Bank Competition, Fire-sale and Financial Stability

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Abstract

For more than two decades, researchers have been trying to figure out how bank competition is related to financial stability. However, their results are inconsistent and ambiguous, in both theoretical and empirical studies. Although the conclusions of previous studies are very diverse, most of the mentioned papers have a common feature. These previous studies mainly focus on the asset risks of financial institutions. However, these asset risks only cover the risks that are originated from one side of the banks' balance sheet. How the risks that come from the other side, the funding structure of banks, affect the relationship between bank competition and financial stability has been rarely discussed. The objective of this paper is to explore how the funding structure of banks affect the relationship between bank competition and financial stability. This paper applies a simple liquidity modeling framework and shows that fire-sale, which has rarely been included in the discussion of the debated topic, plays an important role. An important finding in this paper is that, the existence of fire-sale can create an incentive for banks' excessive risk-taking. This incentive is originated from the fact that in a multi-bank economy, a bank can take advantage of other banks in fire-sale by choosing riskier funding structure. This paper also shows that this excessive risk-taking incentive increases with the number of banks in the economy.

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1 Introduction

For more than two decades, researchers have been trying to figure out how bank competition is related to financial stability. However, their results are inconsistent and ambiguous, in both theoretical and empirical studies. The traditional view of the literature suggests the competition-fragility view (also called "franchise-value" paradigm) that bank competition has a negative impact on financial stability. The argument lies in the fact that bank competition leads to smaller franchise value (Keeley, 1990 [17]), this motivates banks to take excessive risk, resulting in higher bank losses when there are economic or financial distresses. Empirically, the competition-fragility view is supported by the evidence that, when bank competition increases there are a larger proportion of non-performing loans (NPLs), a smaller capital-to-asset ratio, and/or a higher frequency of financial crisis (Beck, Demirgus-Kunt and Levine, 2006 [2]; Jimenez, Lopez and Saurina, 2007 [15]).

Recently, some literature suggests otherwise. The opposite view (competition-stability view) argues that banks, when facing little competition in the market, are usually inefficient and demands higher loan rates; this creates a risk-shifting effect (Boyd and De Nicolo, 2005 [6]) to the borrowers and causes a higher probability of loan defaults, which can also be detrimental to financial stability. The competition-stability view is also supported by some empirical evidences (Boyd, De Nicolo, and Jalal, 2006 [7]; Amidu and Wolfe, 2011 [1]).

To add confusion to the debated topic, some studies suggest more diverse and complicated results. For example, Berger, Klapper and Turk-Ariss (2009) [3] find evidence that, when market power of banks grows, although loan risk of banks increases, overall bank risk decreases. Boyd and Runkle (1993) [8] suggest that failure probabilities are essentially unrelated to bank size. Molyneux and Nguyen-Linh (2008) [20], based on the data of South-East Asian banking industry, find no evidence to support bank competition can lead to risk-taking behaviour. Martinez-Miera and Repullo (2010) [19] predict with their model that a U-shaped pattern can also be a possible answer to the debated question; however, soon after that, Jimenez, Lopez and Saurina (2007) [15] show that there is no evidence to support the U-shaped pattern.[1]

Although the conclusions of previous studies are very diverse, most of the mentioned papers have a common feature. These previous studies mainly focus on the asset risks of financial institutions. These risks come from the choice of investment portfolio, the profit margin from asset returns, and the probability of defaults. However, these asset risks only cover the risks that are originated from one side of the banks' balance sheet. How the risks that come from the other side, the funding structure of banks, affect the relationship between bank competition and financial stability has been rarely discussed. One key difference between the asset risks and funding-structure risks is that these risks belong to very different risk categories. Most of the asset risks are market risks, they rely on the market prices, asset returns, and business cycles. These are different from the funding-structure risks which comes more often from the supplies of funding and liquidity risks.

The objective of this paper is to explore how the funding structure of banks affect the relationship between bank competition and financial stability. This paper applies a simple liquidity modeling framework and shows that fire-sale, which has rarely been included in the discussion of the debated topic, plays an important role. One key feature of fire-sale is that it is very often based on a systemic

[1] The criticism of Jimenez, Lopez and Saurina (2007) is based on the working paper in 2007 of Martinez-Miera and Repullo (2010).

basis.[2] For this reason, it is difficult for individuals to accurately evaluate fire-sale costs given the incomplete information that they have. In the latest paper of Shleifer and Vishny (2011) [21], they explain that asset *fire-sales* can deplete the balance sheets of financial institutions and aggravate the fragility of the financial system; they also point out the problem of fire-sales in the 2007-2009 financial crisis in their paper. This supports the necessity to include the role of fire-sale in the studies of financial stability. And this is why this paper attempts to extend the discussion of the debated topic to characterise fire-sale in the proposed model.

An important finding in this paper is that, the existence of fire-sale can create an incentive for banks' excessive risk-taking. This incentive is originated from the fact that in a multi-bank economy, a bank can take advantage of other banks in fire-sale by choosing riskier funding structure.

This is because fire-sale price depends on the aggregate amount of asset sold in the economy; it decreases when the amount of asset sold in the economy increases. When a bank chooses a riskier funding structure than the other banks, it needs to sell more asset than the other banks when the economy is at bad states. However, since all other banks are selling a smaller amount of asset, the fire-sale price is relatively high to the riskier bank. On the contrary, the fire-sale price is relatively low to the safer banks. This can be interpreted as a subsidy from safer banks to the riskier bank.

In this paper, I show that this result holds even when the economy is in equilibrium. Moreover, I show that this excessive risk-taking incentive increases with the number of banks in the economy (which is the measure for bank competition in the proposed model). With this result, I can conclude that, based on the model framework of this paper, banking competition leads to financial instability; in order words, my results support the traditional view of the mentioned topic (that bank competition weakens financial stability), with a different aspect on the source of risks.

This paper also discusses policy interventions to control the excessive risk-taking in funding structures. I show that capital requirement is a good way to restore the banks' funding structure to the socially optimal level. However, since capital requirement has been widely used for the control of asset risks in banks, it may be difficult to apply the same policy to the liquidity risks in funding structure. Therefore, I also discuss two other policies: reducing the gap between the costs of deposit and equity, and applying a fire-sale penalty. The numerical results of this paper show that reducing the gap between the costs of different sources of funding can effectively restore the optimal funding structure. But fire-sale penalty seems to have limited effect and can be outrun by the excessive risk-taking incentive.

This paper builds on the recent studies of Stein (2011) [22], and Kashyap and Stein (2011) [16]. In their papers, they construct a model to show that banks have an intention to create excessive short-term debt, because the loss from asset fire sales is not fully internalised by individual banks. Although the motivations are different, this paper is similar in spirit to Stein and Kashyap's papers.

In this paper, the banks are assumed to be able to raise funding by issuing deposit contract and equity stock. The deposit contract is guaranteed to be risk-free by banks, and therefore is a cheaper source of funding to banks compared with the risky equity stock. This paper extends Kashyap and Stein's papers by introducing liquidity risk to banks. This liquidity risk comes from the randomness of the proportion of early (late) production (similar to Diamond and Rajan, 2005 [13]), and the liquidity demand that comes from pre-deterministic deposit withdrawals from the households (similar in spirit to Diamond and Dybvig, 1983 [12]). If banks cannot satisfy the withdrawal of deposit contract with the output from production on an interim date, the banks will have to sell their assets (bank loans) to

[2] Besar et al. (2011) [4] examines how systemic risk can impact the entire financial system and explores its disturbances in the banking sector.

external investors at a discount (fire-sale price). The discount depends on the aggregate size of asset sold in the fire-sale; the more bank assets need to be sold, the lower is the fire sale price.

The aim of the banks is to maximise their net expected return to their equity holders. In this paper, I begin with the monopoly-bank economy, the construction of the monopoly-bank decision problem is straightforward because the decision of the monopoly bank does not depend on others. The funding structure chosen by the monopoly bank has the highest level of equity stock (which implies lowest level of liquidity risk) compared with the funding structure chosen in the multi-bank economy under symmetric equilibrium, and is therefore considered as the socially optimal choice.

I then construct the decision problems in a two-bank economy, which is more complicated because a bank decision needs to depend on the conjectures of the decision made by the other bank. The model shows that there exists a unique equilibrium in which the two banks choose the same funding structure. The choice of funding structure under this symmetric equilibrium acts as a basis for the later analysis and comparison in the model.

The model is then further extended from the two-bank economy to a n-bank economy for generalisation. With these specifications, I prove that banks choose riskier funding structures in economies with more banks.

This paper contributes to the literature in several aspects. First, to the best of my knowledge, this is the first study that incorporates fire-sale to the discussion of the relationship between bank competition and financial stability. Second, this study applies a liquidity framework for the modeling; this is rare in the literature, because most literature focuses on the credit-risk or market-risk aspect (return randomness) for their model construction. The advantage of the liquidity framework is that it can easily capture the existence of liquidity shortfalls and characterise the cost of fire-sale. Third, most literature defines financial instability by measuring loan risk and/or bank overall risk; this neglects the existence of liquidity risk and systemic risk, which can have significant impact in financial crises. This study introduces liquidity shortfall as a source of liquidity risk and fire-sale as a source of systemic risk to study the impact of these risks in financial distresses.

The rest of the paper is divided into six sections. Section 2 reviews important literature which discusses bank competition and financial stability. Section 3 presents the model specifications, in which the role of each agent is carefully explained and the model background is constructed. Section 4 discusses the decision problems of banks step by step, beginning from a monopoly-bank economy, and then a two-bank economy, and finally a n-bank economy. Section 5 illustrates the results from a numerical example based on the proposed model. Section 6 discusses the policy interventions that can help control the funding structure of banks based on the proposed model framework. Section 7 concludes.

2 Literature Review

Numerous research has tried to determine how bank competition can affect financial stability. This section reviews some of these important studies. However, due to the extensive previous research, this literature review is incapable to include all the works that have been done. For a better understanding of previous works, please refer to Vives (2010) [23] and Jimenez, Lopez and Saurina (2007) [15] for excellent and comprehensive reviews on the topic.

In this section, theoretical models and empirical works are discussed separately. The reviewed literature is categorised into two groups according to their results: studies that support *competition fragility*, and those that support *competition stability*.

2.1 Theoretical Models

2.1.1 Competition-Fragility

The traditional view of the debated topic suggests that excessive bank competition has a negative impact on financial stability. Following Furlong and Keeley (1987) [14] and Marcus (1984) [18], Keeley (1990) [17] proposes a 2-dated model, in which a bank faces a random asset return that can take two values (two states). The model shows that if the expected charter (franchise) value is high, the bank chooses a high capital level and a low asset risk to guarantee solvency in both states. Otherwise, the bank makes a riskier decision and bankruptcy is observed in the bad state. The model shows the importance of franchise value in determining bank (overall) risk.

Wagner (2010) [24] proposes a generalised version of the Boyd and De Nicolo (2005) model (reviewed in the following subsection of competition-stability models) and argues that their conclusion may not hold. Wagner shows that the conclusion of Boyd and De Nicolo's model is strongly based on the assumption that bank risk is entirely determined by the borrowers, and the bank has no control over its own risk.

Wagner relaxes the assumption and proposes the following 2-dated model: On date 1, there exist a continuum of entrepreneurs that have different risk-return portfolios; the entrepreneurs are given the right to decide the riskiness of its own project; however, it is the modelled bank who chooses the entrepreneur to be financed. Therefore, it can be interpreted that the bank chooses its own preferred level of risk in the model. Bank competition is measured by the cost of switching; Specifically, the cost of switching reduces when bank competition gets more intense; this limits the maximum loan rate determined by the modelled bank. Wagner proves the following mechanism: when bank competition gets more intense, loan rate decreases; this encourages the financed entrepreneur to reduce its project risk.[3] However, this lower level of risk is below the preferred level of risk chosen by the bank. As a result, the bank chooses another entrepreneur (instead of the one that it picks in the less competitive banking industry) who has a higher level of risk than the original one. This unambiguously leads to the weakening of financial stability.

2.1.2 Competition-Stability

Recently, Boyd and De Nicolo (2005) [6] challenges the traditional competition-fragility view by comparing two simple models. In the first *base* model, banks competes only in deposit market, and they maximise their expected return by choosing an optimal level of asset risk and deposit taking. Their paper shows that as the number of bank increases, the asset risk chosen by banks strictly increases. This result supports the traditional view of competition fragility. However, in the *extended* second model, when banks extend their competition to both loan market and deposit market, the role of borrowers is taken into consideration; borrowers are given the right to decide the asset risk they prefer, leaving the banks with only the decision on the amount of deposit taking (which is equal to the amount of loans the banks choose to grant). The authors show that the asset risk is decreasing in the number of banks;

[3] A lower loan rate increases the return to the entrepreneurs, leading to a choice of a lower level of risk to prevent the failure of the project.

this is because higher bank competition leads to lower loan rates, and in turn lower the moral hazard of borrowers, resulting in a choice of lower asset risk. This result supports the alternative view of competition stability. Their paper argues that previous studies fail to consider the risk-shifting effect from banks to borrowers, and the loan market channel is as important as the deposit market channel.

Following Boyd and De Nicolo's work, Martinez-Miera and Repullo (2010) [19] point out that the earlier work fails to consider that lower loan rates can also reduce bank returns. And if this effect is taken into account, the relationship between bank competition and bank risk can be U-shaped. In the latter model, the probability of default is determined endogenously by the borrowers, whose investments are imperfectly correlated. Their paper finds that there is a margin effect together with Boyd and De Nicolo's risk-shifting effect. The margin effect originated from the fact that more competition leads to lower loan rates, which in turn lower the return of non-defaulting loans to banks; this reduces the buffer against loan losses, resulting in the existence of riskier banks.

Boot and Thakor (2000) [5] studies intra-bank competition and also competition from capital market, based on an alternative aspect: the roles of relationship banking and the welfare of borrowers. They proposes a four-dated model and specifies the roles of borrowers, depositors, banks and underwriters (capital market). The model characterises intra-bank competition in the form of competitive bidding of loan offers between the banks; relationship banking is defined as a costly sector specialisation investment by the banks, which increases the probability of having successful borrowers' projects. Boot and Thakor show that more intensive intra-bank competition increases the welfare of borrowers with good-quality (high probability of success) projects; however, the welfare of the other borrowers with poor-quality (low probability of success) projects are ambiguous. The empirical study of Degryse and Ongena (2005) [10] finds evidence to support the prediction of Boot and Thakor.

2.2 Empirical Studies

2.2.1 Competition-Fragility

There are a lot of empirical studies supporting the traditional competition-fragility view; however, the measures that these papers have applied for quantifying bank competition and bank risk are quite different.

Keeley (1990) [17] aims to show that deregulation in the United States in mid-1960's and the expanded powers of thrifts in the early 1980's increased the competition in banking industry, which in turn eroded banks' charter (franchise) values. This encouraged the U.S. banks to take excessive risks, causing the sharp increase of bank failures after 1980's. Using the data of 85 largest bank holding companies (BHCs) in the U.S. between 1970-1986, Keeley finds that banks with greater market power (measured by market-to-book asset ratio) have lower overall bank risk (larger capital-to-asset ratios) and lower default risk (measured by the loan risk premium on certificate of deposit).

Demsetz, Saidenberg and Strahan (1996) [11] discuss the relationship between franchise value and bank risks, based on the market value and accounting data of 100 bank holding companies (BHC) during 1986-1994. They define franchise value as the difference between market value and replacement cost of a bank, which after normalisation is in the form of Tobin's q ratio. Their paper shows significant results of (1) a negative relationship between franchise value and bank risks (all-in risk, systematic risk and firm-specific risk), and (2) the high-franchise-value BHCs reduce their risk by increasing the capital-to-asset ratio and shifting to a less risky and more diversified asset portfolio.

Beck, Demirguc-Kunt and Levine (2006) [2] conduct the first empirical test that uses cross-country data. In their paper, they use bank data across 60 countries between 1980-1997. They measure financial stability using the frequency of crises (which is defined by non-performing loans of at least 10 percent, and/or requiring government interventions to restore market order). The measure for market concentration is the share of assets of the three largest banks in the countries. They find that crises are less likely in economies with more concentrated systems. However, their paper has been criticised by some latter studies that market concentration is not a proper measure for competition; Classens and Laeven (2004) [9] point out that there is no evidence for a negative relationship between bank concentration and bank competitiveness.

Jimenez, Lopez and Saurine (2007) [15] also find evidence to support competition-fragility view. They use unique dataset from Spanish banking system, which allows them to calculate Lerner index (their measure for market power) from the marginal interest rates charged by each bank for several banking products. They also extract the risk premium from the marginal interest rates and obtain the non-performing loan ratios (NPLs) as their measures for bank risk. They find a negative relationship between market power and bank risk. Another important contribution from their paper is that the authors have used some standard proxies of market concentration, including Herfindahl-Hirschmann indexes (HHI) and the number of banks operating in the market, to conduct the empirical tests; and they find that the measures of market concentration do not affect the bank risk measure. This finding supports Classens and Laeven (2004) [9] that bank concentration is not a proper measure for bank competitiveness. However, their paper fails to consider the deposit market of banking industry, and this weakens their evidence on competition-fragility view.

2.2.2 Competition-Stability

In contrast, to support their risk-shifting model, Boyd, De Nicolo and Jalal (2006) [7] use z-score and the ratio of equity to total assets as their measures of overall bank risk, and Herfindahl-Hirschmann index (HHI) as their measure of market concentration, to study two data samples: a cross sectional sample of 2500 banks in the United States, and a panel sample of 2700 banks in 134 countries. The results from both samples are consistent, and suggest more concentrated banking industries are associated with more bank failures. However, this empirical literature has the same shortcoming as in Beck, Demirguc-Kunt and Levine (2006): market concentration (HHI) is not a good measure for the degree of bank competition.

Yaldiz and Bazzana (2010) [25] study the role of market power in both the loan risk and overall bank risk, using the data from Turkish banks during 2001-2009 to conduct their empirical tests. They use the non-performing loans to total loans ratio (NPL) as a measure of loan risk for the Turkish banks, and Z-score as the measure of bank overall risk. For the measures of market competitiveness, they use Lerner index and the ratio of the difference between the total revenues and total costs to the total revenues. Their results suggest that (1) as market power of a bank increases, bank risk increases, and (2) as market power decreases, competition creates less risky banks, which in turn contributes to the stability of the whole banking system.

Amidu and Wolfe (2011) [1] empirically investigate the significance of diversification in the relationship between bank competition and financial stability. They employ three-stage-least-squares-estimate techniques to a panel dataset of 978 banks during the period 2000-2007. They use a number of measures for bank risk (Z-score, capital ratio, and non-performing loans ratio (NPL)) and for proxies of market power (H statistics and Lerner index). For the measure of revenue diversification, they calculate Herfindahl-Hirschmann index (HHI) for each bank. The core finding for their paper is that as bank competition increases, diversification across and within both interest and non-interest income generating activities increases, and this increases financial stability. Their paper also points out that funding structure plays an important role on financial stability, which is consistent to the suggestion of this paper.

3 Model Specifications

3.1 Model Framework

The framework of the model is similar to the one proposed in Diamond and Rajan (2005) [13]. The model begins with an economy with a three-dated time horizon, date 0, 1, and 2. All contracts are drawn under uncertainties on date 0, and all uncertainties are resolved on date 1. The framework of the model characterises the choice of funding structure (deposit contract verse equity stock) when a bank maximises its shareholders' value, taking into consideration of a random liquidity risk and fire-sale loss. Under this framework I determine how the choice of funding structure changes as the number of banks in the economy increases.

There are four types of agents in this model: firms, households, banks and external investors. The following time line summarises their relationship and the story of the model. All notations that are used in this model is tabulated in the following Table 1 for reference.

Date 0	Date 1	Date 2
Households receive an endowment of goods. They invest their endowment in banks in the forms of deposit contract and equity stock.	Some firms (α) produces their output early, and pay their promised returns to banks. Other firms continue their productions if they can extend their loans. If not, their productions are confiscated by the banks.	All the remaining firms ($1-\alpha$) with loan extensions produce their output, and pay their promised returns to the banks.
Banks absorb the endowment from the households, and provide short-term bank loans to the firms.	Households withdraw some deposits (β) from the banks for their consumption.	The banks pay for the date-2 deposit withdrawals, and share the remaining profit equally between the equity holders.
Firms invest in their production technology with the funding from bank loans.	If the return from the firms is higher than the date-1 deposit withdrawals, the banks extend the bank loans to all firms; otherwise, some productions are confiscated and sold at a fire-sale price to cover the liquidity shortfalls.	The households consume everything they have on date 2.

Figure 1: The time line of the model.

3.2 Firms

There are a large number (continuum) of perfectly competitive and independent firms in the economy. They are assumed to have no endowments, but all of them have an identical constant-return-to-scale production technology, which requires an initial input of endowment (good) on date 0, and produces

Notations	Definitions
n	Number of banks in the economy
α	Random proportion of firms that produces on date 1
β	Proportion of deposits withdrawn on date 1
ρ	Constant return to production technology
r_t	Return to deposit withdrawn on date t, $t = 1, 2$
d_k	Proportion of deposit in Bank k
$1 - d_k$	Proportion of equity in Bank k
q_n	Amount of endowment invested in *one bank* in a n-bank economy
h	Fire-sale-price coefficient
x_{jk}	Per unit production sold by Bank k in a j-bank fire-sale
α_k^*	Liquidity-shortfall threshold for Bank k

Table 1: The Table of Notations

ρ units of goods on either date 1 or date 2; $\rho > 1$ represents the exogenous gross rate of return from production. The randomness of the production maturity depends on a uniform-distributed random variable $\alpha \in (0, 1)$: a proportion of α of the firms produces output on date 1 and the rest $(1 - \alpha)$ produces output on date 2; α is realised on date 1, and this is a public information available to all agents.[4]

As the firms have no endowment, they need to obtain funding for their production from the banks on date 0. This model assumes that the firms lack the technology to collect funding from the households; therefore, financing is only possible through the financial intermediation services provided by the banks (which are assumed to have the technology to collect households' endowment). The banks are assumed to provide only short-term bank loans (with maturity on date 1) to the firms. These bank loans can be extended to date 2 for the firms with late production, subject to banks' approval. If the banks do not agree to extend the bank loans, the on-going production on date 1 can be fully or partially confiscated by the banks as a repayment for the bank loans. On-going production that is confiscated does not produce any output on date 2.

3.3 Households

Households are assumed to be a group of identical and continuum individuals. They have a total of one unit of endowment (good) on date 0. This good can be used for either production or consumption. Each household needs to consume on both date 1 and date 2. This model assumes that the households withdraw a constant proportion (β) of the date-1 withdraw-able investment (deposit contract) for their consumption on date 1, and the returns to the remaining investment (non-withdrawn deposit and equity stock) are left until date 2 for consumption. The utility function for a consumer (household) can be represented by

$$U(C_1, C_2) = u(C_1) + u(C_2)$$

where C_1 and C_2 are the consumption on date 1 and date 2 respectively, and $u(\cdot)$ is a neoclassical utility function (increasing, concave, and twice continuously differentiable).

There are two possible investments available to the households. The first one is to deposit their

[4]It is worth pointing out that although the production maturity is random on date 0, there is no uncertainty in the production return (ρ) over time.

endowment in banks; the deposit is risk-free. The second one is to invest in equity stock issued by the banks; these investments in banks are described in details in section 3.4. For simplicity, I assume that the households do not store the endowments themselves because the banks provide a higher expected return to both the deposit contract and the equity stock compared with the storage technology. I also assume that the households' endowment are equally invested in the banks; therefore, the endowment invested in a bank in a n-bank economy is given by $q_n = 1/n$.

3.4 Banks

Banks are financial intermediaries between firms and households. They are assumed to have a banking technology which allows them to collect endowment from the households, so that bank loans can be created to finance the firms' production. As the firms are perfectly competitive, they are willing to produce with zero profit. Therefore, the returns to the banks are $\alpha\rho$ on date 1 and $(1-\alpha)\rho$ on date 2, if no on-going production is confiscated on date 1.[5]

As mentioned, the banks provide only short-term bank loans (with maturity on date 1) to the firms on date 0; the maturity for the short-term loans can be extended to date 2 for the firms with on-going production on date 1 (late-producing firms). However, whether the short-term loans can be extended depends on the banks' liquidity on date 1. If the proportion of early-producing firms (α) is sufficiently large, such that the banks have enough liquidity to satisfy their date-1 deposit withdrawals, all bank loans to late-producing firms are extended. Otherwise, the banks have liquidity shortfalls; some short-term loans for the on-going production have to be discontinued, and their productions are confiscated and sold to external investors in fire-sale; the proceeds from fire-sale is used to fill up the liquidity shortfall. The banks try to avoid a fire-sale if possible because they always suffer some losses due to the low fire-sale price.

To obtain the household endowment on date 0, the banks issue both deposit contract and equity stock to raise funding for the bank loans. Deposit contract is a risk-free debt contract which can be withdrawn any time by the depositors. The returns to deposit contract depend on the date of withdrawal: the per-unit gross returns to the deposit withdrawn on date 1 and 2 are exogenous in this model, and are denoted as r_1 and r_2 respectively, where $1 \leq r_1 \leq r_2$.[6] Equity stock provides a return to its holders only on date 2, and it is not risk-free. The risk to equity stock comes from the uncertain proportion of early production (α). If the return from early production turns out to be too small to satisfy the withdrawal of short-term debt, there is fire-sale loss which affects the return to the equity holders. For this reason, the banks are not able to provide promised returns for equity stock; instead, the equity holders equally share the value of the banks on date 2. The proportion of funding structure in Bank k is characterised by the decision variable d_k: d_k represents the proportion of deposit contract in Bank k and $1 - d_k$ represents the proportion of equity stock.[7]

[5] It can also be assumed that the return to production technology is $\rho + y$, where y is an exogenous rent given to the firms. This specification does not affect the following model.

[6] Note that due to the risk-free nature of deposit contract, the returns to deposit contact have to be the same for all banks; otherwise, households will only deposit in the bank(s) with the highest returns.

[7] In this model, I do not assume that the banks possess storage technology because this can be redundant. One can easily figure out that the only purpose for banks to store liquid asset (good) is to increase their liquidity on date 1; this reduces the possibility of liquidity shortage in the fire-sale. However, this approach is not necessary because the same result can be obtained by by choosing a less risky funding structure. Moreover, raising extra equity stock is also costly (the cost of equity is explained in section 4.) Therefore, storage technology is not necessary for banks to achieve their preferred risk level. This further implies that in this model, the total amount of endowment collected from the households is equal to the amount of bank loans provided to the firms.

In this model, I assume that in a multi-bank economy, the banks are not aware of the actual funding structures of each other. Therefore, their decisions are based on some reaction functions that depend on the conjectures of others' decision variables. With these reaction functions, I aim to find out the optimal funding structure of the banks under a symmetric equilibrium, which is in nature very similar to the standard Cournot equilibrium. I then use this symmetric equilibrium as a foundation to analyse how the number of banks can affect the banks' funding structure under symmetric equilibrium, which in turn affects financial stability. Further details for the reaction functions and the symmetric equilibrium are explained in Section 4.

3.5 External Investors and Fire-sale

When banks have liquidity shortfalls on date 1, they need to sell their confiscated production to external investors. The external investors determine the fire-sale price based on the aggregate amount of asset sold in the economy. The fire-sale price of a j-bank fire-sale in a n-bank economy is assumed to be $1 - h(\sum_{k=1}^{j} q_n x_{jk})$, where $q_n x_{jk}$ is the amount of on-going production sold by Bank k to the external investors in a j-bank fire-sale, in a n-bank economy; $h < 1$ is an exogenous coefficient for the fire-sale price. By definition, the fire-sale price is smaller than one whenever there is a fire-sale; therefore, there must be a loss to the banks as the gross rate of return is smaller than one.

In a n-bank economy, the total proceeds for Bank k in a j-bank fire-sale is given by

$$q_n x_{jk} \left(1 - h \left(\sum_{k=1}^{j} q_n x_{jk} \right) \right)$$

The proceeds must be equivalent to the liquidity shortfall of Bank k, because no bank is willing to liquidate more assets than necessary due to the fire-sale loss. This is given by

$$q_n x_{jk}(1 - h(\sum_{k=1}^{j} q_n x_{jk})) = q_n(\beta r_1 d_k - \alpha\rho)$$

or simply

$$x_{jk}(1 - h(\sum_{k=1}^{j} q_n x_{jk})) = \beta r_1 d_k - \alpha\rho$$

The right-hand side of the equation represents the liquidity shortfall in Bank k; the first term refers to the deposit withdrawal on date 1, and the second term refers to the return to bank from early production. It is worth mentioning that fire-sale does not necessarily imply a huge loss to the banks. In fact, the loss in a small-scale fire-sale is minimal. In this model, I assume that the loss in the fire-sale is not big enough to affect the guaranteed returns to the deposit contract; therefore, the deposit contract is risk-free and the fire-sale loss is absorbed by the equity holders of the distressed banks.

Whether Bank k has a liquidity shortfall on date 1 depends on a threshold α_k^*; this is given by

$$q_n \alpha_k^* \rho = q_n \beta r_1 d_k,$$

or

$$\alpha_k^* = \frac{\beta r_1 d_k}{\rho}$$

When $\alpha \in [\alpha_k^*, 1]$, Bank k has no liquidity shortfall and does not need to liquidate its asset in fire-sale; otherwise, when $\alpha \in [0, \alpha_k^*]$, Bank k has a positive liquidity shortfall and some on-going production needs to be discontinued and sold in a fire-sale at a loss. It is worth mentioning a smaller α_k^* (lower threshold) implies that Bank k has a smaller liquidity risk, because it is less likely for α to be smaller than the (lower) threshold.

4 Decision Problems

In this section, I construct the decision problems for banks. The objective for a bank is to choose a funding structure in order to maximise the expected *net* return to their equity holders. For Bank k, this is given by

$$q_n(1-d_k)([E_\alpha[R(\alpha)]-c)$$

where $R(\alpha)$ is the rate of return to the equity holders (after all returns to deposit contract are deducted), and c is the (per unit) cost of equity. I assume that the cost of equity is higher than the cost of deposit (c^*) due to the existence of market friction[8]; this is represented by

$$c > \beta r_1 + (1-\beta) r_2 \equiv c^*$$

Note that both $R(\alpha)$ and c are the values based on *per unit* of equity; therefore, both of them have to be multiplied by the actual size of equity stock in Bank k, which is $q_n(1-d_k)$, to generate the absolute values in Bank k.

In the following subsections, I begin with the construction of the decision problem of a monopoly bank. This is the simplest model because the decision of a monopoly bank does not rely the conjectures of other banks. Then, I construct the decision problems under the more complicated two-bank economy, in which conjectures and reaction functions are required to determine the optimal funding structure. After that, I extend the framework of the two-bank economy to a generalised framework of a n-bank economy.

4.1 Monopoly-Bank Economy

In an economy with one (monopoly) bank, the decision problem for the bank is quite straightforward. It needs to choose its funding structure by maintaining a balance in the trade-off of the (lower) cost and the (higher) risk of deposit contract. Recall that due to the existence of market friction, the cost of equity is higher than the cost of deposit. For this reason, it is relatively cheaper for a bank to fund its asset (bank loans) by deposit contract. However, a higher proportion of deposit also means that a bank has to face a higher liquidity risk. The higher risk comes from the higher level of deposit withdrawal on date 1, which increases the threshold for liquidity shortfall (a higher probability of having liquidity shortfall) and the fire-sale losses.

In the following, the returns to the monopoly bank are specified under two scenarios: (Scenario 0) there is no liquidity shortfall; the return to bank does not depend on α; and (Scenario 1) there is a positive liquidity shortfall; the return to bank decreases in α. It is worth mentioning that the fire-sale in a monopoly-bank economy is not an externality to the monopoly bank because the bank

[8]A common example of market friction is the taxation benefit of debt (deposit).

fully internalises the cost of fire-sale within its decision problem.

Scenario (0): No fire-sale. When there is no fire-sale, the gross return to the bank consists of the returns from early and late production, which sum up to ρ. The total cost of deposit is $c^* = \beta r_1 + (1-\beta) r_2$. Therefore, the rate of return to equity is simply

$$R_0 = \frac{q_1 \left[\rho - (\beta r_1 + (1-\beta) r_2) d_1\right]}{q_1(1-d_1)}$$

or equivalently

$$R_0(1-d_1) = \rho - c^* d_1$$

where the subscript of R_0 corresponds to Scenario (0). Note that the Scenario (0) equity return is independent of α. The condition for having Scenario (0) is given by $\alpha \geq \alpha_1^* = \frac{\beta r_1 d_1}{\rho}$.

Scenario (1): Fire-sale. When there is a liquidity shortfall on date 1, fire-sale is necessary. The amount of asset that the monopoly bank (or Bank 1) needs to sell in the one-bank fire-sale is denoted as x_{11}; this value is obtained by solving

$$x_{11}(1 - h q_1 x_{11}) = \beta r_1 d_1 - \alpha \rho$$

As part of the on-going production is confiscated and sold in the fire-sale, the date-2 return is reduced. The date-2 return (after fire-sale) is expressed by $(1 - \alpha - x_{11})\rho$.

The rate of equity return, conditional on a positive liquidity shortfall on date 1, is

$$R_1 = \frac{q_1 \left[\alpha\rho + x_{11}(1 - h q_1 x_{11}) - \beta r_1 d_1 + (1 - \alpha - x_{11})\rho - (1-\beta) r_2 d_1\right]}{q_1(1-d_1)}$$

The first three numerator terms within the square bracket on the right-hand side are the pay-off to the bank on date 1: the first term is the return from early production; the second term is the proceeds from fire-sale; the third term is the payment to date-1 deposit withdrawal. These three terms, by definition, sum up to zero; however, I keep this terms in the equation for further simplification of algebraic expression. The last two numerator terms correspond to the date-2 pay-off: the fourth term is the return from *remaining* late production after the fire-sale; the fifth term is the payment to date-2 deposit withdrawal. The above expression can be simplified as

$$R_1(1-d_1) = R_0(1-d_1) - h q_1 x_{11}^2 - (\rho - 1) x_{11}$$

The term $h q_1 x_{11}^2$ represents the loss in the fire-sale due to the low fire-sale price (captured by the fire-sale price coefficient h); the term $(\rho - 1)x_{11}$ represents the reduction in date-2 return due to the fire-sale of on-going production on date 1. The condition for having Scenario (1) is given by $\alpha < \alpha_1^*$.

4.1.1 Decision Problem

The decision problem of the monopoly bank is given by[9]

$$\max_{d_1} q_1 (1-d_1)(E_\alpha[R] - c)$$

[9]Note that $(R_0 - c)(1-d_1) = \rho - c^* d_1 - c(1-d_1) = (\rho - c) - (c - c^*)(1-d_1)$

Substituting $q_1 = 1$, and the expressions of R_0 and R_1 into R, one can get the following expression.

$$\begin{aligned}
&\max_{d_1} \int_{\alpha_1^*}^{1} (R_0 - c)(1 - d_1) d\alpha + \int_0^{\alpha_1^*} (R_1 - c)(1 - d_1) d\alpha \\
=&\max_{d_1} (R_0 - c)(1 - d_1) - h \int_0^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{11} d\alpha \\
=&\max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - h \int_0^{\alpha_1^*} x_{11}^2 d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{11} d\alpha
\end{aligned}$$

The first-order condition (FOC) for the decision problem with respect to d_1 is given by

$$(c - c^*) - 2h \int_0^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha = 0$$

where

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2h x_{11}} > 0$$

The second-order condition (SOC) is given by

$$-\int_0^{\alpha_1^*} \left[2h \left(x_{11} \frac{\partial^2 x_{11}}{\partial d_1^2} + \left(\frac{\partial x_{11}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{2h(\beta r_1)^2}{(1 - 2h x_{11})^3}$$

To have a stable FOC that maximises the objective function, the SOC has to be negative for all values of α. The sufficient condition for negative SOC is $h < \frac{1}{2}$. This ensures that $\frac{\partial^2 x_{11}}{\partial d_1^2} > 0$ even when x_{11} is at its maximum value of one[10].

To avoid unstable optimisation results, this model assumes that $h < \frac{1}{2}$ throughout the paper.[11]

4.2 Two-Bank Economy

The two-bank economy is more complicated. Since the two banks do not know each other's decision, they have to choose their funding structure based on the conjectures of the funding structure of the other bank. In this model, I assume that the two banks make their decision based on a reaction function. In order to specify the reaction function correctly, one needs to know that the specification for the banks' return depends on the risk levels of the two banks. Specifically, the return specification for Bank 1 being the riskier bank (based on the conjectures of a safer funding structure in Bank 2) is different from the return specification for Bank 1 being the safer bank (based on the conjectures of a riskier funding structure in Bank 2). In the following, I discuss these two types of return specifications separately. To avoid cumbersome, I discuss the decision making and the scenario analysis only from the point of view of Bank 1; the discussion for Bank 2 is exactly the same, and is therefore omitted.

[10] The derivations of $\frac{\partial x_{11}}{\partial d_1}$ and $\frac{\partial^2 x_{11}}{\partial d_1^2}$ are shown in the appendix.

[11] I show in the coming subsections that this sufficient condition is also sufficient to ensure the stableness of all FOC's in the multi-bank economies.

4.2.1 Being a Riskier Bank

Recall that the liquidity risk from the random firm production is a common shock to all banks in the economy; in other words, all banks have the same level of liquidity shock, which is given by $\alpha\rho$ and $(1-\alpha)\rho$. Therefore, the condition for Bank 1 being riskier than Bank 2 comes from the funding structure, $d_1 \geq d_2$. If Bank 1 has a higher proportion of deposit funding, it also has a higher date-1 deposit withdrawal, and therefore it has a higher liquidity risk compared with Bank 2. Equivalently, this is represented by the thresholds that $\alpha_1^* \geq \alpha_2^*$.[12]

With Bank 1 being the riskier bank, its return specification is subdivided into three scenarios: (Scenario 0) there is no liquidity shortfall in both banks; (Scenario 1) there is a positive liquidity shortfall in Bank 1, but not in Bank 2, causing a one-bank fire-sale; (Scenario 2) there are positive liquidity shortfalls in both banks, causing a two-bank fire-sale.

Scenario (0): No fire-sale. Similar to the monopoly-bank economy, when there is no fire-sale, the rate of return to Bank-1 equity holders is simply

$$R_0 = \frac{q_2\left[\rho - (\beta r_1 + (1-\beta) r_2) d_1\right]}{q_2(1-d_1)}$$

Or equivalently

$$R_0(1-d_1) = \rho - (\beta r_1 + (1-\beta) r_2) d_1$$

Again, the return to Bank 1 is not affected by the random proportion of early production α. The condition for having Scenario (0) is given by $\alpha \geq \alpha_1^*$.

Scenario (1): One-bank fire-sale. If there is a liquidity shortfall in Bank 1 but not in Bank 2, there is a one-bank fire-sale. The amount of asset needs to be sold by Bank 1 is determined by the following equation.

$$x_{11}(1 - hq_2 x_{11}) = \beta r_1 d_1 - \alpha\rho$$

where x_{11} is the amount of asset liquidation for Bank 1 in a one-bank fire-sale. The reduced date-2 return for Bank 1 (after the fire-sale) is specified by $(1-\alpha-x_{11})\rho$.

The rate of equity return for Bank 1 in Scenario (1) is specified by,

$$R_1 = \frac{q_2\left[\alpha\rho + x_{11}(1-hq_2 x_{11}) - \beta r_1 d_1 + (1-\alpha-x_{11})\rho - (1-\beta) r_2 d_1\right]}{q_2(1-d_1)}$$

or in a simpler way

$$R_1(1-d_1) = R_0(1-d_1) - hq_2 x_{11}^2 - (\rho-1)x_{11}$$

The condition for having Scenario (1) is given by $\alpha_2^* < \alpha \leq \alpha_1^*$.

Scenario (2): Two-bank fire-sale. If there are liquidity shortfalls in both banks, there is a two-bank fire-sale. The amount of asset needs to be sold by Bank 1 and Bank 2 in a two-bank fire-sale are determined by the following equations respectively.

$$x_{21}(1 - hq_2(x_{21} + x_{22})) = \beta r_1 d_1 - \alpha\rho$$

$$x_{22}(1 - hq_2(x_{21} + x_{22})) = \beta r_1 d_2 - \alpha\rho$$

[12] Recall that in this model, a higher threshold corresponds to a higher probability of having liquidity shortfall.

The reduced date-2 return for Bank 1 is specified by $(1-\alpha-x_{21})\rho$.

The rate of equity return for Bank 1 in Scenario (2) is specified by,

$$R_2 = \frac{q_2\left[\alpha\rho + x_{21}(1-hq_2(x_{21}+x_{22})) - \beta r_1 d_1 + (1-\alpha-x_{21})\rho - (1-\beta)r_2 d_1\right]}{q_2(1-d_1)}$$

or in a simpler way

$$R_2(1-d_1) = R_0(1-d_1) - hq_2(x_{21}^2 + x_{21}x_{22}) - (\rho-1)x_{21}$$

The condition for having Scenario (2) is given by $\alpha < \alpha_2^*$.

4.2.2 Decision Problem for Riskier Bank

The decision problem for Bank 1 being the riskier bank in the two-bank economy can be expressed by (with substitution of $q_2 = 1/2$)

$$\max_{d_1} \int_{\alpha_1^*}^{1} (R_0 - c)(1-d_1)d\alpha + \int_{\alpha_2^*}^{\alpha_1^*} (R_1 - c)(1-d_1)d\alpha + \int_{0}^{\alpha_2^*} (R_2 - c)(1-d_1)d\alpha \tag{1}$$

$$= \max_{d_1}(\rho - c) - (c-c^*)(1-d_1) - \frac{h}{2}\int_{\alpha_2^*}^{\alpha_1^*} x_{11}^2 d\alpha - (\rho-1)\int_{\alpha_2^*}^{\alpha_1^*} x_{11} d\alpha$$

$$-\frac{h}{2}\int_{0}^{\alpha_2^*} (x_{21}^2 + x_{21}x_{22})d\alpha - (\rho-1)\int_{0}^{\alpha_2^*} x_{21} d\alpha$$

Note that given different values for d_2 (conjectures on Bank 2), which affects the values for x_{22} and α_2^* in the expression, there are different values for the optimal decision variable d_1. Therefore, the above optimisation problem is in fact a reaction function (based on the value of d_2).

The first-order condition (FOC) for the decision problem with respect to d_1 is given by

$$(c-c^*) - h\int_{\alpha_2^*}^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho-1)\int_{\alpha_2^*}^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha$$

$$-h\int_{0}^{\alpha_2^*} (x_{21} + \frac{1}{2}x_{22}) \cdot \frac{\partial x_{21}}{\partial d_1} d\alpha - (\rho-1)\int_{0}^{\alpha_2^*} \frac{\partial x_{21}}{\partial d_1} d\alpha = 0$$

where

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - hx_{11}} > 0$$

$$\frac{\partial x_{21}}{\partial d_1} = \frac{\beta r_1}{1 - h(x_{21} + \frac{1}{2}x_{22})} > 0$$

The second-order condition (SOC) is given by

$$-\int_{\alpha_2^*}^{\alpha_1^*} \left[h\left(x_{11}\frac{\partial^2 x_{11}}{\partial d_1^2} + \left(\frac{\partial x_{11}}{\partial d_1}\right)^2 \right) + (\rho-1)\frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha$$

$$-\int_{0}^{\alpha_2^*} \left[h\left((x_{21} + \frac{1}{2}x_{22})\frac{\partial^2 x_{21}}{\partial d_1^2} + \left(\frac{\partial x_{21}}{\partial d_1}\right)^2 \right) + (\rho-1)\frac{\partial^2 x_{21}}{\partial d_1^2} \right] d\alpha$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{(1 - h x_{11})^3}$$

and

$$\frac{\partial^2 x_{21}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{(1 - h(x_{21} + \frac{1}{2}x_{22}))^3}$$

To ensure that SOC is negative such that FOC always maximises the objective function, the sufficient condition is $h < \frac{2}{3}$. This ensures that both $\frac{\partial^2 x_{11}}{\partial d_1^2}$ and $\frac{\partial^2 x_{21}}{\partial d_1^2}$ are positive even when x_{11}, x_{21}, and x_{22} are equal to their maximum value of one.[13] Note that the sufficient condition in the two-bank economy is weaker than the one in one-bank economy because $h < 1/2$ is a stronger condition than $h < 2/3$.

4.2.3 Being a Safer Bank

If Bank 1 chooses a safer funding structure than Bank 2, then the model must have $d_1 \leq d_2$. Or equivalently, $\alpha_1^* \leq \alpha_2^*$. There are only two scenarios for Bank 1: (Scenario 0) there is no liquidity shortfall for Bank 1; and (Scenario 1) there are liquidity shortfalls in both banks. The reason that one-bank (Bank 2) fire-sale is irrelevant is because the return to Bank 1 is not affected by the liquidity shortfall of Bank 2 in a one-bank fire-sale.

Scenario (0): No fire-sale *needed by Bank 1.* When there is no fire-sale, the rate of return to Bank-1 equity holders is again

$$R_0 = \frac{q_2 \left[\rho - (\beta r_1 + (1 - \beta) r_2) d_1\right]}{q_2(1 - d_1)}$$

Or equivalently

$$R_0(1 - d_1) = \rho - (\beta r_1 + (1 - \beta) r_2) d_1$$

The condition for having Scenario (0) is given by $\alpha \geq \alpha_1^*$.

Scenario (1): Two-bank fire-sale. Note that when there is a liquidity shortfall in Bank 1, there must also be a liquidity shortfall in Bank 2, due to the definition of $\alpha_1^* \leq \alpha_2^*$. In a two-bank fire-sale, the rate of equity return for Bank 1 is specified by,

$$R_1 = \frac{q_2 \left[\alpha\rho + x_{21}(1 - h q_2(x_{21} + x_{22})) - \beta r_1 d_1 + (1 - \alpha - x_{21})\rho - (1 - \beta) r_2 d_1\right]}{q_2(1 - d_1)}$$

or in a simpler way

$$R_1(1 - d_1) = R_0(1 - d_1) - h q_2(x_{21}^2 + x_{21}x_{22}) - (\rho - 1)x_{21}$$

The condition for having Scenario (1) is given by $\alpha < \alpha_1^*$.

[13] The derivations of $\frac{\partial x_{11}}{\partial d_1}$, $\frac{\partial x_{21}}{\partial d_1}$, $\frac{\partial^2 x_{11}}{\partial d_1^2}$, and $\frac{\partial^2 x_{21}}{\partial d_1^2}$ are shown in the appendix.

4.2.4 Decision Problem for Safer Bank

The decision problem for Bank 1 being the safer bank in the two-bank economy can be expressed by

$$\max_{d_1} \int_{\alpha_1^*}^{1} (R_0 - c)(1 - d_1) d\alpha + \int_{0}^{\alpha_1^*} (R_1 - c)(1 - d_1) d\alpha \tag{2}$$

$$= \max_{d_1} (\rho - c) - (c - c^*)(1 - d_1) - \frac{h}{2} \int_0^{\alpha_1^*} (x_{21}^2 + x_{21}x_{22}) d\alpha - (\rho - 1) \int_0^{\alpha_1^*} x_{21} d\alpha$$

Again, given different values for d_2 (conjectures on Bank 2's funding structure) which affect the values for x_{22} in the expression, there will be different optimal values for d_1. Therefore, the above optimisation problem is a reaction function based on the value of d_2.

The first-order condition (FOC) for the decision problem with respect to d_1 is given by

$$(c - c^*) - h \int_0^{\alpha_1^*} (x_{21} + \frac{1}{2} x_{22}) \cdot \frac{\partial x_{21}}{\partial d_1} d\alpha - (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{21}}{\partial d_1} d\alpha = 0$$

The second-order condition (SOC) is given by

$$-\int_0^{\alpha_1^*} \left[h \left((x_{21} + \frac{1}{2} x_{22}) \frac{\partial^2 x_{21}}{\partial d_1^2} + \left(\frac{\partial x_{21}}{\partial d_1} \right)^2 \right) + (\rho - 1) \frac{\partial^2 x_{21}}{\partial d_1^2} \right] d\alpha$$

The expression for $\frac{\partial x_{21}}{\partial d_1}$ and $\frac{\partial^2 x_{21}}{\partial d_1^2}$, and the sufficient conditions are the same as in Section 4.2.2.

4.2.5 Equilibrium

As there can be infinite conjectures on the funding structure, this model focuses on the analysis of funding structure in symmetric equilibrium. I define *symmetric equilibrium* in the following.

Definition 1 *In a two-bank economy, given one bank chooses the funding structure d_s, the other bank has no intention to deviate from this funding structure and also chooses d_s as its funding structure, d_s is said to be the funding structure in symmetric equilibrium.*

Mathematically, this is expressed as follows. Given $d_2 = d_s$, the optimal decision variable d_1 derived from both objective functions, eq.(1) and eq.(2), are equivalent and are both equal to d_s. Then the funding structure of Bank 1 and Bank 2 are said to be in symmetric equilibrium. The proof of the following proposition is given in the appendix of this paper.

Proposition 1 *In symmetric equilibrium, the proportion of deposit funding chosen in a two-bank economy is higher than the proportion chosen in a one-bank (monopoly) economy.*

This proposition is very similar to the Cournot equilibrium in standard microeconomics in which a firm decides how much to produce based on the conjectures of the other firm in duopoly. In this model, the choice of the monopoly bank can be interpreted as the socially optimal funding structure for bank(s), because this choice generates the lowest fire-sale loss in the banking industry (equivalently, highest net expected returns to the equity holders) compared with all other symmetric choices. Why do the banks in the two-bank economy choose a riskier funding structure? The reason is that when there are two banks, there exists a motivation for one bank to take advantage of the other bank by choosing a riskier funding structure; by doing so, the safer bank subsidises the riskier bank in the two-bank fire-sale. Specifically, the riskier bank is selling more asset than the safer bank in the two-bank fire-sale; however, the fire-sale price is not too low for the riskier bank because the safer bank is selling less asset. On the other hand, the fire-price is too low for the safer bank because the riskier bank is selling more asset. This can be interpreted as a subsidy from the safer bank to the riskier bank. Due to this motivation, both banks choose a riskier funding structure under symmetric equilibrium.

The above phenomenon shows that, even under symmetric equilibrium, the externality from fire-sale in a two-bank economy cannot be fully internalised as in the monopoly-bank economy, because the funding structure chosen by one bank does not take into account of the fire-sale cost of the choice of its funding structure imposed on the other bank.

4.3 N-Bank Economy

Based on the framework of the two-bank economy, the model can be easily extended to a n-bank economy. As the model analysis is studied based on the optimal decision under symmetric equilibrium, without loss of generality, I simplify the model by assuming that when Bank 1 makes its decision, it assumes that all other banks have a symmetric funding structure; in order words, the conjectures on all other banks are the same. In the following, I use Bank 2 as a representative for the other (n-1) banks. Therefore, the variables for Bank 2 is the same as the corresponding variables of the other banks (except for Bank 1). For example, the liquidity-shortfall threshold of Bank 2 (denoted by α_2^*) is also the threshold for all other banks except for Bank 1.

As the explanation for the scenarios and decision problems in the n-bank economy is very similar to those in the two-bank economy, I present the following subsections in a briefer way to avoid cumbersome repeats in explanation.

4.3.1 Being a Riskier Bank

When Bank 1 is the riskier bank, compared with all other (n-1) banks in the economy, its return specification is subdivided into three scenarios: (Scenario 0) there is no liquidity shortfall in all banks; (Scenario 1) there is a positive liquidity shortfall in Bank 1, but not in the other (n-1) banks (i.e. a one-bank fire-sale); (Scenario 2) there are positive liquidity shortfalls in all banks (i.e. a n-bank fire-sale).

Scenario (0): No fire-sale. When there is no fire-sale, the rate of return to Bank-1 equity holders is

$$R_0 = \frac{q_n\left[\rho - (\beta r_1 + (1-\beta) r_2) d_1\right]}{q_n(1-d_1)}$$

Or equivalently

$$R_0(1-d_1) = \rho - (\beta r_1 + (1-\beta) r_2) d_1$$

The condition for having Scenario (0) is given by $\alpha \geq \alpha_1^*$.

Scenario (1): One-bank fire-sale. If there is a liquidity shortfall in Bank 1 only, there is a one-bank fire-sale. The amount of asset needs to be sold by Bank 1 is determined by the following equation.

$$x_{11}(1 - h q_n x_{11}) = \beta r_1 d_1 - \alpha\rho$$

where x_{11} is the amount of asset liquidation for Bank 1 in a one-bank fire-sale.

The rate of equity return for Bank 1 in Scenario (1) is specified by,

$$R_1 = \frac{q_n\left[\alpha\rho + x_{11}(1 - hq_n x_{11}) - \beta r_1 d_1 + (1-\alpha-x_{11})\rho - (1-\beta) r_2 d_1\right]}{q_n(1-d_1)}$$

or in a simpler way

$$R_1(1-d_1) = R_0(1-d_1) - hq_n x_{11}^2 - (\rho-1)x_{11}$$

The condition for having Scenario (1) is given by $\alpha_2^* < \alpha \le \alpha_1^*$.

Scenario (2): n-bank fire-sale. If there are liquidity shortfalls in all banks, there is a n-bank fire-sale. The amount of asset needs to be sold by Bank 1 and Bank 2 in a n-bank fire-sale are determined by the following equations respectively.

$$x_{n1}(1 - hq_n(x_{n1} + (n-1)x_{n2})) = \beta r_1 d_1 - \alpha\rho$$

$$x_{n2}(1 - hq_n(x_{n1} + (n-1)x_{n2})) = \beta r_1 d_2 - \alpha\rho$$

The rate of equity return for Bank 1 in Scenario (2) is specified by,

$$R_2 = \frac{q_n\left[\alpha\rho + x_{n1}(1 - hq_n(x_{n1} + (n-1)x_{n2})) - \beta r_1 d_1 + (1-\alpha-x_{n1})\rho - (1-\beta) r_2 d_1\right]}{q_n(1-d_1)}$$

or in a simpler way

$$R_2(1-d_1) = R_0(1-d_1) - hq_n(x_{n1}^2 + (n-1)x_{n1}x_{n2}) - (\rho-1)x_{n1}$$

The condition for having Scenario (2) is given by $\alpha < \alpha_2^*$.

4.3.2 Decision Problem for Riskier Bank

The decision problem for Bank 1 being the riskier bank in the n-bank economy can be expressed by

$$\max_{d_1} \int_{\alpha_1^*}^{1} (R_0 - c)(1-d_1)d\alpha + \int_{\alpha_2^*}^{\alpha_1^*} (R_1 - c)(1-d_1)d\alpha + \int_{0}^{\alpha_2^*} (R_2 - c)(1-d_1)d\alpha \tag{3}$$

$$= \max_{d_1}(\rho - c) - (c - c^*)(1-d_1) - hq_n \int_{\alpha_2^*}^{\alpha_1^*} x_{11}^2 d\alpha - (\rho-1)\int_{\alpha_2^*}^{\alpha_1^*} x_{11} d\alpha$$

$$-hq_n \int_0^{\alpha_2^*} (x_{n1}^2 + (n-1)x_{n1}x_{n2})d\alpha - (\rho-1)\int_0^{\alpha_2^*} x_{n1} d\alpha$$

The first-order condition (FOC) for the decision problem with respect to d_1 is given by

$$(c - c^*) - 2hq_n \int_{\alpha_2^*}^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha - (\rho-1)\int_{\alpha_2^*}^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha$$

$$-hq_n \int_0^{\alpha_2^*} (2x_{n1} + (n-1)x_{n2}) \cdot \frac{\partial x_{n1}}{\partial d_1} d\alpha - (\rho-1)\int_0^{\alpha_2^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha = 0$$

where

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hq_n x_{11}} > 0$$

$$\frac{\partial x_{n1}}{\partial d_1} = \frac{\beta r_1}{1 - hq_n(2x_{n1} + (n-1)x_{n2})} > 0$$

The second-order condition (SOC) is given by

$$-\int_{\alpha_2^*}^{\alpha_1^*} \left[2hq_n \left(x_{11}\frac{\partial^2 x_{11}}{\partial d_1^2} + \left(\frac{\partial x_{11}}{\partial d_1}\right)^2 \right) + (\rho - 1)\frac{\partial^2 x_{11}}{\partial d_1^2} \right] d\alpha$$

$$-\int_{0}^{\alpha_2^*} \left[hq_n \left((2x_{n1} + (n-1)x_{n2})\frac{\partial^2 x_{n1}}{\partial d_1^2} + 2\left(\frac{\partial x_{n1}}{\partial d_1}\right)^2 \right) + (\rho - 1)\frac{\partial^2 x_{n1}}{\partial d_1^2} \right] d\alpha$$

where

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{2hq_n(\beta r_1)^2}{(1 - 2hq_n x_{11})^3}$$

and

$$\frac{\partial^2 x_{n1}}{\partial d_1^2} = \frac{2hq_n(\beta r_1)^2}{(1 - hq_n(2x_{n1} + (n-1)x_{n2}))^3}$$

To ensure that SOC is negative such that FOC always maximises the objective function, the sufficient condition is $h < \frac{n}{n+1}$. This ensures that both SOC's are positive even when x_{11}, x_{n1}, and x_{n2} are equal to their maximum value of one. Note that the sufficient condition in the n-bank economy is weaker than the one in one-bank economy, because $\frac{1}{2} < \frac{n}{n+1}$ for $n \geq 2$; therefore, the assumption of $h < 1/2$ is sufficient to guarantee the stableness of all FOC's, regardless of the number of banks in the economy.[14]

4.3.3 Being a Safer Bank

If Bank 1 chooses a safer funding structure than all other (n-1) banks, then the model must have $d_1 \leq d_2$. Or equivalently, $\alpha_1^* \leq \alpha_2^*$. There are only two scenarios in the decision-making for Bank 1: (Scenario 0) there is no liquidity shortfall for Bank 1; and (Scenario 1) there are liquidity shortfalls in all banks.

Scenario (0): No fire-sale *needed by Bank 1.* The rate of return to Bank-1 equity holders when there is no fire-sale is

$$R_0 = \frac{q_n\left[\rho - (\beta r_1 + (1-\beta)r_2)d_1\right]}{q_n(1-d_1)}$$

Or equivalently

$$R_0(1-d_1) = \rho - (\beta r_1 + (1-\beta)r_2)d_1$$

The condition for having Scenario (0) is given by $\alpha \geq \alpha_1^*$.

Scenario (1): n-bank fire-sale. In a n-bank fire-sale, the rate of equity return for Bank 1 is specified by,

$$R_1 = \frac{q_n\left[\alpha\rho + x_{n1}(1 - hq_n(x_{n1} + (n-1)x_{n2})) - \beta r_1 d_1 + (1 - \alpha - x_{n1})\rho - (1-\beta)r_2 d_1\right]}{q_n(1-d_1)}$$

[14] The derivations of $\frac{\partial x_{11}}{\partial d_1}$, $\frac{\partial x_{n1}}{\partial d_1}$, $\frac{\partial^2 x_{11}}{\partial d_1^2}$, and $\frac{\partial^2 x_{n1}}{\partial d_1^2}$ are shown in the appendix.

or in a simpler way

$$R_1(1-d_1) = R_0(1-d_1) - hq_n(x_{n1}^2 + (n-1)x_{n1}x_{n2}) - (\rho-1)x_{n1}$$

The condition for having Scenario (1) is given by $\alpha < \alpha_1^*$.

4.3.4 Decision Problem for Safer Bank

The decision problem for Bank 1 being the safer bank in the n-bank economy can be expressed by

$$\max_{d_1} \int_{\alpha_1^*}^{1} (R_0 - c)(1-d_1)d\alpha + \int_0^{\alpha_1^*} (R_1 - c)(1-d_1)d\alpha \tag{4}$$

$$= \max_{d_1} (\rho - c) - (c - c^*)(1-d_1) - hq_n \int_0^{\alpha_1^*} (x_{n1}^2 + (n-1)x_{n1}x_{n2})d\alpha - (\rho-1)\int_0^{\alpha_1^*} x_{n1}d\alpha$$

The first-order condition (FOC) for the decision problem with respect to d_1 is given by

$$(c - c^*) - hq_n \int_0^{\alpha_1^*} (2x_{n1} + (n-1)x_{n2}) \cdot \frac{\partial x_{n1}}{\partial d_1} d\alpha - (\rho-1)\int_0^{\alpha_1^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha = 0$$

The second-order condition (SOC) is given by

$$-\int_0^{\alpha_1^*} \left[hq_n \left((2x_{n1} + (n-1)x_{n2}) \frac{\partial^2 x_{n1}}{\partial d_1^2} + 2\left(\frac{\partial x_{n1}}{\partial d_1}\right)^2 \right) + (\rho-1)\frac{\partial^2 x_{n1}}{\partial d_1^2} \right] d\alpha$$

The expression for $\frac{\partial x_{n1}}{\partial d_1}$ and $\frac{\partial^2 x_{n1}}{\partial d_1^2}$, and the sufficient conditions are the same as in Section 4.3.2.

4.3.5 Equilibrium

I define *symmetric equilibrium* for the n-bank economy in the following.

Definition 2 *In a n-bank economy, given (n-1) banks choose the funding structure d_s, the remaining (one) bank has no intention to deviate from this funding structure and also chooses d_s as its funding structure, d_s is said to be the funding structure in symmetric equilibrium.*

Mathematically, this is expressed as follows. Given $d_2 = d_s$, the optimal decision variable d_1 derived from the two objective functions, eq.(3) and eq.(4), are equivalent and are both equal to d_s. Then the funding structure of all banks are said to be in symmetric equilibrium. The proof for the following proposition is given in the appendix of this paper.

Proposition 2 *In symmetric equilibrium, the proportion of deposit funding chosen in a n-bank economy is higher than the proportion chosen in a (n-1)-bank economy, for $n \geq 2$.*

From Proposition 2, one can observe that the liquidity risk for the banks is higher in an economy with more banks. The reason is that each bank has a stronger incentive to take excessive risk due to the larger subsidy from the larger number of *other banks*. Under symmetric equilibrium, the banks choose a higher proportion of deposit (higher liquidity risk) to attempt to obtain this subsidy from each other. The result leads to weaker financial stability. As this paper uses the number of banks in the economy as a measure for bank competition. Proposition 2 implies that bank competition leads to financial instability.

4.4 Further Discussion

It is interesting to discuss an alternative framework for this paper: what if the banks can acquire assets from each others, does it affect the risk-taking incentives in the banking system? To answer this question, one has to be aware that the discussion of this question is not possible based on a symmetric equilibrium, because when all banks choose the same funding structure, they have liquidity shortage at the same time, and no banks has excess liquidity to acquire assets from the other banks.

Therefore, to address this question, one needs to focus on an asymmetric (coordinated) equilibrium. The asymmetric equilibrium can be defined as a situation in which the banks are divided into two groups, with one group having a safer funding structure than the other. Under this asymmetric equilibrium, it is possible (under some occasions) for the safer group to have excess liquidity when the riskier group is in need of liquidity. The safer group can compete with the external investors to acquire the fire-sale assets from the riskier group. For simplicity of the analysis, I assume that the safer group of banks are price-takers and therefore they cannot alter the fire-sale price.

Based on this framework, there exists a trade-off when the banks choose their funding structures: (1) choosing riskier funding structure allows them to take advantage of other banks when all banks require asset fire-sale at the very bad economy states; (2) choosing safer funding structure allows them to acquire assets from the riskier banks when the economy is bad but not too bad (riskier banks have liquidity shortage while safer banks still have excess liquidity), at the cost of the expensive equity funding.

It is obvious that in order for (2) to be a possible choice, the expected profit from acquiring assets has to be larger than the higher cost of equity. I would like to point out that it is quite unlikely, because the cost of equity is independent from the economy states, but the profit from fire-sales only occurs at a narrow range of (bad-but-not-too-bad) economy states. If the cost of equity dominates the fire-sale profit, then (2) is not an optimal option, and the asymmetric framework boils down to the symmetric framework introduced in this paper.

What if the cost of equity is lower than the expected profit from fire-sales? The situation will then become more complicated. The magnitude of two forces becomes the key issue for determining bank decisions. Recall that the main factor that affects both (1) and (2) is the fire-sale price, which is determined by the fire-sale coefficient h in this model. A high value of this coefficient implies higher loss in fire-sale (lower fire-sale price). If $h = 0$ (meaning that there is no loss in fire-sale), the profit from acquiring assets during fire-sale is zero; in this case no bank will choose to be safer than others. If h is relatively high (meaning that the fire-sale loss is relatively high), the profit from acquiring assets is high; in such case some banks may choose to be safer to earn a profit from acquiring assets in fire-sale. The relationship between bank competition and financial stability is therefore more ambiguous under this asymmetric framework.

5 Numerical Results

To generate numerical results, I apply the following parameters for the numerical calculations.

$$\rho = 1.5, \quad \beta = 0.5, \quad r_1 = r_2 = 1, \quad c = 1.05, \quad h = 0.25 \quad n = 1, 2, 4, 6, 8, 10, \infty$$

The results are shown in the Figure 2[15]. One can see that as the number of banks in the economy

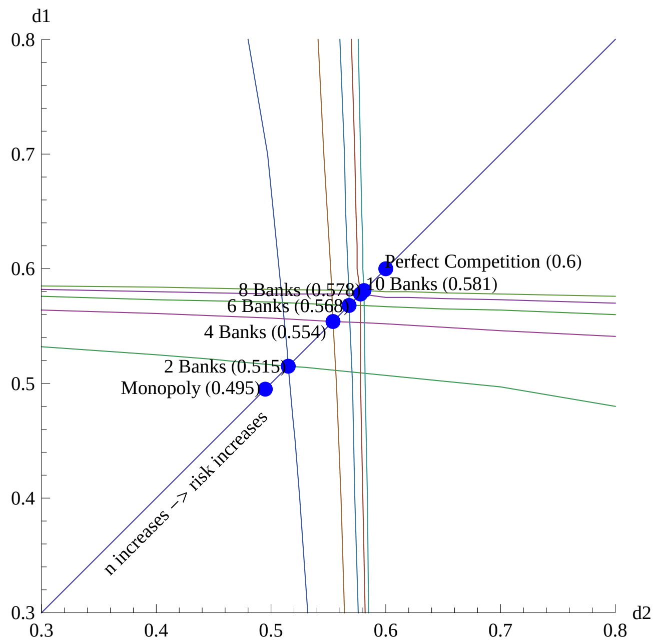

Figure 2: Proportion of deposit funding in an economy with different number of banks.

increases, the (symmetric) proportion of deposit contract increases. This observation is consistent with Proposition 1 and 2; one can conclude that due to the motivation to take advantage of other banks in the fire-sale, a larger number of banks in the economy lead to further weakening of financial stability, due to the riskier funding structure chosen by the banks.

To guarantee the risk-free nature for deposit contract in the numerical results, the proportion of deposit funding has to be constrained. I make sure that the numerical results satisfy the risk-free nature of deposit contract by adding the following constraint to the programming.

$$(1 - \beta) r_2 d_1 \leq \rho (1 - x_{n1})$$

This constraint ensures that even in the worst scenario ($\alpha = 0$), the return from fire-sale is enough to repay all deposit contracts on date 2 ($(1 - \beta) r_2 d_1$); this, of course, also implies all deposit contracts on date 1 are fully repaid. Other minor details for the coding are explained in the appendix of this paper.

[15]Due to the numerical limit, it is impossible to input $n = \infty$ into the coding; instead I use $n = 1,000,000$ to generate the result for perfect competition.

6 Policy Discussion

In this section, some policy interventions that may help restoring the socially optimal funding structure are discussed. As mentioned, the socially optimal funding structure in this paper is the structure chosen by the monopoly bank. As the number of banks increases, the (symmetric) funding structure becomes more risky and are socially suboptimal. Three policies are discussed in this section: (1) restricting the minimum capital requirement; (2) bridging the gap between the costs of equity and deposit; and (3) fire-sale penalty. In the following, I apply the numerical parameters used in Section 5 to generate supporting results for the policy discussion.

6.1 Minimum Capital Requirement

Setting a minimum capital requirement is the most straightforward policy according to this model. From the numerical results, it is clear that the socially optimal funding structure is characterised by $d_s = 0.495$, meaning that the optimal proportions for deposit and equity are 0.495 and 0.505 respectively. If the central authority can set a capital requirement to force all banks in the multi-bank economy to hold a proportion of equity of at least 50.5 percent of the total assets, then the socially optimal funding structure is restored.

However, capital requirement has already been commonly used in reality as a control for the asset risk of banks. Specifically, many central authorities set a minimum capital requirement such that bank equity has to be at least equal to a certain proportion of the risk-weighted assets, in order to protect the depositors. For this reason, it is hardly possible to adjust the minimum capital requirement to control for both the risks from asset portfolio and funding structure. Therefore, two other policies which can also control the funding structure of banks are studied.

6.2 Bridging the Gap of the Costs of Funding

According to the model specifications, the motivation for banks to accept deposit comes from the assumption that the cost of deposit is lower than the cost of equity. This difference between the two costs is defined as a market friction. A common example for this market friction is the taxation benefit from deposit (debt): as the interest expense from deposit is tax-deductible, funding bank loans with deposit is comparatively cheaper than funding with equity stock.

If the central authority can reduce the gap between the two sources of funding, it motivates the banks to choose a higher proportion of equity. Yet, according to this model, it is not a good idea to remove the gap between these two sources of funding entirely, because by doing so the banks will have no motivation to accept any deposit from households. How much subsidy should the central authority provide such that the banks can choose a funding structure that is socially optimal? I use the parameters from the numerical results in Section 5 to generate the following table.

The following table shows the cost of equity that can restore the symmetric funding structure to $d_s = 0.495$. One can see that the maximum subsidy is 0.00875 per unit of equity in an economy with infinite number of banks.[16] This subsidy is not particularly high, but it is very effective for restoring socially optimal funding structure.

In reality, this explicit subsidy can be implemented by limiting the tax-deductible interest expense in deposits; this also reduces the market friction between the costs of deposit and equity, and helps

[16] 1.05-1.04125=0.00875

Number of Banks (n)	Cost of Equity (c)
1	1.05
2	1.0477
4	1.0443
6	1.0432
8	1.0427
10	1.0424
∞	1.04125

Table 2: The Cost of Equity for Restoring Optimal Funding Structure.

control the bank risk to the central authority's target.

6.3 Fire-Sale Penalty

Is it possible for the central authority to punish the banks for asset fire-sale, in order to control the funding structure of banks? According to the model specifications in this paper, penalty is not an efficient tool to control bank risk (or funding structure) because the incentive for excessive risk-taking can outrun the penalty in an economy with many banks. Again, I use the parameters from the numerical results to generate the following table.

Number of Banks (n)	Fire-Sale Price Coefficient (h)
2	0.325
4	0.65
6	0.97
8	> 1
10	> 1
∞	∞

Table 3: The Fire-Sale Penalty for Restoring Optimal Funding Structure.

A penalty to fire-sale can be interpreted as an increase in the fire-sale-price coefficient (h). When h increases, the fire-sale price drops, causing larger loss to the banks. These extra losses can be interpreted as the penalty from the central authority for banks who have liquidity shortfalls. The table shows the value of h that can restore the funding structure to $d_s = 0.495$.

One can observe from the table that, h has to increase a lot in order to restore the socially optimal funding structure. In fact, some of the h have already violated the model constraint of $h < \frac{n}{n+1}$. What do these results imply? It implies that, according to this model, penalty can be outrun by the incentive for excessive risk-taking in a multi-bank economy. In extreme cases, the central authority needs to set penalty to infinite when the number of banks approaches infinite; this is neither realistic nor possible. In fact, from the numerical results shown in the table, one can see that it is hardly possible for the central authority to restore optimal funding structure even in a four-bank economy, because the central authority will almost need to triple h to achieve the goal (increase h from 0.25 to 0.65). Therefore, according to the numerical results, it is inefficient or even impossible to use penalty to restore optimal funding structure.

7 Conclusion

This paper studies the long-debated question, does bank competition lead to the weakening of financial stability? I examine this question from with a different perspective compared with the existing literature. Instead of the commonly-discussed asset risks, I apply a liquidity-risk framework to study how the interaction of competition and funding structure of banks can affect financial stability. I show that the existence of fire-sale plays an important role in the determination of funding structure of banks. In this paper, fire-sale creates an incentive for a bank to take advantage of other banks by choosing a riskier funding structure. Based on this, I prove that banks choose riskier funding structures in an economy with more banks in symmetric equilibrium. This implies that higher bank competition (characterised by more banks in the economy) leads to higher financial instability.

In the numerical simulations, I show that capital requirement and subsidising for the difference between the costs of different funding are efficient policies to restore the banks' funding structure to the socially optimal level, but fire-sale penalty seems to have limited effect and can be outrun by the excessive risk-taking incentive. These results suggest some room for further studies in both empirical and theoretical research, not on the widely-discussed asset risk aspects, but on the liquidity risk aspects for the relationship between bank competition and financial stability.

8 Appendix

8.1 Proof of Proposition 1

Proof. Step 1: In symmetric equilibrium, $d_1 = d_2 = d_s$ in the two-bank economy; therefore, $\alpha_1^* = \alpha_2^*$ and $x_{21} = x_{22}$ also hold. Substitute these results into the two FOC's in the two-bank economy, one can find that the two FOC's converge to the same result, which is

$$(c - c^*) - \left[\frac{3h}{2} \int_0^{\alpha_1^*} x_{21} \frac{\partial x_{21}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{21}}{\partial d_1} d\alpha \right] = 0 \tag{5}$$

Rewrite the FOC of the one-bank economy in the following.

$$(c - c^*) - \left[2h \int_0^{\alpha_1^*} x_{11} \cdot \frac{\partial x_{11}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{11}}{\partial d_1} d\alpha \right] = 0 \tag{6}$$

Step 2: I prove that there is a contradiction if $x_{11} \geq x_{21}$ (which implies the d_1 in the one-bank economy is greater than or equal to the d_1 in the two-bank economy); therefore, the model must have $x_{11} < x_{21}$ (which implies the d_1 in the one-bank economy is less than the d_1 in the two-bank economy).

If $x_{11} \geq x_{21}$, then

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hx_{11}} > \frac{\beta r_1}{1 - \frac{3h}{2} x_{21}} = \frac{\partial x_{21}}{\partial d_1}$$

Therefore, the second term of eq.(6) must be bigger than the second term of eq.(5).

As both FOC's equal zero, the threshold of eq.(6) must be smaller than the threshold of eq.(5) for the zero sum to be true. However, this is a contraction because if $x_{11} \geq x_{21}$, the threshold for eq.(6) must be bigger than the threshold for eq.(5).

Therefore, the model must have $x_{11} < x_{21}$, implying that the proportion of deposit funding in the one-bank economy is smaller than the proportion in the two-bank economy. ■

8.2 Proof of Proposition 2

Proof. Step 1: In symmetric equilibrium, $d_1 = d_2 = d_s$ in the n-bank economy; therefore, $\alpha_1^* = \alpha_2^*$ and $x_{n1} = x_{n2}$ also hold. Substitute these results into the two FOC's in the n-bank economy, one can find that the two FOC's converge to the same result, which is (with substitution of $q_n = 1/n$)

$$(c - c^*) - \left[\frac{h(n+1)}{n} \int_0^{\alpha_1^*} x_{n1} \frac{\partial x_{n1}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{n1}}{\partial d_1} d\alpha \right] = 0 \tag{7}$$

The FOC of the (n-1)-bank economy is

$$(c - c^*) - \left[\frac{hn}{n-1} \int_0^{\alpha_1^*} x_{(n-1)1} \frac{\partial x_{(n-1)1}}{\partial d_1} d\alpha + (\rho - 1) \int_0^{\alpha_1^*} \frac{\partial x_{(n-1)1}}{\partial d_1} d\alpha \right] = 0 \tag{8}$$

Step 2: I prove that there is a contradiction if $x_{n-1,1} \geq x_{n1}$ (which implies d_1 in the (n-1)-bank economy is greater than or equal to d_1 in the n-bank economy); therefore, the model must have $x_{n-1,1} < x_{n1}$ (which implies d_1 in the (n-1)-bank economy is less than d_1 in the n-bank economy).

If $x_{n-1,1} \geq x_{n1}$, then

$$\frac{\partial x_{n-1,1}}{\partial d_1} = \frac{\beta r_1}{1 - hq_{n-1}(2x_{n-1,1} + (n-2)x_{n-1,2})} > \frac{\beta r_1}{1 - hq_n(2x_{n1} + (n-1)x_{n2})} = \frac{\partial x_{n1}}{\partial d_1}$$

The second term of eq.(8) must be bigger than the second term of eq.(7).

As both FOC's equal zero, the threshold of eq.(8) must be smaller than the threshold of eq.(7) for the zero sum to be true. However, this is a contraction because if $x_{n-1,1} \geq x_{n1}$, the threshold for eq.(8) must be bigger than the threshold for eq.(7).

Therefore, the model must have $x_{(n-1)1} < x_{n1}$, implying that the proportion of deposit funding in the (n-1)-bank economy is smaller than the proportion in the n-bank economy. ■

8.3 The Differentiation of x_{jk} with Respect to d_1

In the following, I show the differentiation of x_{jk} with respect to d_1.

8.3.1 One-Bank Fire-Sale

In a n-bank economy, the liquidity shortfalls of Bank 1 in a one-bank fire-sale is given by

$$x_{11}(1 - hq_n x_{11}) = \beta r_1 d_1 - \alpha\rho$$

The change of x_{11} with respect to d_1 is therefore

$$(1 - 2hq_n x_{11})\frac{\partial x_{11}}{\partial d_1} = \beta r_1$$

or

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hq_n x_{11}}$$

The second derivative can be calculated from the following chain rule.

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{\partial \frac{\partial x_{11}}{\partial d_1}}{\partial x_{11}} \cdot \frac{\partial x_{11}}{\partial d_1} = \frac{2hq_n \beta r_1}{(1 - 2hq_n x_{11})^2} \cdot \frac{\beta r_1}{1 - 2hq_n x_{11}} = \frac{2hq_n(\beta r_1)^2}{(1 - 2hq_n x_{11})^3}$$

Substituting $n = 2$ and $q_n = 1/2$, one can obtain the differentials in the two-bank economy, which are

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - hx_{11}}$$

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{(1 - hx_{11})^3}$$

Substituting $n = 1$ and $q_n = 1$, one can obtain the differentials in the monopoly-bank economy, which are

$$\frac{\partial x_{11}}{\partial d_1} = \frac{\beta r_1}{1 - 2hx_{11}}$$

$$\frac{\partial^2 x_{11}}{\partial d_1^2} = \frac{2h(\beta r_1)^2}{(1 - 2hx_{11})^3}$$

8.3.2 N-Bank Fire-Sale

Similarly, the liquidity shortfalls of Bank 1 in a n-bank fire-sale is given by

$$x_{n1}[1 - hq_n(x_{n1} + (n-1)x_{n2})] = \beta r_1 d_1 - \alpha\rho$$

The change of x_{n1} with respect to d_1 is therefore

$$[1 - hq_n(2x_{n1} + (n-1)x_{n2}]\frac{\partial x_{n1}}{\partial d_1} = \beta r_1$$

or

$$\frac{\partial x_{n1}}{\partial d_1} = \frac{\beta r_1}{1 - hq_n[2x_{n1} + (n-1)x_{n2}]}$$

The second derivative calculated from the following chain rule is given by

$$\frac{\partial^2 x_{n1}}{\partial d_1^2} = \frac{2hq_n\beta r_1}{[1 - hq_n(2x_{n1} + (n-1)x_{n2})]^2} \cdot \frac{\beta r_1}{1 - hq_n[2x_{n1} + (n-1)x_{n2}]}$$

$$= \frac{2hq_n(\beta r_1)^2}{[1 - hq_n[2x_{n1} + (n-1)x_{n2}]]^3}$$

Substituting $n = 2$ and $q_n = 1/2$, one can obtain the differentials in the two-bank economy, which are

$$\frac{\partial x_{21}}{\partial d_1} = \frac{\beta r_1}{1 - h(x_{21} + \frac{1}{2}x_{22})}$$

$$\frac{\partial^2 x_{21}}{\partial d_1^2} = \frac{h(\beta r_1)^2}{[1 - h(x_{21} + \frac{1}{2}x_{22})]^3}$$

as shown in Section 4.

8.4 Minor Details in Numerical Programming

In order to generate the numerical programming for the model, better expressions for x_{11}, x_{n1}, and x_{n2} are needed. Recall that the definition of x_{11} is given by

$$x_{11}(1 - hq_n x_{11}) = \beta r_1 d_1 - \alpha\rho$$

or

$$hq_n x_{11}^2 - x_{11} + (\beta r_1 d_1 - \alpha\rho) = 0$$

The solutions for x_{11} are

$$x_{11} = \frac{1 \pm \sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$$

I need to rule out one solution in the numerical programming. I do so by showing that there is a contradiction if $x_{11} = \frac{1+\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$; therefore, the unique solution should be $x_{11} = \frac{1-\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$.

By definition, $x_{11} \geq 0$. If $x_{11} = \frac{1+\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$ is the solution, then

$$\sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha\rho)} \geq -1$$

which results in a complex number for x_{11}. This violates the definition of x_{11} of being a positive real number. Therefore, $x_{11} = \frac{1+\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$ is not a real-number solution.

If $x_{11} = \frac{1-\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$ is the solution, then

$$\sqrt{1 - 4hq_n(\beta r_1 d_1 - \alpha\rho)} \leq 1$$

which shows that $x_{11} = \frac{1-\sqrt{1-4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$ is the real-number solution, and I apply this solution to the numerical programming for the model.

Similarly, using the same technique, one can rule out a complex-number solution for x_{n1} and x_{n2}, and get the following unique real-number solution.

$$x_{n1} = \frac{(1 - hq_n(n-1)x_{n2}) - \sqrt{(1 - hq_n(n-1)x_{n2})^2 - 4hq_n(\beta r_1 d_1 - \alpha\rho)}}{2hq_n}$$

$$x_{n2} = \frac{(1 - hq_n x_{n1}) - \sqrt{(1 - hq_n x_{n1})^2 - 4hq_n(n-1)(\beta r_1 d_2 - \alpha\rho)}}{2hq_n(n-1)}$$

I apply these solutions to the numerical programming for the model.

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