Skew Index Option Pricing

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Joint Work with King Wang
The Skew Index.
Remarks on Option Pricing.
Hedging with Options on Related Assets.
Marginal Distributions of Returns to Option Maturity.
Joint Distributions for Hedged and Hedging Returns.
Distribution of Hedged Return Conditional of Hedging Asset Return.
Pricing of Residual Risk.
Function to be Hedged in the Related Asset Market.
Pricing at Cost of Hedge.
Skew Index Details.
Examples JPM via XLF.
VIX via SPY.
Skew via SPY.
VIX and Skew of Skew.
The VIX Index

- The Index construction follows the principles of the VIX.
- The VIX is the forward price quote for receiving the sum of squared returns for a month.
- For horizon $h$ this value is the following risk neutral expectation

$$VIX_t = \sqrt{\frac{1}{h} E^Q \left[ -2 \ln \left( \frac{S(t+h)}{S(t)} \right) \right]}.$$

- A portfolio of stock options hedges twice the negative of the log contract to deliver the VIX.
- There is a substantial market in options on the VIX index.
The Skew Index

- The Skewness of returns is the centered third moment divided by the variance to the power 1.5.
- For return to horizon $h$,
  \[ R = \ln \left( \frac{S(t+h)}{S(t)} \right), \]
- The skewness is
  \[ s = \frac{E^Q[(R - E^Q(R))^3]}{E^Q[(R - E^Q(R))^2]^{1.5}}. \]
- All powered return expectations required may be hedged to determine the skewness $s$.
- It is typically negative for equity assets and the Skewness Index is
  \[ S = 100 - 10s \]
- where $s$ is the skewness of the one month return on the S&P 500 index.
The VIX shot up past 80 in COVID time with an increase of 500% before coming down.

The Skew Index however, has steadily risen to 148.27 in late June from a mid March low of 113.54.

Currently there are no options trading on the Skew Index.

We shall price such options here.
Remarks on Option Pricing

- The Black and Scholes (1973), and Merton (1973) theory taught us how to price options at the cost of a replicating hedge.
- It solved a complicated and difficult problem of defining option values that converged to the kinked payoff at maturity.
- Under strong assumptions replication was delivered by dynamic trading in the stock.
- The result was a formula for pricing options valid under assumptions that turned out to be false.
- The formula was soon abandoned for pricing options and is used mainly to quote prices and manage risks.
The intervening years saw the development many parametric option pricing models.

They were at best technologies for dimensional reduction inferring the prices of now thousands of options from a handful.

In particular, there isn’t an associated hedge and/or the cost of such a hedge.
Back, prior to 1973, options did not trade as extensively as they do now.

Hedges were sought in the liquid market for the stock.

Today one may seek to hedge the risk of say an option on the Skew or VIX index by positions in options for the stock.

But replication is not to be expected under realistic assumptions.

Strong assumptions delivering replication may suffer the same fate of being false.

In the absence of replication, residual risk needs to be directly priced.

One has to price the risk that still remains after the hedge is put in place.
The Agenda Before Us

- Describing the joint risk of the hedged and hedging returns.
- Describing the residual or post hedge risk.
- Pricing the residual risk.
- Constructing the hedge.
- Pricing at the cost of the hedge.
- Applying the methodology and evaluating the results.
Univariate Return Distributions

- In appreciation of Gauss attention is restricted to limit laws.
- In recognition of the principle of likelihoods entropy is maximized.
- All the limit laws are the self decomposable laws and the selected distribution is in this class.
- Entropy maximization delivers the Gaussian density for a known variance.
- Entropy maximization for a random variance delivers the variance gamma model.
- The variance gamma is a difference of two independent gamma processes with the same variance rate or speed.
- Recognizing that up moves are both more frequent and smaller than down moves leads to the bilateral gamma (BG) model.
- The BG is a four parameter limit law calibrating freely the drift, volatility, skewness and kurtosis.
- Four parameters are a minimal requirement given that there are separate drifts and volatilities for the up and down moves.
Let $\gamma_p(t), \gamma_n(t)$ be two independent standard gamma processes with unit mean and variance rates.

The $BG$ process $X(t)$ parameters $b_p, c_p, b_n, c_n$ is defined as

$$X(t) = b_p \gamma_p (c_p t) - b_n \gamma_n (c_n t),$$

where $b_p, b_n$ are the scale parameters and $c_p, c_n$ are the speed parameters for the positive and negative moves.
The characteristic function of the Bilateral Gamma at time $t$ is given by

$$\phi_{BG}(u, t) = \left(\frac{1}{1 - iub_p}\right)^{c_p t} \left(\frac{1}{1 + iub_n}\right)^{c_n t}.$$
The Bilateral Gamma density at unit time, for $x > 0$, is given by

$$f_{BG}(x) = \left( \frac{1}{b_p} \right)^{c_p} \left( \frac{1}{b_n} \right)^{c_n} \left( \frac{1}{(1/b_p + 1/b_n)^{(c_p+c_n)/2}} \Gamma(c_p) \right) \times \right.$$  

$$x^{(c_p+c_n)/2-1} \exp\left(-x/2 \left(1/b_p + 1/b_n\right)\right) \times$$  

$$\text{whittaker W} \left( \frac{c_p - c_n}{2}, \frac{c_p + c_n - 1}{2}, \left( \frac{1}{b_p} + \frac{1}{b_n} \right), x \right).$$

For $x < 0$ the roles of $b_p, c_p$ and $b_n, c_n$ are reversed with an evaluation at $|x|$. 
The Bilateral Gamma Lévy density is given by

\[ k_{BG}(x) = c_p \frac{\exp \left( -\frac{x}{b_p} \right)}{x} 1_{x>0} + c_n \frac{\exp \left( -\frac{|x|}{b_p} \right)}{|x|} 1_{x>0} \]

As \(|x| k_{BG}(x)\) is decreasing in \(|x|\) the unit time law is self decomposable and thus is a limit law.
The bilateral gamma model fits have been reported for daily returns in time series data and at option maturities risk neutrally. The interest here is for the physical return distribution at the option maturity. Empirically the returns are constructed by bootstrapping data on immediately prior price histories. We present a graph of the model fit to five, ten and 21 day bootstrapped returns.
We present a Table of the root mean square error percentiles across 195 underlying equity assets.

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<tr>
<th>Percentile</th>
<th>5 day return</th>
<th>10 day return</th>
<th>21 day return</th>
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The fit is better at the shorter maturities.
### TABLE 2
Representative Bilateral Gamma Parameter Values

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<th>bp</th>
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<th>proportion</th>
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<td><strong>21 days</strong></td>
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<td></td>
<td></td>
<td></td>
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Just as the multivariate normal distribution is a joint distribution with Gaussian marginals there is multivariate law consistent with prespecified bilateral gamma marginals.

The multivariate law is that of multivariate variance gamma plus independent bilateral gamma laws.

Multivariate variance gamma is multivariate Brownian motion with drift \( \theta \) and covariance matrix \( \Sigma \) time changed by a gamma process with unit mean and variance rate \( \nu \).

The dependency parameters are just \( \nu \) and the correlation matrix \( C \) of the Brownian motions.

The drifts and variances are given by the marginal laws.
Let the marginal parameter vectors be $b_p, c_p, b_n, \text{ and } c_n$.

For dependency parameters $\nu, C$ define

$$\theta_i = \frac{b_{pi} - b_{ni}}{\nu}$$

$$\sigma_i^2 = \frac{2b_{pi}b_{ni}}{\nu}$$

and set

$$\Sigma = \Delta(\sigma) C \Delta(\sigma).$$

Let $Y$ be a vector of independent bilateral gamma variates with parameters $b_p, c_p - 1/\nu, b_n, c_n - 1/\nu$.

The multivariate bilateral gamma (MBG) variate $X$ has the distribution of

$$X = \theta g + \Delta(\sigma) \sqrt{g} Z + Y$$

where $g$ is has a gamma distribution with unit mean and variance $\nu$. 

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Skew Index Options

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Empirical Characteristic Function Matching

- The dependency parameters may be estimated in higher dimensions by matching the theoretical characteristic function to the empirical characteristic function.
- The MBG characteristic function is given by

\[
\phi_{MBG}(u) = E \left[ \exp(iu'X) \right] = \left( \frac{1}{1 - bu'\theta v + \frac{v}{2} u'\Sigma u} \right)^{\frac{1}{v}} \times \prod_j \left( \frac{1}{1 - iu_j b_{pj}} \right)^{c_{pj} - 1/v} \left( \frac{1}{1 + iub_{nj}} \right)^{c_{nj} - 1/v}.
\]

- The correlation matrix may be parameterized by the lower triangular part of its matrix logarithm Archakov and Hansen (2018).
The multivariate arrival rates or Lévy density $k(x)$ for the multivariate bilateral gamma model may be specified as follows.

$$k(x) = \tilde{m}(x) + \sum_{j=1}^{n} k_j(x_j) \prod_{l=1, l \neq j}^{M} 1_{x_l=0},$$

where

$$\tilde{m}(z) = \frac{\exp \left( \theta^T \Sigma^{-1} x \right)}{\nu (2\pi)^{n/2-1} \sqrt{\nu} \sqrt{x^T \Sigma^{-1} x} \exp \left( -\sqrt{\left( \theta^T \Sigma^{-1} \theta + \frac{2}{\nu} \right)(x^T \Sigma^{-1} x)} \right)} \times$$

$$k_j(x_j) = \frac{c_{nj}^{-1/\nu}}{|x_j|} \exp \left( -\frac{|x_j|}{b_{nj}} \right) 1_{x_j < 0}$$

$$+ \frac{c_{pj} - 1/\nu}{x_j} \exp \left( -\frac{x_j}{b_{pj}} \right) 1_{x_j > 0}.$$
The residual risk of $y$ given $x$ may be simulated from the conditional density of $y$ given $x$.

For this we require the joint density of two multivariate bilateral gamma distributed variates.

The two variates are specified as

\[
X = \theta_x g + \sigma_x \sqrt{g} z_1 + y_1 \\
Y = \theta_y g + \sigma_y \sqrt{g} z_2 + y_2
\]

where $g$ is gamma distributed and $y_1, y_2$ are independent bilateral gamma variates.
The joint density of \((X, Y)\) conditional on \(g, y_1, y_2\) is

\[
f_{MBG}(x_1, x_2; g, y_1, y_2) = \frac{1}{2\pi\sigma_x\sigma_y g \sqrt{1 - \rho^2}} \times \exp \left( -\frac{(x - \theta_x g - y_1)^2}{2\sigma_x^2 g (1 - \rho^2)} - \frac{(y - \theta_y g - y_2)^2}{2\sigma_y^2 g (1 - \rho^2)} + \frac{\rho(x - \theta_x g - y_1)(y - \theta_y g - y_2)}{\sigma_x \sigma_y g (1 - \rho^2)} \right)
\]

The variates \(g, y_1, y_2\) may be integrated out by Monte Carlo.
The MBG law may be used on \((x, y)\) but when there is a strong dependence as occurs for the VIX and the SPX one may first perform a regression

\[ y = a + bx + u \]

If the only dependence is the regression then one models \(u\) as a bilateral gamma. This is the model MBGIR.

Alternatively one may model the pair \((x, u)\) as MBG. This the model MBGR.
The bilateral gamma marginals may be put together into a joint law using a copula.

In addition we employ a Gaussian and a ’t’ copula, both of which incorporate a correlation coefficient.

Four models of dependence are employed, $MBGR$, $MBGIR$, and a Gaussian and a ’t’ copula.
Copulas concentrate attention on return correlations.

The introduction of common time changes for correlated Brownian motions helps differentiate return correlations from squared return correlations.

The parameter $C$ drives return correlations while $\nu$ controls squared return correlation.
Residual Risk

- Suppose the return on the hedging asset at maturity turns out to be $x$.
- The option to be priced has a payout $C$ for strike $K$ and $w$ equal to unity for a call and zero otherwise of
  \[ C = (1 - w) (K - S_y(0)e^y)^+ + w(S_y(0)e^y - K)^+. \]
- Further suppose it is organized that conditional on the hedging return being $x$ we shall receive the funds $L(x)$.
- The residual risk on selling the option is then
  \[ R = L(x) - C. \]
- If $R$ is positive we have no issue, we payout $C$ and keep the difference.
- The residual risk is then an acceptable risk.
- Our interest then is in the smallest value $L(x)$ such that $R$ is an acceptable risk.
The theory of acceptable risks developed in Artzner, Delbaen, Eber and Heath (1999) defines acceptable risks more generally as a convex cone of random variables that contains the nonnegative random variables, as these are clearly acceptable.

Equivalently it is shown that there exists a convex collection $\mathcal{M}$ of test probabilities $Q \in \mathcal{M}$ such that a risk $R$ is acceptable if and only if

$$E^Q[R] \geq 0, \text{ for all } Q \in \mathcal{M}.$$ 

It follows that the smallest value $L(x)$ is given by

$$L(x) = \sup_{Q \in \mathcal{M}} E^Q[R].$$

This is the ask price for selling and option. The bid price for buying it is

$$B(x) = \inf_{Q \in \mathcal{M}} E^Q[R].$$
Distorted Expectations I

- The test probabilities may be defined by a probability altering rule.
- For a concave distribution function $\Psi(u)$, $0 \leq u \leq 1$ define
  \[ M = \{ Q | Q(A) \leq \Psi(P(A)), \text{ all } A \} \]
  where $P$ is the original probability measure. Then
  \[ B(x) = \int_{-\infty}^{\infty} rd\Psi(F_R(r)), \]
  is a distorted expectation and $F_R$ is the distribution function of the risk.
- It may be observed to be below the expectation as it is an expectation under an altered probability that reweights losses upwards and gains downwards.
- The gap between the two depends on the concavity of $\Psi$. 
Distorted expectations may be computed from a simulation of cash flows $C_n$.

Let $C(n)$ be the sequence of cash flows in ascending order. The distorted expectation

$$D(C) \approx \sum_{n=1}^{N} C(n) \left( \Psi \left( \frac{n}{N} \right) - \Psi \left( \frac{n-1}{N} \right) \right).$$

From the relationship between inf and sup we have that

$$L(x) = -D(-C).$$
Cherny and M. (2009) introduced the distortion termed minmaxvar defined by a single stress parameter $\gamma$ as

$$
\Psi^{(\gamma)}(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}.
$$

- The concavity increases with $\gamma$ and the reweighting of large losses is infinite with large gains being discounted down to zero.
- $\Psi^{(\gamma)'}(u) \to \infty$ as $u \to 0$ and $\Psi^{(\gamma)'}(u) \to 0$ as $u \to 1$. 
For a small stress level of $\gamma = 0.05$ we present a graph of the residual risk ask price $L(x)$ as a function of the price of $XLF$ at the maturity of one month for a put option on JPM struck at 57.5 with the spot price at 59.69.

The four curves are for the four joint laws used in constructing the residual risk from the required conditional distributions.
Residual Ask Price for JPM on 20160302

- Gaussian Copula
- MBGIR
- MBGR
- t-copula

Spot Level vs. Ask Price for Residual
Hedging the Residual Risk Function

- The Residual Risk Function may be hedged by least squares regression of the target cash flows onto the cash flows delivered by the hedge instruments.
- Let $S_x$ denote the price of the hedging asset at maturity with the target being $L(S_x)$ as evaluated by the residual risk pricing function.
- For hedging assets of a bond, the stock, and options with strikes $K_i$ and $w_i$ unity for calls and zero otherwise the hedge cash flows for positions $\alpha, \beta, \eta_i$ are

$$H(S_x) = \alpha + \beta S_x + \sum_i \eta_i \left[ (1 - w_i) (K_i - S_x)^+ + w_i (S_x - K_i)^+ \right]$$

- For a grid of values for $S_x$ one may evaluate the matrix of payouts to each hedge instrument and regress $L(S_x)$ to determine the hedge positions $\alpha, \beta, \eta_i$.
- The cost of the hedge may then be computed.
- For the JPM put hedged by XLF options the hedge costs were 1.3877, 1.2730, 1.6435 and 2.1357 for the Gaussian Copula, MBGIR, MBGR and $0_t$ copula.
The CBOE Skew Index

- The Skew Index is a linear transformation of the risk neutral skewness.
- For a bilateral gamma density the skewness may be analytically be obtained from the derivatives of the logarithm of the characteristic function.
- Explicitly we have

\[
\begin{align*}
    d &= b_p c_p - b_n c_n \\
    \nu &= \sqrt{b_p^2 c_p + b_n^2 c_n} \\
    \tilde{s} &= 2 \frac{b_p^3 c_p - b_n^3 c_n}{\nu^3} \\
    \tilde{k} &= 3 + 6 \frac{b_p^4 c_p + b_n^4 c_n}{\nu^4}.
\end{align*}
\]

- for the drift, volatility, skewness and kurtosis.
- We present a graph of the CBOE Skew and the Bilateral Gamma calibrated skewness.
For every ten days between January 4, 2016 and December 31, 2019 call options on the VIX that were between five and fifteen percent out of the money were priced at four hedge costs for the four models.

There were 56 days and a total of 96 strikes.

The stress level was 0.025.

A table presents percentiles of the hedge cost price premia over market.
<table>
<thead>
<tr>
<th>Percentile</th>
<th>Gaussian</th>
<th>MBGIR</th>
<th>MBGR</th>
<th>t-copula</th>
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The same methodology was used to price skew index option calls that were similarly out of the money for a monthly maturity.

The ‘t’ copula was dropped as it priced options a bit too high.

A Table presents a sample Skew Index Options...
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<th>Date</th>
<th>SPY</th>
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<th>Strike</th>
<th>Gaussian</th>
<th>BGIR</th>
<th>MBG</th>
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VIX and Skew of the Skew Index itself

- One may fit a bilateral gamma model to out of the money skew options to infer the VIX and Skew Index of the Skew Index.
- We present a graph of such a fit.
- The VIX of the Skew was 5.72% and the skew index was 101.56.
- Also presented is the implied volatility curve for the Skew options.
Skew Option Price 20191231 MBGR S5 K2.5 Stress 0.025

bp = 0.0156

cp = 41.4629

bn = 0.0262

cn = 42.5175
Conclusion

- Joint laws are formulated for the pair of returns on an asset and a related asset with an active option market.
- The joint laws are used to simulate the conditional return on the asset to be hedged given the return of the hedging asset.
- This residual risk is priced using pricing to acceptability to generate a function of the hedging asset price that must be earned to cover the residual risk exposures.
- The function of the hedging asset price is earned using a portfolio of bond, stocks and options.
- Options of the hedged asset are then priced at the cost of this hedge.
- The methodology is illustrated by pricing JPM options using XLF and VIX options using SPY.
- The methods are then applied to price options on CBOE Skew Index that currently does not have an active option market.