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Tobin-like Taxes and Market Crashes

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Abstract

Recent crisis revived interest in financial transaction taxes (FTTs) as a means to offset negative risk externalities. However, up-to-date academic research does not provide sufficient insights into the effects of transaction taxes on financial markets, as the literature has here-to-fore been focused too narrowly on variance as a measure of volatility. In this paper we argue that it is imperative to understand the relationship between price jumps, variance, and FTTs. While variance is not necessarily problem in itself, the non-normality of return distribution caused by price jumps affects not only the performance of many risk-hedging algorithms but directly influences the frequency of catastrophic market events. To study the aforementioned relationship we use an agent-based model of financial markets. Its results show that FTTs may increase the variance while decreasing the impact of price jumps. This result implies that regulators may face a trade-off between variance and price jumps when designing optimal tax.

Keywords: Price jumps, Tobin tax, Agent-based modeling, Monte Carlo, Volatility.

J.E.L. Classification Number: C15, C16, C61, G17, G18
Highlights

- We model the artificial financial market using the agent-based model and Monte Carlo.
- We study the impact of the financial transaction tax (FTT) on the price process.
- We focus on the overall volatility and Gaussian distinguish noise from price jumps.
- Increasing FTT is increasing overall volatility while decreasing rate of price jumps.
- Increasing FTT is making the markets more stable and more hedgeable.

1 Introduction

Nobel Laureate James Tobin first proposed a tax on spot conversions of one currency into another (Tobin, 1978) in the aftermath of the Bretton-Woods system’s break-up as a way to mitigate short-term financial round-trip excursions into another currency. His intention was “to throw some sand in the wheels of our excessively efficient international money markets.” He and his co-authors offered more arguments in favor of the tax in Eichengreen, Tobin, and Wyplosz (1995). But Tobin’s idea was just a specific application of Keynes’s idea of a tax on transactions mitigating the effect of speculation on financial markets (Keynes, 2006). However, the name ‘Tobin tax’ is today often used to denote not only foreign exchange transaction taxes, but financial transaction taxes in general as well.

The debate on the merits of Tobin-like taxes has not so far reached a definite conclusion. The proponents of the tax claim that increased transaction cost affects short-term high volume trading (speculation) more than long-term positions, decreasing market volatility and thus potential for crashes. In this regard the tax can be thought of as a Pigovian tax on a negative risk externality, as increased volatility can decrease welfare and efficiency. The opponents of the Tobin tax generally claim that it can, in fact, increase volatility by decreasing market liquidity or that speculative trading serves to stabilize prices around long-run equilibrium.

Although widely discussed in policy and political circles, the debate there is often driven by ideology and politics rather than academic research. The academic debate was historically driven mostly by theoretical models, although more recently simulation and empirical studies have been gaining some ground. However, theoretical and empirical evidence is so far mixed.
Historically, one branch of the literature used macroeconomic models of market bubbles. The arguments against the tax are often based on efficient market hypothesis (EMH from now on; due Fama, 1965), which implies that speculators cannot destabilize market, as they would eventually run out of money and be driven out of the market. Furthermore, based on EMH one can argue that speculative trading provides liquidity and helps to incorporate new information into the prices. Opposing models argue that externalities and imperfect information may be a cause of inefficiencies and that in these cases taxes can help economy reach the second best outcome.

Another strand of literature is focused on microeconomic behavior of the agents of the financial markets. Earlier examples of heterogenous agent models include Palley (1999), who combined noise traders (which were shown in prior literature to increase volatility, see e.g. De Long, Shleifer, Summers, and Waldmann, 1990) with the literature analyzing Tobin tax. He identified conditions under which such a tax drives out noise traders, thus benefiting fundamental traders and lowering volatility, and leading to higher efficiency. Also, he concluded that there is a trade-off between costs and benefits, because Tobin tax may discourage fundamental traders, as well. Westerhoff (2003) used model with fundamentalist and chartist traders in the framework of a foreign exchange market, and its outputs were consistent with the stylized facts of foreign exchange markets (unit roots in exchange rates, fat tails for returns). In his model, a low tax rate first crowds out chartism, but higher rates lead to misalignments due to decreasing number of fundamentalist. Mathevet and Steiner (2012) use a dynamic global game to show that in the imperfect information setting transaction taxes may stop sudden investment reversals under certain conditions, thus increasing welfare.

The empirical evidence on this issue is scant (one of the reasons is that it has never been adopted in its true form as a global tax) and, as we will argue, methodologically problematic. Few papers that tried to estimate the effect empirically (estimating effect of transaction taxes either on local foreign exchange or financial markets) offer support for all possible sides of the debate. The side that found evidence against the transaction tax includes Umlauf (1993) who, based on time series data on equity returns in Sweden, found that by introducing transaction tax the volatility measured by the conditional variance went up and trading volumes down. Moreover, the author argued that trading moved to London. However, it must be noted that Swedish transaction tax of 1% (later increased to 2%) was higher than what Tobin pro-

\[\text{Note although the tax rate was initially 0.5\% and later increased to 1\%, this tax was nominally borne by both sides of the}\]
posed originally (0.5%), and the author himself notes that “appropriate theoretical foundations are lacking” making the estimation imprecise and warns against “generalizing from a single data point” (ibid. p. 239). Aliber, Chowdhry, and Yan (2002) examined the effect of transaction costs in general on volatility (defined as standard deviation of prices) of foreign exchange rates for four different currencies, and found positive relationship, as well. The opposite result, in support of proponents of the Tobin tax, can be found in Liu and Zhu (2009) who found that lowering of transaction costs in Japan lead to higher volatility, implying negative correlation between transaction costs and volatility. Finally, in the third group there is a study by Hu (1998), who found no significant effect of stock transaction tax on market volatility and turnover taking advantage of 14 tax changes that occurred in stock markets in Hong Kong, Japan, Korea, and Taiwan during the period 1975-1994.

We see two major issues that are left rather unexplored. First, these studies do not take into account a scale effect (Tobin tax was meant to be a global tax). Small markets like Sweden does not have a significant impact on the world economy, so the speculative trading moves abroad, it does not alter the volatility on these foreign markets, but may very much hurt trade volumes domestically. However, if the market is large enough, there will be an impact on the foreign market, as well. Second, we want to argue that the studies choose their focus in a wrong way by focusing on conditional variance as a single measure of volatility. Concerning the first point, not much work has been done. Westerhoff and Dieci (2006) studied the phenomenon on a model with heterogeneous agents who can trade in different markets and can choose a trading strategy (e.g. fundamentalist vs. chartist). Importance of strategies evolve over time according to their fitness. They find that the tax decreases volatility in the market where it was imposed while increasing it on the other. The opposite effect of transaction tax on volatility in the two market framework was obtained by Mannaro, Marchesi, and Setza (2008), who used the methodology of agent-based models (ABMs). They used four types of traders with different strategies, who can trade on the maximum of two markets. However the relative share of strategies is exogenously input by the authors, but agents may choose where to trade and whether to trade at all. On the other hand, one of the few most recent studies, Bianconi, Galla, Marsili, and Pin (2009), concluded that transaction tax decreases volatility. Their ABM based on Minority Game framework used again fixed strategies that were randomly distributed across agents transaction implying the overall tax rate of 1% and 2%, respectively.
at the beginning of the simulation.

Our second – perhaps more important – point is that all of these studies are focused on the conditional variance as a measure of volatility. They ignore additional source of information: price jumps. Generally, the literature suggests (Merton, 1976, or Giot, Laurent, and Petitjean, 2010) that volatility of most financial instruments can be decomposed into two parts: a regular Gaussian component and a price jump component. Many models aim to estimate conditional variance, such as various GARCH models, ignore the price jump component while allowing the realized variance to deviate from the Gaussian distribution. However, as we show in this paper, the link between price jumps and conditional variance is not that straightforward – the measure of one may rise, while the measure of the other may decrease. Higher conditional variance does not have to be a problem per se, because it does not necessarily lead to leptokurtic return distribution. Fat tails, which have become a stylized fact of financial markets, are better explained by price jumps, so even if the transaction tax increases conditional variance, its effect on price jump frequency may be opposite, thus making the distribution less fat-tailed. If this is the case, it would not only improve the prediction power of standard asset pricing models that use normal distribution but, given that catastrophic events are non-normal in nature, it would lead to higher stability of financial markets. However, the relationship between transaction taxes and price jumps has not here-to-f ore been studied extensively in the literature.

This paper argues that it is crucial to understand the effect of the Tobin tax on price jumps. As Andersen, Benzoni, and Lund (2002) and Andersen, Bollerslev, and Diebold (2007) show, price jumps are present in majority of price time series, therefore their presence should be a subject of research. Price jumps can have serious adverse impact on predictive power of the pricing formulas and calculation of the estimates of the financial variables. Moreover, price jumps are the source of non-normality and may cause black-swan events on financial markets.

While the presence of price jumps in the data is well-established, the literature disagrees on their origin. One branch of literature (Merton, 1976; Lee and Mykland, 2008 or Lahaye, Laurent, and Neely, 2011) considers new information a primary source of price jumps, while other authors, like Joulin, Lefèvre, Grunberg, and Bouchaud (2008) and Bouchaud, Kockelkoren, and Potters (2006), conclude that price jumps are mainly caused by a

\[ \text{For an overview see Hamilton (1994)} \]

\[ \text{For illustrations of changes in the pricing formulas caused by price jumps see Pan (2002) or Broadie and Jain (2008). Brooks, Černý, and Miffre (2011) discuss the effect of higher moments on optimal allocations within utility-based framework.} \]
local lack of liquidity with news announcements having a negligible effect. The third branch – behavioral finance literature (e.g., Shiller, 2005) – suggests that price jumps are caused by the behavior of market participants themselves. For analyzing the two latter views the ABM methodology is especially appropriate, since it allows for explicit modeling of interactions among market participants.

The principal contribution of this paper is to study the relationship between price jumps and variance, and how transaction taxes affect them in an established model. To this end we employ a slightly modified version of the model used by Mannaro, Marchesi, and Setzu (2008), who find the positive correlation between tax rate and variance.

The paper is organized as follows. In Section 2, we describe the agent based model for simulation of the artificial financial markets. In Section 3, we model the impact of the financial transaction tax on the price process and provide estimators to quantify this effect. In Section 4, we provide results of our analysis. Section 5 concludes.

2 The Agent-based model

In this paper, we model the impact of the financial transaction tax on the artificial financial markets using the agent-based models (abbrev. ABMs). In our opinion ABMs are especially appropriate for the study of the questions concerned with transaction taxes, because:

1. They allow for explicit modeling of said transactions (interactions), not relying on market clearing assumption thus allowing us to study behavior out of steady state;

2. They allow for modeling of each agent independently, and every agent can pick their strategies endogenously according to the evolution of the modeled system. This implies that these agents are allowed to be heterogeneous both ex ante and ex post.

Our basic treatment follows closely the methodology of Mannaro, Marchesi, and Setzu (2008) with some minor modifications. We define four types of agents based on their behavior: noise traders, fundamentalist traders, momentum traders, and contrarian traders. We use their values of parameters that were calibrated so that the price series generated match the usual stylized facts of financial markets. The agent-based
modeling procedure itself is performed as follows (similar to Lavička, Lin, and Novotný, 2010): We set initial conditions of the model including number of interacting agents and various model-specific parameters described below. Then we let the economy to evolve step by step until a predetermined number of steps (or trading days) are reached. Every step we record closing price, overall traded amount of assets, amount of assets sold and bought by each trader group, total demand and total supply by each trader group, wealth in each trader group, and tax revenue. We repeat this simulation certain number of times to obtain statistically robust results. For the actual simulation we use modified Zarja C++ environment for agent-based modeling developed in Lavička (2010) and downloadable from http://sourceforge.net/projects/politeconomy/.

2.1 Trader types

Our artificial market consists of traders distributed evenly into four groups (random, fundamentalists, momentarian, and contrarian) described below.

Random traders These zero intelligence traders denoted as $R$ do not follow any particular strategy, they issue a buy or a sell order with equal probability. They are a proxy for traders that trade for their private reasons independent of the market situation, or who follow irrelevant information. If they buy (sell) the limit price of their buy (sell) order is determined as:

\[ l^b_i = p(t) \cdot N(\mu, s_i), \]
\[ l^s_i = \frac{p(t)}{N(\mu, s_i)}, \]

where $N(\mu, s_i)$ is a Gaussian distribution.

\[ s_i = k \cdot \sigma_i(\tau_i), \]

where $\sigma_i(\tau_i)$ is the variance computed based on window length $\tau_i \sim U[2,5]$. $k$ was set to 1.9. As Mannaro, Marchesi, and Setzu (2008) argue, the dependence on past variance simulates a GARCH model trading.
The problem may arise when $s_i$ becomes so large that the realization of $N(\mu, s_i)$ becomes negative. We solve this problem by setting the sell or buy order to zero in these cases.

Fundamentalists  Fundamentalists trade based on their beliefs about the fundamental price of assets. If fundamentalists decide to buy or sell, they buy/sell a fraction $q$ of their inventory, which depends on current ($p(t)$) and fundamental ($p_f$) price of the asset. The parameter $k$ is the same as in the noise traders’ case.

$$q = k \cdot \frac{|p(t) - p_f|}{p_f}.$$  

Momentum traders  Denoted as $T$, these traders follow trend - they buy when the price goes up and sell when it goes down. They are a proxy for traders using technical analysis or herd behavior. They look back at the history based on a time window $\tau_i$, which is randomly drawn at the beginning of the simulation. If they decide to issue an order, the limit price $l_i$ is computed as:

$$l_i = p(t) \cdot \left[1 + k \cdot \frac{p(t) - p(t - \tau_i)}{\tau_i p(t - \tau_i)}\right],$$  

where $k$ is the same parameter as before. In this case $\tau_i \sim U[3, 20]$. Conditional on the decision to sell (buy) the exact quantities are computed as follows:

$$q_i^s = a_i(t) \cdot U(0, 1) \cdot \left[1 + k \cdot \frac{|p(t) - p(t - \tau_i)|}{\tau_i p(t - \tau_i)}\right],$$

$$q_i^b = a_i(t) \cdot U(0, 1) \cdot \left[1 + k \cdot \frac{|p(t) - p(t - \tau_i)|}{\tau_i p(t - \tau_i)}\right],$$

where $q_i^s$ is the quantity to sell and $q_i^b$ is the quantity to buy. Naturally, $q_i^s \leq a_i(t)$ and $q_i^b \leq c_i(t)$.

Contrarian traders  Similarly to momentum traders, they follow technical analysis of the trend, however they expect that if the price is rising it is going to fall soon, so they try to sell near the maximum and vice versa. This implies that their behavioral rules are the same as those of momentum traders, only in the opposite direction. These traders are analogous to the arbitrageurs discussed above: they play against
2.2 Price Clearing Mechanism

Each of $U$ buy orders is defined by quantity $a_u^b$ and limit price $b_u$, $u = 1, ..., U$. Similarly, $V$ selling orders are defined by $(a_v^s, s_v)$, $v = 1, ..., V$. Demand and supply are then defined by following two functions:

$$f_{t+1}(p) = \sum_{u|b_u \geq p} a_u^b,$$

$$g_{t+1}(p) = \sum_{v|s_v \leq p} a_v^s.$$  \hspace{1cm} (8) \hspace{1cm} (9)

Market clearing price $p^*$ is then determined by intersection of (8) and (9). More specifically, the orders are sorted by price: sell orders that satisfy $s_v \leq p^*$ from lowest to highest, and buy orders that satisfy $b_u \geq p^*$ from highest to lowest. Then they are being matched from the bottom of the list while there is at least one pair to be matched. Thus the last (buy or sell) order may be satisfied only partially. Variables $a_i$ and $c_i$ are then updated accordingly.

3 The Model of the Financial Transaction Tax

In this section, we model the effect of the financial transaction tax on the price process and propose methods to analyse the impact.

3.1 The Model of Price Process

The dynamics of the proposed model defines the price generating process, whose log-returns have extensive skewness and kurtosis. In this part, we focus on the market price of the traded asset and model its dependence of the prices on the imposed Tobin tax. We assume that the one-dimensional asset log-price process $X$ takes the form of the Ito semimartingale described by the stochastic differential equation

$$dX_t = \mu_t dt + \sigma_t dB_t + \int_{\mathbb{R}} x \mu_t (dt, dx),$$  \hspace{1cm} (10)
where $B(t)$ is a standard Brownian motion. The spot volatility $\sigma_t$ is cadlag process bounded away from zero almost surely. The drift $\mu_t$ is in our case identically equal to zero. $\mu(dt, dx)$ is an integer-valued random measure that captures the jump in $X_t$ over a time interval $[t, t + dt)$. This implies that the jump arrives the market, whenever $\Delta X_t \equiv X_t - X_{t^-} \neq 0$. Further, let us denote the jump intensity for $\mu(dt, dx)$ as $dt \otimes \nu_t(dx)$, where $\nu_t(dx)$ is some non-negative measure with following constraint $\int_R (x^2 \land 1) \nu_t(dx) < \infty$.

In particular, we assume large finite activity price jumps, which means that for any fixed interval $[0, T]$, there is a finite number of time moments $\tau$, such that $\Delta X_\tau \neq 0$.

For a certain fixed interval $[0, T]$, the jump term with corresponding jump intensity $\nu_t$ gives rise to a finite number of price jumps, or, there exists a finite number of $t_i \in [0, T]$, such that $c_i \equiv \Delta X_{t_i} > 0$ in the limit, with $i = 1, \ldots, N_T$. In this case, there is exactly $N_T$ price jumps. Therefore, the term $\nu_t$ affects both the $c_i$ and the grid $T_T = \{t_1, \ldots, t_{N_T}\}$ including its cardinality.

The Tobin tax imposed in the model is changing the trading mechanism and thus the random processes in the equation (10). In particular, the process driving the volatility and the jump measure depends on the Tobin tax:

\[
\begin{align*}
\sigma_t &\rightarrow \sigma_t(\tau) \\
\nu_t &\rightarrow \nu_t(\tau)
\end{align*}
\]

Estimation of the functional dependence of the spot processes expressed in (11) as function of the Tobin tax is not a feasible task. The randomness in the spot processes—keep in mind that $\sigma_t$ itself is a random process with a structure similar to the log-price equation—would spoil the information and any test would therefore require to compare the random processes, which depends also on the current state of the world. To tackle these issues, we work with integrated variables.

The two integrated variables are of particular interest in this context: the Quadratic Variance and the Integrated Variance. For the log-price $X_t$ specified by equation (10) defined over a time interval $[0, T]$, we define Quadratic Variance as

\[
QV_T(\tau) = \int_0^T \sigma_s^2(\tau)ds + \sum_{i=1}^{N_T} c_i^2(\tau),
\]

\footnote{We fix the fair price to be constant and therefore assume the case with zero deterministic interest rates.}
where we keep the explicit functional dependence on the Tobin tax. The Quadratic Variance captures both the contribution from the continuous-time of the spot volatility $\sigma_t(\tau)$ and the contribution from the jumps. The Integrated Variance, on the other hand, is defined as

$$IV_T(\tau) = \int_0^T \sigma_s^2(\tau) \, ds,$$

is composed of the integral over the square of the continuous-time spot volatility.

For all practical purposes, we have to have consistent estimators of the $QV_T$ and $IV_T$ under the log-price process \( \{ \} \). In particular, for the log-price process $X_t$ defined over a time interval $[0, T]$, sampled into $M$ equidistant time intervals, we know that realized quadratic variance, defined as

$$\hat{RV}_{M,T} = \sum_{i=1}^{M} \left( Y_{i\frac{T}{M}} - Y_{(i-1)\frac{T}{M}} \right)^2,$$

for any sequence of non-stochastic partitions $0 = t_0 < t_1 < \cdots < t_M = T$, which in the limit $M \to \infty$ satisfies $\sup (t_i - t_{i-1}) \to 0$, has the property

$$\hat{RV}_{M,T} \xrightarrow{P} QV_T.$$ (12)

Further, Barndorff-Nielsen and Shephard (2004) showed that realized bipower variance, defined as

$$\hat{BV}_{M,T} = \mu_1^2 \sum_{i=2}^{M} \left| Y_{i\frac{T}{M}} - Y_{(i-1)\frac{T}{M}} \right| \left| Y_{(i-1)\frac{T}{M}} - Y_{(i-2)\frac{T}{M}} \right|,$$ (13)

where $\mu_\alpha = E(|z|^\alpha)$, with $z \sim N(0, 1)$, in particular, $\mu_1 = \sqrt{2/\pi}$, and for the same sequence of partitions as above has the property

$$\hat{BV}_{M,T} \xrightarrow{P} IV_T.$$ (14)

Conveniently, the small sample correction coefficient $\frac{M}{M}$ is introduced in the definition \( \{ \} \). The literature provides a plethora of estimators of the Integrated Variance, see, for example Dumitru and Urga (2012) for a comprehensive overview.
The difference of the $IV_T$ and $QV_T$ can be utilized as a basis for the test statistics to detect the presence of price jumps, see, Barndorff-Nielsen and Shephard (2006).

3.2 The effects of the Tobin tax

We study the effect of the Tobin tax on the financial markets through the change of the Quadratic Variance and Integrated Variance as a function of $\tau$. The convergence of estimators in (12,14) is achieved for every process satisfying the assumptions stated at the equation (10). We assume that for a reasonable region of the Tobin tax considered in this study, $\tau \in [0, \bar{\tau}]$, these assumptions holds. Therefore, for every imposed Tobin tax the asymptotic convergence of (12,14) takes the form

$$\forall \tau \in [0, \bar{\tau}]: \frac{\hat{RV}_{M,T}(\tau)}{\hat{BV}_{M,T}(\tau)} \overset{p}{\rightarrow} \frac{QV_T(\tau)}{I V_T(\tau)}. \quad (15)$$

Due to the well known properties of the self-organized systems, we do not impose any assumptions on the continuity or smoothness of the spot variance $\sigma_t$ and the jump process captured by the jump intensity $\nu_t$ as a function of the Tobin tax. The system may develop phase transitions of the first or second kind, as was demonstrated in the socio-economic system by Lavicka, Lin, and Novotny (2010). As a consequence, the derivative of the $QV_T(\tau)$ and $IV_T(\tau)$—or their respective estimators—with respect to $\tau$ may not necessarily exist.

Due to the asymptotics (15) for any fixed $\tau \in [0, \bar{\tau}]$, we analyze the dynamics of contribution of price jumps to the total quadratic variance through the $\hat{RV}_{M,T}(\tau)$ and $\hat{BV}_{M,T}(\tau)$, respectively. The variable of interest is the relative contribution of the price jumps to the quadratic variance as a function of $\tau$

$$\hat{JS}_{M,T}(\tau) \equiv \frac{\hat{RV}_{M,T}(\tau) - \hat{BV}_{M,T}(\tau)}{\hat{RV}_{M,T}(\tau)}. \quad (16)$$

Then, having in hand the knowledge $\hat{RV}_{M,T}(\tau)$ and $\hat{JS}_{M,T}(\tau)$, we may assess the effect of the Tobin tax on the market volatility. In general, we depict the possible options as depicted in the Table.
Table 1: Sensitivity of the $\hat{RV}_{M,T}(\tau)$ and $\hat{JS}_{M,T}(\tau)$ as a function of $\tau$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta \hat{JS}_{M,T}(\tau_0)$</th>
<th>Case</th>
<th>$\Delta \hat{RV}_{M,T}(\tau_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\leq 0$</td>
<td>D</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>B</td>
<td>$&gt; 0$</td>
<td>E</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>C</td>
<td>$&lt; 0$</td>
<td>F</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

point $\tau_0 \in (0, \bar{\tau})$.

3.2.1 The economic intuition

The sensitivity of the volatility to the Tobin tax can be then expressed through the combination of $\hat{RV}_{M,T}(\tau)$ and $\hat{JS}_{M,T}(\tau)$ in the Table 1. In particular, the combination of Case A with the Case D, or (C,D), corresponds to no sensitivity of the volatility process—both the diffusion and the price jumps—to the Tobin tax. All other combinations suggest any form of sensitivity to the Tobin tax.

Among those, the combinations of cases (B,F) and (C,E) play special role. In the former one, the overall realized Quadratic Variance, usually perceived as market volatility, is decreasing, while the corresponding contribution of price jumps is increasing. This means that imposing the Tobin tax leads to decreasing of volatility, while the volatility is less Gaussian. In the later case, the mechanism is reversed. The changing contribution of price jumps to the overall volatility than has serious consequences.

3.3 Estimating numbers of price jumps

To test for a presence of a price jump, we employ a test developed by Lee and Mykland (2008). As Hanousek, Kočenda, and Novotný (2012) argue, this test is optimal with respect to the Type-I error. It is based on the bipower variation introduced by Barndorff-Nielsen and Shephard (2004), who discuss the role of the standard variance, or the second central moment in the models where the underlying process follows eq. (10). In these cases standard variance captures the contribution from both the noise and the price jump process. The authors further show that a definition exists for the realized variance which does not take into account the term with price jumps. Such a definition is called the realized bi-power variance. The difference between the standard and the bi-power variance can be used test for the presence of a price jump. Barndorff-Nielsen and Shephard (2004) define the bipower variation as
\[ \hat{\sigma}_t^2 = \frac{1}{T-1} \sum_{\tau=t-T+1}^{t-1} |r_\tau| |r_{\tau-1}| . \]  (17)

This definition is used in a test statistics developed by Lee and Mykland (2008) as:

\[ \max_{\tau \in A_n} \frac{|\mathcal{L}_\tau| - C_n}{S_n} \to \xi , \]  (18)

where \( A_n \) is the tested region with \( n \) observations and \( \mathcal{L}_t = r_t / \hat{\sigma}_t \), \( C_n = \frac{(2 \ln n)^{1/2}}{c} - \frac{\ln \pi + \ln(\ln n)}{2c(2 \ln n)^{1/2}} \), \( S_n = \frac{1}{c(2 \ln n)^{1/2}} \), and \( c = \sqrt{2/\pi} \). Lee and Mykland (2008) show that under the null hypothesis of no price jump, the random variable \( \xi \) follows the standard Gumbel distribution function \( P(\xi \leq x) = \exp(e^{-x}) \). The number of price jumps detected in this way is then counted for a given window, in our case 120 days.

3.4 TVaR

The sensitivity of markets to extreme events can be measured by a widely-used measure of risk – tail value at risk (TVaR).\(^5\) Since the prices in our model can be seen as stochastic deviations from a deterministic trend (which can be thought of as a risk-free instrument price), we can construct the following portfolio. A hypothetical investor borrows 50 dollars at time \( t \) — the “fair” price of stocks in our model — and uses them to buy stocks. At time \( T \), he sells the stocks and returns the money. Therefore, the difference \((P_T - P_t)\) can be perceived as the value of the zero investment portfolio over the time horizon \( T - t \). This difference is exactly the return \( r_{*,(T-t)} \).

The cumulative distribution function \( F \) of the return \( r_{*,(T-t)} \) can be used to define the quantile function as \( Q = F^{-1} \). Then, the TVaR over a period \((T - t)\) is defined as:

\[ TVaR_p(r) = E \left[ r_{*,(T-t)} \mid r_{*,(T-t)} \leq VaR_p \right] = \]

\[ = \frac{1}{p} \int_0^P Q(1-u) \, du = \frac{1}{p} \int_0^P VaR_u \, du , \]  (19)

\(^5\)Although value-at-risk, and by extension TVaR, is convenient to use, it is sometimes criticized for failing to meet criteria for a coherent measure of risk (homogeneity, monotonicity, translation invariance and subadditivity). However, Danielsson, Jorgensen, Samorodnitsky, Sarma, and de Vries (2013) show that these measures, in fact, do satisfy these conditions in many practical cases, making them a reasonable choice for our application.
The TVaR is an equally weighted average of the returns in the VaR region. It takes into account the magnitude of the losses and therefore it incorporates more information than the pure VaR measure. The TVaR was computed for p’s of 95%, 99% and 99.9%. To evaluate the integral in Eq.(19) we use numerical integration with 50 points.

4 Results

In this section, we provide the results of the simulation study.

4.1 Simulation procedure

We have performed an extensive Monte Carlo, where we have simulated the operations of the financial markets. The financial market is populated with 800 traders evenly splitted into four groups: random traders, fundamentalists, momentarian traders, and contrarian traders. Initial wealth of an agent $i$ both in cash and stocks was set as follows: Traders were divided into groups of 20 and initial cash $c_i(0)$ was distributed according to Zipf’s law with parameter 1 and mean of 50,000 within each group. Number of each agent’s assets $a_i(0)$ averaged at 1000 in all cases.

The simulation itself is performed as follows: First, we populate the market with traders as described above. Second, we set the level of the Tobin tax, where we consider rates varying from 0% to 3% in 0.05 percentage point increments. For every simulation, the tax rate is fixed. In the next step, we perform a simulation of the market operations. Every simulation run is composed of 3600 trading sessions, or trading days, which corresponds to 15 years, each composed of 15 trading years. First 5 years of market operations are then considered as an initialization period and those data are not taken into account. Every simulation run is then repeated 300 times.

Then, for every level of the Tobin tax, we have 3000 trading years of data, which is large enough to obtain robust statistical inference. During the simulations, we collect the information at the end of every trading day, which consists of the market price of the traded asset, daily traded volume as well as the characteristics of the four different groups of traders’ types. Having generated the artificial market data as a function of the imposed Tobin tax, we calculate several characteristics based on the data.
4.2 The aggregate market data

First, we focus on the analyses of the price time series and traded volumes as a function of the imposed Tobin tax. The decrease in the liquidity due to the increased taxation is one of the main arguments, which goes in line with the Tobin tax proposals.

Figure 1 depicts the dependence between the traded volumes and the tax rate. The results clearly show that the traded volume is not a monotonic function of the tax rate but rather it is maximized around the tax rate of 0.15% imposed on the markets. This result is counter intuitive, therefore, we analyse the response of the supply and demand to the imposed tax rate.

Figure 2 shows the average daily levels of supply and demand as a function of the tax rate. The demand for the assets is highly fluctuating with a moderately decreasing trend. On the other hand, the supply of the assets initially drops with an introduction of the Tobin tax and then consequently decreases with the tax rate. The supply for assets is less volatile compared to the demand and the relation between the supply and the tax rate is more robust. The results therefore show that the relative decrease in the supply of assets leads to the increased traded volumes.

Figure 1: Volumes traded

![Graph showing the dependence between the traded volumes and the tax rate.](image)
Then, we analyze the daily log-return time series. Graphs of the first four central moments and median of the daily log-returns in the Figure 3 show two important results: First, standard deviation of returns goes up (which is similar to the results of [Mannaro, Marchesi, and Setzu, 2008]). Second and more important feature is that skewness and kurtosis decrease in absolute value, making the distribution more “normal”. While there is no significant correlation between mean of returns and tax rate, median increases with higher tax rate. The increase in normality is also supported by the weekly and monthly returns (results available upon request).

The basic analysis of log-returns therefore suggests that the deviation of the price process from normality is decreasing with the increasing tax rate. Using the language of the Table 1, we may assess that the measure of the overall variation is increasing with rate, or the Case E, while the relative contribution of the price jumps has to be properly analyzed, which follows in the following subsection.

4.3 Quadratic Variation vs. Integrated Variation

We use the daily frequency and calculate the relative contribution of price jumps to the total variance provided in equation (16). First, we plot in the left panel of Figure 4 the Quadratic Variation as estimated by the Realized Variance $\hat{RV}_{Daily}$ in daily levels.

The figure suggests an increase in volatility due to the imposed tax. However, for very few values of
Figure 3: Moments of returns as a function of the tax for daily log-returns

- Mean
- Standard deviation
- Skewness
- Kurtosis
- Median

Figure 4: Realized Variance $\hat{RV}_{Daily}$ and ratio $\hat{JS}_{Daily,T} (\tau)$ in daily levels

- Quadratic Variation, daily
- JS, daily
the tax rate, we see that there is a decrease in volatility, therefore we may claim that a small level of the tax rate is decreasing overall volatility of the markets. This observation holds for the weekly and monthly returns as well.

Right panel of Figure 4 depicts the ratio \( \hat{J} S_{Daily,T}(\tau) \) — the relative contribution of the price jumps to the Quadratic Variation. The picture clearly shows that the contribution of price jumps to the overall variance steadily decreases as tax rate increases. Since our price process is fluctuating around the fair price, which is set fixed, the \( \hat{J} S_{Weekly,T}(\tau) \) and \( \hat{J} S_{Monthly,T}(\tau) \) are more close to each other and the effect of the diminishing price jump contribution to the overall quadratic variation is not significant.

In terms of Table 1, the imposed Tobin tax tends to increase the overall Quadratic Variation of the prices, i.e., the case E capturing \( \Delta \hat{RV}_{M,T}(\tau_0) > 0 \), while it tends to decrease the contribution of price jumps to the Quadratic Variation, i.e., the case C describing \( \Delta \hat{J} S_{M,T}(\tau_0) \). The increasing tax rate is therefore making the prices more volatile and more Gaussian. Indeed, there is a tradeoff between an increase in quadratic variance and decrease in number of price jumps and non-normality. This intuition is in line with decreasing kurtosis and increasing variance in Figure 3.

In the following, we explore the rate of price jump arrivals in more details. We employ the test statistics in (18) with 95% confidence interval and identify the price jumps in the entire sample for every tax rate. The size of sample sample is 3000 trading years, which gives 720,000 observations of daily prices. In addition, we further distinguish the upward price jumps and the downward price jumps.

Figure 5 depicts the number of identified price jumps as a function of the tax rate. The upward and downward price jumps are plotted separately. The overall rate of price jump arrivals is decreasing with the increasing tax rate, where the decrease is convex. The subsequent figures for upward and downward price jumps explain that the convexity comes from the upward price jumps, which suggests a saturation of the price jump arrivals around 2.5% tax rate. On the other hand, the downward price jumps are decaying in a linear way and there is no suggestion for a saturation effect. In conclusion, the tax rate is decreasing the number of price jump arrivals, where the suppressing effect is more significant for downward price jumps.

In addition, we estimate formulate the empirical model capturing the dependence between the number of price jumps and the imposed tax rate as
Figure 5: Number of jumps – one-day returns

(a) Overall

(b) Upward jumps

(c) Downward jumps
\[ N_\tau = \alpha_1 + \alpha_2 \tau + \alpha_3 \tau^2 + \epsilon_\tau. \]  
(20)

Table 2 summarizes the results of the estimation. We perform regression for the overall numbers of price jump arrivals as well as separately for arrivals of the upward and downward price jumps. The estimated coefficients support the empirical results, where in particular \( \alpha_3 \) for upward price jumps is significantly higher than \( \alpha_3 \) for the downward price jumps. Further, the figures show that price jumps represents 6.8% percent of all returns, which drops to 6.4% at highest considered tax rate. Overall high rates of price jump arrivals are given by the 95% confidence interval in the employed test statistics. The results for the weekly and monthly returns are in line with the daily returns.

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<tr>
<th>VARIABLES</th>
<th>Overall N</th>
<th>Upward N</th>
<th>Downward N</th>
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<td>-7.720***</td>
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<td>(0.980)</td>
<td>(0.667)</td>
<td>(0.934)</td>
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<td>0.0170***</td>
<td>0.00260</td>
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<td></td>
<td>(0.00316)</td>
<td>(0.00215)</td>
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<td>(63.60)</td>
<td>(43.30)</td>
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<td>Observations</td>
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<td>61</td>
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<td>R-squared</td>
<td>0.973</td>
<td>0.922</td>
<td>0.937</td>
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</table>

Standard errors in parentheses  
*** p<0.01, ** p<0.05, * p<0.1

4.4 Risk of the Portfolio – TVaR

We calculate the risk of the portfolio of holding an asset over a period of time as captured by the TVaR measure defined in (19). In particular, the change in the TVaR due to the variation of the tax rate is affected by two effects. First, the Quadratic Variation is increasing as the tax rate grows, which is causing the increase of the TVaR. Second, the rate of price jumps is decreasing with increasing tax rate and thus
the TVaR tends to decrease with the growing tax rate. Thus, the final response of the TVaR will be the trade-off between these two effects.

Figures depicts the TVaR for maturities ranging from the short term to the long term. We depict 6 horizons to maturity: 1 day, 1 week, 1 month, 1 quarter, 10 years, and 15 years, respectively. For every investment horizon, we \( p = 0.001 \), \( p = 0.01 \), and \( p = 0.05 \). In all the cases, the TVaR is a decreasing function of the imposed tax rate. This means that losses projected with our measure and in any investment horizon considered in this analysis are increasing with the increasing tax rate. We can therefore conclude that from the TVaR perspective, the variance is growing faster than the price jumps are decreasing and therefore an investor tends to suffer higher losses due to the imposed Tobin tax.

### 4.5 Market Microstructure

Last but not least, we the market micro-structure of our artificial market. In particular, we focus on the change in the aggregate behavior of the four trading groups due to the variation in the tax rate. Figure reports the average daily inventories—the assets and money—for the four groups as a function of the Tobin tax. For Random traders and Trend followers, the increasing tax rate has negative effect on both the asset and the money stocks. These two groups are therefore directly negatively affected by the imposed tax. The Contrarians experience the increase in the asset stocks as the tax rate is imposed, where the positive trend soon reverse and the inventory decreases with the tax rate. In the case of money stocks, the average amount of money held is steeply decreasing with increasing tax rate. Finally, the Fundamentalist are prospering from the imposed Tobin tax. The amount money held are positively affected by the tax rate, while average amount of assets are initially negatively affected by the Tobin tax, which is immediately reversed. The evidence suggests that Tobin tax rate affects the Fundamentalists’ and Contrarians’ asset stocks in an opposite way, where the Tobin tax works in favor of the Fundamentalists.

The effect of the imposed tax on the price process and rate of price jumps is directly connected to liquidity on the markets. Figure reports the daily averages of the supply and demand of the assets by the traders.

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6This follows from the fact that a fixed \( p \)-level and fixed Quadratic Variation, adding one large price jump to the very left tail of the distribution, is decreasing the (negative) TVaR, while the \( p \)-th quantile will increase.
Figure 6: Tail value at risk

(a) 1-day losses
(b) 5-day losses
(c) 20-day losses
(d) 60-day losses
(e) 10-year losses
(f) 15-year losses
Figure 7: Inventories by traders

(a) Assets

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<tr>
<th>Tobin tax (%)</th>
<th>R</th>
<th>F</th>
<th>T</th>
<th>C</th>
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<tr>
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<tr>
<td>Tobin tax (%)</td>
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(b) Money

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</tr>
<tr>
<td>Tobin tax (%)</td>
<td></td>
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</table>
group. First, the supply of assets, which are traders willing to trade, is negatively affected by the imposed Tobin tax for all four groups. The supply of assets supplied by Fundamentalists immediately drop by the imposed tax and then only very slightly decreases with increasing tax rate. For the remaining three groups, the supply increases when a very low tax rate is imposed and this effect immediately reverses with increasing tax rate.

On the other hand, the demand has a different response to the imposed tax. The demand by the Random traders is highly volatile while mildly affected by the imposed tax. The demand by the Trend followers is first positively boosted by a small change in the Tobin, where the trend reverses and the demand decreases with the increased imposed tax rate. The demand by Contrarians is monotonously decreasing with the imposed Tobin tax, which dramatically drops when the tax is introduced and then slowly decreases with increasing tax rate. The demand by the Fundamentalists, on the other hand, is positively affected by the imposed tax rate. The demand is monotonously increasing with the tax rate, where the small introduction of the tax is boosting the demand by around 50%. The response of the Fundamentalists and the Contrarians are in opposite and the effect of the Tobin tax seems to have exactly opposite impact on two groups.

Finally, Figure 9 reports the average amount of assets sold and purchased by individual traders. The pattern of response to the imposed Tobin tax is similar to all four groups. First, there is an increase in the activity as small tax rate was imposed and then the amount of sold and purchased assets is decreasing with increasing tax rate. The initial increase and then the rate of decrease varies across the traders’ groups though.

Finally, we extend the model (20) and include some additional information about the traders’ group behavior. First, the only exogenous variable in our model is the Tobin tax and therefore by including any other variable measured in the system would lead to a possible issue of endogeneity. The reason why to include the endogenous variables is to reflect the fact that the different traders’ group respond to the imposed tax in a very non-linear way and therefore the trade-off for endogeneity is to capture the non-linear response.

We therefore propose the following model, where are in particular interesting on the effect of liquidity

$$N_{\tau} = \alpha_1 + \alpha_2 \tau + \alpha_3 \tau^2 + \sum_{i=R,T,F,C} \alpha_{4(i)} s^{(i)} + \sum_{i=R,T,F,C} \alpha_{5(i)} d^{(i)} + \epsilon_{\tau},$$  \hspace{1cm} (21)
Figure 8: Market order book by traders

(a) Supply

(b) Demand
Figure 9: Market activity by traders

(a) Sold assets

(b) Purchased assets
where \( s^{(i)} \) and \( d^{(i)} \) is a share of assets sold by group \( i \), respectively, or, \( s^{(i)} = S^{(i)} / \sum_{i=R,T,F,C} S^{(i)} \), with \( S^{(i)} \) being the average volumes of assets sold by every group every trading day, and \( d^{(i)} = D^{(i)} / \sum_{i=R,T,F,C} D^{(i)} \), with \( D^{(i)} \) being the average volumes of assets demanded by every group every trading day. Table 3 summarizes the results of the estimation. Since we use relative shares, only three of the four groups are present in the regression since the trivial identity holds

\[ 1 = s^{(F)} + s^{(T)} + s^{(R)} + s^{(C)} \]

and analogously for the shares of demand \( 1 = d^{(F)} + d^{(T)} + d^{(R)} + d^{(C)} \). First, the results of the full specification suggests that number of price jumps up are not significantly affected by any of the variables. However, the large errors suggests an endogeneity to take place. On the other hand, the total number of price jumps and the number of price jumps downward share the same dependence on the tobin tax and amount of assets sold by the trend followers. When we restrict the model and include the demand only, we see that the relative demand by the Fundamentalists play a crucial role and in particular, \( \alpha_5^{(F)} \) is significantly small compare to other \( \alpha_5 \)'s for the remaining groups. Since the demand for assets by Fundamentalists is increasing with the imposed tax, the results show that the relative demand for assets by Fundamentalists is (seemingly) explaining the decrease in the number of price jumps with increasing tax rate.

5 Conclusion

The main goal of this paper was to open discussion on the heretofore ignored relationship between financial transaction taxes and price jumps. We have argued that looking at the effect of FTTs on realized variance as a measure of volatility is insufficient, as it does not convey enough information. We argued that an increase in variance itself does not necessarily mean less stable markets, because realized variance can be decomposed into two parts—Gaussian variance and price jumps. As we have shown, the variance may go up through an increase in Gaussian variance, while at the same time the contribution of price jumps may go down, decreasing the kurtosis of the return distribution. Given that there is a sizeable literature on hedging against Gaussian variance, this result implies that such a tax may improve effectiveness of these formulas, and through that, functioning of markets. In the future work, we aim to apply the model in the empirical setup and test the hypotheses implied by this model using the asset prices data.
Table 3: The effects of the tax rate and average shares of sold volumes by traders group on the number of jumps for daily returns.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) $N_\tau$</th>
<th>(2) $N_\tau^{Down}$</th>
<th>(3) $N_\tau^{Up}$</th>
<th>(4) $N_\tau$</th>
<th>(5) $N_\tau^{Down}$</th>
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<td>(798.6)</td>
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<td>$\tau^2$</td>
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<td>106.1***</td>
<td>101.3***</td>
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<td>(14,822)</td>
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<td>$d(F)$</td>
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<td>9,730***</td>
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<td>-8,893</td>
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Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
6 Acknowledgements

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References


