Developing Equity Release Markets: Risk Analysis for Reverse Mortgage and Home Reversion

Daniel Alai, Hua Chen, Daniel Cho, Katja Hanewald, Michael Sherris
Introduction

- **Home equity release products**
  - Allow retirees to convert a previously illiquid asset into cash payments which can be used for home improvements, regular income, debt repayment, aged care and medical treatments etc.

- **Reverse mortgage**

- **Home reversion**
  - Little research has been done on the risk analysis of other equity release products, such as home reversion contracts.
Introduction

- **US**
  - Reverse mortgage products dominate.
  - HECM accounts for 95% of the market (Ma and Deng, 2006).

- **UK**
  - Reverse mortgages, home reversion and other equity release products have been available for between 10 and 30 years
  - Reverse mortgage: 75%; home aversion: 25% (ASIC 2005).

---

### Availability of product

<table>
<thead>
<tr>
<th>Features</th>
<th>US</th>
<th>UK</th>
<th>Australia</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled-up interest</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest-only mortgage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annuity scheme</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shared appreciation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home reversion schemes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed-rate products</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Widely available**
- **Not available**

Source: FSA Factsheet – UK Equity Release Market, FSA, Equity Release Schemes in the UK; Datamonitor; Equity Release Schemes in Australia, Datamonitor, Oliver Wyman analysis
Reverse Mortgage
- The provider lends the customer cash and obtains a mortgage charge over the customer's property (or a share of the property).
- The contract is terminated upon the death or permanent moveout of the customer, at which time the property is sold and the proceeds are used to repay the outstanding loan.
- Typically, a no negative equity guarantee (NNEG) is included in the contact.

Home Reversion
- The provider purchases the ownership right over the customer's property (or a share of the property).
- The home is sold at discount, and the contract includes a lease for life agreement.
Major Risks in RM and HR

- Risks of RM
  - Termination Risk
  - Longevity risk, mobility risk, refinancing risk
  - Loan Interest Rate risk
  - House Price Depreciation Risk

- Risks of HR
  - Termination Risk
  - Longevity risk, mobility risk, refinancing risk
  - Rental yield appreciation risk
  - House Price Depreciation Risk
Markov Termination Model

- Assume a single, female policyholder.

- Contract termination is determined by two proportionality constants on female population mortality.

\[ \mu_c^x = (\theta + \rho)\mu_x \]

where \( \theta \) is the at-home mortality proportionality constant
\( \rho \) is non-mortality driven proportionality constant

- Assume a Gompertz structure for the population force of mortality

\[ \mu_x = \lambda \exp(\gamma x) \]
Parameter Calibration

- $\lambda$ and $\gamma$
  - Use data from the Human Mortality Database: ages 50-105 and calendar years 1950-2009 from Australian females.
  - Fit both an ordinary linear regression (LR) on the log-transformed mortality rates as well as a Poisson regression (PR) on death counts with an appropriate exposure offset.
  - The results are very similar; we use PR results
    \[
    \hat{\lambda}^{(LR)} = 0.000217; \hat{\lambda}^{(PR)} = 0.000171; \hat{\gamma}^{(LR)} = 0.0993; \hat{\gamma}^{(PR)} = 0.1017
    \]

- $\theta$ and $\rho$
  - Values are taken from Ji et al. (2012)

<table>
<thead>
<tr>
<th>Age</th>
<th>$\theta_f$</th>
<th>$\rho_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 70$</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>80</td>
<td>0.20</td>
<td>0.90</td>
</tr>
<tr>
<td>90</td>
<td>0.33</td>
<td>0.85</td>
</tr>
<tr>
<td>$\geq 100$</td>
<td>0.46</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Termination Probabilities

Female, age 50 at inception: $\rho + \theta = 1$

Female, age 50 at inception: $\rho + \theta = 1.3$
VAR Economic Model

- Raw data: quarterly data from June 1993 to June 2011
  - zero-coupon interest rates (3-month and 10-year)
  - standard variable mortgage rate (MR)
  - Sydney house price index (HI)
  - Sydney rental index (RI).
  - Sydney GDP

- the mortgage rate and the 3-month zero coupon rates are highly correlated
VAR (1)

\[ z_{t+1} = c + Bz_t + \sum \zeta_{t+1} \]

where \( \sum \) is a lower triangular matrix,

\[ \zeta_{t+1} \sim N(0, I) \]

\( Z \) includes the following variables:

- 3-month zero coupon rate (mortgage rate can be expressed as the 3-month zero coupon rate plus a fixed margin)
- Term structure spread = 10-year ZC rate – 3-month ZC rate
- \( d\ln(HI) \)
- \( d\ln(RI) \)
- \( d\ln(GDP) \)
We define the stochastic discount factor as (Cochrane and Piazzesi 2005)

\[ m_{t+1} = \exp \left( \delta_0 + \delta_1 z_t + \frac{1}{2} \lambda_t' \Lambda_t + \lambda_t' \zeta_{t+1} \right) \]

- \( \delta_0 + \delta_1 z_t \) is the short rate in the VAR model where \( \delta_0 = 0 \) and \( \delta_1 = (1,0,0,0,0) \)

- \( \frac{1}{2} \lambda_t' \Lambda_t + \lambda_t' \zeta_{t+1} \) relates shocks in the state variables to the pricing kernel where \( \lambda_t = \lambda_0 + \Lambda_t z_t \) is a time-varying vector of market price of risk

The price of an asset at time \( t \) : 

\[ P_t = E_t \left[ m_{t+1} X_{t+1} \right] \]

Recursively, 

\[ P_0 = E \left[ \prod_{s=0}^{t} m_s(X_t) \right] \]
Stochastic Discount Factors

SDF

SDR vs The Short Rate

SDR
ERP
dlnZC025I

(Years)
Estimation Procedure

VAR parameters \((c, B, \text{ and } \Sigma)\) are estimated by maximum likelihood.

Risk parameters \((\lambda_0 \text{ and } \Lambda_1)\) can be estimated by minimizing the sum of the squared differences between the fitted yields of the term structure model and historical zero coupon yields.

\[
\min_{\{\hat{\lambda}_0, \Lambda_1\}} \sum_{t=1}^{T} \sum_{n=1}^{N} \left( \hat{y}_t^{(n)} - y_t^{(n)} \right)^2
\]

where \(\hat{y}_t^{(n)} = -\frac{A_n}{n} - \frac{B_n'}{n} z_t\)

and \(A_n\) and \(B_n\) can be solved recursively

\[
A_n = -\delta_0 + A_{n-1} + B_{n-1}'(c - \Sigma \lambda_0) + \frac{1}{2} B_{n-1}' \Sigma \Sigma' B_{n-1}
\]

\[
B_n = -\delta_1 + (B - \Sigma \Lambda_1)' B_{n-1}
\]
Non-Negative Equity Guarantee in RM

- For a fixed termination time $t$
  \[ NN_t = \max (L_t - H_t, 0) \]

- For random termination time
  \[ NN = \sum_{t=0}^{\omega-x-1} E \left[ \left( \prod_{s=0}^{t} m_s \right)_t p_x q_{x+t} \max (L_t - H_t, 0) \right] \]

- Cash flow analysis
  \[ RM = \sum_{t=0}^{\omega-x-1} t p_x q_{x+t} e^{-r_t} \left[ \min (L_t, H_t) - L_0 e^{ct} \right] \]
Lease for Life in HR

- For a fixed termination time $T$

$$LL_0 = \sum_{t=0}^{T} E\left[\left(\prod_{s=0}^{t} m_s\right) H_t g_t\right]$$

- For random termination time

$$LL = \sum_{t=0}^{\omega-x-1} E\left[\left(\prod_{s=0}^{t} m_s\right)_t p_x H_t g_t\right]$$

- Cash flow analysis

$$HR = \sum_{t=0}^{\omega-x-1} p_x q_{x+t} e^{-r_t t} \left[H_t - (H_0 - LL) e^{ct}\right]$$
# Developing the Equity Release Markets

<table>
<thead>
<tr>
<th>Age</th>
<th>H0</th>
<th>LTV/Acq</th>
<th>Reverse Mortgage</th>
<th>Home Reversion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>NN</td>
<td>Premium</td>
</tr>
<tr>
<td>65</td>
<td>480,000</td>
<td>64%</td>
<td>19,947</td>
<td>0.5465%</td>
</tr>
<tr>
<td>65</td>
<td>540,000</td>
<td>64%</td>
<td>22,440</td>
<td>0.5465%</td>
</tr>
<tr>
<td>65</td>
<td>600,000</td>
<td>64%</td>
<td>24,933</td>
<td>0.5465%</td>
</tr>
<tr>
<td>75</td>
<td>480,000</td>
<td>70%</td>
<td>18,086</td>
<td>0.6792%</td>
</tr>
<tr>
<td>75</td>
<td>540,000</td>
<td>70%</td>
<td>20,346</td>
<td>0.6792%</td>
</tr>
<tr>
<td>75</td>
<td>600,000</td>
<td>70%</td>
<td>22,607</td>
<td>0.6792%</td>
</tr>
<tr>
<td>85</td>
<td>480,000</td>
<td>76%</td>
<td>16,637</td>
<td>1.042%</td>
</tr>
<tr>
<td>85</td>
<td>540,000</td>
<td>76%</td>
<td>18,717</td>
<td>1.042%</td>
</tr>
<tr>
<td>85</td>
<td>600,000</td>
<td>76%</td>
<td>20,797</td>
<td>1.042%</td>
</tr>
</tbody>
</table>
Equity release LTVs
Across products and countries

- Australia
- UK
- Fannie Mae
- Financial Freedom
- HECM

LTV

Source: HUD, FF mortgage calculator, Datamonitor
Payoff Comparison

Payoff Comparison: HR&RM

$AUD(,000)
### Risk Measures

#### Reverse Mortgage

<table>
<thead>
<tr>
<th></th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>VaR 99.5%</th>
<th>CVaR 95%</th>
<th>CVaR 99%</th>
<th>CVaR 99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A65H48L64</td>
<td>-49.12</td>
<td>-113.30</td>
<td>-126.23</td>
<td>-88.11</td>
<td>-134.51</td>
<td>-149.18</td>
</tr>
<tr>
<td>A65H54L64</td>
<td>-55.26</td>
<td>-127.46</td>
<td>-142.01</td>
<td>-99.12</td>
<td>-151.32</td>
<td>-167.83</td>
</tr>
<tr>
<td>A75H48L70</td>
<td>-43.07</td>
<td>-112.87</td>
<td>-128.19</td>
<td>-85.36</td>
<td>-137.80</td>
<td>-159.35</td>
</tr>
<tr>
<td>A75H54L70</td>
<td>-48.45</td>
<td>-126.98</td>
<td>-144.21</td>
<td>-96.03</td>
<td>-155.03</td>
<td>-179.26</td>
</tr>
<tr>
<td>A75H60L70</td>
<td>-53.83</td>
<td>-141.09</td>
<td>-160.24</td>
<td>-106.69</td>
<td>-172.26</td>
<td>-199.18</td>
</tr>
<tr>
<td>A85H48L76</td>
<td>-29.68</td>
<td>-89.94</td>
<td>-118.16</td>
<td>-64.63</td>
<td>-120.82</td>
<td>-141.05</td>
</tr>
<tr>
<td>A85H54L76</td>
<td>-33.39</td>
<td>-101.19</td>
<td>-132.93</td>
<td>-72.71</td>
<td>-135.92</td>
<td>-158.68</td>
</tr>
<tr>
<td>A85H60L76</td>
<td>-37.10</td>
<td>-112.43</td>
<td>-147.70</td>
<td>-80.79</td>
<td>-151.02</td>
<td>-176.32</td>
</tr>
</tbody>
</table>

#### Home Reversion

<table>
<thead>
<tr>
<th></th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>VaR 99.5%</th>
<th>CVaR 95%</th>
<th>CVaR 99%</th>
<th>CVaR 99.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A65H48L64</td>
<td>-137.02</td>
<td>-178.09</td>
<td>-186.37</td>
<td>-161.97</td>
<td>-191.66</td>
<td>-201.05</td>
</tr>
<tr>
<td>A65H60L64</td>
<td>-171.27</td>
<td>-222.61</td>
<td>-232.96</td>
<td>-202.46</td>
<td>-239.58</td>
<td>-251.32</td>
</tr>
<tr>
<td>A75H48L70</td>
<td>-123.11</td>
<td>-171.97</td>
<td>-182.70</td>
<td>-152.71</td>
<td>-189.43</td>
<td>-204.50</td>
</tr>
<tr>
<td>A75H54L70</td>
<td>-138.50</td>
<td>-193.47</td>
<td>-205.53</td>
<td>-171.80</td>
<td>-213.10</td>
<td>-230.07</td>
</tr>
<tr>
<td>A75H60L70</td>
<td>-153.89</td>
<td>-214.96</td>
<td>-228.37</td>
<td>-190.89</td>
<td>-236.78</td>
<td>-255.63</td>
</tr>
<tr>
<td>A85H48L76</td>
<td>-97.19</td>
<td>-140.96</td>
<td>-155.19</td>
<td>-122.44</td>
<td>-161.31</td>
<td>-176.83</td>
</tr>
<tr>
<td>A85H54L76</td>
<td>-109.33</td>
<td>-158.58</td>
<td>-174.58</td>
<td>-137.75</td>
<td>-181.47</td>
<td>-198.93</td>
</tr>
<tr>
<td>A85H60L76</td>
<td>-121.48</td>
<td>-176.20</td>
<td>-193.98</td>
<td>-153.05</td>
<td>-201.63</td>
<td>-221.03</td>
</tr>
</tbody>
</table>
Conclusions

- Comparison between RM and HR
  - RM: accumulation of debt over the life of the contract
  - HR: debt-free.

- RM: senior homeowners bear various risks, including longevity risk, interest rate risk and property value risk
  - HR: these risks are remitted to the provider with the transfer of the title.

- RM: providers want house price to rise, while homeowners do not care
  - HR: The interests of investors and consumers are aligned: both want the value of the home to rise

- Consumers may prefer home reversion product, but providers bear higher risk and need more stringent capital requirement.