Hedging using longevity-linked securities: Costs, benefits and systemic risks

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Annuity providers may incur significant losses if mortality improves by more than expected. This is driving the development of new markets of assets with cash-flows linked to the longevity of an underlying population. In 1970s Black-Scholes option pricing model enabled the growth of new markets in derivative assets. Over time the market price of options adjusted to reflect volatility of underlying assets and systemic constraints. E.g. Since 1987, market implied volatility for options of low strike prices are higher than high strike prices. Similarly market price of longevity derivatives should reflect volatility of underlying mortality rates and Systemic constraints e.g. Solvency Capital Requirements.
Research Issues

- SCR affect companies’ willingness to pay for securitization
- Similarly, capital relief under SCR will affect insurers’ willingness to pay for longevity bonds
- Profit-maximizing insurer will only buy a longevity bond for hedging if this is cost-effective
- It is unclear which hedging strategies are cost-effective under Solvency II framework
- It is unclear how cost-effective hedging strategies differ from risk-reducing hedging strategies
Research questions

- How will a profit-maximizing insurer use LBs?
  - Trade off between the cost of the LB and benefit from holding the LB, which is cost of capital saving
  - Assume decision is made based on PV of all future costs vs. benefits at $t = 0$

- How does the profit-maximizing hedging strategy influence financial and systemic risks?
  - Expected shortfall of reserves to meet annuity payments
  - Insurer’s probability of default
The annuity book

- At time $t = 0$ the insurer receives a single premium $P = BEL$
- Each year the insurer must pay out $t \cdot p_{65}$
- Insurer must also maintain SCR under Solvency II
Solvency Capital Reserve

- Insurer holds technical provisions and SCR
- Technical provision = BE Liabilities + Risk Margin i.e. amount insurer needs to immediately transfer its obligations
- SCR is the capital required to ensure 99.5% VaR over 1 year

Model set-up:
- Each year the insurer tops up the technical provisions and holds the SCR
- So the annual cost of maintaining the technical provision is = (Cost of Capital)*(Loss + SCR)
- SCR is $\Delta NAV$ from permanent reduction to BE mortality of 20% for all ages
SCR with hedging

- Buying a T year longevity bond changes cash flow

<table>
<thead>
<tr>
<th></th>
<th>No hedge</th>
<th>Buy a T year longevity bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payments</td>
<td>( t p_x )</td>
<td>( E(t p_x) ) in years 1 to T ( t p_x ) year T+1 on</td>
</tr>
<tr>
<td>Loss</td>
<td>( t p_x - E(t p_x) )</td>
<td>0 in years 1 to T ( t p_x - E(t p_x) ) from T+1 on</td>
</tr>
<tr>
<td>Capital required</td>
<td>( K(t) + t p_x - E(t p_x) )</td>
<td>0 in years 1 to T ( K(t) + t p_x - E(t p_x) ) from T+1 on</td>
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</table>

- Benefit of hedging = cost of capital saving for the T years over which longevity risk is hedged
  - E.g. 6\%(\( K(t) + t p_x - E(t p_x) \)) in years 1 to T
  - Minimise cost of capital + cost of hedging (i.e. cost of LB)
Cost-benefit analysis

PV at t=0 of LB cost vs saving in cost of capital

- **LB cost**
- **Cost of cap saving**

Term of bond in years

7
Pricing the LB

- q-forward exchanges realized mortality rate at some future date, for a fixed mortality rate agreed at inception
- the fixed mortality rate agreed at inception will be forecast mortality rate adjusted for risk premium
- the risk premium and price of longevity derivatives is driven by volatility of the underlying mortality rates $\sigma_x$
Pricing the LB

q-forward can be priced using a Sharpe Ratio:

\[ q^F_{x,t} = (1 - SR\sigma_x t)E(q_{x,t}) \]

Coupon paying LB can be priced using approximation:

\[ S_{x,t} = \prod_{i=0}^{t-1} (1 - q^F_{x,i}) - (q_{x,i} - q^F_{x,i}) \]

\[ \approx \prod_{i=0}^{t-1} (1 - q^F_{x,i}) - \sum_{i=0}^{t-1} (q_{x,i} - q^F_{x,i}) \prod_{j=0,j\neq i}^{t-1} (1 - q^F_{x,j}) \]

So hedge \( S_{x,t} \) by holding:

\[ -v^{t-1} \prod_{j=0,j\neq 0}^{t-1} (1 - q^F_{x,j}) \] units of the 1-yr q-forward

\[ -v^{t-2} \prod_{j=0,j\neq 1}^{t-1} (1 - q^F_{x,j}) \] units of the 2-yr q-forward

... 

\[ \prod_{j=0,j\neq t-1}^{t-1} (1 - q^F_{x,j}) \] units of the t-yr q-forward
Data and assumptions

- Australian male age 65 purchases life annuity for $100,000
- Insurer BE basis is 2009 rates rolled forward using improvement factor based on last 25 years
- Annual payment of $\approx 8,900$ not indexed from EOY 1
- Analysis does not allow for investment risk, basis risk, Solvency IIs counterparty risk requirements or loss of diversification benefits

Other assumptions for pricing:
- Insurer’s annual cost of capital is 6% (+)
- No profit loading, tax or frictional costs (+)
- Sharpe ratio of 0.20 (+/-)
- Assume 100% capital relief for hedged position (-)
Discounting

Figure 1: Australian government bond yields 1 July 2009

[Graph showing Australian government bond yields 1 July 2009 with a blue line for yield and a red line for spot rate.]
Mortality assumptions

For forecasting insurer’s experience:

- Lee-Carter model was fit to Australian mortality rates 1970 to 2009 and used to forecast mortality
- Assume actual experience follows LC forecast

For pricing the LB and hedge:

- $\sigma_x$ calculated as standard deviation of smoothed (5 year rolling average) annual percentage change in $q_{x,t}$
- Also use LLMA (2012) smoothing method to smooth crude rates (cubic spline with 5 year age knots) then calculate $\sigma_x$
### Sensitivity tests

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<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Cost of capital</td>
<td>6.0%</td>
<td>8.5%</td>
<td>6.0%</td>
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- Sharpe ratios for LB used in past studies: 0.20 Ngai and Sherris (2010), 0.25 Loeys et al. (2007), ≈0.12 Bauer et al. (2009)
- For smoothing mortality rates, LLMA (2012) uses cubic splines with knots at every 5 years from 0-100+
Results: Base case

PV at t=0 of LB cost vs saving in cost of capital
Results: Frictional costs

PV at t=0 of LB cost vs saving in cost of capital

- Blue line: LB cost
- Red line: Cost of cap saving

Term of bond in years

0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26  27  28  29  30

Values range from 0 to 700.
Results: Low Sharpe Ratio
Results: High Sharpe Ratio

PV at t=0 of LB cost vs saving in cost of capital

- Blue line: LB cost
- Red line: Cost of cap saving

Term of bond in years
Results: 50% Capital relief

PV at t=0 of LB cost vs saving in cost of capital

Term of bond in years

- LB cost
- Cost of cap saving
Results: Smoothing of $q_x$

PV at $t=0$ of LB cost vs saving in cost of capital

Term of bond in years

- LB cost
- Cost of cap saving
## Summary of results

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<td><strong>LB T</strong></td>
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<td>7</td>
<td>6</td>
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Contributions:

- Framework to quantify the trade-off between the cost of buying a longevity bond and the benefit from holding it in terms of reduced SCR
- LBs with term over 25 years are not cost-effective
- Market-based risk transfer mechanisms for oldest ages likely to be expensive
- Insurers should consider in-house risk management e.g. diversifying across cohorts

Limitations and further directions:

- Sharpe ratio is an approximation to the market price of LB, as market evolves other pricing models should be used
- Sensitivity analysis for volatility of $q_x$
Questions?
References


