiple breaks on neutrality results, using annual data on real output and monetary aggregates for the US (1869-2000), the UK (1871-2000), and Mexico (1932-2000). In particular, we empirically verify, using the Fisher and Seater (*American Economic Review, 1993, 83(3), 402-415)*) tests, whether the monetary neutrality propositions remain addressable (and if so, whether they hold or not), when unit root tests are carried out allowing for (possibly) multiple structural breaks in the long-run trend function of the variables. It is found that conclusions on monetary neutrality are sensitive not only to whether there is a break or not, but also to the *number* of breaks allowed. In order to interpret the evidence for structural breaks, we utilize a notion of *deterministic* monetary neutrality, which naturally arises in the absence of permanent *stochastic* shocks to the variables.

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1 Introduction

Economists care about long-run monetary neutrality (LRN) because most theoretical models of money predict that money is neutral in the long-run; that is, the real effects of an unanticipated, permanent change in the level of money, tend to disappear as time elapses[2]. They also care about LRN because LRN is often used as an identification assumption (i.e. the large literature using Blanchard-Quah (1989) decompositions). On the other hand, the case for monetary superneutrality has limited theoretical support[3]. As summarized by Bullard (1999), "if monetary growth causes inflation, and inflation has distortionary effects, then long-run monetary superneutrality should not hold in the data. On the contrary, a permanent shock to the rate of monetary growth should have some long-run effect on the real economy; why else should we worry about it?" (p.59). In fact, central banks around the world pursue long-run price stability, due to the distortionary effects of inflation, caused by monetary growth (see the *Federal Reserve Bank of Kansas City Symposium* "Achieving Price Stability" (1996)).

Empirical results based on the reduced-form tests of Long Run Neutrality and Long Run Superneutrality (LRSN), derived by Fisher and Seatler (1993) (henceforth, FS), depend on the order of integration of both real output and the money aggregates. Identifying the order of integration of each of these variables is crucial for testing these long-run propositions. A number of recent papers examine the validity of these key macro propositions using long annual data and the reduced form tests of FS[4]. In this literature, the orders of integration are identified through the application of Augmented Dickey-Fuller (ADF) tests (see Dickey and Fuller (1979) and Said and Dickey (1984)) to real output and the money aggregates. For instance, LRN finds empirical support in the studies of Boschen and Otrok (1994, US data), Haug and Lucas (1997, Canadian data), Serletis and Krause (1996, international data set), Wallace (1999) and Noriega (2000) (Mexican data), and Bae and Ratti (2000 Brazilian and Argentinean data).

It is well known that long spans of data are very likely to include structural breaks, due to both domestic and external shocks, such as wars, economic crises, and changes in institutional arrangements. Under different methods to locate breaks, several empirical studies demonstrate the prevalence of (infrequent) parameter variation in the trend function of time series models of macroeconomic variables, as well as the impact of such structural breaks on unit root testing. In many applications, it is found that the order of integration for a time series is reduced once structural breaks are allowed[5]. This in turn affects results when

[2] Gottardi (1994) shows, on the other hand, that the phenomenon of non neutrality is associated with the effects of monetary policy on the assets' payoffs, due to incomplete markets.

[3] In the literature on monetary growth theory, there are very few available models which embody some form of monetary superneutrality. See for instance, Sidrauski (1967), Hayakawa (1995) and Faria (2001).

[4] Bae and Jensen (1999) examine these propositions by extending FS long-run neutrality requirements to long-memory processes. An alternative econometric perspective of LRN and LRSN is presented in King and Watson (1997).

[5] Empirical examples with macro time series can be found in Perron (1989, 1992, 1997),

testing economic propositions, such as LRN and LRSN. For instance, Serletis and Krause (1996) and Serletis and Koustas (1998), in testing neutrality using the Backus and Kehoe (1992) data set, find that the orders of integration of money and (particularly) real output do change (decline) when the Zivot and Andrews (1992) unit root test allows for a single structural break in the trend function.

Serletis and Koustas (1998) argue that the issue of whether neutrality results hold under the presence of structural breaks -an issue that has not been resolved yet in the literature- depends on how big shocks are treated. If they are treated like any other shock, then there is no need to account for them in interpreting neutrality results. If, on the other hand, they are regarded as (infrequent) big shocks that need to be accounted for, then conclusions on neutrality may change, because such shocks may induce lower orders of integration for output and money. FS use the convention that if a variable is stationary around a *linear* trend then it is treated as trend-stationary, that is, integrated of order zero. Extending FS's idea, one can say that if a variable is stationary around a *broken* trend then it is also integrated of order zero. This is precisely the interpretation followed by Serletis and Krause (1996), and Serletis and Koustas (1998). Under their approach, however, the number of structural breaks allowed in the deterministic trend function is fixed to one. This selection may not be inconsequential[6]. Furthermore, there are several recent methods that allow the estimation of the number of breaks using sample information.

In this paper, we analyze the issue of the impact of multiple breaks on neutrality results, using annual data on real output and monetary aggregates for the US (1869-2000), the UK (1871-2000), and Mexico (1932-2000). In particular, we empirically verify, using FS´s tests, whether the monetary neutrality propositions remain addressable (and if so, whether they hold or not), when unit root tests are carried out allowing for (possibly) multiple structural breaks in the long-run trend function of the variables. It is found that conclusions on monetary neutrality are sensitive not only to whether there is a break or not, but also to the *number* of breaks allowed. In order to interpret the evidence for structural breaks, we utilize a notion of *deterministic* monetary neutrality, which naturally arises in the absence of permanent *stochastic* shocks to the variables. For the UK for instance, LRN fails under linear trends, and becomes unaddressable under broken trends. However, it is interesting to note that, apart from the 1918 break, UK output's long-run trend remained unaltered,

Lumsdaine and Papell (1997), Ohara (1999), Mehl (2000), Noriega and De Alba (2001), and Gil-Alana (2002); Perron and Vogelsang (1992), Culver and Papell (1995), and Aggarwal et.al. (2000) for real exchange rates; Raj (1992) and Zelhorst and Haan (1995), for real output; Clemente, et. al. (1998) for interest rates. Theoretical and/or experimental results can be found in Perron (1989, 1997), Rappoport and Reichlin (1989), Reichlin (1989), Chen and Tiao (1990), Hendry and Neale (1990), and Noriega (1993).

[6] In a recent paper, Arestis and Biefang-Frisancho Mariscal (1999) conclude that "...unit root tests that do not account sufficiently for the presence of structural breaks are misspecified and suggest excessive persistence"(p.155). It is well documented by now that structural breaks in the trend function of macro series are responsible for the 'apparent' unit root behaviour which results from ignoring them in the model's specification.

even though two big shocks hit the level (1938), and level and trend (1970) of money. The absence of breaks in real output following these two shocks in money is what we refer to as "deterministic" LRN (with respect to the break in level), and LRSN (with respect to the second break). We have chosen this data set because it shows the extent to which breaks affect neutrality conclusions. Results range from no effect of breaks (US), to reversing conclusions regarding monetary neutrality (UK, Mexico). It also offers the possibility of formulating the notion of deterministic neutrality[7].

We start by applying the asymptotically consistent sequential unit root testing procedure introduced by Pantula (1989), in order to identify the number of unit roots in each individual series. In this way we estimate the number of unit roots, instead of simply applying Dickey-Fuller tests based on the assumption of at most one unit root. We then analyze the behavior of these orders of integration under different trend specifications, which allow for an increasing number of structural breaks in the long-run trend function under the alternative hypothesis. Note that under broken trend-stationary models, permanent changes are deterministic, as opposed to stochastic. This allows the possibility of investigating any potential relationship between the estimated break dates and historic events. The identified breaks can be analyzed through careful examination of the particular economic and political environment surrounding them.

We arrive at different conclusions in relation to previous empirical findings. Our results, all based on bootstraped critical values, indicate, for instance, that LRN and LRSN are not addressable for the US and the UK. On the other hand, using a notion of deterministic neutrality, both LRN and LRSN hold for the UK.

The rest of the paper is organized as follows. Next section briefly presents the FS reduced-form tests for LRN and LRSN of money. Then, a description of the methods for testing stationarity while allowing for an unknown number of structural breaks in the trend function of the data is provided. Our econometric methodology is based on procedures and methods of Bai (1997b), Bai and Perron (1998a, b), and Noriega (1999). Section 3 reports the empirical results of applying Pantula's (1989) procedure, and of testing for unit roots under multiple structural breaks to the extended Friedman-Schwartz (1982) and Wallace (1999) data sets (see appendix). Section 4 offers some discussion of the results, in which it is suggested a deterministic interpretation of monetary neutrality, and presents some conclusions.

2 Econometric Methodology

2.1 Tests of LRN and LRSN

Consider the following stationary invertible bivariate Vector Autoregression (VAR) in money (m) and output (y):

[7] Details on the sources of data are presented in the Appendix.

$$
\begin{aligned}
a(L)\Delta^{\langle m\rangle}m_t &= b(L)\Delta^{\langle y\rangle}y_t + u_t \\
d(L)\Delta^{\langle y\rangle}y_t &= c(L)\Delta^{\langle m\rangle}m_t + w_t
\end{aligned} \tag{1}
$$

where $a(L)$, $b(L)$, $c(L)$ and $d(L)$ are polynomials in the lag operator L, with $a_0 = d_0 = 1$, $\Delta = (1-L)$, and the symbol $\langle x\rangle$ stands for the order of integration of x; i.e. $\langle x\rangle = 1$, means that x is integrated of order one. The solution, or impulse-response representation of system (1) is given by:

$$
\begin{aligned}
m_t &= \Delta^{\langle -m\rangle}\left[\alpha(L)u_t + \beta(L)w_t\right] \\
y_t &= \Delta^{\langle -y\rangle}\left[\gamma(L)u_t + \delta(L)w_t\right]
\end{aligned} \tag{2}
$$

where $\alpha(L) = d(L)/\det A$, $\beta(L) = b(L)/\det A$, $\gamma(L) = c(L)/\det A$, $\delta(L) = a(L)/\det A$, with $\det A = a(L)d(L) - c(L)b(L)$. As in FS, the neutrality of money is measured through the long-run elasticity, or Long-Run Derivative (LRD) of output with respect to permanent stochastic exogenous changes in money:

$$
LRD_{y,m} \equiv \lim_{k\to\infty} \frac{\partial y_{t+k}/\partial u_t}{\partial m_{t+k}/\partial u_t} \tag{3}
$$

where u represents a zero-mean stationary stochastic process. The limit of the ratio in (3) measures the ultimate effect of a (stochastic) monetary disturbance on real output relative to that disturbance's ultimate effect on the monetary variable. The definitions used by FS of LRN and LRSN are as follows:

LRN: Money is long-run neutral if $LRD_{y,m} = \lambda$, where $\lambda = 1$ when y is a nominal variable and $\lambda = 0$ when y is a real variable or the nominal interest rate.

LRSN: Money is long-run superneutral if $LRD_{y,\Delta m} = \mu$, where $\mu = 1$ when y is the nominal rate of interest and $\mu = 0$ when y is a real variable.

As noted in FS, the definition of LRSN only applies to those variables y for which LRN implies $LRD_{y,m} = 0$. Using (2), it is easy to show that the computation of the *LRD* depends on the order of integration of each variable:

$$
LRD_{y,m} = \frac{(1-L)^{\langle m\rangle - \langle y\rangle}\gamma(L)\mid_{L=1}}{\alpha(1)}
$$

From this formula it is straightforward to derive the relevant values for λ and μ under LRN and LRSN, summarized in Table 1

TABLE 1

Clearly, the order of integration of money should be at least equal to one for LRN to make sense, otherwise there are no stochastic permanent changes in money that can affect real output. A similar logic applies to LRSN. On the other hand, there are no permanent changes to analyze when the order of integration of real output is zero. For the cases when $\langle y \rangle < \langle m \rangle$, the determination of LRN is immediate. This discussion makes clear that proper determination of the orders of integration of y and m is crucial in assessing LRN and LRSN of money, and we turn to this issue in the next subsection. For the other cases in the Table, an estimate of $c(1)/d(1)$ is given by $\lim_{k\to\infty} b_k$, where b_k is the coefficient from the OLS regression

$$\left[\sum_{j=0}^{k} \Delta^{\langle y \rangle} y_{t-j}\right] = a_k + b_k \left[\sum_{j=0}^{k} \Delta^{\langle m \rangle} m_{t-j}\right] + \varepsilon_{kt}. \tag{4}$$

2.2 Unit Root testing under multiple structural breaks

Recent literature on multiple structural change (Bai (1997a,b), Bai and Perron (1998a,b)) studies the issue of estimating the number of breaks based on a parameter constancy test in a *sequential* fashion. In these papers, the criterion is to add structural breaks to the model until the null hypothesis of parameter constancy is not rejected[8]. Bai (1999) proposes a Likelihood Ratio-type test for multiple structural changes. As opposed to the tests mentioned above, this test *globally* identifies multiple breaks. Nevertheless, both test procedures rely on the same parameter constancy principle. In general, the phenomenon of structural change can be related to different forms of nonstationarity[9]: shifts in mean or variance of a discrete-parameter stochastic process, changes in the ARMA structure of the process, changes in the probability law of the ARMA errors, or unit roots in AR models. As Granger and Newbold (1986) argue, "there are unlimited ways in which a process could be nonstationary" (p.38). Hence, the stopping rule of parameter constancy is only one possibility among many. In this paper we establish a relationship between the phenomenon of structural breaks and unit root behavior. Broadly speaking (see below for details), the stopping rule we use is: stop adding breaks when the *unit root* form of nonstationarity is rejected, instead of: stop adding breaks when the *parameter variation* form of nonstationarity is rejected. We call this the Unit Root Rejection-Stopping Rule (URR-SR). When the true data generating process is of the trend-stationary class with deterministic breaks, this criterion allows the possibility of separating stationary cyclical movements from a long run broken trend. In other words, it allows the identification of those dates which are responsible for (apparent) unit root behavior.[10]

[8] In Bai (1997a) and Bai and Perron (1998a) it is shown that this stopping rule yields a consistent estimation of the true number of breaks, provided the size of the test slowly converges to zero.

[9] See for instance the Monte Carlo experiment in Epps (1988).

[10] Monte Carlo results in Noriega (1999) show advantages of the unit root rejection stopping rule over the parameter constancy- and minimum BIC- stopping rules in identifying the true

We present the procedure for testing the presence of a unit root with an unknown number of structural breaks in the deterministic trend function, making use of the URR-SR. Let us start with the no break case, denoting by Y_t the logarithm of the observed series. The first step is to estimate (by OLS) the following Trend Stationary (TS) and Difference Stationary (DS) models, respectively:

$$\Delta Y_t = \mu + \beta t + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t \tag{5}$$

$$\Delta Y_t = \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t, \tag{6}$$

for $t = 1, 2, \ldots, T$, where T is the sample size and ε_t is an *iid* process. In the TS model (5), $\alpha < 0$, so that Y_t generates stationary fluctuations around a deterministic linear trend. When $\alpha = 0$ (the null hypothesis), then Y_t does not generate stationary cycles. The DS specification in (6) represents the latter, where no deterministic components are considered. The reason is that interest centers on the autoregressive parameter and its associated t-statistic estimated from (5), both of which are invariant with respect to the parameters μ and β for any sample size[11]. The determination of the autoregressive order is discussed below.

Next we simulate, as in Rudebusch (1992), and Diebold and Senhadji (1996), the distribution of the t-statistic for the null hypothesis of a unit root ($\alpha = 0$ in (5)), called $\widehat{\tau}$, under the hypotheses that the true models are the TS model (5) and the DS model (6), both estimated from the data[12]. That is, under the TS (DS) model we use the estimated parameters from (5)((6)), and the first $k+1$ observations as initial conditions ($\Delta Y_2, ..., \Delta Y_{k+1}$) to generate 10,000 samples of ΔY_t, $t = 2, ..., T$, with randomly selected residuals (with replacement) for each ΔY_t, $t = k+2, ..., T$ from the estimated TS (DS) model. For each sample thus generated, regression equation (5) is run and the corresponding 10,000 values of $\widehat{\tau}$ are used to construct the empirical density function of this statistic under the TS (DS) model, labeled $f_{TS}(\widehat{\tau})$ ($f_{DS}(\widehat{\tau})$)[13].

Now consider a Broken Trend Stationary (BTS) model with m structural breaks in both level and trend:

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t, \tag{7}$$

number of structural breaks in the trend function, for the cases of one, two, and three structural breaks, and lower values of the autoregressive parameter (≤ 0.5).

[11] The same invariance holds when considering below alternative hypotheses allowing for structural breaks, see for example Perron (1989, p.1393).

[12] A similar apprach is used by Kuo and Mikkola (1999) for the US/UK real exchange rate series.

[13] The 10,000 fitted regressions utilize the estimated value of k (see below), under the TS (DS) model. All calculations were carried out in GAUSS 3.6.

where DU_{it} and DT_{it} are dummy variables allowing changes in the trend's level and slope respectively, that is, $DU_{it} = \mathbf{1}(t > T_{b_i})$ and $DT_{it} = (t - T_{b_i})\mathbf{1}(t > T_{b_i})$, where $\mathbf{1}(\cdot)$ is the indicator function and T_{b_i} is the unknown date of the ith break. This equation is a generalization to m breaks of the Innovational Outlier Model, used by Perron (1989) and others[14]. For reasons explained above, the relevant DS model is again, for each value of m, equation (6).

In order to simulate, via Monte Carlo, the empirical densities of $\widehat{\tau}$ under the estimated BTS DGP (7) and DS DGP (6), $f_{TS_m}(\widehat{\tau})$, $(m = 1, 2, ...)$ and $f_{DS}(\widehat{\tau})$, respectively, we need an estimation of the number (m) and location (T_{b_i}) of breaks. Once these estimates are obtained (see below), the Monte Carlo experiments for generating $f_{TS_m}(\widehat{\tau})$ and $f_{DS}(\widehat{\tau})$, follow the same steps as in the no breaks case.

In order to determine the location of breaks, we use the criterion which selects the break dates, from all possible combinations of m break dates, that minimize the residual sum of squares from (7). The occurrence of a break has to be restricted to the following intervals. For $m = 1$, $k+1+h \leq T_{b_1} \leq T - mh$, for the two breaks case, $k+1+h \leq T_{b_1} \leq T - mh$ and $T_{b_1} + h \leq T_{b_2} \leq T - (m-1)h$, for the three breaks case, $k+1+h \leq T_{b_1} \leq T - mh$, $T_{b_1} + h \leq T_{b_2} \leq T - (m-1)h$, and $T_{b_2} + h \leq T_{b_3} \leq T - (m-2)h$, etc., where h represents the smallest possible size for an interval or segment[15]. This criterion is called $\min RSS$.[16] Note that this criterion implies simultaneous determination of m breaks via a global search.

We follow Noriega and De Alba (2001) in the determination of the type of breaks and lag length allowed in the Innovational Outlier Model (7).[17] We first fix an arbitrary maximum value for k, labeled $k\max$. Then we estimate equation (7) with OLS for each of the three types of Innovational Outlier models (change in level only, change in level and trend, and change in trend only), over all possible values of T_{b_1}, and choose, for each model, the break date for which the residual sum of squares (RSS) is minimized, as explained above. The Akaike Information Criterion (AIC) is then calculated for each of the three regressions corresponding to the estimated break dates. If the coefficient on the $k\max^{th}$ lag is not significant for the model which yields the smallest AIC, then we estimate the three versions of equation (7) again, over all possible values of T_b with $k\max - 1$ lags of the differenced dependent variable. Again, we choose the break date corresponding to the smallest RSS, and compute the AIC for the three regressions corresponding to the newly estimated break

[14] The only difference is that (7) does not include a pulse variable, called $D(TB)_t$ by Perron (1989). This is also the approach in Zivot and Andrews (1992).

[15] This representation for h is based on the dynamic programming algorithm introduced by Bai and Perron (1998b) to obtain global minimizers of the RSS.

[16] This criterion for estimating break points is discussed in Bai (1997a,b), and Bai and Perron (1998a,b).

[17] Perron (1993) argues that, although a model allowing changes in both level and slope of trend is the most general one (it encompasses models with breaks in level alone, or with breaks in slope alone), there are power gains by estimating a model without irrelevant regressors. For example, model (7) with $\theta_i = 0$ would be more appropriate if it were apparent from the data that the type of break involved no change in level but only in trend.

dates. Continuing in this fashion, we select the combination '*model type/lag length*' which corresponds to the model which yields the smallest value of the AIC (amongst the three models) *and* a corresponding significant lag (called $\hat{k}$), using a two-sided 10% test based on the asymptotic normal distribution. Note that if there are no significant lags, then $\hat{k} = 0$, which implies an AR(1) model for equation (7). If this is the case, the selection of the model follows simply from the lowest value of the AIC.

In order to determine the number of breaks, we equip the above procedure with the URR-SR, which indicates the termination of the search. Under the URR-SR, we proceed sequentially: after we estimate equation (5), the relevance of both the null (a unit root) and alternative (a TS model with $m = 0$) hypotheses is analyzed in terms of the position where the sample estimate of the t-statistic for testing a unit root ($\overset{\wedge}{\tau}_{sample}$) lies relative to the empirical densities of $\widehat{\tau}$ under the estimated BTS DGP (7) and DS DGP (6), $f_{TS_m}(\widehat{\tau})$, $m = 0, 1, ...$, and $f_{DS}(\widehat{\tau})$, respectively. These positions are calculated as the probability mass to the left of $\overset{\wedge}{\tau}_{sample}$, denoted $p_{TS_m} \equiv \Pr[\widehat{\tau} \leq \widehat{\tau}_{sample} \mid f_{TS_m}(\widehat{\tau})]$ and $p_{DS} \equiv \Pr[\widehat{\tau} \leq \widehat{\tau}_{sample} \mid f_{DS}(\widehat{\tau})]$. If as a result it is concluded that the null can not be rejected, or that it is not possible to discriminate between hypotheses, then we allow the procedure to search and locate one structural break, and again the relevance of both the null of a unit root (model (6)) and the alternative of a BTS model with a single structural break (model (7) with $m = 1$) is analyzed. This process continues until the null hypothesis is rejected and the alternative is most supported by the data, for any number of structural breaks. If this happens, we suggest analyzing the results from allowing one additional break. That is, comparing the relevance of both the null and alternative hypotheses under two different trend specifications. If a conflict should arise between two consecutive values for m, the final decision is made on the basis of which of the consecutive values of p_{TS_m} is closer to the middle of the corresponding empirical distribution. As can be seen, this is a sequential procedure which globally searches for an increasing number of structural breaks.

3 Neutrality and Superneutrality Results

As mentioned above, conclusions about long run monetary neutrality depend on the order of integration of real output and money. Table 2 collects results from the application of Pantula's (1989) procedure for determining the number of unit roots in each series, for the three countries. We allow a maximum of three unit roots in each variable, and perform unit root tests downwards, starting with a test of the null hypothesis H_3: exactly three unit roots (or a unit root in the second differences of the data). We then test the null H_2: exactly two unit roots, against the alternative H_1: one unit root in the autoregressive representation of the series. If H_3 and H_2 are rejected, we then test for the presence of a single

unit root, H_1, against the trend-stationary alternative.[18]

TABLE 2

As can be seen, both real GDP and $M1$ for Mexico are $I(1)$, while $M2$ is $I(2)$. This implies that LRN is testable for $M1$ while LRSN is testable for $M2$. The upper panel of Figure 1 shows LRN results for $M1$, while the lower panel LRSN results for $M2$. The graphs show that the 95-percent confidence interval around $\widehat{b}_k$, estimated from equation (4), indicates rejection of both LRN for $M1$ and LRSN for $M2$. Note that LRN for $M2$ holds. For the UK, both real GDP and the money stock are $I(1)$, making LRN testable. As can be seen from Figure 2, the 95-percent confidence interval around $\widehat{b}_k$ includes 0 for $k \leq 27$, and positive values for $k \geq 28$, suggesting that LRN does not hold. For the US, real output is $I(0)$, while the money stock rejects a unit root the 5% level, but not at the 1% level. If we assume the later, LRN holds, otherwise it is unaddressable, since the long-run derivative is undefined (see Table 1). Results are summarized in Table 3.

TABLE 3

We now review these results in the light of (endogenously determined) structural breaks in the long-run trend function of each variable. FS use the convention that if a variable is stationary around a linear trend then it is treated as trend-stationary, that is, integrated of order zero. Extending this idea, we argue that if a variable is stationary around a broken trend then it is also integrated of order zero. We discuss below the implication of such extension.

We begin by testing the null of a unit root against the alternative hypotheses of a TS specification allowing an increasing number of $m \geq 0$ structural breaks using the unit-root rejection stopping rule. Results are given in Tables 4 and 5. The first column in each table indicates the number of breaks allowed in the trend function, m. The second column refers to the estimated lag length, $\hat{k}$. In the empirical applications $k\max$ is set at 5. The next columns report the estimated break dates. The type of break allowed in the trend function is reported in parenthesis. Column labeled AC reports the $p-$values for the Lagrange Multiplier test of the null hypothesis that the disturbances are serially uncorrelated against the alternative that they are autocorrelated of order one. The next column reports the value of the $t-$statistic for testing the null hypothesis of a unit root, estimated from equation (7). The probabilities under each of the simulated DS and BTS specifications are presented in the last two columns of the table.

[18] Pantula (1989) shows that this is an asymptotically consistent sequential procedure for testing the *number* of unit roots present in the data, starting from an arbitrary upper value.

TABLE 4

TABLE 5

Results for Mexico (Table 4) point to a broken-trend stationary model for real output with three structural breaks, in 1953, 1981, and 1994; p_{TS_3} lies close to the middle of the distribution (0.6), while p_{DS} indicates that it is unlikely that the estimated $t-$statistic for testing a unit root in real GDP could have been generated by a DS model. Note that the associated probabilities for $m = 0$, $m = 1$ (break in 1981), and $m = 2$ (breaks in 1953 and 1981), do not indicate rejection of the DS model. Although the DS null is also rejected for the four breaks model, the alternative of three breaks seems to be more plausible (p_{TS_3} lies closer to the centre of the distribution). As far as monetary aggregates is concerned, we analyze the (log) levels of $M1$, and the first differences of $M2$, since two unit roots could not be rejected for this series from the application of the Pantula (1989) procedure[19]. Results for $M1$ in Table 5 indicate that it is not possible to discriminate between a unit root process and a stationary alternative with (up to) five breaks. Note that the case of four breaks clearly rejects the null, but the alternative is not supported either. Therefore, we maintain the hypothesis of an $I(1)$ process for $M1$. The picture for $\Delta M2$ is quite different. As can be seen, the unit root can be rejected for all the broken trend cases analyzed. A stationary model with two breaks (1976 and 1987, both significant at conventional levels) however, seems to be the most plausible one, according to the reported probabilities. Hence, under the convention that if a variable is stationary around a broken trend then it is integrated of order zero, we conclude that for $M1$, LRN holds and LRSN is unaddressable, while for $M2$, both LRN and LRSN hold, since detrended output is $I(0)$ while detrended $M1$ is $I(1)$, and detrended $M2$ is still $I(2)$. [20]

For US real output (Table 4), the unit root is strongly rejected without any structural break in the long-run trend function, implying that fluctuations around a linear trend spanning 133 years, have been stationary[21]. Note that the procedure does detect a (significant) change in the level of trend in 1929, and the corresponding model displays slightly better probabilities than the linear TS

[19] We also applied the URR-SR to the (log) *levels* of $M2$, for an increasing number of structural breaks $m = 0, 1, 2, 3, 4, 5$ and 6. In no case was it possible to discriminate between the DS and the BTS hypotheses. This implies that the order of integration for $M2$, $I(2)$, cannot be reduced to a BTS ($I(0)$) process with (up to 6) structural breaks.

[20] There is another possiblitity. Since detrended $\Delta M2$ is $I(0)$, one can write $\Delta M2 \sim I(0) + \boldsymbol{breaks}$, or $M2 \sim I(1) + \boldsymbol{breaks}$. If this was the case, then $M2 - \boldsymbol{breaks} \sim I(1)$, in which case LRSN would become unaddressable. This is equivalent to assuming that the two unit roots in $M2$ can be removed by a combination of detrendeing and (first) differencing.

[21] Results in Diebold and Senhadji (1996) also point to the rejection of a unit root for the case of no breaks. Murray and Nelson (2000) apply a battery of unit root tests to the real output series of Maddison (1995), to find a strong rejection of the unit root hypothesis against both linear and broken trends. They, however, call this inference into question on the grounds of size distortions due to the endogeneity of the lag length, and potential heterogeneity of the process generating real output.

model. For the US money aggregate (Table 5), the unit root is again rejected (at the 2.2% level) for the case of no breaks. However, the (significant) drop in level detected in 1928 allows both a stronger rejection of the null, and a more plausible model under the alternative. It is interesting to note that neither of these two breaks were big enough so as to induce unit root behavior in the data. Thus, both output and money are stationary, implying that monetary neutrality and superneutrality are unaddressable.

For UK real output in Table 4, the unit root can not be rejected against the alternative of trend-stationarity without breaks. When we allow for a drop in level and an increase in slope of trend in 1918, the unit root is strongly rejected[22]. Note that allowing for a drop in level as a second break (1902), does not change the results. For the money aggregate (Table 5), the unit root is rejected after two breaks are estimated: 1938 (level only), and 1970 (level and slope). This two-breaks specification is the most supported by the data, according to the reported probabilities. Hence, when no breaks are allowed in the trend function, LRN is testable and, according to Figure 2, it only holds for a period of 26 years, after which the effect of the permanent shock to money on output becomes positive. Allowing for breaks, LRN becomes unaddressable, since there are not permanent stochastic changes in money.

We gather results in Table 6. As can be seen, long-run (stochastic) monetary neutrality and superneutrality are both unaddressable propositions for the US, whether we allow for breaks or not. The same result holds for the UK under the presence of structural breaks. For Mexico, the long-run impact of money on output does depend on whether we allow for the presence of structural breaks. Under the later, both money aggregates are long-run neutral, and $M2$ is superneutral.

TABLE 6

Our results conflict with the ones obtained by Boschen and Otrok (1994, US), Serletis and Krause (1996, US and UK), Bae and Jensen (1999, UK), Wallace (1999) and Noriega (2000) (both Mexican data), and some others surveyed in Bullard (1999). We believe that the differences in the results arise, by and large, because the above papers find higher orders of integration of the relevant variables. Next section offers a different interpretation of the above results, which again lead to different conclusions on monetary neutrality.[23]

[22] These results are in line with those obtained by Duck (1992) and Noriega (1992).

[23] Since long-run neutrality tests are inefficient in the presence of cointegration (see King and Watson 1992), we tested for cointegration using Johansen's ML approach (Johansen, 1991, 1995). Although our results show some conflict among the Trace test and the Max-eig test, they generally confirm those of Serletis and Koustas (1998): there is no cointegration between money and output for the US and the UK. The same coclusion was reached for the case of Mexico.

4 Discussion and Concluding Remarks

This paper empirically documents the impact of (endogenously determined) changes in the long-run trend of money and real output, on the Fisher and Seater (1993) tests of LRN and LRSN, using long annual data for Mexico, the US and the UK. It was found that conclusions on monetary neutrality are sensitive to the number of breaks allowed in the long-run trend of the relevant variables, and generally conflict with previous results reported in the literature. For Mexico for instance, the broken trend-stationary model found for real output, together with an $I(2)$ process for $M2$, leads one to conclude that superneutrality holds for this money aggregate, contrary to the findings in Noriega (2000).

The issue remains of whether real output is neutral (and superneutral) to the deterministic shifts identified in the trend function of money. In order to interpret the results, we utilize an heuristic notion of deterministic monetary neutrality, which naturally arises in the absence of permanent stochastic shocks to the variables.

For the UK for instance, there is no support for LRN when no breaks are allowed. On the other hand, when allowing for breaks, LRN becomes unaddressable, since both output and money become (broken) trend-stationary and, therefore, $I(0)$ processes. It is interesting to note, however, that even though two upward shifts in the trend function of the money aggregate were identified (which eliminated the unit root behavior of the data), the long-run deterministic trend of output does not show any kind of change after these breaks took place, making itself neutral to these two monetary shifts. This seems to indicate that some form of deterministic LRN holds for the UK economy. Furthermore, since the long-run trend function of output did not change after the break *in trend* of money occurred in 1970, one could conclude that deterministic LRSN holds for the UK, for the last 30 years[24]. Hence, permanent shocks to the rate of monetary growth did not translate into distortionary effects on the UK real economy. For the US, although both monetary propositions are unaddressable, real output underwent a small but significant drop in level (1929) just after a similar shock was detected in the monetary aggregate (1928).

For Mexico, even though the evidence (with or without breaks) seems to indicate that $M2$ is neutral, the identified changes in money growth (1976, 1987) could be behind the breaks found in the long-run trend of real output (1981, 1994). As far as the money aggregates is concerned, 1970 saw the end of the "Stabilizing Development" economic strategy, supplanted by a populist development strategy, which included the second (for $M1$) and third (for $M2$) largest upward trend shift in money of the century[25]. The resulting inflation and the rigid exchange rate policy, lead to a 76% peso devaluation between 1976 and 1977. In theory (see Marty 1994), anticipated inflation would lead people to economize on real balances, affecting the payment matrix and, therefore, the allocation of resources. After the (second) break in 1987, $M2$ growth rapidly

[24] It is illustrative to note that the coefficient of the trend function of UK money multiplied by nearly 5 after 1970.

[25] The first one ocurred around 1931, see Noriega and De Alba (2001).

declined, lowering the inflation rate, thus inducing the representative agent to devote less leisure time in acquiring commodities. This again affects the allocation of resources. Therefore, it is theoretically possible that these breaks had an influence on those registered for real output in 1981 and 1994. If this was the case, then it could be argued that permanent deterministic breaks in aggregate money are neither neutral nor superneutral for Mexico.[26] These phenomena can be analyzed using the recently developed theory (and testing for) co-breaking, introduced in Hendry and Mizon (1998), and Clements and Hendry (1999, chapter 9). Based in this, the reduced rank technique developed by Krolzig and Toro (2000) yields information on how breaks are related through economic variables and across time. We hope to report results in this direction in a separate paper.

Our results suggest that a distinction should be made between reactions to deterministic and stochastic shocks. The FS test measures the correlation between permanent stochastic shocks in money and output data. Our findings suggest that it could be useful to broaden the notion of LRN by allowing for deterministic and stochastic LRN. For the UK for instance, stochastic LRN does not hold, or is unaddressable (allowing for breaks), while deterministic LRN does.

Finally, the use of smooth transition models (Leybourne, et.al. (1998), Sollis, et.al. (1999)) to test for a unit root would help refining our results, since the break dates in these models are not restricted to be instantaneous, but allowed to occur along a number of periods in a smooth way. This might shed some light on the issue of co-breaking for the US and Mexico, by establishing more accurately the beginning and end of the breaks found for money and real output.

[26] It is interesting to note that the 1953 upward trend break in real output coincides with the reduction in $M2$ volatility. When monetary volatility went up again in the late 70s, real output experienced a persistent slowdown, from 1981 onwards. A related phenomenon is documented in Ramey and Ramey (1995), who find that countries with high public expenditure volatility have lower growth. Santaella (1998) analized possible causes of the Mexico production slowdown starting in 1982. He argues that the evolution of both public spending and the inflation rate are consistent with the hypthesis that macroeconomic instability caused by expansionary policies is behind the slowdown in real GDP. See Santaella for further discussion.

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Table 1

Values for λ and μ under LRN and LRSN

	$LRD_{y,m}$			$LRD_{y,\Delta m}$		
$\langle y \rangle$	$\langle m \rangle = 0$	$\langle m \rangle = 1$	$\langle m \rangle = 2$	$\langle m \rangle = 0$	$\langle m \rangle = 1$	$\langle m \rangle = 2$
0	undefined	$\equiv 0$	$\equiv 0$	undefined	undefined	$\equiv 0$
1	undefined	$c(1)/d(1)$	$\equiv 0$	undefined	undefined	$c(1)/d(1)$

Source: Adapted from Fisher and Seater (1993).

Table 2

Order of Integration of Real GDP and Money Aggregates

Regression:

$$\Delta^r X_t = \mu + \beta t + a_r \Delta^{r-1} X_{t-1} + \sum_{j=1}^{k} b_j \Delta^r X_{t-j} + \varepsilon_{t;} \quad r = 1, 2, 3.$$

Variable	$H_3(\mu = \beta = 0)$	$H_2(\beta = 0)$	H_1
Mexico (1932-2000)			
Real *GDP*	-6.17** (4)	-6.26** (0)	-0.09 (0)
$M1$	-8.86** (1)	-3.89** (0)	-1.41 (3)
$M2$	-9.32** (1)	-2.01 (2)	—
United States. (1869-2000)			
Real *GDP*	-10.17** (4)	-6.75** (4)	-4.38** (1)
M	- 7.87** (4)	-6.24** (1)	-3.78* (5)
United Kingdom. (1871-2000)			
Real *GDP*	-8.09** (5)	-7.41** (2)	-1.67 (3)
M	-6.58** (5)	-3.74** (1)	-0.94 (2)

Notes: M for the US and the UK stands for monetary stock.
* and ** stand for significant at the 5%, and 1% levels, respectively. Results for Mexico are taken from Noriega (2000).

Table 3

Summary of LRN and LRSN Results

Country	LRN	LRSN
Mexico		
$M1$	fails	unaddressable
$M2$	holds	fails
UK	fails	unaddressable
USA	unaddressable*	unaddressable

* Result obtained since M is $I(0)$ at the 0.05 level. At the 0.01 level, however, M is $I(1)$, in which case LRN holds.

Table 4

Broken Trend Models for Real *GDP*

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t$$

m	$\hat{k}$	$Tb1$ $Tb2$ $Tb3$	AC	$\hat{\tau}_{sample}$	p_{TS_m}	p_{DS}
		Mexico (1932-2000)				
0	1		.969	−0.45	.943	.960
1	0	1981(L)	.950	−2.95	.746	.266
2	3	1953(T) 1981(LT)	.634	−4.09	.967	.174
3	5	1953(T) 1981(T) 1994(LT)	.759	−8.16	.603	.000
4	3	1953(T) 1981(T) 1985(L) 1994(LT)	.463	−9.25	.785	.000
		United States (1869-2000)				
0	1		.765	−4.38	.683	.002
1	1	1929*(L)*	.709	−5.85	.674	.000
		United Kingdom (1871-2000)				
0	3		.927	−1.67	.933	.742
1	1	1918*(LT)*	.913	−9.14	.764	.000
2	1	1902(L) 1918(L)	.778	−9.77	.763	.000

*L, T , and LT stand for: level, trend, and level and trend, respectively.

Table 5

Broken Trend Models for Real Money Supply

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t$$

m	$\hat{k}$	$Tb1$ / $Tb2$ / $Tb3$ / $Tb4$ / $Tb5$	AC	$\widehat{\tau}_{sample}$	p_{TS_m}	p_{DS}
		Mexico (1933-2000). "*M1*"				
0	3		.984	−1.40	.867	.737
1	1	1991(L)	.384	3.45	1.00	1.00
2	4	1971(T) 1991(LT)	.869	−2.52	.702	.187
3	4	1944(LT) 1971(T) 1991(LT)	.174	0.642	.598	.974
4	1	1942(LT) 1974(T) 1982(LT) 1991(T)	.726	−10.1	.929	.000
5	3	1942(LT) 1948(LT) 1971(T) 1986(LT) 1993(LT)	.378	−0.63	.765	.874
		Mexico (1933-2000). "*M2*" in first differences.				
0	2		.590	−1.79	.940	.683
1	0	1980*(LT)*	.800	−6.13	.883	.000
2	0	1976(T) 1987(LT)	.714	−8.06	.569	.000
3	5	1959(LT) 1977(T) 1986(LT)	.890	−6.22	.996	.000
		United States (1869-2000). Logs. of Real Money Supply.				
0	5		.782	−3.77	.683	.022
1	5	1928*(L)*	.671	−4.88	.657	.002
2	5	1928(LT) 1932(LT)	.462	−3.80	.899	.143
		United Kingdom (1871-2000) Logs. of Real Money Supply.				
0	2		.926	−0.94	.839	.902
1	2	1970*(LT)*	.911	−3.12	.853	.227
2	1	1938(L) 1970(LT)	.144	−5.53	.833	.002
3	1	1913(L) 1939(LT) 1970(LT)	.480	−7.57	.917	.000

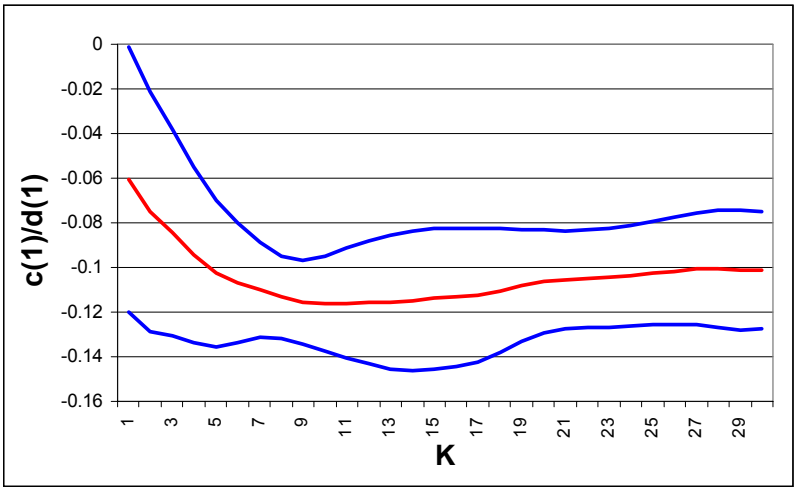
*L, T, and LT stand for: level, trend, and level and trend, respectively.

Table 6

Summary of LRN and LRSN Results

	MEXICO M1	M2	USA M	UK M
LRN				
$m = 0$	fails	holds	unad.	fails
$m > 0$	holds	holds	unad.	unad.
LRSN				
$m = 0$	unad.	fails	unad.	unad.
$m > 0$	unad.	holds	unad.	unad.

Figure 1
a) Point Estimates and 95% Confidence Bands
for LRN test
$M1$, Mexico



b) Point Estimates and 95% Confidence Bands
for LRSN test
$M2$, Mexico

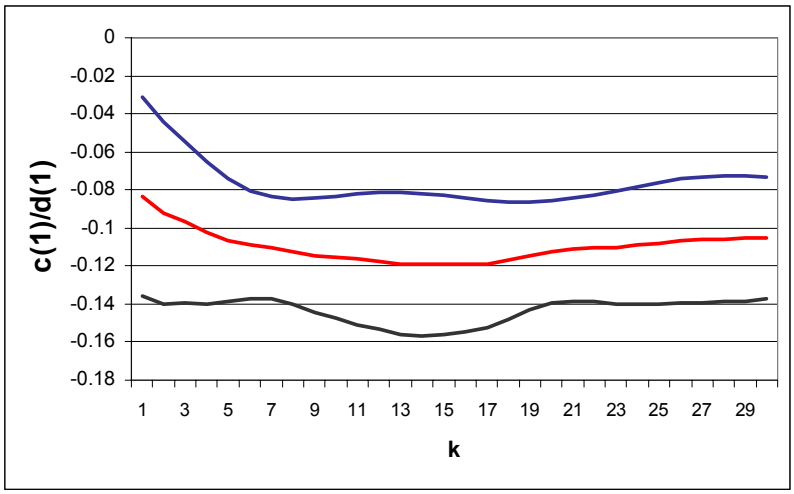
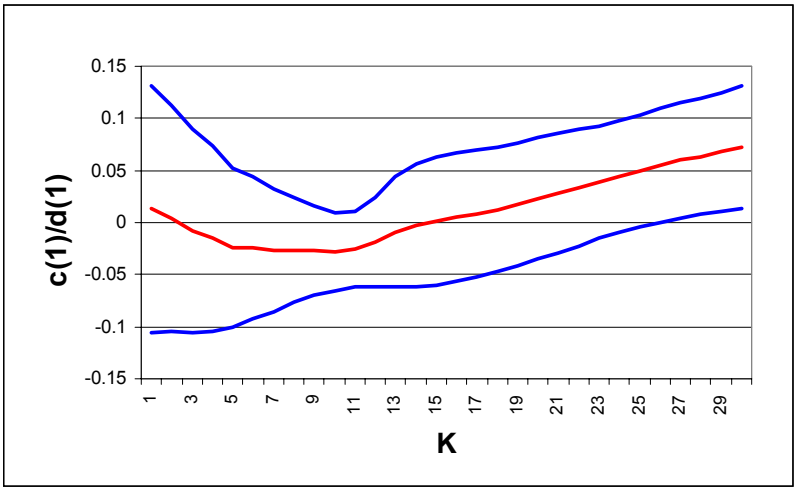


Figure 2
Point Estimates and 95% Confidence Bands for
LRN, United Kingdom



6 Appendix

Annual data for this paper´s empirical results come from the following sources:

Mexico:

Real GDP: 1932-1970 (Instituto Nacional de Estadistica, Geografia e Informatica (INEGI), "Estadisticas Historicas"). 1971-2000 (World Economic Outlook (WEO) Database December 2001), available from:
http://www.imf.org/external/pubs/ft/weo/2001/03/data/index.htm
$M1$, $M2$: 1932-1992: Wallace (1999), which comes from INEGI. 1993-2000: Banco de México, at:
http://www.banxico.org.mx/eInfofinanciera/FSinfoFinanciera.html

United States:

Real GDP: 1869-1975: Friedman and Schwartz (1982). 1976-2000: IMF, World Economic Outlook (WEO) Database December 2001, at:
http://www.imf.org/external/pubs/ft/weo/2001/03/data/index.htm
$M1$: 1869-1975: Friedman and Schwartz (1982). 1976-2000: Federal Reserve, Historical DATA, at:
http://www.federalreserve.gov/releases/H6/hist/

United Kingdom:

Real GDP: 1871-1975: Friedman and Schwartz (1982). 1976-2000: IMF, World Economic Outlook (WEO) Database December 2001, at:
http://www.imf.org/external/pubs/ft/weo/2001/03/data/index.htm
M4: 1871-1975: Friedman and Schwartz (1982). 1976-2000: Bank of England, at:
http://www.bankofengland.co.uk/mfsd/abst/ab1t1.xls.