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A NON-LINEAR ANALYSIS OF EXCESS FOREIGN EXCHANGE RETURNS*

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In this paper we explore the dynamics of US dollar excess foreign exchange returns for the G10 currencies and the Swiss franc, 1976–97. The non-linear framework adopted is justified by the results of linearity tests and a parametric bootstrap likelihood ratio statistic which indicate threshold effects or differential adjustment to small and large excess returns. Impulse response analysis suggests that the effect of small shocks to excess returns inside the no-arbitrage band exhibits most persistence. Large shocks outside the band decay most rapidly and also exhibit overshooting. These phenomena are explained in terms of noise trading strategies and transaction costs.

1 Introduction

Lewis (1995) has highlighted deviations from uncovered interest parity (UIP) or excess foreign exchange returns (hereafter ER) as one of the major puzzles in international finance. ER represent the profit from speculation in foreign currency and are linked by covered interest parity (CIP) to the forward premium. Thus the above puzzle can be restated as the forward premium anomaly in which future exchange rate movements are predicted with the wrong sign (Engel, 1996). Although in theory predicted ER should be zero, the puzzle is that they are significantly different from zero, exhibit considerably more variability than spot returns and change sign frequently. Since UIP is a central building block in international finance, it is not surprising that this puzzle has generated a huge literature. One strand investigates the long-run time series properties of both ER and the forward premium. Another focuses on tests for a possibly time-varying risk premium since, assuming rational expectations, ER can be decomposed into a risk premium and a white noise forecast error term.

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In this paper we seek to investigate the suggestion of Clarida and Taylor (1997), Peel (1993) and others that risk premia, and by implication ER, are likely to be non-linear. Using weekly data 1977–93 for the US dollar series for Germany, Japan and the UK, Clarida and Taylor (1997) establish that the term structure of forward premia contains significant information about subsequent spot returns. They conclude that their linear vector error correction framework might usefully be extended to examine non-linearities. Peel (1993) analyses seven monthly sterling exchange rates for the 1976–90 period and shows that their ER or *ex post* forecast errors can be satisfactorily modelled as bilinear processes. These studies motivate the use of non-linear specifications to explore the behaviour of ER.

The paper makes two contributions to the literature. First, rather than finding the best non-linear model, we seek to extend previous linear analysis by combining frequency aspects, or how long UIP deviations last, with amplitude aspects, or the contrasting nature of small and large deviations. To this end, we employ a threshold autoregressive (TAR) framework which provides estimates of and associated confidence intervals for a no-arbitrage band. While the latter traditionally has been interpreted as a transaction cost band, we propose a complementary rationale based on noise trading activity which can explain some of the persistence observed in ER.[1] The non-linear framework adopted is justified by the results of linearity tests and a parametric bootstrap likelihood ratio (LR) statistic.

Second, we suggest that ER dynamics are more complex and subtle than those dictated by a linear autoregression (AR) structure.[2] Non-linear impulse response (IR) functions show that, while ER are mean reverting overall—which rehabilitates the UIP condition—their time profile in the wake of a shock is complex. It depends, *inter alia*, on whether ER are inside or outside the no-arbitrage band when the shock arises, the size of the shock and the width of the no-arbitrage band. These time profiles show that large shocks to outer-band ER decay most rapidly while small shocks to inner-band ER are most persistent. They also indicate that outer-band ER tend to overshoot following large innovations which is consistent with the sign changes observed in ER. The paper is organized as follows. Section 2 motivates non-linearities in ER. In Section 3 we discuss the inference and estimation results. Section 4 summarizes the IR analysis while a final section concludes.

[1]Baillie and Bollerslev (1994, 2000), Byers and Peel (1996) and Maynard and Phillips (2001) use fractional integration to explain persistence in the forward premia as a long-memory phenomenon. However, while the results in Byers and Peel (1996) are mainly consistent with stationarity, the opposite implication emerges from the other studies.

[2]We are indebted to an anonymous referee for this suggestion.

2 Non-linearities in ER

CIP implies that the interest rate differential equals the forward premium:

$$i_t - i_t^* = f_t - s_t \tag{1}$$

where i_t and i_t^* are the domestic and foreign interest rates, and s_t and f_t are the log spot and one-month forward rates, respectively.[3] Now suppose that at time t we borrow at interest rate i_t, change into foreign currency to invest at i_t^*, and use the realized exchange rate at $t+1$ to convert the proceeds back to domestic currency. The actual profit from this speculation is called ER or the *ex post* forecast error. It can be written as the spot return, typically a short-memory stationary process, minus the interest differential or forward premium under (1):

$$\mathrm{ER}_{t+1} = s_{t+1} - f_t = (s_{t+1} - s_t) - (f_t - s_t) \tag{2}$$

Ex ante or predicted excess returns (PER) based on time t information give deviations from UIP and can be defined as

$$\mathrm{PER}_t = E_t s_{t+1} - f_t = (E_t s_{t+1} - s_t) - (f_t - s_t) \tag{3}$$

where $E_t(\cdot)$ is the expectations operator conditional upon current information. Assuming rational expectations, we have

$$\mathrm{ER}_{t+1} = \mathrm{PER}_t + \varepsilon_{t+1} \tag{4}$$

where $\varepsilon_{t+1} = s_{t+1} - E_t s_{t+1}$ is a white noise forecast error term. In principle, PER should be insignificantly different from zero but, as Lewis (1995) shows, they are not. This may be explained for instance by non-rational expectations or alternatively by interpreting PER as a time-varying risk premium. According to the latter interpretation *ex post* ER can be decomposed into a risk premium (ρ_t) and a stationary error term:

$$\mathrm{ER}_{t+1} = s_{t+1} - f_t = \rho_t + \varepsilon_{t+1} \tag{5}$$

Hence, from (5) ER can only be stationary if the risk premium is stationary and vice versa.

In this paper we seek to model ER dynamics as a no-arbitrage band of persistent small deviations around long-run equilibrium and an outer band of large deviations with rapid adjustment. The no-arbitrage band is usually justified by the idea that transaction costs prevent profitable arbitrage within its confines. The latter may be minimally proxied by the bid–ask spread as in Balke and Wohar's (1998) non-linear analysis of deviations from CIP. An alternative justification for a no-arbitrage band relates to the nature of trading and is consistent with near rational behaviour in the sense of Caballero (1995) and with the conjecture of Mark

[3]Spot and forward rates are defined as the domestic price of foreign currency.

and Wu (1998) that the ER puzzle may be explained by noise trading.[4] Here one can distinguish between noise or chartist and fundamentalist trading. Allen and Taylor (1990) find from survey evidence that these types are mainly complementary and not mutually exclusive in foreign exchange markets even if chartism tends to predominate at short horizons and fundamentalism at long horizons. Thus, in a schematic trading model, one may expect to observe predominantly noise trading such as positive feedback trading in a no-arbitrage band. Alternatively, fundamentalist trading may predominate in an outer band where one observes large deviations from equilibrium.

Our theoretical motivation for a non-linear framework is supported by both standard linearity tests and a bootstrap LR statistic which provide clear evidence of (threshold) non-linearities in conditional mean for our sample of ER. While regime-switching dynamics could be modelled by a smooth transition autoregression (STAR) with a continuum of states between regimes, we follow Balke and Wohar (1998) in opting for the parsimony of a TAR model which does not require specification of a transition function.[5] They show that the dynamics of deviations from CIP for UK/US daily data 1974–99 can be satisfactorily captured by TAR models where the thresholds are taken as the bid–ask spread for borrower and lender arbitrage. By contrast, in our study of (*ex post*) deviations from UIP, the thresholds and their confidence intervals are estimated along with the other parameters. TAR dynamics is also consistent with Aït-Sahalia's (1996) parametric and non-parametric stochastic differential equation models suggesting that interest rates follow a random walk around their mean but revert when far away from it.

Taking the sample mean of ER as a proxy for the long-run equilibrium or attractor (μ), the following specification is proposed for $z_t = (s_t - f_{t-1}) - \mu$ or demeaned ER at time t:[6]

$$\Delta z_t = I_t\alpha(L)z_t + (1 - I_t)\beta(L)z_t + \varepsilon_t \qquad \varepsilon_t \sim \text{iid}(0, \sigma^2)$$
$$I_t = \begin{cases} 1 & \text{if } |z_{t-d}| > \theta \\ 0 & \text{if } |z_{t-d}| \leq \theta \end{cases} \tag{6}$$

where $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_p L^p$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q$ represent the outer- and inner-band dynamics, respectively, I_t is the Heaviside indicator function, θ is a threshold parameter and d is a

[4]Mark and Wu base their analysis on the De Long *et al.* (1990) noise trader model but they rely on survey findings for their empirical study.

[5]TAR models, first proposed by Tong (1978), are nested within the more general STAR family. See Öcal and Osborn (2000) for a STAR application to UK consumption and production and Kapetanios (1999) and Pesaran and Potter (1997) for TAR analysis of US output.

[6]Note that the estimated sample mean is very close to zero for all currencies. It ranges from −0.0021 in the case of the Italian lira to 0.0009 for the Japanese yen.

(positive integer) delay parameter or threshold lag. Model (6) represents a generalization of the equilibrium (EQ-) TAR specification of Balke and Fomby (1997).[7] It allows for asymmetric dynamics related to the amplitude of the (lagged) disequilibrium $|z_{t-d}|$.[8] Therefore, it can parsimoniously model a process exhibiting non-stationarity or persistent behaviour for small disequilibria but rapid mean reversion whenever the threshold is exceeded ($|z_{t-d}| > \theta$). Finally, this process has the appealing feature that its global stationarity depends on the outer-band dynamics only.

3 Testing and Estimation

The data consist of spot and one-month forward exchange rates for the G10 countries plus Switzerland from Datastream. We employ end-of-month averages of bid and ask rates denominated in American dollars for the 22 year floating exchange rate period, 1976:1–1997:12.[9] For Japan the data commence in 1978:6. The data span covering the full current floating rate regime addresses one recurring problem in the ER literature. With a few exceptions, the sample span of existing studies ends in the early 1990s and so is dominated by episodes such as the late 1980s when, although forward markets implied appreciation by currencies relative to the US dollar, the latter actually appreciated.

3.1 Unit Root and Non-linearity Tests

Given that non-stationarity may lead to spurious rejections of linearity as indicated by Hinich and Patterson (1985), we first address this issue. The Phillips–Perron (PP) and augmented Dickey–Fuller (ADF) unit root tests applied to the spot and forward rates in levels and first differences confirm earlier findings that these series are I(1) processes.[10] Noting the Balke and Fomby (1997) suggestion that non-parametric

[7]Though this model is related to other switching models such as Markov-switching models, the regime changes in the latter are determined by an unobservable process. See Hall *et al.* (1999) for an interesting application of these models and Granger and Teräsvirta (1993) for a more general analysis of non-linear models.

[8]A more general four-regime EQ-TAR model allowing also for differential dynamics for positive ($z_{t-d} > 0$) and negative ($z_{t-d} \leq 0$) deviations was initially considered. The nested sign-symmetric TAR model (6) was tested for our ER sample by means of the bootstrap LR test proposed by Coakley and Fuertes (2000a). The statistics suggested no evidence against this null at conventional 5 per cent and 10 per cent significance levels.

[9]The Interim Committee of the IMF ratified the Rambouillet agreement which formally implemented the floating exchange regime in January 1976.

[10]These and other non-reported results are available from the authors on request. For our empirical analysis we employed GAUSS 3.2 and the EasyReg package of Bierens (1999). We are grateful to Herman J. Bierens for kindly making available the EasyReg software.

cointegration methods may be more effective than standard linear methods when the true generating process is non-linear, we also use the semiparametric PP test and the non-parametric multivariate Bierens (1997) method.

The Engle–Granger single-equation approach assuming $(1, -1)$ as cointegrating vector indicates that ER stationarity is strongly supported by the PP test results but not to the same extent by the ADF results. The results from the Johansen approach are at best mixed. Cointegration is supported for eight currencies and the $(1, -1)$ null is accepted in six cases. These results are consistent with the Pippenger and Goering (1993) and Balke and Fomby (1997) findings that linear cointegration methods may lose power in the presence of threshold non-linearities.

The non-parametric Bierens (1997) approach is based on the generalized eigenvalues of a pair of random matrices A_m and $B_m + n^{-2}cA_m^{-1}$ where n is the sample size and c and m are arbitrary scalars such that $c > 0$ and $m = q$, with q the dimension of the system.[11] To estimate the number r of cointegrating vectors we use the $\lambda_{\min}$ test which is in the same spirit as Johansen's $\lambda_{\max}$ test and we also compute the $\hat{g}_m(r)$ statistic which gives $\hat{r} = \min_{r \leq 2}[\hat{g}_m(r)]$. Table 1 reports the results. Both statistics unanimously indicate a unique cointegrating relation for all currencies though the Canadian dollar is a marginal exception for the $\lambda_{\min}$ statistic. The reported λ_{trace} statistics suggest that the $(1, -1)$ cointegrating null, or ER mean reversion, cannot be rejected for any currency at even the 20 per cent significance level.

A range of tests is applied to detect potential non-linearities in both the conditional mean and conditional variance. An AR(p) model is fitted to each ER series to filter out linear dependence and the estimated residuals are tested for neglected non-linearity. A lag length $p = 1$ is duly selected for all currencies using Akaike's criterion (AIC) and the Ljung–Box statistic. The Thursby and Schmidt (1977) and Tsay (1991) general non-linearity tests reject the AR null for seven and nine out of 10 currencies, respectively. Both the Tsay (1989) and Petruccelli and Davies (1986) TAR non-linearity tests suggest threshold behaviour in eight currencies, mostly for $d = 1$.[12] The Saikkonen and Luukkonen (1988) Lagrange multiplier (LM) test for first- and second-order bilinear non-linearity gives two rejections only. Finally, the McLeod–Li (1983) test for autoregressive conditional heteroscedasticity (ARCH) type non-linearity

[11]Following Bierens, we choose $c = 1$ and m from his tabulated values so as to maximize the lower bound of the power function for each test. See Coakley and Fuertes (2001) for details and an application to real exchange rates.

[12]Only delay parameters $d \leq 3$ are entertained in the context of liquid foreign exchange markets. Increasing and decreasing ordered ARs are used for the Tsay tests and the Petruccelli and Davies test and the minimal p values are reported.

Table 1
Bierens (1997) Non-parametric Cointegration Test Results

	BG[a]	CN	FR	GE	IT	JP	NH	SD	SW	UK
$\lambda_{\min}$[b]										
$r=0/r=1$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$r=1/r=2$	0.426	0.011	0.661	0.162	0.186	0.203	0.194	0.180	0.085	0.344
$\hat{g}_m(r)$[c]										
$r=0$	58.9e + 5	16.3e + 10	14.5e + 6	12.1e + 7	7.3e + 5	67.2e + 5	36.2e + 6	44.5e + 5	96.2e + 7	85.6e + 6
$r=1$	64.3e − 3	40.0e − 4	10.8e − 3	21.6e − 3	34.6e − 2	19.7e − 2	50.2e − 3	47.6e − 2	98.2e − 4	67.9e − 4
$r=2$	80.1e + 1	28.9e − 3	32.5e + 1	38.8	82.2e + 1	44.0e + 1	13.0e + 1	10.6e + 2	48.9e − 1	55.1
$[\hat{\beta}_1 \hat{\beta}_2]$[d]	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	−1.017	−0.980	−1.012	−1.028	−0.997	−0.976	−1.032	−0.954	−1.036	−0.964
λ_{trace}[e]	1.12	1.25	1.04	1.32	1.19	1.25	1.33	1.24	1.57	1.13

Notes: [a] BG, Belgium; CN, Canada; FR, France; GE, Germany; IT, Italy; JP, Japan; NH, Netherlands; SD, Sweden; SW, Switzerland; UK, United Kingdom.

[b] The critical regions are (0.000, 0.017) and (0.000, 0.005) at the 5 per cent and 10 per cent significance levels, respectively, for the first test, and (0.000, 0.054) and (0.000, 0.111) at the 5 per cent and 10 per cent levels for the second test.

[c] Estimated number of cointegrating vectors given as $\hat{r} = \min_{r\le 2}[\hat{g}(r)]$.

[d] Standardized cointegrating vector.

[e] The null hypothesis is $\beta_1 = 1, \beta_2 = -1$. The critical values are 4.7 (5 per cent) and 2.89 (10 per cent).

Table 2
Non-linearity Test Results

	BG[a]	*CN*	*FR*	*GE*	*IT*	*JP*	*NH*	*SD*	*SW*	*UK*
General non-linearity tests										
Thursby–Schmidt	0.000	0.008	0.005	0.000	0.038	0.674	0.000	0.008	0.084	0.585
Tsay (1991)										
$d=1$	0.000	0.042	0.013	0.000	0.005	0.463	0.000	0.023	0.035	0.000
$d=2$	0.000	0.027	0.027	0.000	0.229	0.423	0.000	0.135	0.117	0.673
TAR non-linearity tests										
Tsay (1989)										
$d=1$	0.000	0.006	0.002	0.000	0.292	0.576	0.000	0.001	0.041	0.387
$d=2$	0.005	0.129	0.028	0.036	0.030	0.461	0.008	0.061	0.130	0.454
Petruccelli–Davies										
$d=1$	0.000	0.003	0.001	0.000	0.043	0.345	0.000	0.000	0.035	0.162
$d=2$	0.049	0.156	0.134	0.090	0.308	0.412	0.030	0.113	0.154	0.572
Other non-linearity tests										
Bilinearity[b]										
$m=k=1$	0.451	0.003	0.236	0.747	0.003	0.383	0.681	0.793	0.086	0.073
$m=k=2$	0.050	0.009	0.256	0.130	0.008	0.921	0.262	0.325	0.466	0.380
ARCH[c]										
$Q(6)$	0.083	0.551	0.777	0.198	0.404	0.437	0.063	0.966	0.630	0.307
$Q(12)$	0.213	0.393	0.912	0.382	0.499	0.828	0.241	0.996	0.060	0.778

Notes: [a] All tests are applied to the residuals of a linear AR(p) filter. The reported figures are the observed significance levels or p values.
[b] Saikkonen and Luukkonen (1988) test for bilinear non-linearity. Two bilinear models BL(p; m, k) are assumed under the alternative for different lag orders m and k.
[c] McLeod–Li (1983) test for ARCH non-linearity.

fails to reject the linearity null for all currencies.[13] Table 2 reports the asymptotic p values of these tests.[14] Overall, these empirical findings alongside our theoretical motivation justify the subsequent analysis of ER in a TAR framework.

3.2 TAR Models for ER

Model (6) is estimated by conditional maximum likelihood via a grid search and the best fit specification is selected by the criterion

$$\mathrm{AIC}(p, q, \theta, d) = (\mathcal{L}_\alpha - p) + (\mathcal{L}_\beta - q) \tag{7}$$

[13]Note that Lee *et al.* (1993) find that this test has low power against TAR non-linearity.
[14]The analysis in Blake and Kapetanios (1999) suggests that inference of non-linearity may arise from neglected moving average (MA) effects. In our ER sample, however, neither the autocorrelation function nor the Akaike or Schwarz information criteria indicated significant MA effects. Exceptions to the latter are the Belgian franc, French franc and Dutch guilder for which an ARMA($1, q$), $q = 2, 1, 1$, respectively, was marginally selected over an AR(1) specification by the AIC. Nevertheless, the same tests carried out on the residuals of an ARMA filter did not overturn the earlier non-linearity conclusion.

where $\mathcal{L}_\alpha$ and $\mathcal{L}_\beta$ represent the maximized log-likelihood functions of the outer- and inner-band ARs, respectively.[15] Table 3 reports the estimated models and residual analysis.

The models identified are relatively parsimonious with lag lengths (p, q) and threshold lag (d) rarely exceeding 1. For all currencies, the characteristic roots of the outer band AR clearly lie within the unit circle, ensuring overall stationarity.[16] This bears out our earlier inference from the non-parametric Bierens (1997) test. These results on mean reversion are consistent with those implied by the recent findings of Byers and Peel (1996), Dibooglu (1998) and Hai *et al.* (1997) but at odds with those of Crowder (1994) and Evans and Lewis (1993). For all currencies, the inner-band AR structure indicates mean reversion—though with more sluggish adjustment than that in the outer band—with the exception of Canada and Japan which contain a (near) unit root.

Following Hansen's (1997) analysis showing that a lack of precision in the threshold estimates may contaminate the distribution of the ordinary least squares/maximum likelihood AR parameters, we construct asymptotically valid confidence intervals for the threshold θ. Fixing p, q and d at their best in-sample fit values using the above AIC, we search for the value $\tilde{\theta}$ that minimizes the residual variance σ^2 of the fitted TAR and use it to construct the convexified interval $\theta^c = [\theta_1, \theta_2]$ with $\theta_1 = \min_\theta \Theta$, $\theta_2 = \max_\theta \Theta$, where Θ is the LR region

$$\Theta = \{\theta\colon \mathrm{LR}_n(\theta) \le c(\alpha)\} \qquad \mathrm{LR}_n(\theta) = n\frac{\sigma^2(\theta) - \sigma^2(\tilde{\theta})}{\sigma^2(\tilde{\theta})} \tag{8}$$

and $c(\alpha)$ is the α-level asymptotic critical value tabulated by Hansen (1997). The minimum residual variance threshold estimates are very close to their maximum AIC counterparts, which is to be expected given the asymptotic equivalence of the two criteria for fixed p, q and d.[17] As Table 3 shows, the 95 per cent confidence intervals are fairly tight, implying that these estimates are quite precise. Moreover, the fitted models seem well

[15]In all cases the threshold search is restricted to between the tenth and ninetieth percentiles of $|z_{t-d}|$. A normalized AIC is used to compare models with different delay parameters. For a discussion of TAR estimation issues, see Coakley *et al.* (2001a, 2001b).

[16]Bearing in mind the potential downward bias of ordinary least squares estimators in AR models we used the median-bias correction in Andrews (1993) for the α parameters (in those cases where $p = 1$) which yielded indistinguishable results. Additionally, standard F tests indicated that the latter are insignificantly different from -1 which is consistent with the frequent sign changes referred to in the ER literature.

[17]The estimates differ in only six cases and the average difference is less than 1 per cent. Note that the above $\mathrm{LR}_n(\theta)$ statistic is asymptotically free of nuisance parameters under the assumption that the threshold effect (the difference between the AR parameters in the two regimes) diminishes as the sample size increases.

Table 3
TAR Estimation and Inference Results

	TAR models [a]						Diagnostic tests			TAR test [b]
	p, q	d	θ ($\times 10^2$) [95% confidence interval]	n_q	$\alpha_1\ \alpha_2 \ldots \alpha_p$ (t ratios)	$\beta_1\ \beta_2 \ldots \beta_q$ (t ratios)	Φ	LM_6 LM_{12}	$ARCH_6$ $ARCH_{12}$	H_0: AR H_1: EQ-TAR
BG	1, 1	1	2.969	176	−1.107	−0.194	0.980	0.176	0.526	26.893
			[2.41, 3.46]		(−16.985)	(−1.215)		0.370	0.530	(0.000)
CN	2, 3	2	0.428	74	−1.043 −0.102	−0.825 0.808 0.010	0.597	0.461	0.981	20.619
			[0.19, 1.18]		(−14.204) (−1.537)	(−7.482) (1.975) (2.098)		0.396	0.387	(0.014)
FR	1, 1	1	3.161	185	−1.074	−0.050	0.857	0.163	0.046	12.024
			[2.43, 4.01]		(−15.553)	(−3.363)		0.104	0.032	(0.102)
GE	1, 1	1	3.298	187	−1.132	−0.115	0.905	0.061	0.753	23.060
			[2.91, 4.33]		(−17.081)	(−2.966)		0.208	0.732	(0.002)
IT	1, 1	1	4.758	223	−1.047	−0.164	0.726	0.666	0.080	27.701
			[3.67, 5.10]		(−10.041)	(−6.972)		0.770	0.225	(0.000)
JP	1, 1	1	0.518	35	−0.904	−2.013	0.987	0.679	0.451	14.916
			[0.38, 0.69]		(−14.040)	(−3.836)		0.474	0.695	(0.031)
NH	2, 1	1	2.636	161	−1.058 0.174	−0.231	0.954	0.143	0.522	24.251
			[2.14, 3.34]		(−16.947) (2.033)	(−1.252)		0.467	0.197	(0.001)
SD	1, 1	1	2.018	149	−0.952	−0.144	0.285	0.384	0.869	26.735
			[1.69, 3.68]		(−12.376)	(−2.330)		0.320	0.958	(0.000)
SW	1, 1	2	3.311	161	−1.147	−0.175	0.866	0.692	0.456	16.740
			[1.13, 4.42]		(−11.669)	(−9.696)		0.521	0.685	(0.017)
UK	1, 3	2	0.955	77	−0.936	−0.752 −0.751 0.259	0.979	0.221	0.424	21.085
			[0.36, 1.75]		(−12.801)	(−6.760) (−1.089) (2.621)		0.396	0.575	(0.009)

Notes: [a] n_q is the number of effective observations in the inner band; Φ is the Lin–Mudholkar statistic for normality (null); LM_6 and LM_{12} are Lagrange multiplier statistics for non-autocorrelated errors up to orders 6 and 12 (null); $ARCH_6$ and $ARCH_{12}$ are the McLeod–Li statistic for no ARCH effects (null); p values reported for all diagnostic tests.
[b] Parametric bootstrap LR statistic for threshold effects; p values in parentheses.

specified in terms of normality of residuals, no autocorrelation or heteroscedasticity.[18]

Finally, an LR test is employed to investigate the (absence of threshold effects) restriction $\alpha(L) = \beta(L)$ in (6), or linear AR dynamics, against TAR behaviour. While the statistic is standard, the presence of nuisance parameters (θ, d) under the null invalidates standard asymptotic inference. Its distribution is approximated using the following parametric bootstrap procedure.[19] The (null) model parameter estimates and normally distributed random disturbances with variance equal to the estimated residual variance are used to generate $B = 1999$ bootstrap samples. The initial 100 observations in each iteration are discarded. We follow Davidson and MacKinnon (1996) in choosing B such that $\alpha(1 + \beta)$ is an integer at conventional significance levels (α) to achieve an exact test. The p values of this test, reported in Table 3, imply that in nine out of the 10 currencies the null is rejected at better than the 5 per cent level, while it is marginal at the 10 per cent level for the remaining case (French franc). We conclude that TAR dynamics provides a reasonable characterization of ER.

4 Analysis of ER Dynamics

Our threshold estimates suggest a mean no-arbitrage band of some 2.4 per cent. The amplitude of the no-arbitrage bands readily exceeds the bid–ask spread for the whole panel and thus cannot be explained on the basis of transaction costs alone. Note, however, that the models for the pound sterling, Canadian dollar and Japanese yen yield an average no-arbitrage band of only 63 basis points containing a small average proportion of observations (24 per cent). The narrower bands for these currencies are plausible given the large and liquid markets in both yen–dollar and sterling–dollar trades and the propinquity of Canada and the USA. We tentatively conjecture that the apparent higher signal-to-noise ratio in the most liquid markets stems from proportionally lower levels of noise trading activity.

Another interesting observation from the estimated TAR models is that most ER observations lie in the inner band. The overall average is some 60 per cent, while for the seven non-liquid currencies the proportion is in the 56–71 per cent range. Figure 1 plots the demeaned ER and

[18]Since the distribution of the Ljung–Box statistic is unknown when applied to the residuals of non-linear models, we employ the Eitrheim and Teräsvirta (1996) LM test for serial correlation in STAR models. This is adapted for the TAR using a logistic function with transition parameter $\gamma = 10^5$ to approximate the Heaviside function.

[19]The simulation study in Coakley and Fuertes (2000a) shows that this bootstrap LR type test of linearity against (TAR) asymmetric adjustment has good size and reasonable power properties. See Hansen (1998) for an interesting alternative approach.

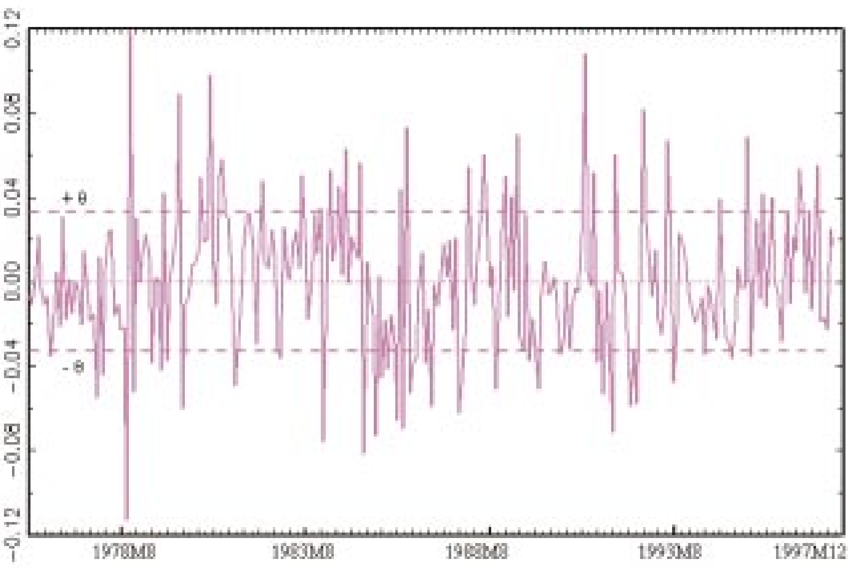


FIG. 1 Demeaned ER and No-arbitrage Band (German Mark)

associated thresholds for the German mark which typifies this average. Since large ER are difficult to reconcile with UIP if CIP holds, the relatively small number of observations in the outer band is consistent with the infrequent CIP violations in the Balke and Wohar (1998) study.

IR functions are employed to analyse the time profile of ER in response to shocks. Here we rely on the Gallant *et al.* (1993), Koop *et al.* (1996) and Potter (2000) non-linear theory which extends the traditional IR analysis. While for linear time series several definitions of IR functions in the macroeconomic literature are informationally equivalent, in the non-linear case they all contain different information. Koop *et al.* (1996) and Potter (2000) resolve this problem indirectly by highlighting the superiority of the generalized IR (GIR) function. In addition, while the GIR of a linear time series can be normalized to produce a non-random function, this is not possible for non-linear time series. Koop *et al.* and Potter tackle the latter by treating the GIR as a random variable on the underlying probability space of the time series. Accordingly, a Monte Carlo simulation technique can be employed to approximate the conditional expectations involved in the GIR function. This approach is implemented for our sample of ER as follows.

For each combination of history $\omega_{t-1} = \{z_{t-1}, z_{t-2}, \ldots\}$ and shock v_t, which act as initial conditions, use the estimated TAR parameters and N (future) random shocks to generate R sets of forecasts for the *shocked* system, $\{z^j_{t+n}(\omega_{t-1}, v_t)\}^N_{n=0}$, $j = 0, \ldots, R-1$. Similarly, generate R sets of *baseline* forecasts $\{z^j_{t+n}(\omega_{t-1})\}_{n=0}^{N}$ using randomly sampled shocks at time t and the same set of $n = 1, \ldots, N$ future shocks. Approximate $\mathrm{GI}_Z(n, \omega_{t-1}, v_t)$ by averaging the difference between the two types of forecasts. This is repeated M times for different initial conditions or combinations (ω_{t-1}, v_t) and the theoretical $\mathrm{GI}_Z(n, \Omega_{t-1}, V_t)$ is then estimated by averaging the individual draws $\{\mathrm{GI}_Z(n, \omega_{t-1}, v_t)\}_{i=0}^{M-1}$.

We condition the analysis on inner- and outer-band histories. More specifically, we compute one inner-band GIR and another outer-band GIR by randomly selecting histories (with replacement) from the observed time series such that ER_t is inside and outside the no-arbitrage band, respectively. The future shocks (and time t shock for the baseline forecasts) are randomly drawn with replacement from the estimated residual vector. We explore the effect of small and large shocks equal to $\upsilon_s = \pm 1$ and $\upsilon_l = \pm 2$ standard deviations of the residuals, respectively. The specifications used are $M = 100$ histories for each band, $R = 500$ Monte Carlo replications for each history and a maximum horizon $N = 10$.

Figure 2 plots the inner- and outer-band GIR for all ER series. Several patterns are apparent. The GIRs suggest that the effect of shocks generally dissipates relatively quickly which is consistent with the earlier inference of overall stationarity for ER.[20] Responses to positive and negative shocks are approximately symmetric in most cases.[21] Interestingly, the time profiles of small and large shocks are not proportional as expected from the non-linear nature of the TAR generating process employed in the simulations. Moreover, excluding Canada, Japan and the UK, the inner- and outer-band GIRs differ. The similar GIRs for the former can be explained by the small no-arbitrage bandwidth relative to the volatility of the innovations. Accordingly, ER are more likely to remain outside this band following both big and small shocks.

For the remaining countries, non-linear forces produce less of a dampening effect in the inner than in the outer band. This is especially apparent in the aftermath of small innovations which tend to maintain ER in the inner band for longer periods. The extent to which inner-band GIRs take longer to die out compared with outer-band GIRs ranges from some 75 per cent (French franc, German mark) to some 30 per cent (Belgian and Swiss francs, Dutch guilder and Swedish kroner) with the Italian lira in between. The clear implication is that shocks to ER in the no-arbitrage band have more persistent effects than shocks to ER outside this band. Moreover, ER in the outer band generally display overshooting behaviour

[20]The relatively rapid mean reversion of ER suggested by the IRs contrasts with that of real exchange rates. See the persistence profile analysis of Coakley and Fuertes (2000b).

[21]Note that this is consistent with the absence of a base currency effect in real exchange rates. See Coakley and Fuertes (2000c).

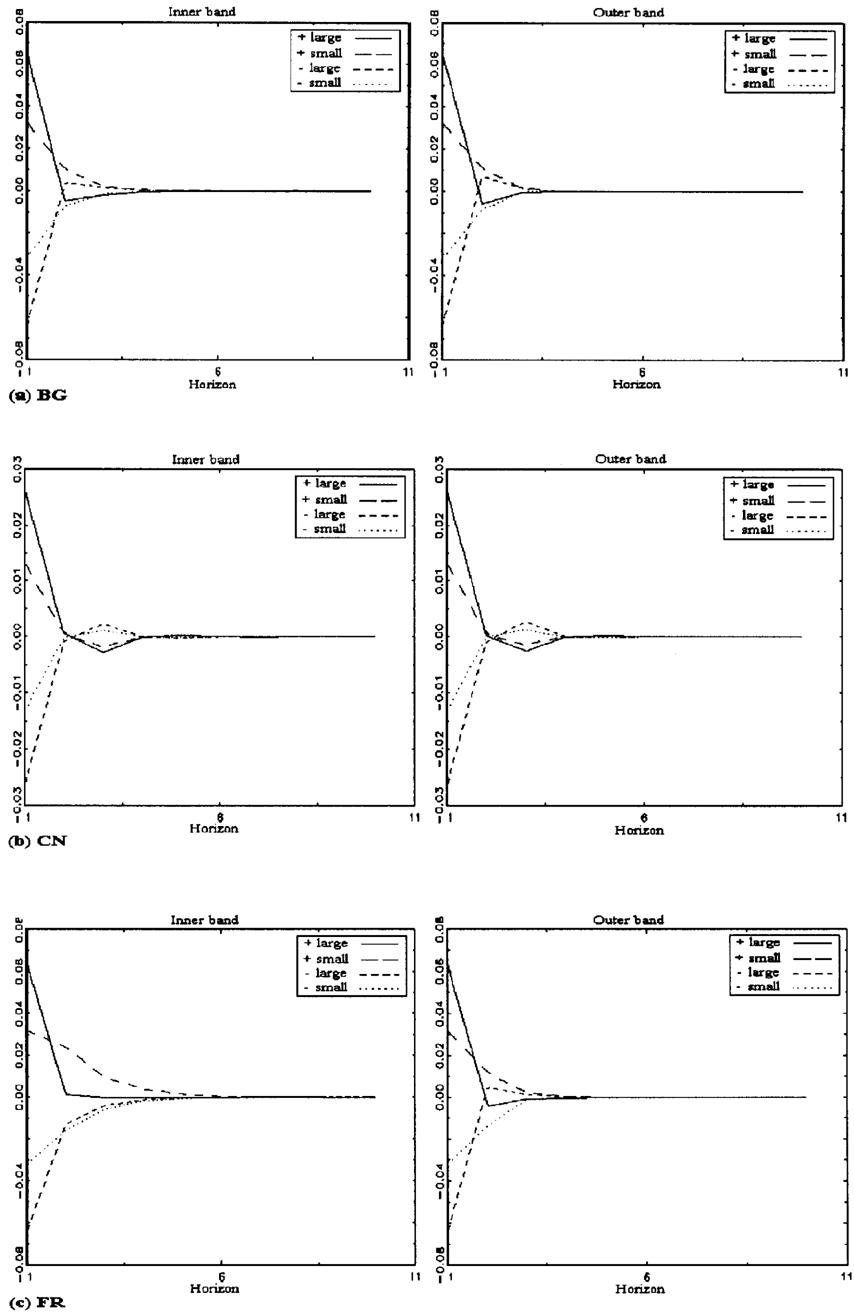


FIG. 2 GIR Functions

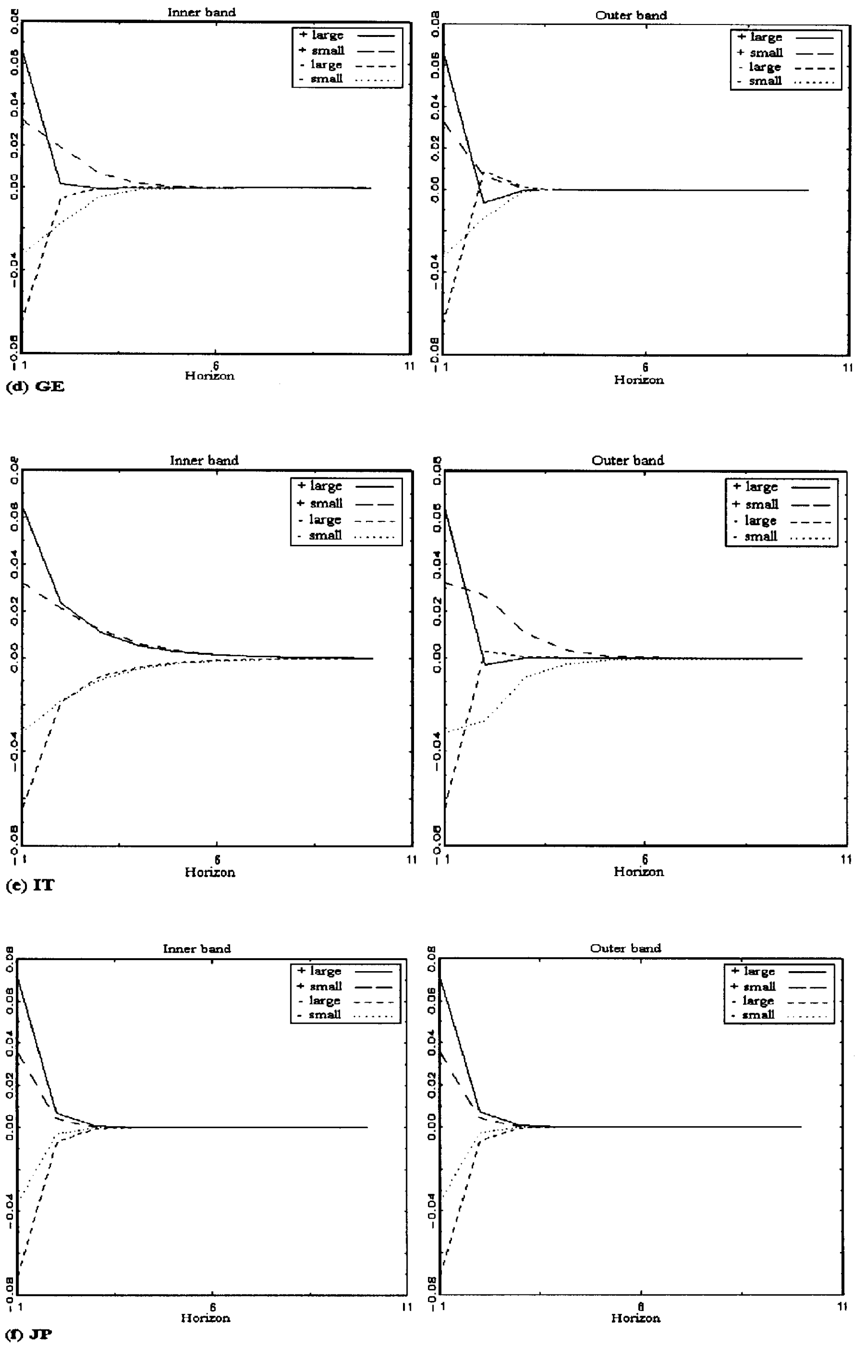
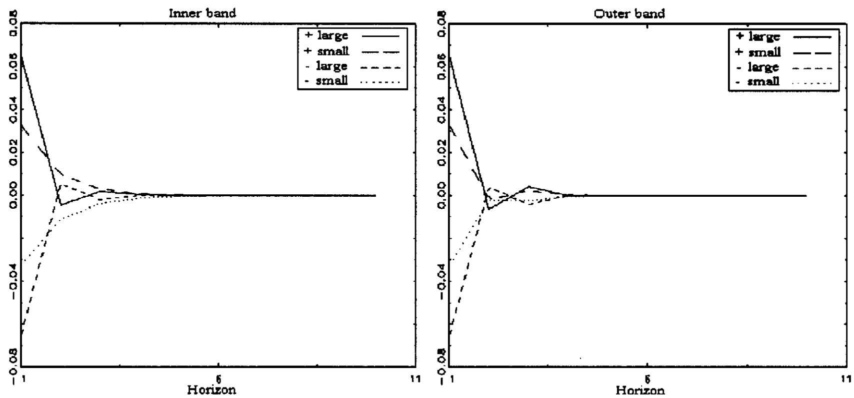
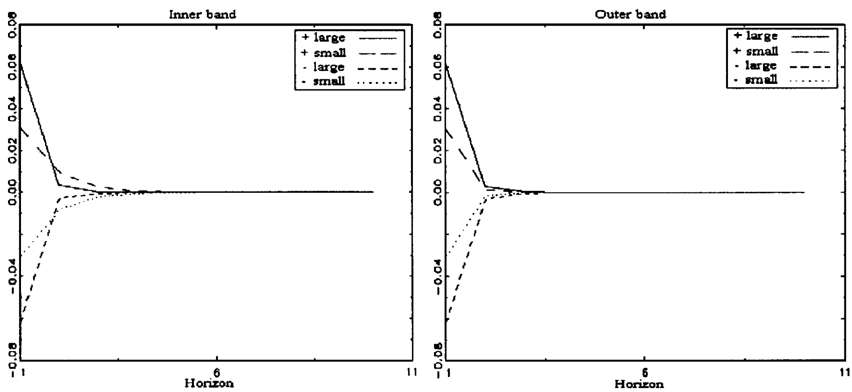


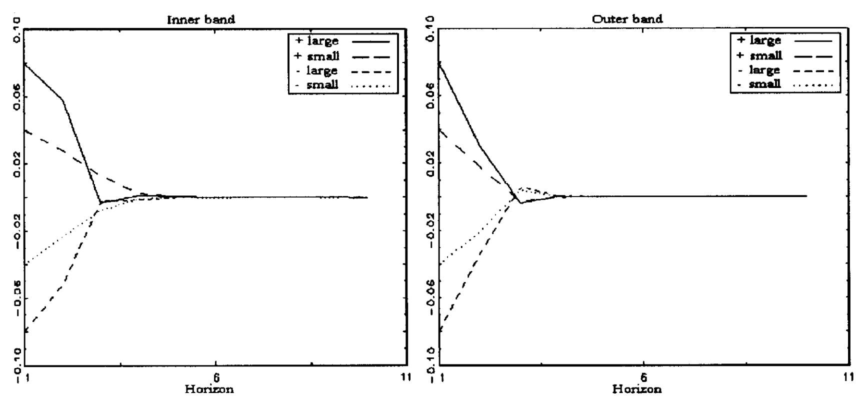
FIG. 2 *Continued*



(g) NH



(h) SD



(i) SW

FIG. 2 *Continued*

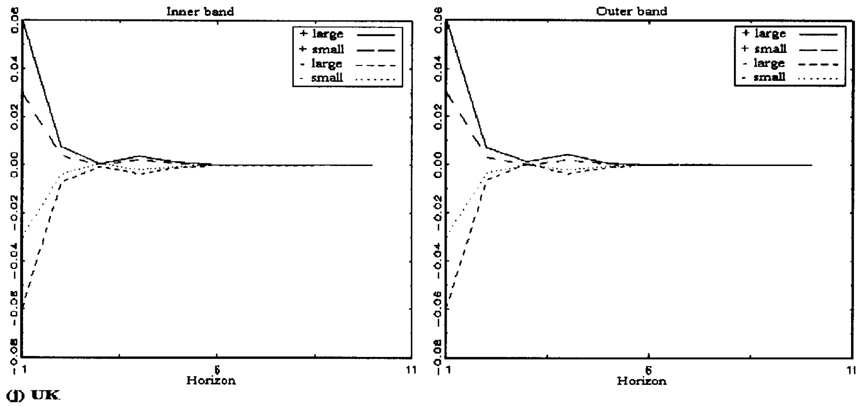


FIG. 2 *Continued*

following large shocks. One reason for this is that large innovations to outer-band ER tend to trigger more arbitrage activity so that the innovation is rapidly discounted, causing the ER to overshoot and change sign before mean reverting again. This is consistent with the observed sign changes in ER.

These findings can be related to both transaction costs and noise trading activity. The relatively more persistent inner-band ER are consistent with the presence of greater noise risk for small innovations and/or small deviations from long-run equilibrium. Correspondingly, the more rapid decay of outer-band ER shocks provides evidence of transitory profitable arbitrage and, relatedly, of the predominance of rational trading activity. These contrasting dynamics complement those for small and large deviations from CIP documented in Balke and Wohar (1998). Finally they also shed some light on the persistence of ER implied in the recent studies of Baillie and Bollerslev (1994, 2000), Bekaert (1995), Byers and Peel (1996), Hai *et al.* (1997) and Maynard and Phillips (2001).

5 Conclusions

In this paper we employ a univariate non-linear time series approach to analyse the dynamics of ER in a US-dollar-denominated panel of the G10 currencies plus the Swiss franc. The results of linearity tests clearly reject linear autoregressive dynamics in favour of threshold effects in the conditional mean of ER. Threshold autoregressive dynamics is consistent with a mean-reverting band for large ER and a no-arbitrage band fostering more persistent behaviour around long-run equilibrium. The suitability of this framework is confirmed by a parametric bootstrap LR test for differential adjustment to the amplitude of deviations.

Our findings shed light on aspects of the ER puzzle. First, both non-parametric test statistics and IRs based on the estimated TAR models suggest that ER are stationary overall. This is consistent with a weaker version of the UIP assumption in which international capital is highly but not perfectly mobile. Second, and despite global stationarity, we observe a no-arbitrage band of deviations from equilibrium whose width is related to the liquidity of the underlying financial market. This imparts a degree of persistence to the underlying ER process which can be explained in terms of noise trading activity and transaction costs. Finally, non-linear forces dissipate the effects of large shocks to ER outside the no-arbitrage band rather quickly producing overshooting behaviour which is consistent with observed sign changes. The implication is that clear arbitrage opportunities from exploiting UIP deviations are rather short-lived.

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