

# The Conditional Price of Basis Risk: An Investigation Using Foreign Exchange Instruments

JOËLLE MIFFRE\*

**Abstract:** This paper uses a conditional multifactor model and shows that the basis of foreign currency instruments includes a time-varying risk premium that is related to the conditional risk of the basis and to the conditional prices of systematic risk present in all assets markets. The result therefore reinforces the view, initially put forward by Bailey and Chan (1993), that the basis is priced rationally in an efficient market. The article also shows that the premium for basis risk increases with the maturity of the instruments used for hedging.

**Keywords:** basis, time-varying price of risk, conditional asset pricing model

## 1. INTRODUCTION

It is commonly agreed that futures markets exist to enable hedgers to transfer price risk to speculators (Keynes, 1930; and Miffre, 2000). By taking opposite positions in the spot and futures markets, hedgers can substantially reduce the volatility of their spot position. However, the hedged position is not totally risk-free. There is a part of the total volatility that cannot

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**Address for correspondence:** Joëlle Miffre, Cass Business School, 106 Bunhill Row, London EC4Y 8TZ, UK.  
e-mail: J.Miffre@city.ac.uk

be diversified away. This is due to the fact that the spread between the spot and futures prices (namely, the basis) at the time the hedge is lifted is uncertain. To state this differently, the price of the futures contract that prevails at the time the position is closed out frequently differs from the price that would yield a perfect hedge. This results in a loss in one market that is not perfectly offset by a commensurate gain in the other market.<sup>1</sup>

Despite its paramount importance to hedgers, basis risk has attracted little interest to date. Research on basis risk and on the presence of a risk premium for basis risk is still at its infancy.<sup>2</sup> Bailey and Chan (1993) were the first to identify time-varying risk premia in the basis of a hedge for a commodity. Baum and Barkoulas (1996) then reported similar results for the basis of foreign exchange instruments. Both papers recognize that there are 'some significant time varying risk premia in the bases' (Baum and Barkoulas, 1996). To show this, the authors decompose the basis into (i) a time-varying risk premium and (ii) the expected change in the spot price. They then follow a two-step approach. First, the authors regress the basis onto a set of predictors used as proxies for systematic risk in the economy. Second, they regress the spot price changes onto these very same instruments. Because the predictability identified in the first step cannot be attributed to the expected spot price change component of the basis it has to reflect the presence of a time varying risk premium in the basis.<sup>3</sup>

The contribution of the paper relative to the existing literature is in the methodology employed to identify the risk premium in the basis. As opposed to Bailey and Chan (1993) and Baum and Barkoulas (1996), we use a one-step approach that relies on the estimation of a conditional multifactor model with time-varying

1 The hedge is totally risk-free when it is lifted at contract expiration. At this time the futures price equals the spot price, resulting in a zero basis and in hedgers completely eliminating price risk. More often than not however, the hedge is lifted prior to maturity and hedgers then only exchange price risk for basis risk.

2 Little attention has been devoted thus far to the issue as to whether the basis contains a risk premium. However, many papers have recently attempted to identify time-varying risk premia in equity markets (Ferson and Harvey, 1993; and Harvey, 1995), in bond markets (Chang and Huang, 1990; and Ilmanen, 1995), and in futures markets (McCurdy and Morgan, 1992; Bessembinder and Chan, 1992; and Miffre, 2000).

3 Indeed the *ex-ante* variables that predict the basis in the first step fail to forecast the change in the spot price in the second step, suggesting therefore that the predictable movements identified in the first step reflects the presence of a risk premium in the basis.

quantities of risk and time-varying prices of systematic risk (Ferson and Harvey, 1993; and Harvey, 1995). We believe this methodology produces more robust inferences on the presence of a conditional price of basis risk than the two-step methodology previously employed.

The bases of interest are for British Pound (BP), Deutsche Mark (DM), Japanese Yen (JY), and Swiss Franc (SF). This specific dataset is chosen to ensure that the characteristics of the spot asset exactly match the characteristics of the underlying asset of the futures contract. Special care is thereby taken to completely eliminate cross-hedging risk and to focus on basis risk alone.

In anticipation of the results, the article shows that the basis includes a time-varying risk premium that is related to the conditional risk of the basis and to the conditional prices of systematic risk present in all assets markets. The results therefore reinforce the view, initially put forward by Bailey and Chan (1993), that the basis is priced rationally in an efficient market. The article also shows that the premium for basis risk increases with the maturity of the instruments used for hedging. The remainder of the article is organised as follows. Section 2 presents the methodology used to measure the conditional price of risk associated with the basis of foreign exchange instruments. Section 3 introduces the data set. Section 4 presents the empirical findings. Section 5 concludes.

## 2. METHODOLOGY

The presence of a conditional price of basis risk is investigated within a conditional multifactor model that allows for variation in the prices of risk associated with the systematic risk factors and for variation in the sensitivities of the bases to these factors. Following Ferson and Harvey (1993) and Harvey (1995), we assume that the time variation in the prices of systematic risk is captured by global instruments and that the time variation in the sensitivities of the basis to the risk factors is captured by local instruments. For each basis the model is as follows:

$$u1_t = F_t - (\gamma_0 + \gamma'Z_{t-1}^G) \quad (1a)$$

$$u2_t = (u1_t u1_t')(\kappa_0 + \kappa'Z_{t-1}^L) - (u1_t B_t) \quad (1b)$$

$$u3_t = B_t - \alpha - (\kappa_0 + \kappa'Z_{t-1}^L)'(\gamma_0 + \gamma'Z_{t-1}^G) \quad (1c)$$

$u1_t$  is a  $K$  – vector of error terms orthogonal to the  $G$  global instruments,  $u2_t$  is a  $K$  – vector of error terms orthogonal to the  $L$  local instruments,  $u3_t$  is a scalar that measures the residual from the conditional multifactor model at time  $t$ .  $Z_{t-1}^G$  is a  $(G - 1)$  – vector of global instruments used to predict the prices of systematic risk one period ahead,  $Z_{t-1}^L$  is a  $(L - 1)$  – vector of local instruments used to capture the variation in the sensitivities of the basis to the risk factors,<sup>4</sup>  $B_t$  is a scalar that represents the basis of the foreign currency at time  $t$ , and  $F_t$  is a  $K$  – vector of excess returns on factor mimicking portfolios. The parameters to estimate are  $\{\gamma_0, \gamma, \kappa_0, \kappa, \alpha\}$ , where  $\gamma_0$  and  $\kappa_0$  are vectors of dimension  $K$ ,  $\gamma$  is a  $((G - 1) * K)$  – matrix,  $\kappa$  is a  $((L - 1) * K)$  – matrix, and  $\alpha$  is a scalar.  $\alpha$  is the conditional counterpart of the Jensen (1968) measure of abnormal performance (see, also, Eckbo and Smith, 1998). The Appendix explicitly derives system (1).

The system of equations outlined in expression (1) does not impose any constraint on the parameters of the conditional multifactor model. It assumes that the change in the sensitivities of the basis to the risk factors, along with the shift in the prices of systematic risk, capture the time varying risk premia present in the basis.

System (1a) is a system of  $K$  equations that defines the conditional prices of systematic risk as the fitted returns from a regression of the excess returns on the factor-mimicking

4 It would be intractable to assume that both global and local instruments capture the variations through time in the conditional prices of systematic risk and in the conditional betas. Such an assumption would require the estimation of  $2K(G + L) + 1$  parameters; almost twice as many as in system (1). In the interest of parsimony, we follow the approach proposed by Ferson and Harvey (1993) and 1 in their studies of conditional risk premia in international equity markets and assume that conditional betas depend on local instruments, while the conditional prices of systematic risk depend on global variables. Because we consider that the conditional prices of systematic risk are not country-specific, we implicitly assume globally integrated capital markets.

portfolios on the global instruments. System (1b) determines the conditional betas  $(\kappa_0 + \kappa'Z_{t-1}^L)$  as the ratio of the conditional covariances  $(u1_t B_t)$  over the conditional variances  $(u1_t u1_t')$ . The conditional betas  $(\kappa_0 + \kappa'Z_{t-1}^L)$  are used along with the conditional prices of systematic risk  $(\gamma_0 + \gamma'Z_{t-1}^G)$  to measure  $(\kappa_0 + \kappa'Z_{t-1}^L)'(\gamma_0 + \gamma'Z_{t-1}^G)$ , the time-varying basis risk premium in (1c). Basically equation (1c) defines  $u3_t$  as the residual from the conditional multifactor model with time-varying risks and time-varying prices of risk. It is derived from the following relationship:

$$E(B_t|Z_{t-1}^G) = E\left[\sum_{j=1}^K (\beta_{jt}|Z_{t-1}^L)(F_{jt}|Z_{t-1}^G)\right] = (\kappa_0 + \kappa'Z_{t-1}^L)'(\gamma_0 + \gamma'Z_{t-1}^G)$$

where  $E(\cdot|Z_{t-1})$  denotes an expectation conditional upon  $Z_{t-1}$  and  $\beta_{jt}$  measures the time  $t$  sensitivity of the basis to  $F_{jt}$ , the excess returns on the  $j^{\text{th}}$  factor-mimicking portfolio.

Within this framework it is easy to test for the presence of a price of basis risk. The basis of the foreign currency will contain a risk premium if the coefficients on the lagged instruments are jointly significant; namely, if the risk and risk premia associated with some pre-specified risk factors change over time. The following hypotheses are tested:  $H_{01}$ :  $\gamma = 0$ ,  $H_{02}$ :  $\kappa = 0$ , and  $H_{03}$ :  $\gamma = \kappa = 0$  for each of the risk factors separately. The third hypothesis is also tested for all of the risk factors simultaneously. A rejection of  $H_{03}$  when all the factors are considered jointly indicates the presence of a time-varying premium for basis risk. Under  $H_{03}$ , the conditional multifactor model reduces to the static OLS multifactor model usually employed in the literature on unconditional asset pricing. Namely, under  $H_{03}$ , the conditional model (1) reduces to:

$$\begin{aligned} v1_t &= F_t - \gamma_0 \\ v2_t &= (v1_t v1_t')\kappa_0 - (v1_t B_t) \\ v3_t &= B_t - \alpha - \kappa_0' \gamma_0. \end{aligned}$$

The price of basis risk estimated from the static model  $\kappa_0' \gamma_0$  is measured as the sum of the cross products of unconditional betas and unconditional prices of systematic risk.

## 3. DATA

*(i) The Basis of Foreign Exchange Instruments*

The data set comprises end-of-month spot and futures exchange rates between the US dollar and the following currencies: British pound (BP), Deutsche mark (DM), Japanese yen (JY), and Swiss franc (SF). The basis is defined as the difference in the logs of the spot and futures prices. This definition of the basis was chosen to reflect the actual positions of US multinational corporations that hedge their long foreign currency exposure by going short futures. To mitigate the problem of non-synchronous trading between the futures and spot markets, the exchange rates at the close of business on the last trading day of the month are collected.

The article tests the sensitivity of the results to the technique employed to link together the futures prices. Two approaches are used to compile the time series of futures prices. First, the futures prices on the contract that is three to six months to maturity are considered.<sup>5</sup> The resulting basis is called the 'nearby basis'. Second, time series of end-of-month closing prices are collected from the contracts with a maturity of six to nine months. The resulting basis is called the 'deferred basis'. The sample covers the period May 30, 1980 to August 31, 2001.

Fama and French (1987) and Bailey and Chan (1993) studied the basis of a hedge for a commodity. Because futures contracts often amalgamate different types of commodities, the characteristics of the underlying asset of the contract do not exactly match the ones of the spot commodity, resulting in the presence of cross-hedging risk as well as basis risk. This matter forced Fama and French (1987) and Bailey and Chan (1993) to consider maturity futures prices as proxies for spot prices. Because futures contracts expire on average every three months, this approach considerably reduces the number of observations that can be used. With currencies however, the spot asset exactly matches the underlying asset of the futures contract. It follows

<sup>5</sup> The January, February and March prices of the June contract are linked together with the April, May and June prices of the September contract. Similarly, the July, August and September prices of the December contract and the October, November and December prices of the March contract are collected.

that one does not need to consider futures prices at maturity as proxies for spot prices. The advantages are twofold. First, cross-hedging risk is no longer a problem and one can therefore focus on basis risk in isolation. Second, one can use monthly data (instead of quarterly data). This produces time series that are sufficiently long for asymptotic theory to be applied to the test statistics.

Table 1, Panel A, presents the correlation between the spot and futures prices for both the nearby and deferred bases. As expected, these correlations are very high but less than perfect. They decrease with the time to maturity: the correlations computed with the deferred futures prices are slightly lower. This is an indication that the risk of the hedge increases with the maturity of the contract. Consequently, it seems legitimate to postulate that the risk premium for basis risk shall also increase with the maturity of the contracts used for hedging.

**Table 1**  
Correlations

<b>Panel A: Correlation Between the Spot and Futures Prices</b>				
	<i>Nearby Basis</i>	<i>Deferred Basis</i>		
BP	0.997998	0.994654		
DM	0.999045	0.997080		
JY	0.999609	0.998926		
SF	0.998876	0.996746		

<b>Panel B: Correlation Across Nearby Bases</b>				
	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>
BP	1.00			
DM	0.54	1.00		
JY	0.69	0.63	1.00	
SF	0.64	0.88	0.71	1.00

<b>Panel C: Correlation Across Deferred Bases</b>				
	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>
BP	1.00			
DM	0.59	1.00		
JY	0.75	0.68	1.00	
SF	0.71	0.92	0.73	1.00

Table 1, Panels B and C, reports correlation matrices across bases. These correlations range from 0.54 to 0.88 for the nearby bases in Panel B, with an average of 0.68 and from 0.59 to 0.92 for the deferred basis in Panel C, with an average of 0.73. These high correlations indicate that there might be some commonalities in the way the bases are priced.

(ii) *The Factor Mimicking Portfolios*

Along with a proxy for the market portfolio, the risk factors that enter into the conditional multifactor model include a proxy for exchange rate risk, shocks to the US term structure of interest rates, a proxy for the risk of default on US bonds, and unexpected inflation. Table 2, Panel A, defines the macroeconomic and financial variables used in this study.

The methodology proposed by Fama and French (1993) is used to construct portfolios that mimic the realisation of the macroeconomic and financial news (see also Chan et al., 1998). To do this, the excess returns on the components of the Russell 2,000 index that were continuously listed over the period March 31, 1975–August 31, 2001, are collected. The approach is as follows. First the stocks' loadings on the factor that is being mimicked are estimated over a 60 month period through regressions of individual excess returns on (i) the factor unexpected component<sup>6</sup> and (ii) the excess return on the Standard and Poor's composite index. Second, the stocks are ranked into five

6 A Kalman filter with time-varying parameters is used to extract from the raw macroeconomic and financial series ( $X_t$ ) the expectations component  $\left(\phi_{0t} + \sum_{j=1}^N \phi_{jt}X_{t-j}\right)$  and the unexpected factor ( $u_t$ ). The model is as follows:

$$X_t = \phi_{0t} + \sum_{j=1}^N \phi_{jt}X_{t-j} + u_t,$$

$$\phi_{jt} = \phi_{jt-1} + \xi_{jt}.$$

The adequacy of the model to generate unexpected factors is judged according to the time-series properties of  $u_t$ . In particular, the condition required is that  $u_t$  is serially uncorrelated. The probability values associated with the Ljung-Box test for serial correlation always exceed 0.03: At least at one percent, the derived factors meet the basic requirement of being serially uncorrelated.



**Table 2**  
The Prices of Systematic Risk

<b>Panel A: The Macroeconomic and Financial Variables</b>					
<i>Risk Factor</i>	<i>Definition</i>				
Market portfolio	Return on the Standard and Poor's composite index in excess of the US one-month Treasury-bill				
Exchange rate	Percentage change in the index: US dollar against major currencies				
Term structure of interest rates	Difference in the yields on US 10 year Treasury-bond and 3 month Treasury-bill				
Default spread	Difference in the yields on BAA and AAA rated US corporate bonds				
Inflation	Percentage change in the US consumer price index				

<b>Panel B: Summary Statistics of the Prices of Systematic Risk</b>					
	<i>Excess Return on the Mimicking Portfolio Associated with:</i>				
	<i>Excess Return on Market Portfolio</i>	<i>Foreign Exchange Risk</i>	<i>Term Structure Shocks</i>	<i>Shocks to Default Spread</i>	<i>Unexpected Inflation</i>
Mean	0.0066	0.0003	0.0013	0.0001	-0.0014
Annualised mean	0.0820	0.0034	0.0151	0.0004	-0.0166
Standard deviation	0.0438	0.0256	0.0282	0.0264	0.0221
Minimum	-0.2212	-0.0764	-0.1221	-0.0717	-0.0636
Maximum	0.1285	0.0830	0.0975	0.0884	0.0796

Table 2 (Continued)

	<i>Excess Return on the Mimicking Portfolio Associated with:</i>				
	<i>Excess Return on Market Portfolio</i>	<i>Foreign Exchange Risk</i>	<i>Term Structure Shocks</i>	<i>Shocks to Default Spread</i>	<i>Unexpected Inflation</i>
Excess return on market portfolio	1.00				
Foreign exchange risk	-0.09	1.00			
Term structure shocks	0.01	-0.09	1.00		
Shocks to default spread	0.03	0.24	0.00	1.00	
Unexpected inflation	-0.09	-0.07	0.14	-0.10	1.00

portfolios according to these loadings. In each of the subsequent 12 months, the difference in the average returns on the highest-ranked and the lowest-ranked portfolios represents the return on the factor mimicking portfolio. The procedure is then rolled forward using the most recent 60 months of data prior to the portfolio formation date.

The above procedure produces 256 estimates of the excess return on the factor mimicking portfolios. Summary statistics of the prices of systematic risk are presented in Panel B of Table 2. The annualised market risk premium equals 8.20%. The prices of risk associated with the macroeconomic factors, however, are less important in economic terms and less volatile than the market portfolio (as evidenced by lower absolute minimums, maximums, and return standard deviations). Panel C in Table 2 reports the correlation matrix in the excess returns on the factor mimicking portfolios. The correlation never exceeds 0.24. Hence multicollinearity is not considered to be a problem.

### *(iii) The Global and Local Instruments*

The set of instruments follows from Ferson and Harvey (1993) and Harvey (1995). The global instruments are expected to predict the systematic risk premia one period ahead. They are the first lag in: the return and dividend yield on a world market index, a measure of the US term structure, and a proxy for the international risk of default on money market instruments.

The local instruments are used to capture the predictable variation in country-specific betas. The set of local instruments include the first lag in: the return and dividend yield on the local stock index, the spread between the yields on long and short term government securities, and the basis under consideration. Details relating to the construction of the global and local information variables are reported in Table 3.

## 4. EMPIRICAL RESULTS

The presence of a time-varying price of basis risk is investigated by testing the significance of the coefficients on the lagged information variables in system (1). The results for the nearby

**Table 3**  
The Set of Instruments

<i>Definition</i>	
<b>Panel A: Global Instruments</b>	
● Market return	Percentage change in Datastream world equity index
● Dividend yield	Dividend yield on Datastream world equity index
● Term structure	Difference in yields on the 30 years US Treasury bonds and the 3 month US Treasury bill
● Default spread	Difference in yields on the 3 month Euro\$ deposit rate and the 3 Month US Treasury bill
<b>Panel B: Local Instruments</b>	
● Market return	Percentage change in Datastream local equity index
● Dividend yield	Dividend yield on Datastream local equity index
● Term structure	Spread in the yields on long and short term securities issued by local governments
● Basis	Basis

bases are reported in Table 4 and the results for the deferred basis are in Table 5. In each table, Panel A reports probability values and  $\chi^2$  tests of the explanatory power of the lagged instruments on the prices of systematic risk; Panel B reports the same information for the measures of risk. Finally, Panel C presents tests of the joint significance of the lagged instruments on both the measures of risk and the prices of systematic risk.

The results in Tables 4 and 5 suggest that the instruments have predictive power over both the measures of risk and the prices of systematic risk. In 75% of the instances, the null hypothesis that the lagged global instruments have no predictive power over the prices of systematic risk is rejected at 5% (Panels A). The inferences are similar for the measures of risk in Panels B: more often than not, the local instruments predict the change in the sensitivities of the basis to the risk factors. Panels C test the joint hypothesis that the prices of systematic risk and the measures of risk are constant for each of the five pre-specified factors. The results in the last column look at the joint significance of both  $\gamma$  and  $\kappa$  when all the five pre-specified factors are considered simultaneously. We consistently reject the null hypothesis that the measures of risk and the prices of systematic risk are constant, for the individual pre-specified factors and also when the five factors are considered jointly (namely; in the last column). This confirms the presence of a time-varying premium for basis risk, first evidenced by Bailey and Chan (1993). The tests overwhelmingly suggest that there is some significant time variation in the basis of foreign exchange instruments and that these variations are related to the risk of the basis and to the prices of systematic risk present in all asset markets.

The remainder of the paper looks more closely at the conditional prices of basis risk. Figures 1 to 4 plot for each of the foreign exchange instruments the time-varying prices of risk associated with the nearby and deferred bases. Three patterns emerge from these graphs. First, the time varying risk premia move in tandem across currencies. Second, the conditional prices of risk associated with the nearby bases are clearly highly correlated with the conditional prices of risk associated with the deferred bases. Third, the risk premia associated with the deferred bases exceed the risk premia associated with the

**Table 4**  
Time Variation in the Nearby Bases

	<i>Excess return on the mimicking portfolio associated with:</i>					
<i>Excess Return on Market Portfolio</i>	<i>Foreign Exchange Risk</i>	<i>Term Structure</i>	<i>Default Spread</i>	<i>Unexpected Inflation</i>	<i>For all Risk Factors, <math>H_{03}: \gamma = \kappa = 0</math></i>	
<b>Panel A: Time Variation in the Prices of Systematic Risk: <math>H_{01}: \gamma = 0</math></b>						
● BP	4.281 [0.37]	5.131 [0.27]	14.980 [0.00]	35.347 [0.00]	33.739 [0.00]	
● DM	4.030 [0.40]	14.965 [0.00]	32.016 [0.00]	47.930 [0.00]	54.165 [0.00]	
● JY	6.551 [0.16]	13.037 [0.01]	25.443 [0.00]	37.833 [0.00]	54.448 [0.00]	
● SF	4.611 [0.33]	15.970 [0.00]	35.186 [0.00]	68.397 [0.00]	72.077 [0.00]	
<b>Panel B: Time Variation in the Measures of Risk: <math>H_{02}: \kappa = 0</math></b>						
● BP	20.101 [0.00]	26.609 [0.00]	23.116 [0.00]	44.559 [0.00]	10.756 [0.03]	
● DM	18.357 [0.00]	3.192 [0.53]	16.153 [0.00]	1.465 [0.83]	5.448 [0.24]	
● JY	14.670 [0.01]	2.769 [0.60]	8.156 [0.09]	10.935 [0.03]	6.035 [0.20]	
● SF	22.482 [0.00]	4.575 [0.33]	6.295 [0.18]	3.874 [0.42]	14.402 [0.01]	

**Panel C: Time Variation in the Prices of Systematic Risk and in the Measures of Risk:  $H_{03}: \gamma = \kappa = 0$** 

● BP	30.116 [0.00]	47.911 [0.00]	72.399 [0.00]	109.495 [0.00]	64.930 [0.00]	397.558 [0.00]
● DM	35.786 [0.00]	25.389 [0.00]	42.406 [0.00]	83.238 [0.00]	90.825 [0.00]	438.969 [0.00]
● JY	148.304 [0.00]	16.912 [0.03]	62.271 [0.00]	69.062 [0.00]	288.675 [0.00]	1068.512 [0.00]
● SF	100.883 [0.00]	33.068 [0.00]	60.031 [0.00]	160.026 [0.00]	100.741 [0.00]	596.064 [0.00]

*Notes:*

This table reports, for each of the risk factors, the  $\chi^2$  values from tests of  $H_{01}: \gamma = 0$ ,  $H_{02}: \kappa = 0$  and  $H_{03}: \gamma = \kappa = 0$  in the conditional multifactor model (1). The tests are distributed as  $\chi^2$  with  $(G - 1)$ ,  $(L - 1)$  and  $(G + L - 2)$  degrees of freedom respectively. The last column tests the joint significance of  $\gamma$  and  $\kappa$  in  $H_{03}$  when all the factors are considered simultaneously. The test is distributed as  $\chi^2$  with  $K(G + L - 2)$  degrees of freedom.  $\gamma$  ( $\kappa$ ) measures the sensitivities of the factor mimicking portfolios (risk measures) to the global (local) instruments. Probability values in brackets. The results relate to the nearby basis.

Table 5

## Time Variation in the Deferred Bases

	<i>Excess Return on the Mimicking Portfolio Associated with:</i>					<i>For all Risk Factors, <math>H_{03}: \gamma = \kappa = 0</math></i>
	<i>Excess Return on Market Portfolio</i>	<i>Foreign Exchange Risk</i>	<i>Term Structure</i>	<i>Default Spread</i>	<i>Unexpected Inflation</i>	
<b>Panel A: Time Variation in the Prices of Systematic Risk: <math>H_{01}: \gamma = 0</math></b>						
● BP	4.156 [0.39]	5.010 [0.29]	17.468 [0.00]	37.239 [0.00]	35.094 [0.00]	
● DM	3.980 [0.41]	16.266 [0.00]	40.467 [0.00]	58.741 [0.00]	70.337 [0.00]	
● JY	6.863 [0.14]	16.434 [0.00]	34.764 [0.00]	49.789 [0.00]	76.473 [0.00]	
● SF	5.353 [0.25]	20.127 [0.00]	46.793 [0.00]	76.463 [0.00]	88.308 [0.00]	
<b>Panel B: Time Variation in the Measures of Risk: <math>H_{02}: \kappa = 0</math></b>						
● BP	28.982 [0.00]	24.104 [0.00]	28.073 [0.00]	59.441 [0.00]	8.831 [0.07]	
● DM	31.987 [0.00]	6.289 [0.18]	14.058 [0.01]	6.347 [0.17]	8.042 [0.09]	
● JY	18.095 [0.00]	6.577 [0.16]	11.358 [0.02]	11.204 [0.02]	4.707 [0.32]	
● SF	37.060 [0.00]	4.006 [0.41]	8.903 [0.06]	6.378 [0.17]	17.303 [0.00]	



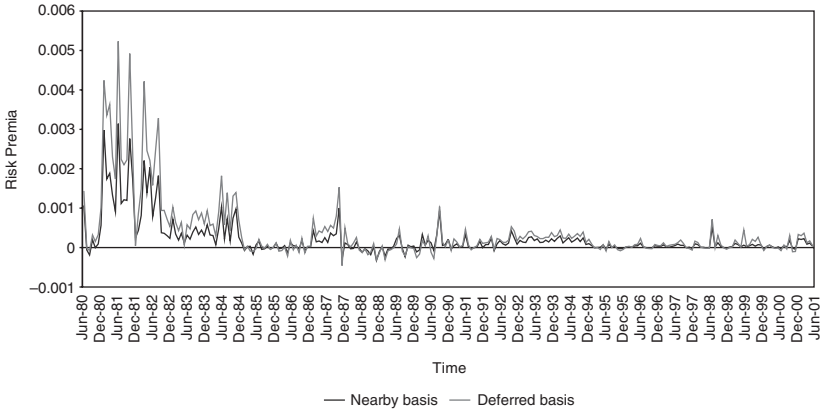
**Panel C: Time-Variation in the Prices of Systematic Risk and in the Measures of Risk:  $H_{03}: \gamma = \kappa = 0$** 

● BP	38.444 [0.00]	45.712 [0.00]	77.085 [0.00]	132.603 [0.00]	71.821 [0.00]	414.872 [0.00]
● DM	82.296 [0.00]	47.883 [0.00]	53.168 [0.00]	100.921 [0.00]	111.644 [0.00]	578.809 [0.00]
● JY	181.189 [0.00]	34.557 [0.00]	98.171 [0.00]	65.322 [0.00]	383.274 [0.00]	1442.954 [0.00]
● SF	167.718 [0.00]	43.573 [0.00]	93.602 [0.00]	243.796 [0.00]	121.909 [0.00]	839.805 [0.00]

*Notes:*

This table reports, for each of the risk factors, the  $\chi^2$  values from tests of  $H_{01}: \gamma = 0$ ,  $H_{02}: \kappa = 0$  and  $H_{03}: \gamma = \kappa = 0$  in the conditional multifactor model (1). The tests are distributed as  $\chi^2$  with  $(G - 1)$ ,  $(L - 1)$  and  $(G + L - 2)$  degrees of freedom respectively. The last column tests the joint significance of  $\gamma$  and  $\kappa$  in  $H_{03}$  when all the factors are considered simultaneously. The test is distributed as  $\chi^2$  with  $K(G + L - 2)$  degrees of freedom.  $\gamma$  ( $\kappa$ ) measures the sensitivities of the factor mimicking portfolios (risk measures) to the global (local) instruments. Probability values in brackets. The results relate to the deferred basis.

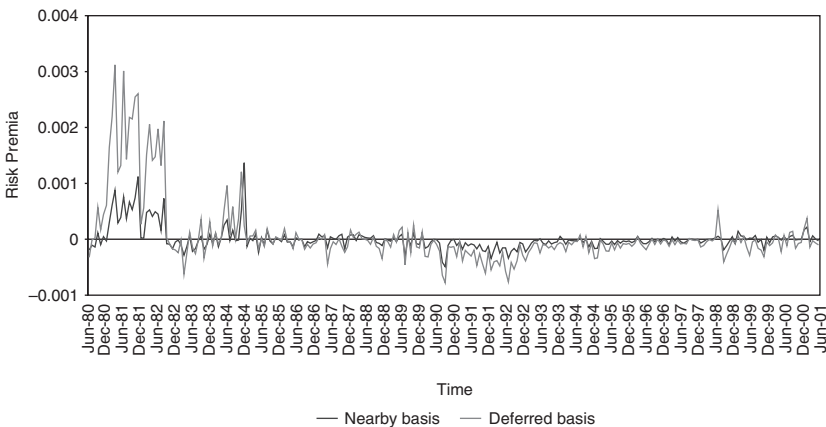
**Figure 1**  
The Conditional Prices of Risk Associated with the BP Bases



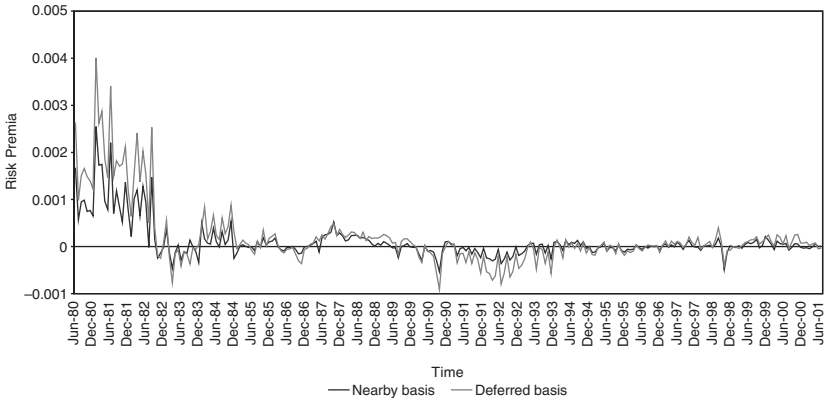
nearby bases. Table 6 further analyses the characteristics of the prices of basis risk and confirms these conclusions.

Table 6, Panel A, presents some summary statistics for both the nearby and deferred bases. The price of basis risk increases with the maturity of the instruments used for hedging. The average annualised risk premium for the nearby bases equals

**Figure 2**  
The Conditional Prices of Risk Associated with the DM Bases

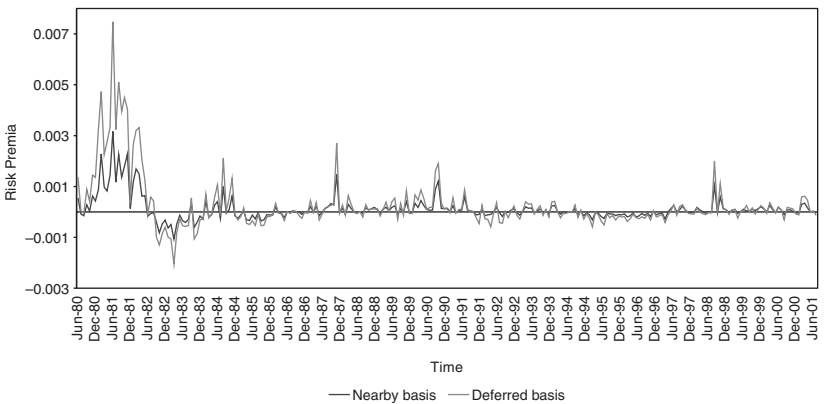


**Figure 3**  
The Conditional Prices of Risk Associated with the JY Bases



0.142%. The average annualised risk premium for the deferred bases equals 0.281%. This suggests that on average the price of risk associated with the deferred basis is almost twice as high as the price of risk associated with the nearby basis. Hedging with a far away maturity contract (six to nine months) is more risky than hedging with a nearby maturity contract (three to six months). Consequently the price of risk associated with distant

**Figure 4**  
The Conditional Prices of Risk Associated with the SF Bases



**Table 6**  
 Characteristics of the Conditional Prices of Basis Risk

	<i>Nearby Basis</i>				<i>Deferred Basis</i>			
	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>
	● Mean	0.000233	0.000010	0.000112	0.000118	0.000400	0.000071	0.000196
● Annualised mean	0.002801	0.000116	0.001340	0.001412	0.004809	0.000857	0.002350	0.003211
● Standard deviation	0.000485	0.000211	0.000399	0.000477	0.000818	0.000590	0.000668	0.001051
● Minimum	-0.000326	-0.000490	-0.000531	-0.001069	-0.000456	-0.000776	-0.000913	-0.002055
● Maximum	0.003145	0.001368	0.002548	0.003167	0.005234	0.003121	0.004009	0.007473

**Panel B: Correlations in the Conditional Prices of Basis Risk**

(The first figure is for the nearby basis, the second figure in parenthesis is for the deferred basis)

	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>
BP	1			
DM	0.64 (0.84)	1		
JY	0.75 (0.77)	0.65 (0.84)	1	
SF	0.69 (0.76)	0.58 (0.78)	0.64 (0.72)	1

**Panel C: The Conditional Price of Basis Risk and the US Dollar**

<i>Nearby Basis</i>				<i>Deferred Basis</i>			
<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>	<i>BP</i>	<i>DM</i>	<i>JY</i>	<i>SF</i>
0.0049 (2.92)	0.0019 (2.65)	0.0021 (1.51)	0.0035 (2.09)	0.0085 (3.00)	0.0075 (3.71)	0.0039 (1.68)	0.0091 (2.49)

*Notes:*

This table reports slope coefficient from a regression of the conditional price of basis risk onto a constant and the percentage change in the US dollar relative to other major currencies. *t* – statistics in parenthesis.

contracts exceeds the premium received on nearby contracts. This is consistent with the idea that the correlation between the spot and futures prices decreases with the maturity of the contract (Table 1, Panel A). This is also in line with the evidence presented in Castelino and Francis (1982), who show that basis risk in agricultural commodity markets declines as maturity approaches.

Table 6, Panel B, reports correlation coefficients across the risk premia measured from the nearby bases. The results in parentheses are for the deferred bases. The correlations range from 0.58 to 0.75, with an average of 0.66 for the nearby bases and from 0.72 to 0.84, with an average of 0.79 for the deferred bases. This reinforces the view that the basis is priced rationally in an efficient market.

The behaviour of the prices of basis risk is further investigated by studying the link between the bases risk premia and the change in the US dollar. The results are reported in Table 6, Panel C. With the noticeable exception of JY, the reported coefficients are positive and statistically significant. The impression is therefore that the risk is higher with an appreciating dollar. The relative higher premium then required by US multinational corporations might reflect the fact that an appreciation of the US dollar relative to other major currencies has a negative impact on the US balance of payment and therefore is compensated by a relatively higher premium.

Summarising the evidence presented here, it appears that the characteristics of the conditional prices of basis risk are comparable to the ones typically found in the stock and bond markets. The prices of basis risk are highly correlated and depend on the systematic risk of the basis and on the prices of systematic risk present in all asset markets. This is consistent with the idea that the basis changes rationally over time.

## 5. CONCLUSIONS

The purpose of the paper is to provide some further evidence on the presence of a time-varying risk premium in the basis of foreign exchange instruments. The contribution of the paper relative to the existing literature is in the methodology employed to identify the risk premium in the basis. While

previous papers made indirect inferences on the presence of a price of basis risk through the use of a two-step methodology, the present paper employs a conditional multifactor model that allows for change in the measures of risk and in the prices of systematic risk. This one-step approach is believed to be more robust.

The main conclusion that emerges from the study is that the basis changes over time in an efficient way and that the risk premium embedded in the basis is related to the conditional risk of the basis and to the conditional prices of systematic risk present in all asset markets. The time varying price of basis risk identified in foreign exchange markets resembles the one typically found in the stock and bond markets. The paper therefore reinforces the view, initially put forward by Bailey and Chan (1993), that the basis includes a time-varying risk premium.

The article also shows that the premium for basis risk increases with the maturity of the instruments used for hedging. Hedgers only interested in minimising price risk should trade near to maturity contracts. However, if risk is not the only parameter considered while hedging and if expected return also enters the decision making process (see, for example, Working, 1953), hedgers might be interested in hedging their spot position with contracts whose maturity is further away. The hedge is then riskier but the additional basis risk is compensated by a premium that is on average twice as high.

The paper takes it for granted that the basis only widens because of an increase in basis risk and thus an increase in the premium for basis risk. We do not investigate any other reasons as to why the basis may change. In particular, we do not consider the impact that segmentation between the futures and spot markets might have on the size of the basis over time. We therefore cannot rule out the possibility that the basis behaves stochastically because market imperfections such as trade restrictions and transaction costs hinder arbitrage between the spot and futures markets. This could result in market segmentation and in a stochastic basis. One might also be tempted to think that the basis of commodities changes with a change in (i) the costs of storage, (ii) interest rates, or (iii) the convenience yields that arise when holding the commodity spot. We do not test these alternative explanations formally and offer them as possible avenues for future research on basis risk.

APPENDIX

**Explicit Derivation of the Conditional Multifactor Model**

The derivation is based on works previously implemented by Harvey (1989) and Ferson and Harvey (1993). Some assumptions are needed to model the time variation in the basis, the prices of systematic risk and the risk measures. Following Ferson and Harvey (1993) and Harvey (1995), we assume that global instruments  $Z_{t-1}^G$  capture the time variation in the prices of systematic risk and in the basis, while local instruments  $Z_{t-1}^L$  capture the time variation in the measures of risk. Finally, we assume a linear relation between (i) the basis, the prices of systematic risk and the risk measures and (ii) the instruments.

The risk premium associated with the basis is measured as the sum of the cross products of the time-dependent measures of risk ( $\beta_{jt}|Z_{t-1}^L$ ) and the time-dependent prices of systematic risk ( $F_{jt}|Z_{t-1}^G$ ):

$$E(B_t|Z_{t-1}) = E \left[ \sum_{j=1}^K (\beta_{jt}|Z_{t-1}^L) (F_{jt}|Z_{t-1}^G) \right],$$

$B_t$  is the basis of the foreign currency at time  $t$  and  $E(\cdot|Z_{t-1})$  denotes an expectation conditional upon  $Z_{t-1}$ .

From the assumptions above, the random shocks in the basis and in the excess returns on the factor mimicking portfolios can be expressed as:

$$\epsilon 1_t = B_t - (\delta_0 + \delta' Z_{t-1}^G), \tag{A1}$$

$$\epsilon 2_t = F_t - (\gamma_0 + \gamma' Z_{t-1}^G), \tag{A2}$$

$\epsilon 1_t$  and  $\epsilon 2_t$  are error terms orthogonal to  $G$  global instruments,  $\epsilon 1_t$  is a scalar,  $\epsilon 2_t$  is a vector of dimension  $K$ ,  $Z_{t-1}^G$  is a  $(G - 1)$  - vector of global instruments,  $F_t$  is a  $K$  - vector of excess returns on factor mimicking portfolios,  $\delta_0$  is a scalar,  $\gamma_0$  is a vector of dimension  $K$ ,  $\delta$  is a vector of dimension  $(G - 1)$ , and  $\gamma$  is a  $((G - 1) * K)$  - matrix (Harvey, 1989).

It follows also from our assumptions and from the definition of beta that  $(\beta_t|Z_{t-1}^L)$ , the  $K$  - vector of time dependent betas, equals:



$$(\beta_t|Z_{t-1}^L) = \frac{\sigma(B_t, F_t|Z_{t-1}^G)}{\sigma^2(F_t|Z_{t-1}^G)}, \tag{A3}$$

$\sigma^2(\cdot|Z_{t-1})$  and  $\sigma(\cdot, \cdot|Z_{t-1})$  denote variance and covariance conditional upon  $Z_{t-1}$ .

By definition of the variance and covariance operators and given (A1), (A2) and (A3),  $(\beta_t|Z_{t-1}^L)$  also equals:

$$(\beta_t|Z_{t-1}) = \frac{E(\varepsilon_{1t}, \varepsilon_{2t}|Z_{t-1}^G)}{E(\varepsilon_{2t}^2|Z_{t-1}^G)}.$$

The forecast error in  $(\beta_t|Z_{t-1}^L)$ , denoted  $\varepsilon_{3t}$ , is defined as:

$$\varepsilon_{3t} = \varepsilon_{2t} \varepsilon_{2t}' (\kappa_0 + \kappa' Z_{t-1}^L) - \varepsilon_{2t} \varepsilon_{1t}, \tag{A4}$$

$\varepsilon_{3t}$  and  $\kappa_0$  are vectors of dimension  $K$ ,  $Z_{t-1}^L$  is a  $(L - 1)$  - vector of local instruments, and  $\kappa$  is a  $((L - 1) * K)$  - matrix. Like equation (A3) before, equation (A4) defines the conditional betas  $(\kappa_0 + \kappa' Z_{t-1}^L)$  as the ratios of the conditional covariances  $\varepsilon_{2t} \varepsilon_{1t}$  to the conditional variance  $\varepsilon_{2t} \varepsilon_{2t}'$ .

Stacking together (A1), (A2) and (A4) and recognising that  $E(\varepsilon_{2t} \varepsilon_{1t} | Z_{t-1}^G) = E(\varepsilon_{2t} B_t | Z_{t-1}^G)$  (Harvey, 1989) yield:

$$\begin{aligned} \varepsilon_{2t} &= F_t - (\gamma_0 + \gamma' Z_{t-1}^G), \\ \varepsilon_{3t} &= (\varepsilon_{2t} \varepsilon_{2t}') (\kappa_0 + \kappa' Z_{t-1}^L) - (\varepsilon_{2t} B_t). \end{aligned}$$

This system is estimated jointly with the equation for the conditional multifactor model. This equation defines the conditional risk premia  $(\kappa_0 + \kappa' Z_{t-1}^L)' (\gamma_0 + \gamma' Z_{t-1}^G)$  as the cross-product of the time-varying measures of risk  $(\beta_t|Z_{t-1}^L)$  and the time-varying prices of systematic risk  $(F_t|Z_{t-1}^G)$ :

$$\varepsilon_{4t} = B_t - \alpha - (\kappa_0 + \kappa' Z_{t-1}^L)' (\gamma_0 + \gamma' Z_{t-1}^G),$$

$\varepsilon_{4t}$  and  $\alpha$  are scalars.  $\alpha$  is the conditional counterpart of the Jensen (1968) measure of abnormal performance.

The resulting system implies the following orthogonality conditions  $E(\varepsilon_{2t} Z_{t-1}^G', \varepsilon_{3t} Z_{t-1}^L', \varepsilon_{4t}) = 0$ . It is estimated through the generalised method of moments (GMM) introduced by Hansen (1982). With  $K$  risk factors,  $G$  global instruments and  $L$  local instruments, there are  $K(G + L) + 1$  orthogonality conditions and  $K(G + L) + 1$  parameters to estimate, leaving the model perfectly identified.

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