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Testing for Co-Jumps in High-Frequency Financial Data: an Approach Based on First-High-Low-Last Prices

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Testing for co-jumps in high-frequency financial data: an approach based on first-high-low-last prices*

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Abstract

This paper proposes a new test for simultaneous intraday jumps (co-jumps) in a panel of high frequency financial data. We utilize intraday first-high-low-last values of asset prices to construct estimates for the cross-variation of returns in a large panel of high frequency financial data, and then employ these estimates to provide a test statistic that can detect co-jumps. Simulations show that a bias corrected version of the test can be used in the presence of microstructure noise. When applied to a panel of high frequency data from the Chinese mainland stock market, our test identifies co-jumps that can be associated with announcements relating to monetary policy and stock market regulations.

Keywords: Covariance, Co-jumps, High-frequency data, First-High-Low-Last price, Microstructure bias, Nonsynchronous trades, Realized covariance, Realized co-range.

JEL classification: C12, C22, C32, G12, G14

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1 Introduction

The documentation and study of jumps (discontinuities) in asset prices is important for many financial decisions, and this has led to a burgeoning literature that tests for jumps and characterizes their properties. The jump test developed by Barndorff-Nielsen and Shephard (2006), henceforth the BN-S test, plays a leading role in this literature, but since this test is designed for a single asset while financial considerations usually involve many assets, there is a need for tests that can detect simultaneous jumps in many assets (co-jump tests) as well. Progress on this front includes the development of tests for co-jumps in a pair of asset returns (Barndorff-Nielsen and Shephard (2003), Gobbi and Mancini (2007) and Jacod and Todorov (2009)), as well as a co-jump test developed by Bollerslev, Law and Tauchen (2008) that is applicable to a large panel of high-frequency returns. The intuition behind this last test (henceforth called the BLT test) is that idiosyncratic noise in individual returns can hide the presence of a synchronous component, so that a test based on the cross products of returns in a panel can avoid this problem while still being sensitive to systematic movements across all stocks.

The BLT (2008) co-jump test relies on a measure of covariation in the panel that is constructed using the average pair-wise cross product of returns. However, a return-based estimator does not necessarily provide the best estimator of this covariation. Its simplicity facilitates very straightforward construction of the test statistic, but the literature on covariance estimation emphasizes efficiency-bias considerations in high frequency settings, and efficient estimation is particularly desirable when constructing tests for use in multivariate contexts. There are now many ways of measuring covariation in high frequency settings, and they vary with respect to their computational difficulty, bias and efficiency. Of particular interest here is the work in Bannouh, van Dijk and Martens (2009) that promotes the use of range based estimators of covariance. Their Monte Carlo illustrates that when there are no market frictions, realized co-range estimators can have variances that are up to five times smaller than returns based realized co-variance estimators, and that simple additive bias corrections can be very effective when microstructure noise is present and trading is not synchronous.

Thus the use of intraday ranges instead of intraday returns can offer large efficiency gains without substantially increasing the computational burden. The large efficiency gains arise from the fact that the range can be very informative, since it is constructed by looking at the entire price process in the sampled time interval.

This paper proposes a new test for co-jumps in panels of high frequency data that follows the intuition behind the BLT test, but uses intraday first-high-low-last (FHLL) price values to capture cross-variation. FHLL price values were first used by Garman and Klass (1980) in the context of estimating daily volatility. These authors suggested using the minimum variance linear combination of the daily ranges and daily returns to estimate daily volatility, so that the information in returns is augmented by the additional information contained in the range. They showed that the asymptotic variance of this new estimator is 7.4 times smaller than the squared daily return and 1.5 times smaller than the squared daily range for daily volatility estimation. Our use of first-high-low-last (FHLL) price values in the intraday context offers the same efficiency gains, but like range based estimators, it involves the need for bias correction because of the effects of microstructure biases in high frequency settings. We use the additive correction proposed by Bannouh et al (2009) for this purpose, because it is simple, easy to implement, and it appears to account for the net effect of the many biases that occur in the high frequency context.

Our proposed test statistic is easy to calculate because it neither relies on estimating the entire covariance structure of returns in the panel, nor on any explicit calculations of bivariate products of prices/returns (that might not necessarily be observed at exactly the same time for different assets). In fact, our test statistic can be calculated using existing univariate methodology, because the average pair-wise cross product term in an n-asset context can also be written in terms of the variance of the equally weighted portfolio and each of the n individual asset variances (see Brandt and Diebold (2006), and BLT (2008)). Like the BLT statistic, our test statistic can not only identify jump days, but it can also explicitly pinpoint co-jump times. Given the relative efficiency gains of FHLL estimators, we expect that co-jump tests based on FHLL prices will be more powerful than the BLT (2008) returns based co-jump test.

Our test is based on a multivariate extension of an FHLL estimator for the covariance between two returns, and since an FHLL covariance estimator is novel in itself, we conduct Monte Carlo simulations to assess and compare it (and an associated bias-corrected version) with corresponding range and return based covariance estimators. We simulate frictionless environments, situations in which bid-ask bounce influences price, and situations in which trades are infrequent and not synchronous, and we vary the intraday sampling frequencies as well. As expected the (uncorrected) FHLL estimator is more biased than its range and return based counterparts. Once bias corrections are employed, the RMSE of the FHLL estimators are lower than the (bias corrected) range and returns estimators, and often substantially so. Having established the superior properties of bias corrected FHLL estimation, we then use it to construct our co-jump test for the panel. We study the test in situations in which microstructure noise is absent (and then present), and demonstrate via simulation that the bias corrections successfully mitigate size distortion when microstructure noise is present. We then demonstrate that the FHLL based cojump test has higher power than analogously constructed co-jump tests based on realized range and realized variance. Finally, we study an empirical example based on a high-frequency panel of forty stocks on the Chinese mainland stock market. We use this data because previous research (Liao (2011)) has found that the jumps in individual stocks of this emerging market are more frequent than those in developed financial markets. We find evidence of several co-jumps per month, and note that about half of these can be linked to announcements about changes in monetary policy or stock market regulations.

The rest of the paper is organized as follows. Section 2 introduces our FHLL price based covariance estimator and analyzes its properties. Section 3 uses this new estimator to develop our FHLL price based co-jump test. Section 4 conducts a Monte Carlo simulation to study the finite sample performance of our new co-jump test, and compares its power properties with those of the existing return-based co-jump test. Section 5 presents our main empirical findings for a panel of stocks from the Chinese mainland stock market. Section 6 concludes.

2 First-High-Low-Last Price Based Estimators

We let p_s denote the log price of an asset, and assume that it evolves as a standard continuous time diffusion process

$$dp_s = \mu(s)ds + \sigma(s)dW_s,$$

where $\mu(s)$ and $\sigma(s)$ denote the drift and volatility respectively, and W_s is a standard Brownian motion.

We assume that high-frequency data are available for each day t which runs from time $t-1$ to t, and that we have prices relating to M intraday periods denoted by $\{t_j\}$ for $j = 1, \ldots, M$, where $t_j \in [t-1, t]$. In addition, we have $m+1$ equally spaced price observations recorded within each intraday time interval $[t_{j-1}, t_j]$ (i.e. at $t_{(j-1)+(0/m)}$, $t_{(j-1)+(1/m)}, \ldots t_{(j-1)+(m/m)} = t_j$).[1] The four extreme price values within each intraday time interval $[t_{j-1}, t_j]$ are:

$p_{t_{j-1}}$: the first (log) price observed during the time interval $[t_{j-1}, t_j]$;

p_{t_j}: the last (log) price observed during the time interval $[t_{j-1}, t_j]$;

$h_{t_{j-1}}$: the highest (log) price observed during the time interval $[t_{j-1}, t_j]$, which is

$max\{t_{(j-1)+(0/m)}, t_{(j-1)+(1/m)}, \ldots t_{(j-1)+(m/m)} = t_j\}$;

$l_{t_{j-1}}$: the lowest (log) price observed during the time interval $[t_{j-1}, t_j]$, which is

$min\{t_{(j-1)+(0/m)}, t_{(j-1)+(1/m)}, \ldots t_{(j-1)+(m/m)} = t_j\}$.

2.1 First-High-Low-Last Price Based Variance Estimator

The most popular approach to estimate the integrated variance $\int_{t-1}^{t} \sigma^2(s)ds$ of the above standard continuous time diffusion process is to use "Realized Volatility", which is constructed using the sum of squared interval returns via

$$RV_t = \sum_{j=1}^{M} r_{t_j}^2 = \sum_{j=1}^{M} (p_{t_j} - p_{t_{j-1}})^2, \qquad (1)$$

[1] M is the number of the intraday periods over a trading day, and each of these M intraday periods is divided into m subintervals.

where the return r_{t_j} of each time subinterval is calculated as the difference between the last price and the first price of that interval. Andersen et al (2001) and Barndorff-Nielsen and Shephard (2004) have proved that realized variance is a consistent estimator for the integrated variation over $[t-1,t]$ in the absence of microstructure noise, and that the asymptotic variance of realized volatility is $2\int_{t-1}^{t}\sigma^4(s)ds$.

The range of an asset's price is defined to be the difference between the highest price and the lowest price during a fixed time interval. The use of the high-low price range in volatility estimation dates back to Parkinson (1980). Recently Christensen and Podolskij (2007) and Martens and van Dijk (2007) have re-considered the use of price range in a high frequency data context to estimate the integrated variation in a standard continuous time diffusion model of (the logarithm of) an asset's price as

$$RRV_t^{M,m} = \frac{1}{\gamma_{2,m}}\sum_{j=1}^{M} s_{p_{t_j}}^2 = \frac{1}{\gamma_{2,m}}\sum_{j=1}^{M}(h_{t_{j-1}} - l_{t_{j-1}})^2, \qquad (2)$$

where $\gamma_{2,m} = E[s_{w,m}^2]$ and $s_{w,m}$ is the range of a standard Brownian motion over $[0,1]$, when we observe m increments of the underlying continuous time process in each sampling interval t_j. The parameter $\gamma_{2,m}$ is monotonically increasing in m with $\gamma_{2,1} = 1$, and $\gamma_{2,m} \to 4\ln 2$ as $m \to \infty$.

Intuitively, the range reveals more information than the return over the same time interval because the highs and lows of asset prices are formed from the entire price evolution path. Parkinson (1980) provided mathematical derivations to show that the daily squared price range is about five times more efficient than the daily squared return for estimating daily volatility. Simulations in Martens and van Dijk (2007) demonstrated that in a frictionless market without microstructure noise, realized range has a lower mean-squared error than realized volatility. This was corroborated by the asymptotic properties derived by Christensen and Podolskij (2007). They deduced the following central limit theorem for realized range, finding that

$$\sqrt{M}(RRV_t^{M,m} - \int_{t-1}^{t} \sigma^2(s)ds) \longrightarrow MN(0, \frac{\gamma_{4,m} - \gamma_{2,m}^2}{\gamma_{2,m}^2} \int_{t-1}^{t} \sigma^4(s)ds),$$

where $MN(.,.)$ denotes a mixed Gaussian distribution, $\gamma_{r,m} = E[s_{w,m}^r]$ and $\lim_{m\to\infty} \frac{\gamma_{4,m}-\gamma_{2,m}^2}{\gamma_{2,m}^2} \approx 0.4073$. They used this theorem to show that $RRV_t^{M,m}$ is an unbiased and more efficient estimator of integrated variance than realized volatility. If $m = 1$, $RRV_t^{M,m}$ is actually equal to realized volatility RV_t and $\frac{\gamma_{4,m}-\gamma_{2,m}^2}{\gamma_{2,m}^2} = 2$, but when $m \to \infty$ and the entire sample path of the price process is available, $RRV_t^{M,m}$ becomes about five times more efficient than RV_t. In practice, inference is typically drawn from discrete data and true ranges are not actually observed. Thus, the efficiency of the $RRV_t^{M,m}$ estimator relative to RV_t depends on how many observations in each intraday period are available for the construction of the high-low price range measures.

The above two estimators are generated by either the intraday first and last prices or the intraday highest and lowest prices, and it is useful to combine these four types of prices together to further improve estimation efficiency. Garman and Klass (1980) did this in a daily data context, by utilizing daily open, high, low and close prices to derive a minimum variance unbiased estimator for daily volatility given by

$$\hat{\sigma_t^2} = 0.511(logH_t - logL_t)^2 - 0.383(logC_t - logC_{t-1})^2$$

$$-0.019((logH_t - logC_t)logH_t + (logL_t - logC_t)logL_t),$$

where H_t, L_t, and C_t are respectively the highest, lowest and close prices during day t. They recommended a simpler version of this estimator for practical use, which is

$$\hat{\sigma_t^2} = 0.5(logH_t - logL_t)^2 - (2log(2) - 1)(logC_t - logC_{t-1})^2,$$

and this latter estimator achieves similar efficiency but eliminates the small cross-product terms.

Their calculations showed that the variance of this estimator is $0.27\sigma^4$, which is 7.4 times more efficient for daily volatility estimation than the daily squared return (whose variance is $2\sigma^4$), and 1.5 times more efficient than daily squared range (whose variance is $0.41\sigma^4$). Simulations by Rogers and Satchell (1991) showed that this estimator (and a modified version of it) performed quite well in a setting that corresponded with typical daily data.

In this paper we replace intraday returns or intraday ranges in high frequency realized variance or realized range estimators with the intraday versions of "Garman and Klass estimators" to construct a first-high-low-last (FHLL) price based estimator for integrated variance given by

$$FHLLV_t = \sum_{j=1}^{M}(0.5(h_{t_j} - l_{t_j})^2 - (2log(2) - 1)(p_{t_j} - p_{t_{j-1}})^2). \tag{3}$$

This estimator is essentially a linear combination of RRV_t and RV_t with weights of $(2ln2)$ and $(1 - 2ln(2))$ respectively.[2] Assuming no microstructure noise and that the entire price path can be observed $(m \to \infty)$, we can derive a central limit theorem for this FHLL variance estimator with respect to M, which is

$$\sqrt{M}(FHLLV_t - \int_{t-1}^{t} \sigma^2(s)ds) \longrightarrow MN(0,\ 0.27\int_{t-1}^{t} \sigma^4(s)ds).$$

This shows that the FHLL price based estimator is a consistent estimator for integrated variance, but it is more efficient than either realized variance or realized range.

2.2 First-High-Low-Last Price Based Covariance Estimator

The fact that the FHLL estimator for integrated variance is more efficient than its realized volatility and realized range counterparts suggests that using first, high, low and last values of asset prices might be advantageous in other settings as well. We now apply this idea to covariance estimation.

Assuming that there are two assets i and l for simplicity, and a portfolio of them with weights

[2] Note that this is an affine combination (weight coefficients add up to 1), but it is not a convex combination because the weight coefficient of RV is negative.

w_i and $w_l = 1 - w_i$, Brandt and Diebold (2006) noted that the daily covariance between asset i and asset l can be obtained from

$$Cov(r_{i,t}, r_{l,t}) = \frac{1}{2w_i w_l}(Var[r_{p,t}] - w_i^2 Var[r_{i,t}] - w_l^2 Var[r_{l,t}]),$$

where $Var[r_{p,t}]$ is the daily variance of the portfolio returns, and $Var[r_{i,t}]$ and $Var[r_{l,t}]$ are the daily variances of assets i and l respectively. Using the realized variance defined in (1) to estimate the three daily variances on the right-hand side of the above equation, realized covariance (see Barndorff-Nielsen and Shephard (2004)) can be calculated using

$$RCV_t = \sum_{j=1}^{M} r_{i,t_j} r_{l,t_j} = \frac{1}{2w_i w_l}(RV_{p,t} - w_i^2 RV_{i,t} - w_l^2 RV_{l,t}) \tag{4}$$

where $RV_{p,t}$ is the realized variance of the portfolio, $RV_{i,t}$ and $RV_{l,t}$ are the realized variances of asset i and asset l, and r_{i,t_j} and r_{l,t_j} are the intraday returns of asset i and asset l. Using the realized range defined in (2) to estimate the three daily variances on the right-hand side of the above equation, realized co-range (see Bannouh, van Dijk and Martens (2009)) can be obtained as

$$RCR_t = \frac{1}{2w_i w_l}(RRV_{p,t} - w_i^2 RRV_{i,t} - w_l^2 RRV_{l,t}), \tag{5}$$

where $RRV_{p,t}$ is the realized range of the portfolio, and $RRV_{i,t}$ and $RRV_{l,t}$ are the realized ranges of asset i and asset l. The Monte Carlo work in Bannouh et al (2009) demonstrates that the realized range is robust to market microstructure noise arising from bid-ask bounce, infrequent trading and asynchronous trading, yet it is also highly efficient, delivering up to fivefold efficiency gains relative to realized covariance. Comparison of the theoretical properties of realized covariance and realized co-range is a subject of on-going research.

The first-high-low-last (FHLL) price based covariance estimator can be generated in an analogous fashion to (4) and (5), by using the FHLL variance estimator defined in (3) to estimate the three daily variances in the covariance equation to obtain

$$FHLLCV_t = \frac{1}{2w_i w_l}(FHLLV_{p,t} - w_i^2 FHLLV_{i,t} - w_l^2 FHLLV_{l,t}). \tag{6}$$

Given the superiority of the FHLL variance estimator over the return-based and range-based competitors, we expect this FHLL covariance estimator to be more efficient than the realized covariance and realized co-range estimators.

2.3 Comparison of the Properties of Covariance Estimators

In this section, we use Monte Carlo simulations to investigate the performance of our FHLL covariance estimator given a variety of underlying asset price process specifications. Throughout, we compare the FHLL estimator with the realized covariance estimator and the realized range estimator in these controlled environments.

2.3.1 Constant Volatility without Microstructure Noise

We firstly study the properties of these estimators for a bivariate Brownian motion process with constant volatility, using the Euler discretization scheme to simulate prices for two correlated assets for 4-hour trading days.[3] For each trading day t, the initial prices for both assets are set equal to one and subsequent log prices for assets are simulated using

$$dp^*_{1,(t-1)+h/K} = \sigma_1 dW_{1,(t-1)+h/K}, \qquad h = 1, 2, ..., K$$

$$dp^*_{2,(t-1)+h/K} = \sigma_2(\rho dW_{1,(t-1)+h/K} + \sqrt{1-\rho^2} dW_{2,(t-1)+h/K}), \; h = 1, 2, ..., K$$

where $p^*_{i,(t-1)+h/K}$ is the log price of asset i at the hth point in the time interval $[t-1, t]$, K is the number of time increments over the day, σ_i is the standard deviation of asset i, W_1 and W_2 are two

[3] We choose to simulate four hour trading days to reflect the trading hours in the Chinese mainland stock market, from which our empirical data are collected.

Brownian motion processes, and ρ represents the contemporaneous correlation of the two assets' prices. We set $\sigma_1 = 0.2$, $\sigma_2 = 0.4$ and $\rho = 0.5$, resulting in a constant covariance between the two asset returns which is equal to 0.04 for each day t, and we let h represent seconds so that we have $K = 14400$ $(4\times60\times60)$ data points for each day. Each of our experiments is based on 5000 simulated days. For the time being our price observations are equidistant and occur synchronously for the two assets.[4] To show the potential merits of using intraday first-high-low-last price for measuring the co-movement of two assets, we compute and compare the bias and root mean squared error (RMSE) of various covariance estimators at different intraday sampling frequencies.

To do this, we divide the trading day t into $\triangle-$minute intervals, which is referred to as the $\triangle - minute$ frequency below. Since we have a four hour trading day, this divides each day into $M = 240/\triangle$ intraday sampling periods. For example, if we sample at a five minute frequency and $\triangle = 5$, then we have $M = 48$ intraday sampling periods per day. In our experiment, we vary the sampling frequency, using 1, 5, 10, 15, 30, 60, and 240 minute intervals, and results are reported in Table 1 panel A. The underlying price process $p^*_{i,(t-1)+h/K}$ in our simulations is assumed to be a pure Brownian motion with constant volatility, but since it is actually discrete (h can only take integer values), we see an "infrequent trading" effect, which leads to a downward bias for all estimators when the sampling intervals are relatively short. We explore the effects of infrequent trading in more detail below. However, the RMSE of the FHLL covariance estimator is always lower than that of realized co-range, and substantially lower than that of realized covariance at the same sampling frequency. Meanwhile, the efficiency improves for all estimators as $\triangle$ decreases and M increases. Figure 1 shows the kernel density graphs of the three covariance estimators at 5-minutes, 10-minutes and 15-minutes sampling frequencies, which further demonstrate that our FHLL estimator is more efficient than the other two, since the kernel density graph of FHLL estimator is narrower than those of the other two estimators.

[4] True and observed prices are denoted by $p^*_{i,\cdot}$ and $p_{i,\cdot}$ respectively. In the next section we set the probability of actually observing the price to be $p_{obs} = 1/\tau$, and use s to denote the bid-ask spread. For the time being, $\tau = 1$ and $s = 0$.

2.3.2 Stochastic Volatility without Microstructure Noise

We now extend the underlying price processes to incorporate stochastic volatility, which is closer to reality. The (log) prices evolve as

$$dp^*_{1,(t-1)+h/K} = \sigma_{1,(t-1)+h/K} dW_{1,(t-1)+h/K}, \qquad h = 1, 2, ..., K$$

$$dln\sigma^2_{1,(t-1)+h/K} = \theta_1(\omega_1 - ln\sigma^2_{1,(t-1)+h/K})dt + \eta_1 dW_{2,t-1+h/K}, \qquad h = 1, 2, ..., K$$

$$dp^*_{2,(t-1)+h/K} = \sigma_{2,(t-1)+h/K} dW_{3,(t-1)+h/K}, \qquad h = 1, 2, ..., K$$

$$dln\sigma^2_{2,(t-1)+h/K} = \theta_2(\omega_2 - ln\sigma^2_{2,(t-1)+h/K})dt + \eta_2 dW_{4,(t-1)+h/K}, \qquad h = 1, 2, ..., K$$

where volatility is a stochastic process that follows a mean reverting Orstein-Uhlenbeck process with parameters θ_1 and θ_2 as the adjustment speeds, ω_1 and ω_2 as the means of the (log) volatilities, and η_1 and η_2 as the volatilities of the (log) volatilities. W_1 and W_2 are standard Brownian motions with a correlation of ρ_1, W_3 and W_4 are standard Brownian motions with a correlation of ρ_2, so that ρ_1 and ρ_2 allow for leverage effects in each asset. The processes W_1 and W_3 are correlated with a correlation coefficient of ρ_3. The initial prices for the two assets are set to be one, the initial values of the two assets' volatilities are set to equal to the mean of the volatilities, and the rest of the simulations are based on the configuration $(\theta_1, \theta_2, \omega_1, \omega_2, \eta_1, \eta_2, \rho_1, \rho_2, \rho_3) = (2,\ 2,\ \frac{0.04}{14400},\ \frac{0.16}{14400},\ \frac{0.4}{120},\ \frac{0.8}{120},\ -0.6,\ -0.4,\ 0.5)$.[5] The data generating process ensures that the daily mean of the covariance between the two assets is 0.04, but it now varies every second. We simulate 5000 days of (log) prices (1 price per second, for $K = 14400$) as before, and then compute and compare the bias and root mean squared error (RMSE)[6] of our various estimators of daily covariance.

[5] We set ω_1, ω_2 and ρ equal to the variances σ_1^2 and σ_2^2 of the two assets' prices, and the correlation ρ between the two assets' prices that we used in last subsection. We set the rest of the parameters according to the simulation study in Aït-Sahalia, Fan and Xiu (2010).

[6] Since the covariance is time-varying in this scenario, the bias reported in Table 1 is actually the mean of the covariance estimation bias.

Table 1 panel B reports the results. Relative to the results in the last subsection, the main difference is that all estimators are now slightly less efficient. From Section 2.1, the asymptotic variance of all three variance estimators is given by $\alpha \int_{t-1}^{t} \sigma_s^4 ds$, where α reflects the relative efficiencies of the different estimators, and this holds true regardless of whether σ_s is constant or time varying. Thus, the ranking of the variance (and hence covariance) estimators in terms of efficiency is unaltered once we have time-variation in volatility. Relative to the constant volatility case, time-variation in asset price volatility tends to increase σ^4 and hence increase the asymptotic variances of all variance (and covariance) estimators and decrease efficiency, but the FHLL covariance estimator is still more efficient than the other two estimators, at any sampling frequency.

2.3.3 Stochastic Volatility with Microstructure Noise

We did not include microstructure noise in the previous experiments, but in this section we compare the three estimators when they are contaminated by the effects of the bid-ask bounce, infrequent trading and asynchronous trading. We compare our FHLL covariance estimator with the realized co-range and realized covariance estimators, both with and without corrections for estimation bias resulting from the presence of microstructure noise.

Following Bannouh et al (2009), we consider the effects of bid-ask bounce by assuming that bid and ask prices occur with equal probability. Hence, the actually observed price $p_{i,(t-1)+h/K}$ is equal to $p^*_{i,(t-1)+h/K} + s/2$ (ask) or $p^*_{i,(t-1)+h/K} - s/2$ (bid), where s is the bid-ask spread and $p^*_{i,(t-1)+h/K}$ is the true price obtained from subsection 2.3.2. Infrequent trading is simulated by filtering the price sample path $p^*_{i,(t-1)+h/K}$ simulated from subsection 2.3.2, so that the price of each asset is observed on average only every τ seconds. Since price observations for the two assets occur independently, we observe prices at different times.

We use the simulations in Table 1 Panel B as a benchmark and change the values of s and τ in our simulations to consider three pairs in which $s = 0.075$ and $\tau = 1$, $s = 0$ and $\tau = 15$, and $s = 0.075$ and $\tau = 15$. The first two pairs of settings are used to examine the separate effects of bid-ask bounce and infrequent trading, while the last setting is used to investigate their joint effects on all the covariance estimators.

It is well known that when continuous underlying price processes are observed only at discrete time points, the intraday range suffers from a downward bias. This is because the observed maximum and minimum prices over a given intraday interval underestimate and overestimate the true maximum and minimum, respectively. Meanwhile, the intraday range also tends to be upward biased due to the presence of bid-ask bounce. For example, when the sampling frequency is relatively high and the intraday time interval is relatively small, the observed high price in a given intraday interval is an ask price and the observed low price is a bid price with probability close to one. The squared intraday range therefore overestimates the true variance of that intraday interval by an amount equal to the squared bid-ask spread s^2. Although univariate intraday returns are not effected much by infrequent trading and bid-ask spread, an important concern in a multivariate setting is the presence of asynchronous trading. As different assets trade at different times, estimates of their covariance are biased toward zero. This is the so-called "Epps effect", which becomes worse with an increase of sampling frequency.

We correct this bias by assuming that the observed log price p_t is equal to the underlying log price p_t^* plus an additive noise term, and then employ an additive bias-correction method discussed in Bannouh et al (2009). These authors define bias-corrected variance estimators as

$$VE_{C,t}^M = VE_t^M + \frac{1}{Q}(\sum_{q=1}^{Q} VE_{t-q}^1 - \sum_{q=1}^{Q} VE_{t-q}^M),$$

where VE_t^1 is the daily squared return or daily squared range or daily "Garman and Klass estimator", and VE_t^M is the realized variance, realized range or our FHLL variance estimator based on M intraday sampling intervals. The number of trading days Q used to compute the correction is a critical choice to make. The RMSE of all the estimators decline as Q increases and we set $Q = 150$, beyond which the RMSEs for the corrected version of all the estimators more or less stabilize.

We set $s = 0$ and $\tau = 15$ to obtain Table 2 panel A, which shows the effects of infrequent (and hence nonsynchronous) trading on all three covariance estimators. As expected, all three non-corrected covariance estimators are downward biased, but realized covariance is downward biased much less than the realized co-range and FHLL covariance estimators. The RMSE first decreases for all the estimators when increasing the sampling frequency and decreasing the length of the sampling interval, but it increases again for higher frequencies because the bias associated with microstructure noise outweighs the increase in information from the higher sampling frequency. Without the bias correction, our FHLL estimator has larger RMSE than the other two estimators at most sampling frequencies. This is not surprising because the FHLL estimator is a linear combination of realized covariance and realized co-range, which is contaminated more by the microstructure noise than the other two estimators. However, the correction scheme eliminates the bias to a large extent and reduces the RMSE of our FHLL estimator substantially. More importantly, the bias-corrected FHLL estimator $FHLLCV_{C,t}$ has the smallest RMSE at all sampling frequencies.

Table 2 panel B demonstrates the influence of bid-ask bounce on the three covariance estimators by setting $s = 0.075$ and $\tau = 1$. We set $\tau = 1$ in this panel, so that results can be compared with those in Table 1 Panel B. Our FHLL covariance estimator and the realized co-range suffer from a strong upward bias in this scenario, which becomes worse with increased sampling frequency, but realized covariance is not affected much by the bid-ask spread. The bias correction reduces the RMSE of the first-high-low-last price based covariance estimator considerably, such that $FHLLCV_{C,t}$ is more accurate than $RCV_{C,t}$ and $RCR_{C,t}$ for most sampling frequencies.

When bid-ask spread and not synchronous trading are jointly present, as in the set-up of Table 2 panel C, we find that our FHLL covariance estimator and the realized co-range still have an upward bias, but it is much smaller than that in the case of bid-ask spread only. This finding is consistent with the discussion in Bannouh, van Dijk and Martens (2009), who suggest that the upward bias due to the presence of bid-ask bounce has been partially offset by the downward bias due to non-synchronous trading. As observed in the last two panels, the bias in all estimators has been largely removed by the correction adjustment. Meanwhile, the bias corrected FHLL estimator $FHLLCV_{C,t}$ has the minimum RMSE at all sampling frequencies.

3 The Co-jump Test

In this section, we review the return-based co-jump test proposed by Bollerslev et al (2008), and then use intraday ranges and FHLL prices to develop range and first-high-low-last (FHLL) price based co-jump test statistics.

3.1 Co-jumps in Portfolio Theory and the Returns Based Co-jump Test

Bollerslev et al (2008) consider a collection of n stock price processes $\{p_{i,s}\}_{i=1}^{n}$evolving in continuous time. Each $p_{i,s}$ evolves as

$$dp_{i,s} = \mu_i(s)dt + \sigma_i(s)dW_i(s) + dL_i(s),$$

where $\mu_i(s)$ and $\sigma_i(s)$ refer to the drift and local volatility, $W_i(s)$ is a standard Brownian motion, and $L_i(s)$ is a pure jump process. The price process is only observed at discrete time points, so they consider a situation in which there are $M+1$ equidistant price observations each day. The jth within-day return of the ith log-price process on day t is then

$$r_{i,t_j} = p_{i,(t-1)+\frac{j}{M}} - p_{i,(t-1)+\frac{j-1}{M}}, \qquad i = 1, 2, ...n, \qquad j = 1, 2, ...M.$$

and the jth within-day return on day t of an equiweighted portfolio of n stocks is

$$r_{EQW,t_j} = \frac{1}{n}\sum_{i=1}^{n} r_{i,t_j}.$$

The daily realized variance for this equiweighted portfolio is given by

$$RV_{EQW,t} = \sum_{j=1}^{M}(\frac{1}{n}\sum_{i=1}^{n} r_{i,t_j})^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{M} r_{i,t_j}^2 + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{l=1,l\neq i}^{n}\sum_{j=1}^{M} r_{i,t_j} r_{l,t_j}, \tag{7}$$

and when this is decomposed into its continuous and jump components, Bollerslev et al (2008) show that most of the jump contribution to $RV_{EQW,t}$ originates from the covariation term (i.e from within $\frac{1}{n^2}\sum_{i=1}^{n}\sum_{l=1,l\neq i}^{n}\sum_{j=1}^{M} r_{i,t_j}r_{l,t_j}$) in (7)) when n is large, while the effects of idiosyncratic jumps (i.e. that originate from the $\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{M} r_{i,t_j}^2$ term) are essentially diversified away. It follows that only co-jumps can cause the price of a portfolio to jump when n is large and it was this observation that motivated Bollerslev et al (2008) to emphasize the cross-product measures associated with a portfolio when they developed their new co-jump test. Their derivation was based on an equiweighted portfolio, but their conclusion that most of the information about cojumps is contained in the covariation between stock returns is valid, given any well-diversified portfolio.

The *zmcp* test statistic proposed by Bollerslev et al (2008) is given by

$$z_{mcp,t_j} = \frac{mcp_{t_j} - \overline{mcp_t}}{s_{mcp,t}}, \; j = 1, 2, ...M, \qquad \text{where} \qquad (8)$$

$$mcp_{t_j} = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{l=i+1}^{n} r_{i,t_j}r_{l,t_j}, \qquad j = 1, 2, \ldots, M, \qquad (8a)$$

$$\overline{mcp_t} = \frac{1}{M}\sum_{j=1}^{M} mcp_{t_j} = \frac{1}{M}[\frac{n}{n-1}RV_{ew,t} - \frac{1}{n(n-1)}\sum_{i=1}^{n} RV_{i,t}], \qquad (8b)$$

$$s_{mcp,t} = \sqrt{\frac{1}{M-1}\sum_{j=1}^{M}(mcp_{t_j} - \overline{mcp_t})^2}, \qquad (8c)$$

and it can be used as a test for common jumps because the jump (but not the continuous) component in the second term in $mcp_t = \frac{n}{n-1}RV_{ew,t} - \frac{1}{n(n-1)}\sum_{i=1}^{n} RV_{i,t} \approx RV_{ew,t} - \frac{1}{n(n-1)}\sum_{i=1}^{n} RV_{i,t}$ is diversified away as n grows large.

It is easy to rearrange the expression for mcp_{t_j} to obtain

$$mcp_{t_j} = \frac{n}{n-1}\left[(\frac{1}{n}\sum_{i=1}^{n} r_{i,t_j})^2 - \sum_{i=1}^{n}(\frac{1}{n})^2 r_{i,t_j}^2\right] = \frac{n}{n-1}r_{ew,t_j}^2 - \frac{1}{n(n-1)}\sum_{i=1}^{n} r_{i,t_j}^2, \qquad (9)$$

and from this we can see that it is possible to calculate the test statistic directly from the squared returns of the equally weighted portfolio and the individual stocks. We work with this alternative expression for mcp_{t_j} in the next subsection, but firstly outline some distributional characteristics of the test statistic.

The z_{mcp,t_j} statistic is not well approximated by any of the standard distributions[7], but it is relatively straightforward to bootstrap its empirical distribution under the null hypothesis of no jumps, to find critical values that are relevant for a given application. Bollerslev et al (2008) did this, and when they checked the sensitivity of their critical values to changes in M, n and the average level of correlation between returns ($\overline{\rho}$) they found strong sensitivity to M, but little sensitivity to n and to $\overline{\rho}$. This sensitivity to the sampling frequency (M) is interesting from our perspective, because it indicates that the amount of price information that is contained in the z_{mcp,t_j} statistic can strongly influence its performance. This leads us to consider the use of more informative measures of daily price (range and first-high-low-last variance measures) when constructing the test statistic.

3.2 Range and FHLL Price Based Co-jump Tests

Theory shows that although the three estimators of daily volatility (RV, RRV and FHLLV) are all consistent when no microstructure noise is present, RV is considerably less efficient than RRV, and RRV is considerably less efficient than FHLLV. Our simulations in sections 2.3.1 and 2.3.2 reflect this, demonstrating that when RV, RRV and FHLLV are used as inputs in the construction of covariance estimators, then all three estimators approach the true covariance and the efficiency rankings of these estimators are the same as those for RV, RRV and FHLLV. Bias can affect each of RV, RRV and FFLLV (and other estimators constructed from RV, RRV and FHLLV) once microstructure noise is present, but our simulations in section 2.3.3 show that the additive bias correction technique works well when corrected versions of RV, RRV and FHLLV are used in the construction of covariance estimators. Further, the efficiency rankings of the estimators that have been corrected for microstructure noise are maintained.

[7] Simulations conducted by Bollerslev et al (2008) show that the distribution of the z_{mcp,t_j} statistic is centred to the left of zero and has a very strong right skew.

The greater efficiency of the (bias corrected) FHLLV estimator (relative to RRV and RV estimators) reflects the more extensive use of price information in the construction of the associated volatility estimate, and we expect this to translate into an increase in power when we use this estimator for constructing a test statistic. We study this via simulation below, and show that this is indeed the case.

A range-based co-jump test statistics can be obtained when we use the realized co-range and intraday squared range instead of the cross-product of intraday returns and realized volatility in the *zmcp* test statistic. We refer to this as the *zmcr* test statistic below. Our range-based co-jump test statistic (*zmcr*) is defined by

$$z_{mcr,t_j} = \frac{mcr_{t_j} - \overline{mcr_t}}{s_{mcr,t}}, \ j = 1, 2, ...M, \qquad (10)$$

where

$$mcr_{t_j} = \frac{n}{n-1} ISR_{ew,t_j} - \frac{1}{n(n-1)} \sum_{i=1}^{n} ISR_{i,t_j},$$

and ISR_{ew,t_j} and ISR_{i,t_j} are intraday squared ranges of the equally weighted portfolio and of each individual stock. These measure the intraday variance of this portfolio and each individual stock on day t, time j, and replace r^2_{ew,t_j} and r^2_{i,t_j} in equation (9). We studentize the mcr_{t_j} statistic using

$$\overline{mcr_t} = \sum_{j=1}^{M} mcr_{t_j} = \frac{1}{M}[\frac{n}{n-1} RRV_{ew,t} - \frac{1}{n(n-1)} \sum_{i=1}^{n} RRV_{i,t}] \text{ and}$$

$$s_{mcr,t} = \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} (mcr_{t_j} - \overline{mcr_t})^2},$$

where $RRV_{ew,t}$ and $RRV_{i,t}$ measure the daily realized ranges of the equally weighted portfolio and each individual stock, and they are obtained by summing all the intraday squared ranges over the whole trading day. The measures of $RRV_{ew,t}$ and $RRV_{i,t}$ will be influenced by the presence of microstructure noise, but one can account for this by using the additive bias correction in Section

2.3.3, as we demonstrate below.

Similarly, we also develop a first-high-low-last price based co-jump test, and do this by using the intraday FHLL measures to replace r^2_{ew,t_j} and r^2_{i,t_j} in equation (9), and the daily FHLL variance estimators to replace $RV_{ew,t}$ and $RV_{i,t}$ in equation (8b). Our FHLL co-jump test statistic ($zmcfhll$) is defined by

$$z_{mcfhll,t_j} = \frac{mcfhll_{t_j} - \overline{mcfhll_t}}{s_{mcfhll,t}}, \ j = 1, 2, ...M, \qquad (11)$$

where

$$mcfhll_{t_j} = \frac{n}{n-1} IFHLL_{ew,t_j} - \frac{1}{n(n-1)} \sum_{i=1}^{n} IFHLL_{i,t_j})),$$

and $IFHLL_{ew,t_j}$ and $IFHLL_{i,t_j}$ are intraday FHLL variance estimators of the equally weighted portfolio and each individual stock. We studentize the $mcfhll_{t_j}$ statistic using

$$\overline{mcfhll_t} = \sum_{j=1}^{M} mcfhll_{t_j} = \frac{1}{M}[\frac{n}{n-1} FHLLV_{ew,t} - \frac{1}{n(n-1)} \sum_{i=1}^{n} FHLLV_{i,t}] \quad \text{and}$$

$$s_{mcfhll,t} = \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} (mcfhll_{t_j} - \overline{mcfhll_t})^2},$$

where $FHLLV_{ew,t}$ and $FHLLV_{i,t}$ are the daily FHLL variance estimators of the equally weighted portfolio and each individual stock, obtained by summing all the intraday FHLL variance estimators over the whole trading day. Like the $RRV_{ew,t}$ and $RRV_{i,t}$ estimators, the measures of $FHLLV_{ew,t}$ and $FHLLV_{i,t}$ will be influenced by the presence of microstructure noise, but one can account for this by using the additive bias correction discussed earlier.

Prior to using these new co-jump test statistics, we conduct a set of Monte Carlo simulations to study their distributions under the null hypothesis of no co-jumps, and compare their finite sample properties with the return-based $zmcp$ test statistic.

4 Monte Carlo Simulation

In this section, we conduct a Monte Carlo simulation to explore the behaviour of the three test statistics under the null hypothesis of no jumps, and then in the presence of co-jumps.

4.1 The Null Distribution via Simulation

We simulate realizations of a logarithmic price diffusion process that has been calibrated to our data set[8] and then calculate the *zmcp* test statistics, the *zmcr* test statistics and the $zmcfhll$ statistics from these simulated values. We use sampling frequencies of 5, 10 and 15 minutes that correspond to $M = 48$, 24, 16 intraday sampling periods per day. Given our discretization grid of thirty seconds, the corresponding number (m) of intraday subintervals used to compute the intraday range and intraday first-high-low-last price for these sampling frequencies is equal to $m = 10$, 20, and 30. We also compute the intraday range and intraday first-high-low-last price by using only half of the available price observations and one third of the available price observations within the 5, 10 and 15-minute intervals, that is, $m = 5, 10, 15$ and $m = 3, 6, 10$ to study the sensitivity of these test statistics to m.[9] It is useful to note that when $m = 1$, then the range equals the absolute return, so that the Bollerslev et al (2008) *zmcp* test statistic can be regarded as a special case of the range-based *zmcr* test statistic. Our initial data generating process is not contaminated by microstructure noise, and we initially calculate the three co-jump test statistics without incorporating any bias corrections.

Figure 2 presents the simulated probability densities of the *zmcp*, *zmcr* and $zmcfhll$ test statistics. All of these distributions are obviously non-Gaussian with a strong right skew, regardless of the sampling frequency. The null distributions of the $zmcfhll$ test statistics have slightly shorter tails and lower peaks than the *zmcr* test statistics, and both of them have much shorter tails and

[8] Implementation involved 480 equally spaced steps per day for 90 days (corresponding to thirty second invervals for four hour trading days over our three month sample period), empirically calibrated to the covariance between the twenty stocks from the Shanghai exchange included in our sample.We replicated the process 1000 times, and thereby worked with about 43.2 million simulated values under the null hypothesis of no jumps.

[9] Christensen and Podolskij (2007) note that the entire sample path of the asset price process is unavailable in practice, so that inference is drawn from discrete data and the true price range is often unobserved. Therefore, the range-based estimator has varying degrees of efficiency over the return-based estimator depending on how many observations (it is m in our paper) that are used to construct the high-low range.

lower peaks than the $zmcp$ test statistics. Meanwhile, the null distributions of the $zmcfhll$ test statistics have shorter tails and lower peaks as M decreases, or as m increases given the same M. This also holds for the $zmcr$ test statistics.[10] Since large values of the test statistics discredit the null hypothesis of no co-jumps, we are mostly interested in the right tails of these distributions.

Table 3 reports the critical values of all the test statistics at the 0.1%, 1% and 5% significance levels. These results suggest that critical values are quite sensitive to the sampling frequency (M) and the number of subintervals (m) involved in forming the intraday high-low prices. In particular, the critical values always rise as the intraday sampling frequency (M) increases, and fall as the number of subintervals (m) used for each calculation of the high-low price range or first-high-low-last price decreases, with some exceptions at the 5% significance level.

The critical values in Table 3 relate to a situation in which no microstructure noise is present and no corrections for microstructure bias have been made, but in practice we would want to cater for possible noise effects. Therefore, we next study how microstructure noise affects the distributions of the test statistics, by adding bid-ask bounce into the data generating process and/or simulating situations in which there is infrequent trading,[11] and then comparing the distributions of bias corrected and uncorrected test statistics with those relating to the no noise and no corrections baselines. The tails of the resulting distributions of test statistics for $M = 48$ and $m = 10$ are illustrated in Figure 3, where each of the nine sub-diagrams in this figure contains three plots that relate to tests when no noise is present and no bias corrections have been made, and then uncorrected and corrected tests when a specified form of microstructure noise is present. Only two plots are readily apparent on each sub-diagram, because the distributions of the bias corrected tests are almost the same as those for uncorrected tests when no microstructure noise is present. This demonstrates that the additive bias corrections are very effective. Although bid-ask bounce can induce upward bias in $\overline{mcp}_t$ ($\overline{mcr}_t$, $\overline{mcfhll}_t$) and hence induce leftwards shifts in the uncorrected distributions of the

[10] We don't provide the results of sensitivity analysis to M and m for $zmcr$ test statistics in Figure 2. They are available upon request.

[11] See Section 2.3.3 for details on how we introduce noise. The values of the noise parameters that we study are indicated on the illustrations of the right hand tails provided in Figure 3.

mcp_{t_j} mcr_{t_j}, $mcfhll_{t_j}$ tests, the corrections move the distributions back to the right, as illustrated in the three left-hand sub-diagrams. Similarly, the three sub-diagrams running down the middle show that although failure to correct when trading is infrequent induces a rightward movement in the test distribution, simple bias corrections can rectify this. The three right-hand sub-diagrams show that the bias corrections can be helpful, even when different sorts of noise partially offset each other.

We repeated the experiments that underlie Figure 3 for different M and m and found analogous results, which suggest that provided that the calculation of the test statistic incorporates an additive bias correction, it will be appropriate to use critical values derived from simulations that relate to a situation in which microstructure noise is not present. Thus, we use the critical values in Table 3 in our power simulations that follow, and in our empirical application in Section 5. Critical values relating to other values of M and m are available from the authors upon request.

4.2 Power Comparisons

In this section, we compare the performance of the three test statistics in terms of their power properties. For power comparisons, we add simulated jump components into the above pure diffusion processes. For idiosyncratic jumps, we simulate 20 independent Gaussian Poisson processes with intensity λ_i[12] and magnitude $N(0, \sigma_i^2)$,[13] and add them to their corresponding pure diffusion processes. For the common jumps, we simulate one Gaussian compound Poisson process with intensity λ and magnitude $N(0, \sigma_J^2)$, and add it to the diffusion process after multiplying each of the twenty components by an estimate of its β_i relative to the portfolio. We calibrate the common jump intensity λ and the common jump size σ_J to empirical data (i.e. $\lambda = 0.05\%$ and $\sigma = 0.005$ in our case), and then change the intensity λ from 0.05% to 1%, and the size of σ_J from 0.005 to 0.1 in order to check the sensitivity of the power of these test statistics to these parameters.

[12] Jump intensity λ_i is defined here as the percentage of price observations that contain a jump, where $\lambda_i \in [0.005\%, 0.01\%]$ in our simulation.

[13] Here $\sigma_i^2 \in [0.0005, 0.001]$ in our simulation.

Table 4 presents power calculations relating to the $zmcp$, $zmcr$ and $zmcfhll$ test statistics under a nominal significance level of 0.1%, but based on different sampling frequencies (M). The number of subintervals (m) used to calculate price range and first-high-low-last price correspond to the maximum possible given M, and co-jump intensities and co-jump sizes are varied. As expected, the $zmcfhll$ tests are the most powerful, followed by the $zmcr$ tests and then the $zmcp$ tests, and all tests have greater ability to find co-jumps as M, λ and σ_J^2 increase.

Table 5 reports the power of the $zmcr$ test statistics and the $zmcfhll$ test statistics, as the sampling frequency M is kept constant,[14] but the number of subintervals (m) used to calculate the price range or first-high-low-last price vary. The co-jump intensities and the co-jump sizes vary as before and the nominal significance level is still 0.1%. As expected, the tests have greater ability to find co-jumps as m, λ and σ_J^2 increase. This finding is important, because it shows that the range based statistics lead to increased power relative to the Bollerslev et al (2008) return-based $zmcp$ test statistic (for which $m = 1$). Furthermore, we find that at all sampling frequencies and all levels of co-jump intensity and co-jump size, the first-high-low-last price based $zmcfhll$ test statistics lead to further power improvement compared with the range-based $zmcr$ test statistics.

5 Empirical Application

5.1 Data

Our empirical analysis is based on intraday data relating to 40 very actively traded stocks in the Chinese mainland stock market.[15] Twenty of these stocks are traded on the Shanghai Stock Exchange (SSE) and the remaining twenty are traded on the Shenzhen Stock Exchange (SZSE). The existing literature relating to jump detection in this market mostly focuses on the univariate situation (see Xu and Zhang (2006), Wang, Yao, Fang and Li (2008) and Ma and Wang (2009)), although

[14] Our reported results relate to the 5-minute sampling frequency ($M = 48$) , but similar tendencies are observed at other sampling frequencies.

[15] There are two official stock exchanges in the Chinese mainland market, i.e. the Shanghai Stock Exchange (SHSE) and the Shenzhen Stock Exchange (SZSE). These were established in December 1990 and July 1991 respectively. All stocks are A-share stocks.

Liao et al (2010) build factor models of jumps to account for simultaneous jumps in more than one stock, and Chen at al (2010) study the microstructure of cross listed A and B shares on the Shanghai exchange. We apply the return-based co-jump test in Bollerslev et al (2008), our range-based co-jump test and our first-high-low-last price based co-jump test to the twenty stocks from the Shanghai Stock Exchange, the twenty stocks from the Shenzhen Stock Exchange and all forty stocks to analyze the co-jump patterns in each stock exchange and co-jumps across the two stock exchanges. The raw transaction prices (together with trading times and volumes) were obtained from the China Stock Market & Accounting Research (CSMAR) database provided by the ShenZhen GuoTaiAn Information and Technology Firm (GTA). Our sample covers the period from July 2nd, 2007 to September 28th, 2007 (three months).

We focus on the active trading period and leave issues associated with overnight jumps for further research. Due to the fact that it is difficult to construct the price sample path of a portfolio from the tick-by-tick data of each individual stock in the case of nonsynchronous trading, we firstly use 30 seconds as the sampling frequency to obtain equally spaced high frequency data for each individual stock, then average the 30-second prices of individual stocks to obtain a price sample path for the equally weighted portfolio.[16] Therefore, the baseline data used in the following analysis is equally spaced high frequency data (observed at thirty second intervals) rather than irregularly spaced tick-by-tick data. We exclude weekends, public holidays and periods when there are firm specific suspensions from our sample, and we avoid market opening effects by only using data from 09:35-11:30 and 13:05-15:00.

Paralleling many previous studies, we attempt to strike a reasonable balance between efficiency and accuracy by using five-minutes as the sampling frequency to construct intraday returns, intraday range, daily realized volatility, daily realized range and daily FHLL estimators of volatility.

[16]Bannouh et al (2009) accounted for nonsynchronous trading by updating their portfolio price each time that they observed a new price for one of the constituent assets. They have only three assets in their portfolio. Their procedure becomes relatively complicated when the number of the constituent assets is large, so in our case, we simply sample the raw tick-by-tick data once every 30 seconds to effectively mimic a synchronous trading scenario in which the portfolio price is updated every 30 seconds. This does not lead to a large loss of information because our prices rarely change much in 30 seconds.

Moreover, when calculating our intraday *zmcp*, *zmcr*, and *zmcfhll* statistics, we employ the additive bias-correction method as discussed in section 2.3.3 to correct for microstructure noise bias in daily realized volatility, daily realized range and daily FHLL variance estimators. Our sample spans 65 trading days, and each trading day has 462 intraday (30-seconds) price observations. Hence, there are $M = 46$ *zmcp*, *zmcr*, and *zmcfhll* test statistics for each trading day, and the number of intraday prices that are used to calculate the range for each five minute interval is $m + 1 = 11$.

5.2 Co-jumps in the Chinese Mainland Stock Market

We simulate the null distributions of the co-jump test statistics for the forty stocks prior to performing the tests. This involves the simulation of 1000 realizations of a 40×1 diffusion process with zero drift and a covariance matrix determined by an unconditional estimate of the covariance matrix of the 30-seconds within-day returns for the relevant 40 stocks. The length of each realization is set equal to the sample size (462 per day for 65 days). We use these simulations to obtain observations on each of the *zmcp* , *zmcr* and *zmcfhll* test statistics. This scheme generated over 30 million simulated values for each of the three test statistics under the null of no jumps, and we used these to obtain critical values at the 0.1%, 1% and 5% significance levels.

Figure 4 plots the three series of test statistics calculated from the panel of 40 stocks, together with horizontal lines that indicate the 0.1%, 1% and 5% critical values determined from the simulated null distributions. It is clear that the empirically observed test statistics exceed their relevant critical values on several occasions, providing evidence of co-jumps. A comparison of the three graphs in the figure shows that the *zmcfhll* tests finds more co-jumps than the *zmcr* and *zmcp* tests regardless of the level of significance of the test.

Table 6 report the outcomes of the co-jump tests the 0.1% significance level. These outcomes include co-jump arrival dates and times. In contrast to previous research that has found frequent jumps in some of the individual stocks (see Liao (2011)), we find relatively few co-jumps in the panels. The return-based test finds six co-jumps on the panel of forty stocks, the range based test finds the same co-jumps as well as an additional three co-jumps (9 in total), and the FHLL test finds all of these co-jumps as well as another six co-jumps (15 in total). Many of the detected co-jumps in this market occurred near the morning opening time or the afternoon closing time of trading sessions. Moreover, the timing of some of co-jumps coincided with the release of news on stock market regulations or monetary policy. These specific events are detailed below Table 6.

We repeated the co-jump tests for our panels of 20 stocks sold on the Shanghai exchange and the 20 stocks sold on the Shenzhen exchange (recalibrating the critical values appropriately in each case). We do not report results here because of space considerations, but note that the results for these separate panels essentially mirrored those for the panel of all 40 stocks. In particular, most of the co-jump times found in the composite panel were also found in each of the two separate panels, and vice-versa.

6 Conclusion

This paper explores the use of first-high-low-last (FHLL) prices in a multivariate high frequency setting. We introduce a first-high-low-last price based covariance estimator and study its properties, and find that after a very simple bias correction, this estimator has lower root mean squared error than counterparts based on realized range and realized variance. We also use FHLL price data instead of returns data in the Bollerslev et al (2008) co-jump test, and find an increase in power. We see a similar but smaller increase in power in an analogous range based co-jump test that we propose and study in conjunction with our FHLL based co-jump test. When we apply our FHLL co-jump test to a panel of high frequency data relating to Chinese mainland stock market, we find co-jumps in the stocks from the two stock market exchanges, and we are able to associate many of these co-jumps with announcements about changes in monetary policy or stock market regulations.

Our FHLL estimator of covariance is quite easy to calculate, since it relies only on univariate methodology (i.e. FHLL measures of the variances of two individual stocks and a portfolio containing those stocks). While the computational burden of estimation might not be a primary consideration when working with just two assets, it becomes an important practical consideration once one works with a large set of assets. Our finding that the use of FHLL measures of variance in covariance estimation can be beneficial relative to the use of return (and range) based measures of variance is likely to encourage researchers to explore the potential of using other univariate measures of variance in covariance estimation further. Similarly, our use of range and FHLL variance estimators in the construction of tests analogous to the Bollerslev et al (2008) co-jump test paves the way for using other variance estimators in such tests. Here, we have focussed on the potential benefits of including information about price ranges (in addition to returns) in both covariance estimation and in a test for co-jumps in a large panel, but further benefits are likely as additional observed information about the price process and noise structure is incorporated into variance estimation, and variance estimates are used for other purposes.

The literature on covariance estimation is growing very rapidly, and it now includes very detailed examinations of the bias effects of microstructure noise and asynchronous trading on covariance estimation, as well as ways of accounting for this. Griffin and Oomen (2011) study several covariance estimators and conclude that the choice between them can depend on the properties of the price process. We have not studied these properties here, but point out that the additive bias correction that we used in this setting of forty stocks was simple and effective, and avoided potential difficulties associated with treating different stocks in the portfolio differently. It would be interesting to experiment with other forms of bias correction in this context, but we leave this for later research.

Thus far we have linked some of the co-jumps that we found to announcements in monetary policy and stock market regulations. We anticipate that it might also be possible to link some of the other co-jumps to political announcements or financial events that occurred overseas. We leave further investigation of possible reasons for co-jumps in China for later work. Meanwhile, the empirical evidence of co-jumps in financial markets suggests that common factor models of jumps have empirical relevance. This lays open the possibility that models of co-jumps might have forecasting potential. We are currently working on this topic and have found encouraging results.

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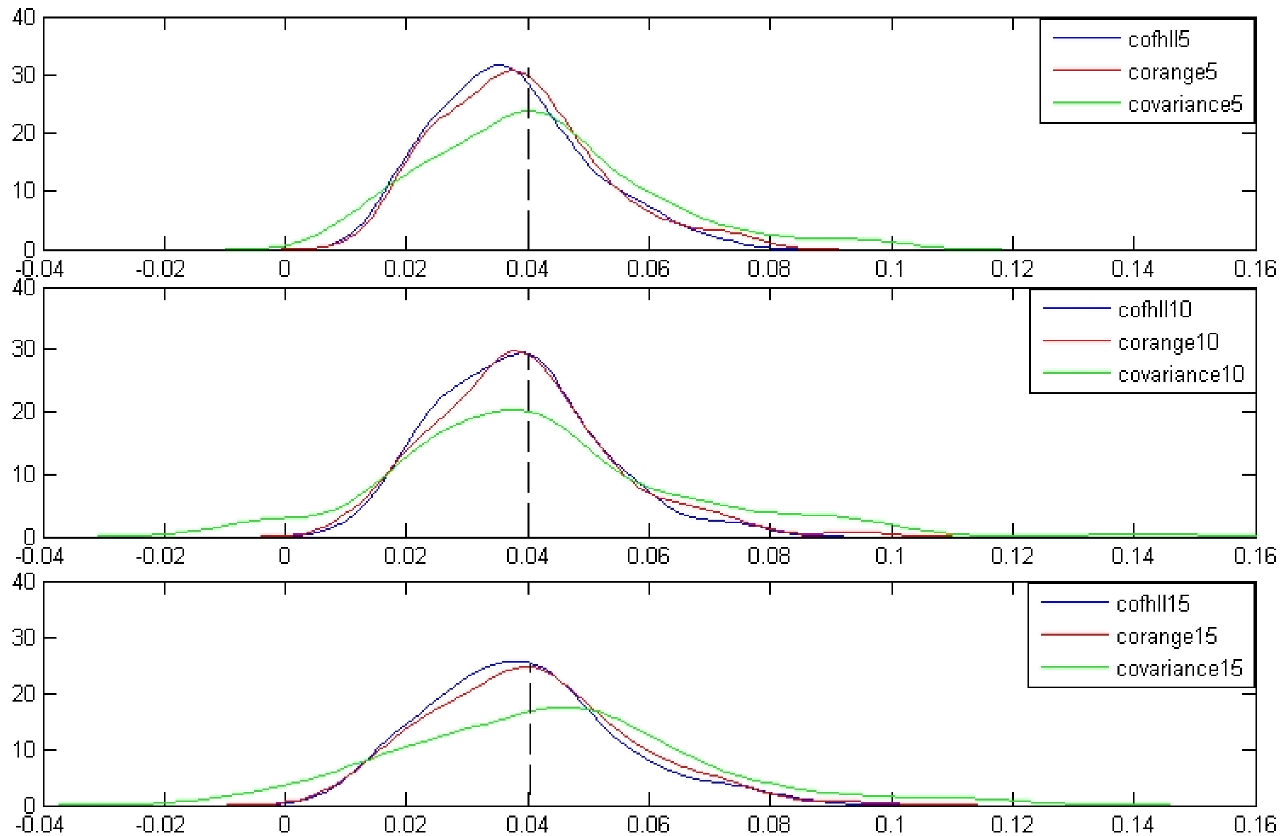
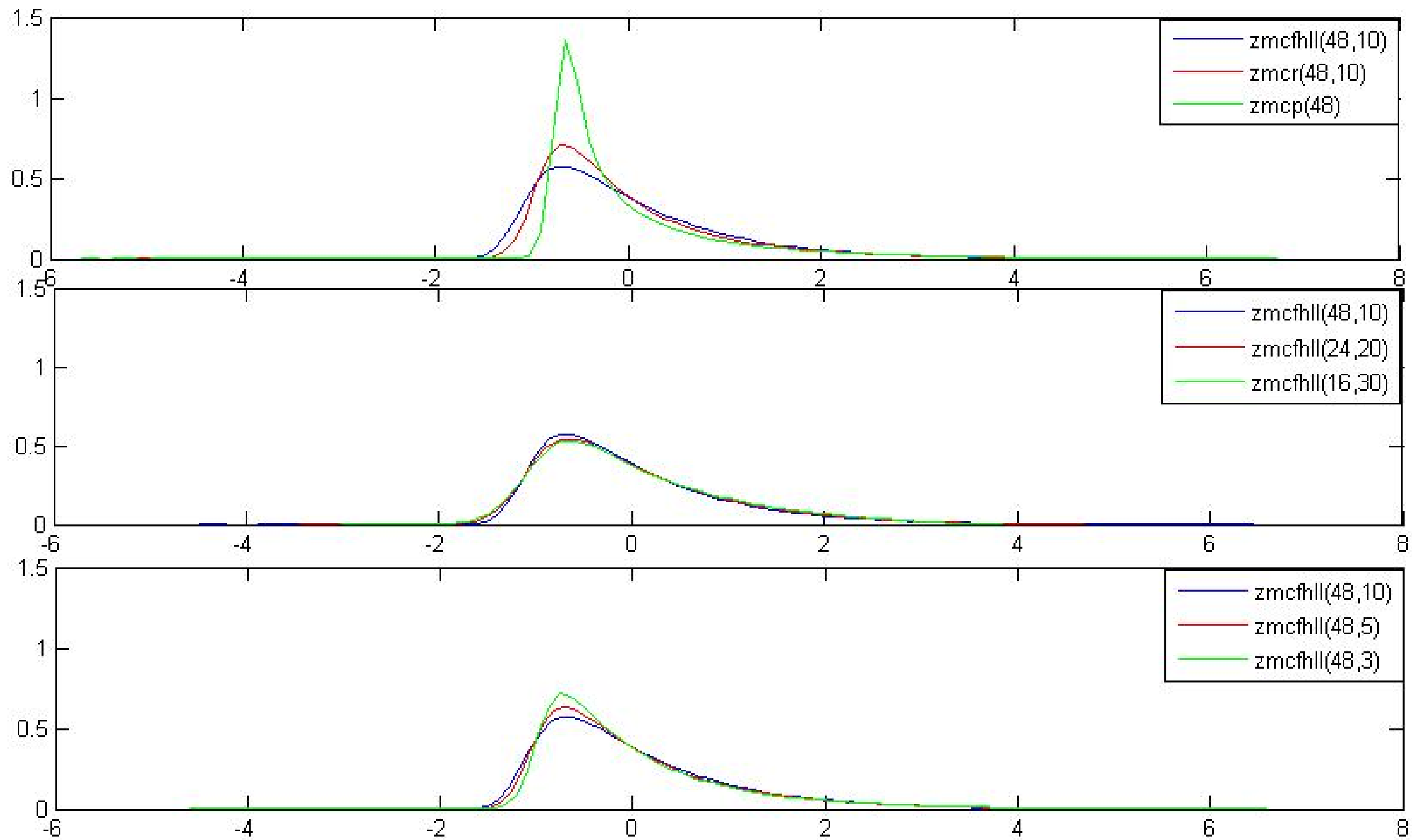
Figure 1: Density Graphs of the Three Covariance Estimators (Constant Volatility and No Microstructure Noise)



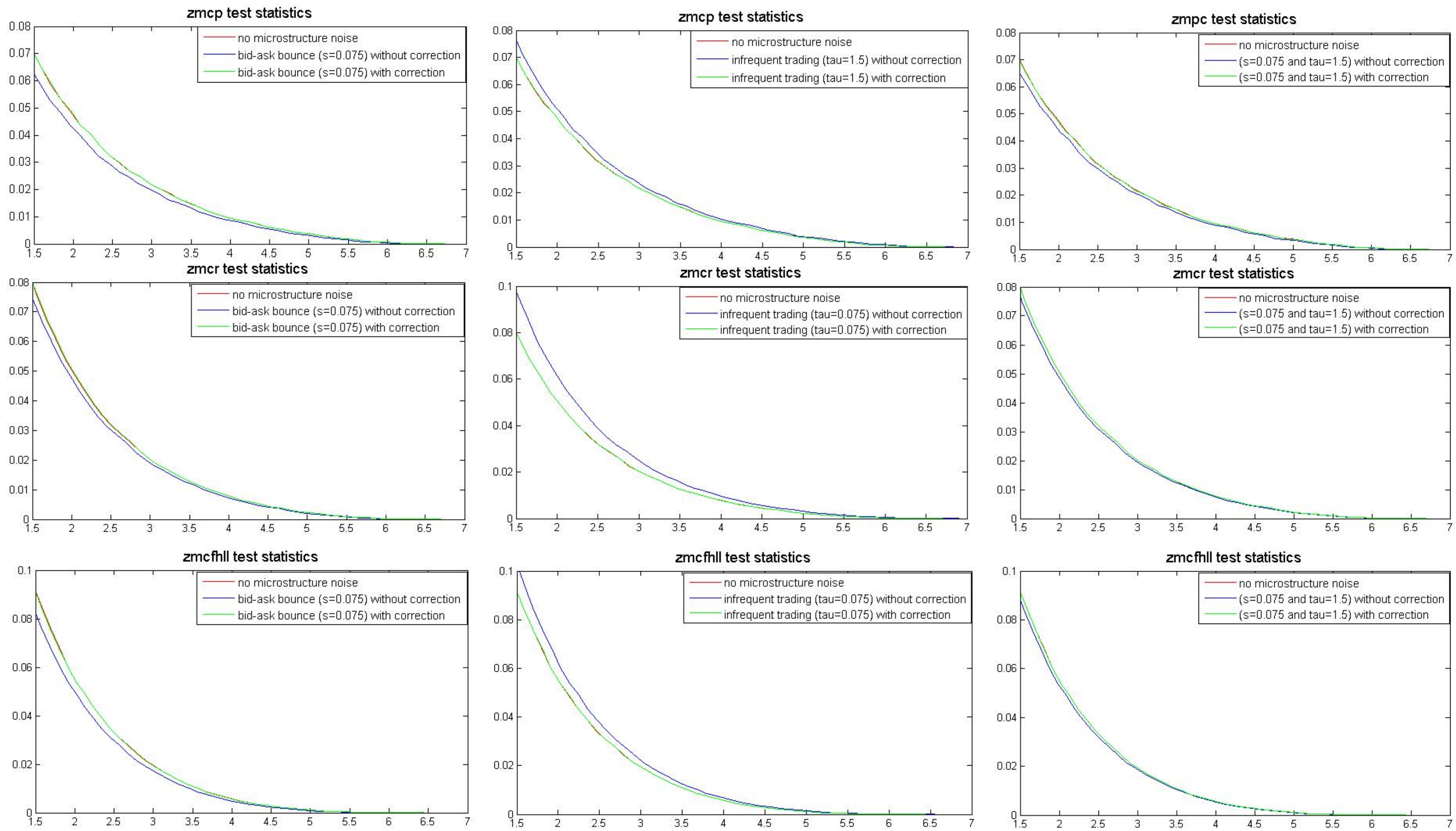
Note: The true covariance is 0.04. The top panel shows the distributions of the three covariance estimators (realized covariance, realized co-range, and first-high-low-last price based estimator) via simulation of 5000 days based on a 5-minute sampling frequency. The middle panel shows the distributions of the three covariance estimators (realized covariance, realized co-range, and first-high-low-last price based estimator) via simulation of 5000 days based on a 10-minute sampling frequency. The bottom panel shows the distributions of the three covariance estimators (realized covariance, realized co-range, and first-high-low-last price based estimator) via simulation of 5000 days based on a 15-minute sampling frequency.

Figure 2: Monte Carlo Simulation of the Null Distributions of the Co-Jump Test Statistics



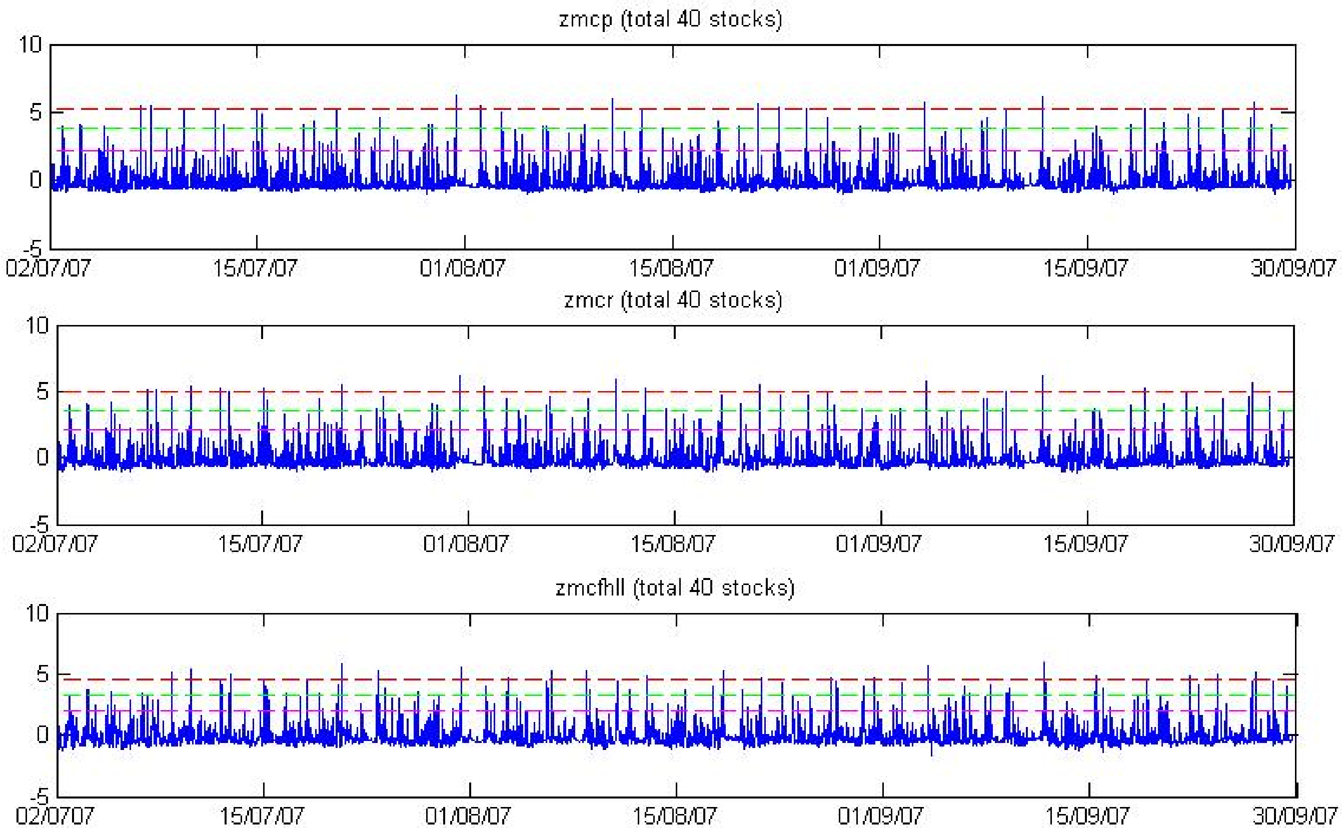
Note: The top panel shows the null distributions of the three test statistics (zmcp, zmcr, and zmcfhll test statistics) when daily variance calculations are based on 5-minute intervals (M=48) and subintervals for range and first-high-low-last price calculations are recorded 10 times (m=10) during each 5-minute interval. The middle panel shows the null distributions of zmcfhll test statistics for different number of intervals (M), when subintervals for range and first-high-low-last price calculations (m) are recorded the maximum possible number of times. The bottom panel shows the null distributions of zmcfhll test statistics for M=48, when the number of subintervals for range and first-high-low-last price calculations (m) within each of these 48 intervals is varied.

Figure 3: The Effects of Microstructure Noise and Bias Corrections on the Right Tail Distributions of the Co-Jump Test Statistics (when $M = 48$, and $m = 10$)



Note: Approximately 10% of the right hand tail of the distribution of the co-jump test statistic is illustrated in each case. The left hand diagrams illustrate the effects of bid-ask bounce, the middle diagrams illustrate the effects of infrequent trading, and the right hand diagrams illustrate the joint effects of bid-ask bounce and infrequent trading. The daily variance calculations are based on 5-minute intervals (M=48) and subintervals for range and first-high-low-last price calculations are recorded 10 times (m=10) during each 5-minute interval. The same tendency is observed when we use other sampling frequencies for the daily variance calculations. Relevant results are available upon request.

Figure 4: Empirical Test Statistics for all 40 stocks



Note: The top plot shows the series of the return-based test statistics (*zmcp*) calculated from the empirical data. The middle plot shows the series of the range-based test statistics (*zmcr*) calculated from the empirical data. The bottom plot shows the series of the first-high-low-last price based test statistics (*zmcfhll*) calculated from the empirical data. The red dotted line represents the 0.1% critical value, the green dotted line represents the 1% critical value, and the pink dotted line represents the 5% critical value.

Table 1: Comparison of Covariance Estimators When There Is No Microstructure Noise

	RCV_t		RCR_t		$FHLLCV_t$	
Sampling Frequency	Bias	RMSE	Bias	RMSE	Bias	RMSE
	Panel A:	Constant Covariance				
1(minute)	-0.0004	0.0229	-0.0007	0.0117	-0.0008	0.0113
5(minutes)	-0.0002	0.0282	-0.0005	0.0130	-0.0006	0.0126
10(minutes)	-0.0001	0.0304	-0.0002	0.0147	-0.0002	0.0132
15(minutes)	0.0001	0.0315	-0.0002	0.0159	-0.0003	0.0153
30(minutes)	-0.0003	0.0367	-0.0003	0.0186	-0.0003	0.0183
60(minutes)	-0.0005	0.0411	-0.0006	0.0204	-0.0006	0.0198
240(daily)	-0.0005	0.0859	-0.0008	0.0411	-0.0009	0.0361
	Panel B:	Stochastic Covariance				
1(minute)	-0.0009	0.0372	-0.0014	0.0134	-0.0016	0.0126
5(minutes)	-0.0006	0.0398	-0.0008	0.0136	-0.0009	0.0127
10(minutes)	-0.0002	0.0420	-0.0006	0.0149	-0.0008	0.0135
15(minutes)	-0.0003	0.0429	-0.0004	0.0154	-0.0004	0.0148
30(minutes)	-0.0003	0.0432	-0.0003	0.0183	-0.0003	0.0177
60(minutes)	-0.0004	0.0487	-0.0005	0.0228	-0.0006	0.0208
240(daily)	-0.0006	0.0881	-0.0007	0.0425	-0.0008	0.0331

Note: The true covariance is 0.04. All points of the simulated data are observed ($\tau = 1$) and there is no bid-ask bounce ($s = 0$). RCV_t is realized covariance, RCR_t is realized co-range, and $FHLLCV_t$ is the first-high-low-last price covariance estimator.

Table 2: Comparison of Covariance Estimators When There Is Microstructure Noise

	RCV_t		$RCV_{C,t}$		RCR_t		$RCR_{C,t}$		$FHLLCV_t$		$FHLLCV_{C,t}$	
Sampling Frequency	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
			Panel A:	Stochastic Covariance ($s = 0, \tau = 15$)								
1(minute)	-0.0188	0.0304	-0.0011	0.0298	-0.0203	0.0317	-0.0013	0.0275	-0.0209	0.0334	-0.0020	0.0239
5(minutes)	-0.0159	0.0289	-0.0009	0.0313	-0.0195	0.0291	-0.0011	0.0281	-0.0209	0.0301	-0.0019	0.0254
10(minutes)	-0.0136	0.0273	-0.0009	0.0398	-0.0152	0.0275	-0.0011	0.0294	-0.0158	0.0290	-0.0017	0.0289
15(minutes)	-0.0099	0.0297	-0.0008	0.0408	-0.0104	0.0279	-0.0009	0.0227	-0.0105	0.0278	-0.0014	0.0215
30(minutes)	-0.0087	0.0335	-0.0005	0.0476	-0.0066	0.0280	-0.0009	0.0209	-0.0059	0.0284	-0.0012	0.0197
60(minutes)	-0.0058	0.0413	-0.0003	0.0592	-0.0035	0.0312	-0.0007	0.0242	-0.0026	0.0301	-0.0011	0.0225
240(daily)	-0.0006	0.0862	-0.0006	0.0862	-0.0009	0.0436	-0.0009	0.0436	-0.0010	0.0386	-0.0010	0.0386
			Panel B:	Stochastic Covariance ($s = 0.075, \tau = 1$)								
1(minute)	-0.0010	0.0367	-0.0001	0.0357	0.0516	0.1133	0.0012	0.0289	0.0719	0.2743	0.0013	0.0322
5(minutes)	-0.0007	0.0402	0.0005	0.0380	0.0570	0.0936	0.0012	0.0299	0.0793	0.2541	0.0013	0.0316
10(minutes)	-0.0005	0.0423	0.0013	0.0488	0.0544	0.0748	0.0011	0.0325	0.0756	0.1678	0.0012	0.0304
15(minutes)	-0.0001	0.0431	0.0015	0.0510	0.0367	0.0654	0.0010	0.0346	0.0509	0.1287	0.0011	0.0312
30(minutes)	0.0007	0.0439	-0.0002	0.0577	0.0255	0.0482	0.0005	0.0389	0.0351	0.0830	0.0008	0.0367
60(minutes)	-0.0001	0.0516	-0.0005	0.0695	0.0173	0.0426	0.0006	0.0414	0.0240	0.0594	0.0007	0.0386
240(daily)	-0.0006	0.0885	-0.0006	0.0885	-0.0006	0.0442	-0.0006	0.0442	-0.0006	0.0429	-0.0006	0.0429
			Panel C:	Stochastic Covariance ($s = 0.075, \tau = 15$)								
1(minute)	-0.0191	0.0428	-0.0012	0.0420	0.0322	0.0987	0.0009	0.0271	0.0520	0.1025	0.0010	0.0258
5(minutes)	-0.0160	0.0412	-0.0010	0.0415	0.0240	0.0845	0.0008	0.0299	0.0394	0.0925	0.0009	0.0272
10(minutes)	-0.0139	0.0429	-0.0009	0.0438	0.0131	0.0612	0.0006	0.0312	0.0235	0.0714	0.0009	0.0294
15(minutes)	-0.0100	0.0508	-0.0009	0.0516	0.0025	0.0558	0.0006	0.0334	0.0074	0.0601	0.0008	0.0316
30(minutes)	-0.0089	0.0576	-0.0006	0.0598	0.0013	0.0471	0.0005	0.0349	0.0053	0.0461	0.0007	0.0335
60(minutes)	-0.0054	0.0692	-0.0004	0.0713	0.0006	0.0458	0.0003	0.0371	0.0029	0.0496	0.0005	0.0352
240(daily)	-0.0004	0.0894	-0.0004	0.0894	-0.0006	0.0463	-0.0006	0.0463	-0.0007	0.0452	-0.0007	0.0452

Note: The true covariance is 0.04. RCV_t is realized covariance, RCR_t is realized co-range, and $FHLLCV_t$ is the first-high-low-last price covariance estimator. $RCV_{C,t}$ is the bias-corrected realized covariance, $RCR_{C,t}$ is the bias-corrected realized co-range, and $FHLLCV_{C,t}$ is the bias-corrected first-high-low-last price covariance estimator.

Table 3: Critical Values for Co-jump Test Statistics

	$\alpha = 0.1\%$			$\alpha = 1\%$			$\alpha = 5\%$		
Co-jump Test Statistics	M=16	M=24	M=48	M=16	M=24	M=48	M=16	M=24	M=48
No microstructure noise ($s = 0, \tau = 30$) without bias correction									
zmcp	3.608	4.296	5.350	3.215	3.546	3.838	2.235	2.205	2.134
zmcr (m=maximum possible)	3.480	4.096	5.041	2.996	3.274	3.563	2.064	2.066	2.042
zmcr (m=half of maximum possible)	3.491	4.125	5.132	3.012	3.308	3.621	2.078	2.082	2.056
zmcr (m=third of maximum possible)	3.505	4.165	5.198	3.029	3.357	3.684	2.088	2.100	2.067
zmcfhll (m=maximum possible)	3.334	3.852	4.611	2.804	3.025	3.285	1.960	1.980	1.988
zmcfhll (m=half of maximum possible)	3.354	3.905	4.783	2.831	3.075	3.398	1.980	2.002	2.013
zmcfhll (m=third of maximum possible)	3.385	3.983	4.979	2.860	3.162	3.512	1.994	2.034	2.035

Table 4: The Power of Co-Jump Test Statistics for Different Sampling Frequencies M

Sampling		$\sigma_J = 0.005$			$\sigma_J = 0.01$			$\sigma_J = 0.05$			$\sigma_J = 0.1$		
Frequency		$zmcp$	$zmcr$	$zmcfhll$	$zmcp$	$zmcr$	$zmcfhll$	$zmcp$	$zmcr$	$zmcfhll$	$zmcp$	$zmcr$	$zmcfhll$
$M = 48$	$\lambda = 0.05\%$	0.0414	0.1032	0.1247	0.0654	0.1765	0.1987	0.1078	0.2654	0.2871	0.1543	0.3421	0.3589
	$\lambda = 0.1\%$	0.0523	0.1421	0.1567	0.0967	0.2325	0.2546	0.1765	0.3217	0.3564	0.2156	0.4236	0.4236
	$\lambda = 0.5\%$	0.1245	0.2144	0.2534	0.1762	0.3524	0.3865	0.2543	0.4762	0.4987	0.3059	0.5543	0.5986
	$\lambda = 1\%$	0.1698	0.3176	0.3985	0.2765	0.4789	0.5321	0.3247	0.5632	0.6543	0.3976	0.6321	0.7653
$M = 24$	$\lambda = 0.05\%$	0.0394	0.0721	0.1087	0.0581	0.1415	0.1523	0.0967	0.2106	0.2456	0.1325	0.2930	0.3452
	$\lambda = 0.1\%$	0.0372	0.1034	0.1324	0.0765	0.1925	0.2357	0.1524	0.2854	0.3067	0.1987	0.3507	0.3783
	$\lambda = 0.5\%$	0.1039	0.2032	0.2462	0.1434	0.2985	0.3234	0.2076	0.3987	0.4357	0.2764	0.4965	0.5346
	$\lambda = 1\%$	0.1355	0.2526	0.2985	0.2076	0.3247	0.3764	0.2877	0.4123	0.4893	0.3088	0.5521	0.6341
$M = 16$	$\lambda = 0.05\%$	0.0297	0.0587	0.0745	0.0455	0.1065	0.1236	0.0731	0.1523	0.1673	0.1087	0.1921	0.2436
	$\lambda = 0.1\%$	0.0308	0.0705	0.0843	0.0692	0.1490	0.1651	0.1123	0.2067	0.2587	0.1568	0.2542	0.2791
	$\lambda = 0.5\%$	0.0935	0.1442	0.1659	0.1120	0.2103	0.2546	0.1567	0.2869	0.3466	0.2034	0.3103	0.3763
	$\lambda = 1\%$	0.1027	0.2042	0.3065	0.1569	0.3024	0.3987	0.2033	0.3976	0.4671	0.2534	0.4576	0.5334
Nominal significance level= 0.1%													

Note: λ denotes co-jump intensity, and σ_J denotes co-jump size. All test statistics have been bias corrected.

Table 5: The Power of Co-Jump Test Statistics for Different m ($M = 48$)

		$\sigma_J = 0.005$		$\sigma_J = 0.01$		$\sigma_J = 0.05$		$\sigma_J = 0.1$	
		$zmcr$	$zmcfhll$	$zmcr$	$zmcfhll$	$zmcr$	$zmcfhll$	$zmcr$	$zmcfhll$
$m = 10$	$\lambda = 0.05\%$	0.1032	0.1247	0.1765	0.1987	0.2654	0.2871	0.3421	0.3589
	$\lambda = 0.1\%$	0.1421	0.1567	0.2325	0.2546	0.3217	0.3564	0.4236	0.4236
	$\lambda = 0.5\%$	0.2144	0.2534	0.3524	0.3865	0.4762	0.4987	0.5543	0.5986
	$\lambda = 1\%$	0.3176	0.3985	0.4789	0.5321	0.5632	0.6543	0.6321	0.7653
$m = 5$	$\lambda = 0.05\%$	0.0487	0.0503	0.0921	0.0990	0.1543	0.1982	0.2178	0.2527
	$\lambda = 0.1\%$	0.0842	0.0967	0.1541	0.1854	0.2367	0.2765	0.3219	0.3588
	$\lambda = 0.5\%$	0.1746	0.1998	0.2543	0.2876	0.3167	0.3543	0.4876	0.4908
	$\lambda = 1\%$	0.2124	0.2554	0.2987	0.3234	0.3542	0.3778	0.5273	0.5879
$m = 3$	$\lambda = 0.05\%$	0.0426	0.0454	0.0743	0.0885	0.1237	0.1534	0.1876	0.1936
	$\lambda = 0.1\%$	0.0654	0.0702	0.1328	0.1534	0.1965	0.2007	0.2378	0.2512
	$\lambda = 0.5\%$	0.1343	0.1538	0.2145	0.2235	0.2976	0.3079	0.3542	0.3774
	$\lambda = 1\%$	0.1978	0.2017	0.2856	0.2998	0.3765	0.3974	0.4589	0.4836
$m = 1$	$\lambda = 0.05\%$	0.0414	0.0414	0.0654	0.0654	0.1078	0.1078	0.1543	0.1543
	$\lambda = 0.1\%$	0.0523	0.0523	0.0967	0.0967	0.1765	0.1765	0.2156	0.2156
	$\lambda = 0.5\%$	0.1245	0.1245	0.1762	0.1762	0.2543	0.2543	0.3059	0.3059
	$\lambda = 1\%$	0.1698	0.1698	0.2765	0.2765	0.3247	0.3247	0.3976	0.3976
Nominal significance level= 0.1%									

Note: λ denotes co-jump intensity, and σ_J denotes co-jump size. All test statistics have been bias corrected. The zmcr and zmcfhll tests are equivalent when m=1.

Table 6: Co-Jumps Dates and Times for the 40 Chinese Stocks Based on the Co-Jump Tests

Return-based Test ($zmcp$)		Range-based Test ($zmcr$)		First-High-Low-Last Price Based Test ($zmcfhll$)	
Co-Jumps Date	Co-Jumps Time	Co-Jumps Date	Co-Jumps Time	Co-Jumps Date	Co-Jumps Time
2007/07/09[a]	09:50-09:55	2007/07/09[a]	09:50-09:55	2007/07/09[a]	09:50-09:55
2007/07/31[d]	10:30-10:35	2007/07/11	09:40-09:45	2007/07/11	09:40-09:45
2007/08/02	13:50-13:55	2007/07/23[c]	09:35-09:40	2007/07/17[b]	09:50-09:55
2007/08/22[e]	09:50-09:55	2007/07/31[d]	10:30-10:35	2007/07/23[c]	09:35-09:40
2007/09/03	14:25-14:30	2007/08/02	13:50-13:55	2007/07/31[d]	10:30-10:35
2007/09/11[f]	14:50-14:55	2007/08/14	09:35-09:40	2007/08/02	13:50-13:55
		2007/08/22[e]	09:50-09:55	2007/08/07	09:35-09:40
		2007/09/03	14:25-14:30	2007/08/14	09:35-09:40
		2007/09/11[f]	14:50-14:55	2007/08/20	09:45-09:50
				2007/08/22[e]	09:50-09:55
				2007/08/27	13:45-13:50
				2007/08/30	09:35-09:40
				2007/09/03	14:25-14:30
				2007/09/11[f]	14:50-14:55
				2007/09/17[g]	09:40-09:45

[a] Regulations of transfer of state-owned shares in the listed companies are issued.

[b] The Insurance Regulatory Commission announced an adjustment to the proportion of insurance funds invested in A-share stocks from 15% to 20%.

[c] The Central Bank announced that the one-year benchmark interest rates are raised by 0.27 percent.

[d] The Central Bank ordered banks to set aside an additional 0.5 percent of their deposits.

[e] The Central Bank announced that the one-year benchmark interest rates are raised by 0.27 percent.

[f] Government announced an issue of 200 billion yuan (26.7 billion U.S. dollars) of special treasury bonds.

[g] The Central Bank announced that the one-year benchmark interest rates are raised by 0.27 percent.