Once Upon a Broker Time?

Order Preferencing and Market Quality [1]

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Abstract

We develop a dynamic, infinite horizon, microstructure model to study how priority rules determine market quality and investor welfare. We compare order preferencing, modeled as price-broker-time priority (PBT), to price-time priority (PT). Priority rules impact investors' choice between limit and market orders. When the tick is tight, trading rates are higher with PBT whereas investor welfare is higher with PT. The opposite holds for a wide tick. PBT endogenously results when brokers individually choose between PT or PBT. Our model has testable implications regarding systematic patterns in order flow, market depth, trade composition, and market fragmentation.

1 Introduction

Assets change hands in a variety of ways. While liquid stocks mostly trade on competing limit order books, other assets, such as corporate bonds, government bonds, derivatives, private equity, and real estate, are traded in over-the-counter (OTC) markets. A natural question that arises is whether there is a "one size fits all" priority rule in trading, or whether priority rules should be adjusted according to the fundamentals of an asset, or the structure of the market.

While U.S. exchanges currently manage their individual order books according to price-time priority (PT), International Exchange (IEX) wanted to follow Canadian markets and implement price-broker-time priority (PBT) as trading rule. PT implies that the first order at a new price becomes the first to trade at that price, and any subsequent orders are executed in the time order in which they are received. With PBT, this time priority is violated as orders from the same broker will execute against each other even if that broker's order was not the first in the queue. In this paper we compare market quality and investor welfare across price-time and price-broker-time priority settings, and study whether brokers have incentives to adopt PBT or prefer PT.

PT has not always been the leading allocation rule in U.S. financial markets. In 1996, while certain U.S. exchanges were still allowed to offer PBT, the U.S. Congress had the U.S. Securities and Exchange Commission (SEC) conduct a study on the effects of the practice. The SEC's "Report on the Practice of Preferencing" found no proof that it had negative effects on the market, but added that the "findings should not be taken to mean that the Commission believes that such adverse effects may not arise in the future". IEX's application to become an exchange has prevented it, for now, to offer PBT.

Regulation NMS in the U.S. and MiFID in Europe, for example, allowed the creation of new trading venues and accordingly trading to become fragmented across venues. Fragmentation implies that PT is broken *across* trading venues and PT only applies within a venue (see, e.g., Foucault and Menkveld 2008; Van Kervel 2015).

PT may not only be violated across venues but also *within* the same venue. PBT is currently in place in some important venues, including the Canadian markets (e.g., Toronto Stock Exchange), Australia (priority crossing), the Nordic countries (e.g., NASDAQ OMX), and continental Europe (e.g., Euronext's Internal Matching Service). More broadly interpreted, some off-exchange trading can also be seen as PBT as such trading may allow traders to jump the queue and give preference to matching within the same broker. Sub-penny trading in the U.S. is another form of PBT where queue-jumping is feasible. Sub-penny trading occurs when a trader provides meaningless price improvement and in this way undercuts orders in the order book to jump the queue and enjoy execution (e.g., Buti et al. 2015). Another example of violation of price-time priority are iceberg orders; these loose time priority on the undisclosed part (e.g., Buti and Rindi 2013). And Blockchain may also impact the priority structure of trading and post-trading as the miners confirming the trades may have different abilities in doing so, or charge different fees according to the speed in which confirmation is required.

In this paper we compare market quality and investor welfare when exchanges function according to PT or PBT. Furthermore, we study which priority rule endogenously prevails in unregulated financial markets by offering brokers individually the choice to adopt PBT or to opt for PT. We further investigate whether this market outcome aligns with the social planner's preferred outcome, and whether regulatory intervention is required. To do so, we build upon the work of Parlour (1998), Foucault (1999), Colliard and Foucault (2012), or Hoffmann (2014), and model a one-tick limit order book with infinite horizon where traders with different valuations for an asset arrive sequentially. We extend these models by allowing limit orders to stay in the order book for two consecutive periods. This requirement is the minimum needed to allow for a meaningful study of the impact of PBT (i.e., allowing that limit orders can jump the queue when from different brokers), and at the same time to keep the complexity of the model as low as possible. We further assume the existence of a dealer market or trading crowd willing to provide liquidity at the minimum tick such that arriving traders can always submit a market order (MO) independent of previous traders having put a limit order (LO) in the book. This design implies that investors' strategies only depend upon the state of the book at their own side. It further allows to study the impact of variation in the minimum tick. There are three relevant states of the order book for an arriving investor under PBT: the absence of a competing LO, the presence of a competing LO submitted by an investor of the same broker, or the presence of a competing LO submitted by an investor of another broker. The limited number of relevant states keeps our model tractable. The infinite time horizon allows us to identify the stationary probability distribution of the system, and to compute trading rates and investor welfare per period.

Our model allows for a meaningful comparison of PT and PBT, and to study whether brokers are willing to adopt PBT or not. In doing so, it generates novel insights about systematic patterns in order flow, market quality (depth, fill rates of LOs, and trading rates), and investor welfare, which may be empirically tested. First, priority rules in the limit order book generate systematic patterns in trade and order flow. These differ across PT and PBT. While LOs at one side of the market are more likely followed by LOs at the same side under PBT, the opposite applies for MOs. With PT, an investor's broker-affiliation does not impact order submission strategies. In contrast, with PBT, two consecutive LOs at the ask are less likely to come from same-broker investors than from different-broker investors. Second, priority rules impact market depth. Markets with PT have a higher average depth than with PBT. Limit order books that are "shallow" (i.e., empty) and "deep" (i.e., depth of 2) are more prevalent with PBT compared to PT. Queue jumping creates more often an empty book but also provides more incentives to join the queue if from another broker.

Third, priority rules determine trading rates, the composition of trades as well as fill rates of LOs. The effect hinges on the size of the minimum tick as this determines arriving traders' decisions to participate in the market as well as their decisions to go for market or limit orders. When the tick is small, trading rates are higher with PBT compared to PT. This result stems from different forces. First, first-in-line investors (i.e., arriving investors not facing a competing LO at the time of arrival) anticipate that their submission of a LO is less likely to be executed due to the potential of queue jumping. This makes a MO more attractive to them resulting in a higher trading rate. Second, and related, as first-in-line investors are less inclined to submit LOs with PBT, the trading rate with the crowd/dealer is higher but trading among investors is lower. Resultingly, priority rules also determine trade composition, i.e., trades with the crowd/dealer, and trades among investors employing the same or different brokers. When the tick is wide, trading rates are higher with PT than PBT, but trading rates among investors are higher. Fill rates of LOs are higher under PBT as first-in-line investors are more likely to submit MOs which is beneficial for the fill rate. Fourth, investor welfare is higher with PT than with PBT when the tick is small whereas the opposite holds for large ticks. With small (large) ticks, the composition of trades is less (more) favorable to generate investor welfare with PBT.

Finally, our model has implications for market design and fragmentation. When brokers can decide whether or not to adopt PBT and assuming brokers maximize their traders' welfare, PBT results as an equilibrium outcome. When the tick is small, brokers' are in a prisonner's dilemma situation. While both jointly would be better off not to offer PBT, it is a dominant strategy to offer PBT. The market outcome then differs from the socially preferred one. For wide ticks, the market outcome and socially preferred outcome coincide as both yield PBT. In a broader perspective, our model also explains how priority rules determine market fragmentation and why there is more off-exchange trading in markets organized by PT than PBT.

Our paper contributes to several strands of literature. First, we extend limit order book models such as Foucault (1999), Handa, Schwartz and Tiwari (2003), Parlour (1998), or Van Achter (2009). These models assume price-time priority and study patterns in order and trade flow. We incorporate an additional priority layer in the trade allocation rule, and extend these models to allow limit orders to stay in the book for two periods. Our results reveal that priority rules substantially shape investors' order submission decisions, order and trade flow patterns, market liquidity and investor welfare.

Second, our paper relates to recent work modeling over-the-counter markets featuring trading through marketmakers (Duffie et al. 2005), bilateral bargaining (Duffie et al. 2007), or a limit order book with random matching (Dugast 2017). Order flow in these setups stems from traders switching between "high" or "low" preference for asset ownership. Similar to these models, we focus on equilibria in which aggregate preferences are in a steady state limiting the dimensionality of the state space. As in Dugast (2017), our setup has a spread equal to the minimum tick, and limit orders queue before executing or being canceled. In order to compare how priority rules impact traders' choices between MOs and LOs, we build upon Parlour (1998) featuring traders' with a continuum of personal valuations for an asset, and where LOs can queue in the book.

Third, our paper relates to recent work on queuing and speed in limit order books as well as on sub-penny trading (i.e., offering a meaningless price improvement to jump the queue in the limit order book). Tick size creates rents for liquidity provision determining the length of the queue and the type of liquidity providers (Chao, Yao and Ye, 2017; Wang and Ye, 2017; Yueshen, 2014). We show that priority rules impact the length of the queue. Buti et al. (2015) investigate how sub-penny trading occurring in a separate sub-penny venue impacts the market quality and welfare on the public limit order book. They find that sub-penny trading is higher when the public book has high liquidity or a high tick-to-price ratio. Sub-penny trading negatively impacts liquidity in the public book. We model the practice of price-broker-time priority of which sub-penny trading is one example. The practice of PBT however is a more prevalent phenomenon, even in the U.S. markets if brokers take limit orders out of the back of the queue in order to fill them through an off-exchange trade. We obtain steady state strategies, study investor welfare, and endogenize the adoption decision of the trade-allocation rule.

Fourth, our work relates to the literature on tie-braking rules when traders are indifferent. One example of this is order preferencing, a practice when a dealer takes priority over same-priced orders or quotations entered prior in time. The SEC (1997) report mentions that "there are numerous practices by which a broker-dealer may obtain time priority over pre-existing customer orders." Past empirical work has found that preferencing could have negative effects on market quality (e.g., Bloomfield and O'Hara, 1998; Chung et al., 2004). Parlour and Seppi (2003) model intermarket competition with preferencing as a tie-braking rule when indifferent. They show that such preferencing for one or the other market substantially impacts the viability of particular market designs. Our paper focuses on PBT within one venue, and shows that tie-braking rules influence market outcomes. We further study the endogenous adoption of tie-braking rules and its consequences on investor welfare. In a broader perspective, our model also explains that priority rules may explain market fragmentation and dark trading, in particular off-exchange reporting of trades.

Our paper has regulatory implications by showing that imposing a unique trading protocol on widely heterogeneous financial markets (ranging from liquid stock markets to trading of illiquid bonds or private equity) is not optimal. While PT leads to greater welfare for markets with high liquidity, allowing for other trading protocols such as PBT may be preferred for markets exhibiting lower market quality.

This paper is organized as follows. Section 2 presents the set up of the model. In Section 3, we analyze the consequences of PT and PBT to market quality and investor welfare. Section 4, studies the endogenous decision of priority rules. Next, in Section 5 we examine the implications of priority rules to off-exchange trading and market fragmentation. Section 6, identifies the testable implications and provides regulatory insights derived from our model. In Section 7 we test the robustness by relaxing some of the assumptions of the model and Section 8 concludes the paper.

2 Model

Price-broker-time priority implies that "queue jumping" may occur: later submitted limit orders are executed first when the arriving market order employs the same broker as the later submitted limit order. However, within brokers time priority prevails. In this section, we build an infinite version of the discrete time model of Parlour (1998) describing the market as an open limit order book. Since our model has an infinite horizon, we are able to identify steady state equilibria and derive traders' optimal decisions regardless the specific time period they arrive to the market. We allow limit orders to stay in the book for two periods which permits us to observe marginal changes in the limit order book. Market orders are executed in the period of submission t, and limit orders execute if an appropriate counterparty willing to trade, arrives within the following two consecutive periods. We investigate and evaluate how endogenous strategic decisions are formed by investors under this environment. In order to better asses these effects, we compare our model operating under PBT to a benchmark case where PT prevails. We derive differences in trading rates and investor welfare and also identify differences in systematic trading patterns. This is of particular interest, since all information is considered to be common knowledge.

We denote by the parameter b, the investor's personal trade-off between submitting a market order today and consume immediately or aim for a better price, and consume in the future but facing the risk of non-execution. Therefore, b represents the agents' willingness to trade. The decision of the arriving investor denoted by $\phi(\beta, s)$, is viewed as a rational action given the state of the book s and her personal valuation β.[1] In contrast to models with a finite horizon trading time (see, e.g., Buti and Rindi 2013; Degryse et al. 2009; Parlour 1998) we obtain the stationary strategy of the trader, i.e., the distribution which describes the states of the book and the actions of the trader independent of the time t. The equilibrium of the model is defined as a vector of decisions, in which the arriving agent, based on the state of the book and her personal valuation of the asset, decides whether she will trade through a market order (against a standing limit order or at the crowd/dealer), submit a limit order or refrain from trading.[2] Our approach captures queue creation, even though our model has an infinite horizon. We identify steady state equilibria combining endogenous choices in PT and PBT. By allowing orders to stay in the book for more than one period we are able to study how depth is affected by PBT.

2.1 Set up

In this section, we introduce an infinite horizon model which captures the dynamic competition between traders. Following Foucault (1999), Colliard and Foucault (2012),

[1] We denote by b the trader's personal valuation i.e., the random draw, and by β the random variable.

[2] We reserve the article *she* for traders and *he* for brokers.

Hoffmann (2014), and Goettler et al. (2009) we model a dynamic market using discrete time intervals. Our market consists of one asset with value $V_t = V$, not subject to innovations, which is a common knowledge. At each time t, a trader arrives, having one unit of endowment to trade (buy or sell) and faces three choices. She may submit a market order, place a limit order or refrain from trading. Once her decision is made it cannot be altered and any order submission cannot be canceled, withdrawn or changed by the trader. Her limit order will either be filled within the next two periods or get removed exogenously after that. Our decision to allow orders to stay in the book for two periods is justified by choosing the least time interval required in order to study the effects of PBT. Traders thus have trading opportunities during two periods and do not have a discount rate within those two periods. Exogenous order cancellation is usual in open limit book models (see, e.g., Biais et al. 2015; Goettler et al. 2005). We denote the ask (bid) price by $A, (B)$ and by V the midquote i.e. the fundamental value of the asset. In order to avoid meaningless undercuts by small amounts and to better model real markets, following Parlour (1998) and Degryse et al. (2009), we assume that intense competition has set prices at the minimum tick Δ (i.e., $\Delta = A - B$).[3] Despite, having a constant tick of size Δ, we investigate the effects that the size Δ has to our model (see, Section 3.2). All agents in our model are assumed to be risk-neutral, maximizing their utility from trading. Since we solve for steady state equilibrium in an infinite time horizon game, we drop the time subscripts for notational simplicity, unless we want to emphasize the time sequence of events.

We assume the existence of two brokers, hereafter X and Y, each having equal market shares. Every trader, seller or buyer, is affiliated to a broker with her affiliation being randomly assigned. In our model 'brokers' should be interpreted as an additional layer that determines the priority among limit orders when at the same price. So brokers are our way of modeling order preferencing.[4] We assume that traders have a private trade-off between immediate and future consumption. This trade-off is captured by the

[3] The choice for a minimum tick is further supported by both theoretical and empirical literature (see, e.g. Dugast, 2017; O'Hara et al., 2015).

[4] For example, in Canadian and Nordic markets, the priority is actually organized as price-broker-time. In other markets, 'broker' could be seen as a way of order preferencing.

private valuation bV of the asset that traders have, where b is a parameter drawn from a uniform distribution β with support $[0, 2]$. A trader, upon arrival, can trade to the market via a market order or opt for a better price by posting a limit order, but face a non-execution risk. A value of b closer to zero is more likely to lead a seller to a MO, since she has almost no private valuation for the asset, while a value b closer to two will create high personal valuation for a buyer and most likely will lead to a MO to buy. Apart from being a seller or a buyer, and the willingness to trade, the decision of the arriving agent is also influenced by the state s of the limit order book. In our model, broker affiliation and the length of the competing queue are two key determinants for her decision, since these affect the execution probability of her limit order, which depends on the priority rules implemented. The arriving trader at time t needs to dynamically solve the problem of her utility maximization, which also depends on the future arriving traders, their type and private valuations. The trader will make her decision about the action she will perform based on her inclination (buyer or seller), her information about the state of the order book, her private valuation, and under PBT her broker affiliation. Then she determines the value b that would make her indifferent between trading with a market, a limit order or refrain from trading. We also assume the existence of a dealer market or a crowd that is willing to provide liquidity at the ask and bid (see, e.g., Parlour and Seppi 2003; Seppi 1997). We view the crowd or dealers as traders having a private valuation of one for the asset and order processing costs equal to $|p - V|$, where p denotes the ask or bid price depending on the inclination of the trader.[5] [6] Since the crowd/dealer are essentially indifferent in the making of liquidity, by assumption, they chose to trade and not refrain from the market. We further assume that limit orders submitted by investors have priority over the crowd/dealer. The private trading gains of arriving investors trading against the crowd/dealer or against a standing limit order

[5]When a dealer or a member of the crowd acts as a counterparty to a trade, her private gains equal her losses due to the order processing costs. Hence her action does not generate social benefits and does not add to investors' welfare.

[6]By a simple argument, we can view the crowd as a dealer market, where the dealer has inventory costs. Then in that case also the welfare of the dealer would contribute zero to the social benefit. Still, if dealer's order processing costs were assumed to be lower than the difference $|p - V|$, then following Colliard and Foucault (2012), we would focus on investors' welfare.

submitted by a trader are identical.

We will set the stage by an example illustrating the timing of events. Assume that the arriving trader is a seller, affiliated with broker X and faces a state s in the book. Given these parameters she solves her decision problem in which she determines a private valuation b_0 which would make her indifferent between placing a market order or submitting a limit order. Since she is a seller, if her private valuation b satisfies the inequality $b \leq b_0$, then she submits a market order to sell. Let $P = P(A, B, X, Y, s)$ be the probability of execution of a limit order then her expected gains from a sell limit order are $P(A - bV)$. We can immediately see that for large values of b the seller will refrain from trading. In particular for any $b > A/V$ she prefers not to trade. For any value of b, in the interval $[b_0, A/V]$, the trader opts for a limit order.

For the following, we denote by $b_k^S(s),\ k \in \{x, y\}$ the private valuation of a seller, affiliated with the broker X or Y who faces a state of the book s, and is indifferent between submitting a market or a limit order. Similar notation will be used for a buyer. We note that the private valuation $b_k^S(s)$ depends also on the trading protocol. Thus every particular state of the book, will create different cut-off values between PT and PBT.[7] We denote the state of the book s by $(\mathbf{q}_1^{\mathbf{i}}, \mathbf{q}_2^{\mathbf{j}})$, where $\mathbf{q}_1^{\mathbf{i}}$ is a vector that represents the orders standing at the bid and the superscript $\mathbf{i}$ denotes the periods for which orders have been in the book as well as the broker affiliation. Thus for example, $\mathbf{i} = (1, k)$ or $(2, k)$, $k \in \{x, y\}$ implies that the order is in the book already for one or two periods, respectively, and was submitted by a trader affiliated to broker k. For notational simplicity we write $\mathbf{i} = k$ instead of $\mathbf{i} = (1, k)$, $k \in \{x, y\}$, for limit orders standing for one period. Similar interpretation is used for $\mathbf{q}_2^{\mathbf{j}}$. Following literature standards, we denote standing orders at the bid with a positive sign and and at the ask with a negative sign. To illustrate, we provide a few examples: $(0, 0)$ denotes an empty book on both sides; $(0, -1^y)$, a book with no limit orders at the bid and a limit order standing at the ask for one period submitted by a seller affiliated to broker Y; $([1^{2,x}, 1^x], 0)$, a book with two limit orders standing at the bid for two and one periods respectively both submitted

[7]For example if s is the empty state, then $b_k^S(empty)$ in PT is different from $b_k^S(empty)$ in PBT. However, it is always clear the trading protocol we are referring to.

through broker X, and no limit orders at the ask. Figure 1, shows a a fraction of the decision problem that a seller faces when she faces no competition at the book.

*** **Please Insert Figure 1 about here*****

Note that under the two period cancellation rule, not all states are feasible. For example $[(1,1),-1]$ is not a feasible state regardless the broker affiliation and the number of periods standing, because it would require for orders to stay in the book for, at least, three periods. The proposition below and its proof follow from the discussion above.[8]

Proposition 1 *A seller's cut-off values b_k^S, $k \in \{x, y\}$ depend on her broker affiliation, the ask and bid prices and the state s of the book at her own side. In particular, they do not depend on the state of the book at the opposite side.*

Proposition 1 is a consequence of our model's assumptions, and in particular the two period lifespan of a limit order and the existence of a crowd/dealer.[9] The importance of Proposition 1 is that it allows us to limit the number of states of interest for each arriving investor. In particular a seller, will form her decision endogenously but independently of the state of the book at the bid.

The cut-off values identify the exact point for which a trader is indifferent between submitting a MO upon arrival or opt for a LO. Thus these points can be expressed as a function of the fundamental value, the tick and the probability of execution. In particular we have the following proposition.

Proposition 2 *Let b_k^S, $k \in \{x, y\}$ be the cut-off value that makes the seller indifferent between a MO and a LO. Let V denote the fundamental value of the asset, P the probability of execution of the LO and Δ the tick size. Then*

$$b_k^S = \frac{(2V - 2PV) - \Delta(1 + P)}{2V - 2PV}$$

[8]The symmetries of our model allows to focus on sellers. All results can be reproduced for buyers.

[9]The model can be extended to more periods, but it does not add to intuition and at the same time makes notation and calculations more cumbersome.

We note that in the above proposition the probability of execution depends on the state of the book. As the probability of execution P increases, the higher the incentives of the trader to submit a LO as opposed to MO. That is reflected in the decrement of b_k^S. As Δ increases, the benefits from submitting a MO decrease and thus b_k^S decreases.

We say that a state of the book s' is irrelevant to s, if $b_k^S(s) = b_k^S(s'),\ k \in \{x, y\}$. Otherwise the states are relevant for the trader. For a given state s of the book, we denote by $-s$ its symmetrical. For example if $s = (1^{2,x}, 0)$ then $-s = (0, -1^{2,x})$.

Proposition 3 *Let $b_k^S(s)$, $(b_k^B(s))$, denote the cut-off value that makes a seller (buyer) indifferent between submitting a LO or trading via a MO, when the state of the book is s. Then for all $k \in \{x, y\}$ the following hold:*

(i) $b_k^B(s) = 2 - b_k^S(-s)$.

(ii) $P(b \geq b_k^B(s)) = P(b \leq b_k^S(-s))$.

The remaining of the section is devoted in depicting the decision problem of the arriving seller. Consider first, the case of PT. Using Proposition 1, we deduce that only two groups of states are relevant: one group comprises states in which she faces no competition in the limit order book and constitutes of the empty book on her side and the irrelevant states to that, and the second group contains the states that place her second in line on her side as well as its irrelevant states to that.[10] Any limit order submission which creates an irrelevant, for the seller, to an empty book state, will result in forming the same decision as if the book was completely empty on her side. Similar reasoning holds when the trader faces competition on the book. Under PBT, the relevant states for a seller are the same as before, but now distinguishes between having competition by a broker of same or opposite affiliation. Thus under PBT the relevant states augment by one. In the following we use interchangeably the terms 'no competition' and being 'first in line', for a trader that faces an empty book state (or irrelevant to that) upon arrival to the market. Respectively, we say that a trader faces 'tough' ('soft') competition if the book contains queue, formed by a trader with the same inclination and employing

[10]Table C2 in the appendix depicts them in detail. We note that a trader in PT is not interested in distinguishing between *same* and *opposite* broker affiliation.

the same (different) broker. For competition under PT, we use the term 'intermediate' competition to describe the states in which the seller finds queue at the book formed by traders with same inclination.[11] [12]

In order to depict the solution to the decision problem of the seller which in term identifies the equilibrium of the model, we need to consider both broker affiliations. The seller needs to solve the following system.

$$\Sigma = \begin{Bmatrix} \Sigma_x \\ \Sigma_y \end{Bmatrix}, \tag{1}$$

where Σ_k defines a set of equations that makes the k-seller, $k \in \{x, y\}$, indifferent between a market and a limit order considering the state of the book upon her arrival. Since brokers have equal market shares, we obtain that $\Sigma_x = \Sigma_y$ and hence we can focus to an x-seller.[13] Its solution generates three cut-off values for the arriving x-seller.

If the arriving trader opts for a limit order, then the execution probability depends on the flow of future traders.[14] For every time period, the arriving trader needs to account for all potential flow of traders when calculating her gain from submitting a limit order. At the same time, she needs to account also for the particular state of the book at her side, at the time of her arrival. Let $G_b = G_b(A, B, X, Y)$ denote the probability of execution of a submitted sell limit order, if the next arriving trader is a buyer taking the action of submitting a limit order, or decline trading and by $G_s = G_s(A, B, X, Y)$ the execution probability of a submitted order, if the arriving trader is seller who either submits a market order, a limit order or refrain from trading.[15] Notice that G_b and G_s do not depend on the state that the seller arrives at t. This is a direct consequence of the two period exogenous cancellation rule. Therefore, each trader needs to account for the

[11]The term *intermediate competition* does not imply that there is some other type of competition for a trader that faces queue under PT priority rules. It refers as opposed to *tough* and *soft* competition that we observe in PBT.

[12]If it is clear by the content, for PT and PBT we may write that a *trader is second in line*, for a trader that faces competition upon arrival to the market.

[13]Because of size symmetry, a seller affiliated to broker X has the same cut-off values with a seller affiliated with broker Y.

[14]For example, for a sell limit order at time t on an empty book, one potential flow would be that at the next period a buyer arrives and submits a limit order to buy. That means that the seller in order to obtain an execution would need at $t + 2$ the arrival of buyer who would submit a market order.

[15]The exact form of G_b and G_s is given in Appendix A.

same values of G_b and G_s. What distinguishes the execution probability of a sell limit order related to the state of the book of the arriving seller, is the action of the subsequent buyer, should she arrives at $t+1$, and whether she would submit a market order. In this case, the state of the book could determine an immediate execution, a preferential execution or neither. Given this discussion, we are ready to define the system Σ_x that the arriving x-seller needs to solve.

$$\begin{cases} (B - b_x^S(0,0)V) = \big(\frac{x}{2}P_{t+1}(b \geq b_x^B(0,-1^x)) + \frac{y}{2}P_{t+1}(b \geq b_x^B(0,-1^x))\big) \\ \qquad (A - b_x^S(0,0)V) + (G_b + G_s)(A - b_x^S(0,0)V) \\ (B - b_x^S(0,-1^y)V) = \frac{x}{2}P_{t+1}(b \geq b_x^B(0,-1^x))(A - b_x^S(0,0)V) \\ \qquad + \frac{y}{2}P_{t+1}(b \geq b_y^B(0,-1^x)) \\ \qquad \big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) + \frac{y}{2}P_{t+2}(b \geq b_y^B(0,0))\big) \\ \qquad (A - b_x^S(0,-1^y)V) + (G_b + G_s)(A - b_x^S(0,-1^y)V) \\ (B - b_x^S(0,-1^x)V) = \big(\frac{x}{2}P_{t+1}(b \geq b_x^B(0,-1^x)) + \frac{y}{2}P_{t+1}(b \geq b_y^B(0,-1^x))\big) \\ \qquad \big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) + \frac{y}{2}P_{t+2}(b \geq b_x^B(0,0))\big) \\ \qquad (A - b_x^S(0,-1^x)V) + (G_b + G_s)(A - b_x^S(0,-1^x)V) \end{cases} \tag{2}$$

In System 2, both x and y are equal to 0.5, but we include them as such in order to demonstrate the required trader flow and broker affiliation needed to obtain execution. In each of the equations, the LHS defines the gains of the seller from submitting a market order depending on the state of the book that she faces. The RHS reports the gains from a limit order to sell. In the first equation, we notice that the seller may get execution at the next period, since she faces no competition, in comparison to the last equation where she needs to stand in the line for two periods. The equation in the middle describes the difference in the execution probability which is due to PBT. The seller, even second in line, can get an execution the following period through her broker flow by the use of preferencial services. Under PT, the arriving seller solves a similar but simplified version of System 2.

The arriving trader, can perform one of three actions. Trade via a market order, submit a limit order or refrain from trading. Her decision depends on the state of the book and her willingness to trade b. The critical values which identify the actions of a trader are determined by the solution of System 2. One of the main implications of our model is that under PBT, a trader faces three different cut-off values in relation to the submission of a LO, while in PT these reduce to two. The reason being that under PBT a trader's decision is also affected by the structure of the queue, and in particular she distinguishes between tough and soft competition. In PT, a trader facing a non empty book on her side, always faces intermediate competition, regardless the structure of the queue. The book is formed based on traders' actions. Since the orders do not stay in the book indefinitely and are canceled exogenously, the states of the book are finite and define a Markov chain. Let $\mathcal{M}$ denote the transition matrix of the Markov chain, i.e. the probability that a trader transits from one state of the book to another. We note that these probabilities depend on the random sequence of the arriving traders and from the willingness to trade. Following Foucault et al. (2005), we derive the stationary probabilities ρ of the system. The stationary distribution shows the probability that the arriving trader faces a specific state of the book independent to the exact time of the arrival. In contrast to Degryse et al. (2016), we identify the likelihood that an arriving trader faces a specific state of the book and not performing a certain action.[16] Tables C1 and C2 in Appendix C depict them in detail. The steady state equilibrium is defined as the left eigenvector of the transition matrix, which is given as the solution of the matrix equation

$$\rho = \rho\mathcal{M}.$$

We need to notice that PT in comparison to PBT differ in both the number of the relevant states as well as the transition matrix for a given trader. In PT, for example, even though being second in line after a trader of the same or opposite affiliation defines two distinct states, the arriving trader treats them as the same as it is immaterial to

[16]Essentially these are equivalent. From the steady state distribution of the book and the distribution of β, we can derive the steady distribution of the actions.

her. The defined Markov chain, in both models is irreducible and aperiodic and thus we obtain unique distribution for both systems (see Appendix B).

At this point, a discussion on the execution probability of a limit order under PBT is in order. Notice that, *ceteris paribus*, a seller arriving on the market who submits a limit order, has higher probability of execution when facing an empty book on her side rather than joining the queue. Respectively, for any particular state that leads to joining the line, if the trader is subject to preferential execution then her execution probability increases. Given this remark, we are able to formulate the following proposition, which identifies the relation between the cut-off values.

Proposition 4 *Assume an x-seller arrives at the market, then the following holds:*

i) $b_x^S(0,0) < b_x^S(0,-1^y) < b_x^S(0,-1^x)$*, under PBT.*

ii) $b_x^S(0,0) < b_x^S(0,-1)$*, under PT.*

iii) $b_x^S(0,0)$ *under PT is less than* $b_x^S(0,0)$ *under PBT.*

iv) $b_x^S(0,-1)$ *under PT is larger than the average of* $b_x^S(0,-1^y)$ *and* $b_x^S(0,-1^x)$ *under PBT.*

Similar relations for the cut-off values of a y-seller hold. Proposition 4 provides an insight on the behavior of traders when facing a queue. According to our intuition, we observe that the longer the queue that a trader faces, the more aggressive she becomes and the more likely is to submit a market order. This is in accordance with both theoretical and empirical findings (see, e.g., Parlour 1998; Ranaldo 2004). In addition Proposition 4 reflects the different value added to limit orders according to the position on the queue. This is reflected also to Dahlmström et al. (2017), where they study endogenous limit order cancellations as response to book changes. In equilibrium, our model endogenously determines the behavior of traders. We are able to identify systematic patterns in the actions of arriving agents even though they arrive independently and have random draws from a uniform distribution related to their personal valuations. Since these actions are determined by the length and the structure of the queue as well as the expected behavior of future arriving agents, the systematic patterns differ according to the priority rule that is in place. This is a novel result and in accordance to Degryse et al. (2009), and creates empirical and testable implications. We solve System 2, for PBT and PT, and obtain the cut-off values for the arriving sellers which define endogenously their actions. These are function of the midquote and therefore of the ask and bid prices. Let $\phi_t(\beta, s)$ denote the strategy of the arriving trader, given her willingness to trade β and the state of the book S that she faces. The following proposition formulates the empirical predictions of our model which are related to the cut-off values and are independent of the level of the ask and the bid prices.

Proposition 5 *PBT predicts different systematic behavior patterns in comparison to PT. In particular the following hold:*

i) There is a higher likelihood at t+1 to observe a limit order at the ask with PBT rather than PT, if the action at t was a limit order at the ask, i.e.,

$$P[\phi_{t+1}(\beta, s) = -1| -1, PBT, t] > P[\phi_{t+1}(\beta, s) = -1| -1, PT, t].$$

ii) There is a higher likelihood at t+1 to observe a market order (transaction) at the ask in PBT rather than PT, if the action at t was a transaction i.e.,

$$P[\phi_{t+1}(\beta, s) = Tr^A| \, Tr^A, PBT, t] > P[\phi_{t+1}(\beta, s) = Tr^A| \, Tr^A, PT, t].$$

iii) Assume at t there was a limit order submission at the ask. Then it is less likely at t+1 to observe a limit order from the same broker in PBT rather than PT, i.e.,

$$P[\phi_{t+1}(\beta, s) = -1^x| -1^x, PBT, t] < P[\phi_{t+1}(\beta, s) = -1^x| -1^x, PT, t].$$

iv) Assume at t there was a limit order submission at the ask. Then it is more likely at t+1 to observe a limit order from different broker in PBT rather than PT, i.e.,

$$P[\phi_{t+1}(\beta, s) = -1^x| -1^y, PBT, t] > P[\phi_{t+1}(\beta, s) = -1^x| -1^y, PT, t].$$

v) In PT, is more likely to have a reversal to a market order on the same side after the submission of a limit order, rather than in PBT, i.e.,

$$P[\phi_{t+1}(\beta, s) = Tr^A| -1, PBT, t] < P[\phi_{t+1}(\beta, s) = Tr^A| -1, PT, t].$$

Similar formulations hold for the bid side. Further empirical predictions of the model are presented in Section 6. From (i) of Proposition 5, we expect that under PBT we are going to observe longer queues on the second level of the PBT book. In particular, with PT it is more likely that there will be depth in the book; with PBT it is more likely that we will have *double depth* in the book i.e., two standing LOs. A more extensive investigation on the depth of the book is presented in Section 3.4.

2.2 Model Equilibrium

In this section, without loss of generality, we normalize the fundamental value V to one and we solve System 2 for a wide variety of tick sizes, but illustrate in more detail the case where $\Delta = 0.2$, implying bid and ask prices of 0.9 and 1.1 respectively.[17] In our analysis we include both PBT and PT, which serves as a benchmark protocol (Butti et al., 2017).[18] Table 1 reports the solution of System 2 for PT and PBT providing the cut-off values for both cases when $\Delta = 0.2$. These are the values for which traders are indifferent between submitting market or limit orders. These values which relate the willingness to trade and the actions of a trader are endogenously determined by our model. Our solution does not depend on the particular tick selection. Figure 2 illustrates the cut-off values for a seller for all tick sizes.[19]

*** **Please Insert Table 1 about here*****

*** **Please Insert Figure 2 about here*****

Table 1 reveals that an arriving trader when facing 'no competition' is more aggressive under PBT than PT. In particular, her willingness to trade b increases, by 14.85 b.p. This increment reflects the probability that being first in line (and thus not facing competition

[17] We solve our model for multiple values of fundamental prices and granular tick sizes, essentially providing *proof by exhaustion*.

[18] As both brokers have identical market shares, the behavior of x and y traders are symmetrical and hence, we report results for a seller affiliated to broker X.

[19] The fundamental value $V = 1$, so the tick ranges in the interval $[0, 2]$. Equivalent the half tick ranges between $[0, 1]$.

from an ex-ante point of view) is subject to losing her line priority by a following opposite-broker trader who may perform queue jumping. PBT has not a uniform effect on traders that face competition. It incentivises traders to join the line after an opposite-broker trader, since they may queue jump later on (i.e., enjoy preferential execution). However, 'second in line' traders behind a same-broker trader, have less motives to join the queue since they cannot perform queue jump, but still in the following periods, may lose their line. Proposition 4 has identified this particularity; a traders decision to join the line is affected by the structure of the queue. This is reflected by the 3.29% increase in the willingness to trade when she faces a line from her broker. In Figure 2, we report how cut-off values for a seller vary across ticks. Panel C identifies the difference in aggressiveness of a trader according to the competition that she faces. In PBT a trader under competition, trades always more with LOs. The opposite holds for traders who are on an empty or irrelevant to empty state. Using the cut-off values, we are able to calculate the transition matrix $\mathcal{M}$, and obtain the steady state probabilities of our system. Table 2 reports the consolidated steady states for $\Delta = 0.2$.

*** **Please Insert Table 2 about here*****

*** **Please Insert Figure 3 about here*****

We observe that the likelihood of observing an empty book in PBT is 12.42 b.p. higher than with PT. This can be attributed to the fact that PBT creates more often an empty book, via queue jumping combined with order cancellation. When we compare the steady states for a trader that faces competition we note that this is 11.41 b.p. lower with PBT than PT, reflecting the benefit from joining the queue under PBT in comparison to PT. Figure 3 depicts the consolidated steady states for all tick sizes under PT and PBT. In Panels A and B we decompose the states that lead to 'no-competition', to the 'empty state' and to the states 'irrelevant to empty'. However, as the tick increases, queues start to form. These results in the increase of the likelihood in observing competition states. Panel D shows the difference between PBT and PT in the probability of observing competition for the arriving investor. We notice that the result is not uniform. For small ticks, it is more likely to observe competition in PT. The opposite holds for wide ticks. This compositional change will be one of the main topics of our next section.

3 Priority Rules, Market Quality and Investor Welfare

In this section we use our model to compare the impact of priority rules on market quality and investor welfare. This is of particular interest to regulators and social planners since it provides an insight on the effects that priority rules have. Should PBT be the new standard in regulating financial markets? To answer this question we focus on market quality by capturing liquidity through trading rates, depth of the limit order book, and fill rates of limit orders. Investor welfare is quantified as investors' gains from trading. We report trading rates and investor welfare for both trading protocols.

3.1 Unconditional and Conditional Trading Rates

We define the unconditional total trading rate, hereafter trading rate as the likelihood that any agent (investor, crowd/dealer) will participate in a transaction.[20] This is equivalent to defining trading rate as the likelihood the arriving investor i.e. a seller, will submit a market order. For the calculations we identify the probability that a trader submits a market order in each state of the book based on the stationary distribution. With a slight abuse of notation we write $P(MO)$ for the more correct $P(MO|\ seller)$.[21] Let S denote the set of all possible states s of the book indexed over a set I. Then the trading rate is defined:

$$TR = P(MO) = \sum_{i \in I} P(MO| \text{ state } s_i) P(s_i).$$

[20] Since the crowd/dealers also participate on transactions via LOs, we account also for them.

[21] We calculate the trading rate of an arriving seller and not of an x- seller, unless explicitly stated. Since sellers arrive through brokers having equal market shares, we obtain that $P(MO|\ seller) = P(MO)$.

Proposition 5 shows that investors' behavior depends on the competition that they face on the book. To obtain a better understanding of the trading rate, we decompose the states of the book that place the seller to an 'empty and irrelevant to empty state' and to those under which she faces 'competition' upon arriving. Let $B_F, (B_S)$ be subsets of indexes of I for which the arriving trader faces no competition (competition) when she arrives to the market and let $TR_F, (TR_S)$ denote the corresponding trading rate, then

$$TR_F = \sum_{i \in B_F} P(MO| \text{ state } s_i)P(s_i), \text{ and } TR_S = \sum_{i \in B_S} P(MO| \text{ state } s_i)P(s_i),$$

and it holds that $TR = TR_F + TR_S$. Table 3, Panel A, summarizes the findings when $\Delta = 0.2$.

*** **Please Insert Table 3 about here*****

We observe that PBT increases the trading rate compared to PT, from 39.688% to 39.708% which corresponds to a 2.11 b.p. increase. This is a direct result from differences in priority rules which have an overall result in the increment of liquidity taking, leading to a less liquid book. Intuitively, we expect that PBT has not a uniform effect on all traders. In particular PBT urges a greater proportion of arriving traders to submit market orders when facing no competition, i.e., when being first in line. Since later arriving traders may jump their LO, they become more aggressive in order to dampen this effect. On the opposite, traders that could join the queue and are therefore second in line, have with PBT more incentives to submit a LO and thus contribute to the making of liquidity.[22] A trader that faces competition under PT has a 1.63% higher probability in submitting a MO in comparison to PBT. In PBT, the investor's broker affiliation is essential to her decision. Therefore, preferencing incentivizes differently traders. To investigate this premise, we calculate the trading rate for a seller given the queue structure where we also account for her broker affiliation. We define the

[22]The terminology first (second) in line refers to a trader that if she choses to submit a LO she will find herself facing no competition (competition) respectively.

conditional trading rate for the first and second in line as the probability that a trader will submit a market order, given her position on the line, i.e.,

$$P(MO|k \text{ in line}) = P(k \text{ in line})^{-1} \sum_{i \in B_j} P(MO| \text{ state } s_i) P(s_i),$$

where $(k, j) \in \{(first, F), (second, S)\}$. We expect to find differences based on the particular formation of the queue. Table 3, Panel B, reports the trading rate of a seller given her position in the line. We observe that traders facing no competition are more aggressive under PBT, and the opposite holds for traders that join the queue. In Panel C, we depict the behavior of a trader that faces competition given the broker affiliation of the trader that submitted the LO standing. Under PT, we do not observe any difference since traders do not distinguish between brokers. However under PBT, an agent who faces competition by a trader having same-broker affiliation, has a 3.29% higher probability to trade via MO as opposed to the case were the trader had different-broker affiliation.

3.2 Trading Rates and Tick Size

Our model presumes that competition has set ask and bid prices to the minimum tick Δ. However, all markets do not have the same degree of liquidity and the same holds for traded assets. U.S. financial markets are highly liquid and for example the average quoted spread on Nasdaq was equal to 1.77 cents (Bessembinder, 2003). In addition, the degree of liquidity does not remain constant in time. Between 2007-2009, there was a significant drop in liquidity in the euro area as stated in ESRB (2016). This section investigates the effects of various tick sizes on the trading rates with PT and PBT. Our model is able to incorporate the heterogeneity which is created by the tick variation. In the two extremes in which there is no tick, or the half tick is equal to the fundamental value, PT and PBT coincide because in the first case traders trade only via MOs against the crowd/dealer and hence there is no queue accumulation in order to observe the structural differences of PBT, and in the second case, essentially the market collapses since traders do not submit any MOs and post only LOs.

Intuitively as the tick is narrow relative to the 'valuation differences between potential buyers and sellers', we expect to observe higher trading rates with PBT rather than PT. The reasoning is that with a tight tick, traders facing no competition, which is the majority in this case, anticipate the possibility of losing their line, so they act more aggressively by posting more often MOs. As a result we have a higher trading rate. However as the tick increases and queues start to form, traders who face competition become increasingly important. Under PBT these traders have more incentives to join the queue, and hence we expect to observe the difference in trading rate between PT and PBT to decrease and after a critical point to reverse. This critical point is expressed in terms of the fundamental value and the support of the β distribution. We define $\tilde{\Delta}$ to be the critical half tick size for which trading rates under PT and PBT are identical. We have that $\tilde{\Delta}$ is approximately equal to $2 \cdot 0.44934$, and we refer to any tick Δ smaller (larger) than $\tilde{\Delta}$ as tight (wide).[23] Figure 4, Panel A and B, illustrates the different levels of trading rates in PT and PBT for all tick sizes as well as the trading rate by traders facing no-competition and competition. Panel C, illustrates the difference between PBT and PT. We observe that trading rate becomes higher in PT when ticks are tight and in PBT when ticks are wide. In Panel D, we observe that there is a change in the behavior of a trader in PT and PBT, under soft competition.

*** **Please Insert Figure 4 about here*****

The following Proposition depicts the change in the trading rates according to the size of the spread.

Proposition 6 *Let Δ denote the tick size. If $\Delta < \tilde{\Delta}$, the total trading rate is higher with PBT in comparison to PT. If the tick is larger than $\tilde{\Delta}$, then the opposite holds.*

[23]For an arbitrary support of the distribution β and a fundamental value V, the critical tick is approximately equal to $2 \cdot 0.44934 \cdot V(b_{max} - b_{mean})$, where b_{max} is the maximum and b_{mean} the mean of the willingness to trade distribution.

3.3 Investor Welfare

In this section we compare investors' welfare between PBT and PT. Following Degryse et al. (2009), our measure of ex ante investor welfare is based on the behavior of rational traders and therefore is equal to the ex post. We first introduce our measure of investor welfare for a dynamic limit order book, then we proceed with our main results. Measuring welfare W, has always been one of the main driving forces in evaluating a policy.[24] Our measure estimates investors' gains from trading excluding the crowd/dealer as in Colliard and Foucault (2012), and Parlour and Seppi (2003). We denote by $\pi(\phi_t(\beta, s))$, the investors' gains from trading at t, following the strategy $\phi_t(\beta, s)$. We note that the strategy depends on the private valuation of the asset, measured by the β parameter and on the state of the book s. Hence,

$$W \equiv E(\pi(\phi_t(\beta, s))).$$

The expectation is taken with respect the product space created by the willingness to trade β and all the steady states of the book s. We calculate the welfare generated in each of the intervals created endogenously by the cut-off values, given in Table 1 (e.g., $A_1 = [0, c^S(0,0)]$, etc.), and let $\pi_i(\phi_t(\beta, s))$ denote the welfare generated over the A_i interval. Then we evaluate the total welfare as follows:

$$W = \sum_{i=1}^{4} E(\pi_i(\phi_t(\beta, s))).$$

We denote by I, the set of indexes that enumerate all states of the book S. Then we further analyse welfare as:

$$W_i \equiv E(\pi_i(\phi_t(\beta, s))) = \int_{A_i} \sum_{j \in I} (w_j \cdot P(s_j)) dF_\beta.$$

We note that w_j depends on the interval on which the welfare is evaluated as well as the state of the book and represents the generated gains. If, for example, (b_j, s_j) is

[24] For welfare measurements in open limit order market see, e.g., Hoffmann (2014); Degryse et al. (2016); Goettler et al. (2005); Butti and Rindi (2013).

such that would lead the arriving trader to submit a MO then w_j is equal to $(B - b_jV)$ while if she opts for a LO then $P(LO_j)(A - b_jV)$, where the subscript j, denotes that the execution probability of the LO depends on the state of the book s_j.

*** **Please Insert Table 4 about here*****

Table 4, reports the results from the welfare analysis for $\Delta = 0.2$. We measure the welfare generated by rational traders arriving to the market and submit either a market order, or a limit order. We also capture the expected gains of a seller given her position in the queue. With PBT, even though the trading rate is higher, investor welfare is decreased by 4.81 b.p. This may seem counter-intuitive at first, but is in accordance with the key elements of our model. When $\Delta = 0.2$ and therefore smaller than $\tilde{\Delta}$, the higher trading rates observed in PBT are mostly trades against the crowd/dealer and not with investors having submitted LOs in previous periods. The rationale lies in the fact that for a small tick, queues do not form easily and thus there is a lower likelihood for a LO to exist.

Opposite results apply for a wide tick. Traders facing no-competition, who are always more aggressive in PBT, generate with their actions higher investor welfare when the tick is large (i.e., larger than $\tilde{\Delta}$) since they trade more often against a LO standing submitted from a trader, which is beneficial as it generates higher investor welfare.[25] Figure 5, Panel A, shows the difference in investor welfare for all ticks. Under a tight tick, PT generates higher welfare than PBT whereas the reverse holds for wide ticks. In Panel C, we measure the difference in the counterparties involved in a transaction between PBT and PT. There are more (less) transactions having as a counterparty investors (the crowd/dealer) in PT (PBT) under tight (wide) ticks rather than in PBT (PT). This shows, as expected, that the main force for investor welfare is transaction between traders. In order to illustrate the compositional change between tight and wide ticks, Panel D, shows the difference in trading rates for different tick sizes, by competition and by the different categories of counterparties involved. We observe that trades by

[25] A similar argument can be found in Colliard and Foucault (2012), where the implementation of a higher fee can have a positive effect on welfare, since it increase the likelihood that two traders will trade directly.

first in line traders are higher in PT in tight spreads, but this rapidly changes in favor of PBT.

*** **Please Insert Figure 5 about here*****

Furthermore, a sell market order against a trader generates $(b_{buy} - b_{sell})V$ while against the crowd/dealer is only the welfare created by the party that submits the market order, i.e. $B - b_{sell}V$. If we decompose B to $V - \Delta/2$ and therefore

$$B - b_{sell}V = (1 - b_{sell})V - \frac{\Delta}{2},$$

then we observe that trading with the crowd/dealer further decreases the welfare by the half-tick. That reduce is more prominent when the tick is large than small. This is an additional force that acts in favor of the welfare generated under PBT with wide ticks. Overall we observe a compositional change between PT and PBT related to welfare. This has regulatory implications since any policy should be evaluated side by side with liquidity of the market. This is elaborated in the following proposition.

Proposition 7 *Let Δ denote the tick size. If $\Delta < \tilde{\Delta}$, then investor welfare is higher under PT than under PBT. If the tick is larger than $\tilde{\Delta}$, investor welfare is higher under PBT than under PT.*

3.4 Fill Rate and the Depth of the Limit Order Book

Proposition 5 identifies systematic trading patterns. This has important implications to fill rates and the depth of the limit order book. Fill rate is defined as the execution probability of a submitted limit order and have been studied both theoretically (Colliard and Foucault, 2012) and empirically (Malinova and Park, 2015). Depth is a major characteristic of market quality, and refers to the ability of the book to sustain large market orders without impacting the price (Kyle, 1985). We note that the higher the fill rate, the higher the incentives for a trader to submit a limit order and therefore actively contribute to the making of liquidity. Thus, higher fill rate implies a more liquid market. We define the fill rate of a limit order as follows:

$$FR = \frac{\sum_{i \in I} P(s_i) P_S(\text{submission of } LO_i) P_S(\text{execution of } LO_i)}{\sum_{i \in I} P(s_i) P_S(\text{submission of } LO_i)},$$

where $P_S(\text{submission of } LO_i)$, $(P_S(\text{execution of } LO_i))$ denote the likelihood of submission (execution) of a limit order when the state of the book is s_i.

*** **Please Insert Table 5 about here*****

The fill rate under PBT and a tight tick of 0.2 is decreased by 8.66 b.p relative to PT. Figure 6, Panel A, shows the difference in fill rate as a function of the half-tick. We observe that for $\Delta < \tilde{\Delta}$, the fill rate for PBT is lower than PT whereas the opposite holds for $\Delta > \tilde{\Delta}$. The fill rates are positively correlated with the trading rates among investors as these lead to the execution of LOs.

In the remaining of the section we compare the average depth supplied by traders under the different trading protocols as well its variation, between average depth and depth of 1 or 2. We define the average depth as a weighted average, using as weights the Markov stationary distribution.

$$\overline{D} = \sum_{i=0}^{2} P(\text{ States Creating a queue of } i) \cdot i.$$

The effects of preferencing on depth vary. PBT incentivizes traders that face soft competition to join the queue, which is reflected in the 1.87% increment of likelihood in observing a 'depth of 2'. However, we observe a drop of 'depth of 1' which is the dominant effect and results in an overall decrease of the average depth by 9 b.p.

*** **Please Insert Figure 6 about here*****

In Figure 6, Panel B we report the average depth in PBT as well as the degree of the depth. The size of the tick and the average depth are positively correlated. This is to be expected since as the tick increases, so the benefit from submitting a LO. Panel C displays the difference in depth between PBT and PT. We observe that under any tick size the depth under PBT is lower, due to queue jumping and simultaneous order cancellations. However, this alters the depth dynamics under the two different trading protocols. Given the existence of a queue of length of 1, then under PBT there is higher likelihood to obtain a 'depth of 2' in the book, as investors have more incentives to join the queue. Nonetheless there is higher probability for this depth to fade out more rapidly in PBT in comparison to PT.[26]

4 Endogenous Adoption of Priority Rules

Many exchanges have policies related to preferencing and queue jumping. Examples include the Canadian and the Nordic financial markets which operate under broker preferencing, and Euronext's internal matching service (2007) basically offering broker's the choice to adopt PBT by adopting this service. Euronext asked for clearance from the regulator before providing this service to the public and brokers responded almost immediately by adopting the new service. This section investigates the endogenous decision on the adoption of priority rules by brokers. The adoption of PBT or PT is a one time decision where we assume that brokers maximize the welfare of their clients anticipating their order submission strategies. While brokers could charge a fee when introducing one priority rule over another, we start from the presumption that Bertrand competition would drive these fees to 0 when investors can choose their broker. We therefore assume that brokers maximize their investors' welfare.[27]

To study the endogenous adoption of priority rules, we have to measure investor welfare when one broker offers PT whereas the other offers PBT. To do so, we modify our main system of indifference equation, presented in Section 2.[28]

Following similar arguments and methodology, we solve the extended version of our model, and derive the equilibrium cut-off values, the corresponding trading rates and the generated welfare. For reasons of completeness, Tables C3, C4 and C5 in Appendix C, provide the results. A trader affiliated to a broker not offering queue jump will most

[26] Under PT, if the ask side has a 'depth of 2', then the likelihood of observing an empty book the following period is 0.

[27] Alternatively, we can make the assumption of perfect elasticity in the mobility of traders through brokers, which drives brokers to maximize their investors' welfare.

[28] The main alteration is that in the indifference equations of this set up, one broker offers preferencing and the other not. That affects the execution probabilities of LOs.

likely act as a taker of liquidity, as a measure of mitigating the effect of losing her line. This is reported also in Table C4, Panel A where we observe that the total trading rate for a y-seller is 13.20 b.p. higher than the one produced by an x-seller. Panel B reports the conditional probabilities on having a market order given the type of the trader and also given his position on the queue. We see that second in line traders, in both cases have higher trading rates compensating for having lower execution probability for their limit orders.

Assume that broker Y, does not offer PBT. *Ceteris paribus*, the welfare of a trader affiliated to broker X is higher when her broker offers PBT and the other broker does not. This stems because of the higher execution probability that her LO has. So in that case, broker X in order to maximize the welfare for his investors would select to offer preferencing services to his clients. The unique Nash equilibrium is when both brokers decide to offer PBT. This theoretical prediction is also verified empirically by the rapid adoption of PBT by Euronext's members.

Proposition 8 *Assume that brokers can unilaterally decide whether to offer PBT or PT in order to maximize their investors' welfare. Then under any tick size, the 2-by-2, non cooperative game has a unique Nash equilibrium, in which both brokers decide to offer PBT.*

Table 6 reports the pay-off matrix from the 2-by-2 non cooperative game, when brokers have equal market shares and ticks are tight i.e., $\Delta = 0.2$. We assume that each player knows the strategies and pay-offs of the other player. Our analysis reveals a unique Nash Equilibrium which shows that brokers are in a prisoner's dilemma situation under tight tick: investor welfare would be higher if they could commit to PT but they have a dominant strategy to introduce PBT. However, in wide ticks where welfare is higher under PBT, brokers follow the social optimum between the two trading systems.

*** **Please Insert Table 6 about here*****

Our analysis thus shows that even though PBT results as a Nash-equilibrium, it does not coincide with the social optimum when the tick is small. The private and socially preferred outcome differ as an individual broker does not internalize the negative impacts its preferred priority rule has on the other broker's investors.

5 Priority Rules and Market Fragmentation

In this section, we study whether priority rules in the limit order book have implications for market fragmentation. We distinguish three potential impacts: fragmentation between LOB and dealer market, the incentives for off-exchange reporting, and the composition of same-broker versus different-broker trades on the LOB.

Priority rules determine the trading rate on a dealer market (i.e., trades against the crowd/dealer) and on the LOB (i.e, trades against investors). Panel C of Figure 5 shows that trading rates against dealers are greater with PBT than with PT when the tick is small; the opposite holds for large ticks. In contrast, the trading rate on the LOB is higher with PT than PBT with small ticks whereas the opposite holds when the tick is wide. Taking together, this implies that the LOB's market share in trading is higher with PT than with PBT when the tick is small whereas it is higher with PBT than with PT for large ticks. Intuitively, with small ticks, first-in-line investors anticipate queue jumping may happen and therefore they are more inclined to turn to the dealer market. With wide ticks, the anticipation of queue jumping also plays a role but now this results in a greater LOB trading rate as the LOB quite often contains standing LOs.

Priority rules may also impact the incentives for off-exchange reporting. A broker could circumvent PT and implement order preferencing by reporting some trades off-exchange. In particular, a broker could induce queue jumping by executing an arriving MO by one of its clients against one of its clients' LO that was standing later in the queue, and report these trades off-exchange. To the extent that this happens, we expect more off-exchange reporting on trading venues that implement PT rather than PBT. The reasoning is that this force is not present with PBT as orders already queue jump in this setting. In order to learn about the magnitude of this force, we compute the trades stemming from queue-jumping in PBT as a share of the total trading rate. Figure 7, Panel B, shows that this share increases as the tick size becomes larger. Our model thus predicts that off-exchange reporting will be larger with PT, a force that is more pronounced when the tick is wide.

Finally, priority rules influence the composition of investor-investor trades, i.e., the share of trades where the counterparties use the 'same-broker' versus 'different-brokers'. With PT, the structure of our model implies that 'same-broker' and 'different-broker' trades are equally important. With PBT, the 'same-broker' trades outweigh 'different-broker' trades stemming from queue-jumping. Figure 7, Panel A, shows how the share of 'same broker' trades as a fraction of all trades where investors form both counterparties evolves as function of the tick size. We observe that the share of 'same-broker' trades increases when the tick becomes larger implying that more queue-jumping occurs as the tick becomes larger.

*** **Please Insert Figure 7 about here*****

6 Empirical and Regulatory Implications

In Proposition 5, we identified empirical implications of our model, and the effects of priority rules on trading rates and depth have been discussed in Sections 3.1 and 3.4. Here, we formulate testable implications of our model which can be of use to empirical researchers. We build on the empirical predictions provided in Parlour (1998) which were stated under PT environment.[29] We first identify implications that are specific to PBT. Next we depict the differences across the two priority rules by focusing on differences in systematic order flow patterns. Trading patterns have been studied extensively and essentially consist in evaluating traders' behavior by studying the submission of market and limit orders, and thus the length of the queue. For example, Biais et al. (1995) and Ranaldo (2004) find that the longer the queue, the higher the likelihood that an arriving trader will opt for a MO.

[29] In Parlour (1998), the trading patterns analyzed also refer to the opposite side of the LOB. In our model this is not feasible because of the two period cancellation rule. Adding one more period to the lifespan of limit orders would allow us to formulate similar claims, but it would clutter our model with algebraic notations and it would not add to the main intuition.

In PBT, a seller under soft competition has more incentives to join the line rather than when she faces tough competition. This is due to the probability of queue jumping. An implication is that the likelihood that two consecutive LOs at the ask coming through the same broker, would be lower as opposed if they were submitted by traders affiliated to different brokers. This implication is specific to PBT and is reflected in the relation that the cut-off values have, provided in Proposition 4.

Testable Implication 1 *In equilibrium in PBT, limit orders are more likely to be followed by limit orders coming from traders affiliated to different brokers in comparison to being affiliated to same brokers.*

From the above implication follows that under PBT, reversion to a MO after a LO is more likely to occur by traders having the same affiliation. Traders facing competition are more reluctant to join the queue as opposed to those who do not. This is true for both trading protocols, but with different intensities. In PBT, competition incentivizes traders under opposite forces. In tough competition, a trader becomes more aggressive in order to compensate for the possibility of losing her position on the queue in the future. Under soft competition, she is more inclined to submit a LO and exploit the possibility of a preferencial execution. However, relative to PT, the dominant force is the latter one.

Testable Implication 2 *In equilibrium under PBT, limit orders are more likely to be followed by limit orders than under PT.*

Next, we turn our focus on the relation between transactions observed and limit order submissions. From our model we obtain that if a trader transacts at one side taking liquidity and reducing the queue, then the arriving trader will exploit this on her favor. Hence limit orders on the ask, are positively correlated to market orders at the ask, transacted at the previous period. The next testable implication depicts this relation.

Testable Implication 3 *In equilibrium under PT, market orders are more likely to be followed by limit orders than under PBT.*

The next implication is of high importance to regulators, and identifies the optimal response of brokers regarding the offer of PBT. Euronext's fast adoption of internal matching service (2007), is in accordance to the predictions of our model and is illustrated in the next implication.

Testable Implication 4 *In equilibrium, under the assumption that brokers maximize their investors' welfare, if given the option, then they will adopt PBT over PT.*

The effects of priority rules to depth of the market were investigated in Section 3.4. In PBT, queue jumping with order cancellation and traders that face no competition affect the overall depth of the book and are the main driving force of the next implication.

Testable Implication 5 *In PT, the average depth of the book is higher in comparison to PBT. The likelihood of observing a queue of 'depth of 2' is larger in PBT rather in PT. However, this depth may reverse to an empty book more rapidly in PBT than PT.*

The novel set up of our model related to priority rules also allows for useful regulatory insights to policy makers. We show that between the two trading protocols, PT is preferred when the tick is tight but PBT is socially preferred under wide ticks. The endogenous market outcome stemming from brokers' priority rules decisions differs from the socially preferred ones when the tick is small.

Testable Implication 6 *Regulators increase investor welfare by prohibiting PBT when the tick is small. For large ticks, regulators increase investor welfare by adopting PBT.*

7 Robustness Checks of the Model

In this section we study the robustness of our model by relaxing some of the modeling assumptions. First, our model so far assumed transparency of broker affiliations. We now relax this to study opacity: traders can still observe whether there are standing LOs but they have no knowledge on the broker affiliation of the trader that placed them. We relax this assumption because we want to investigate whether the possibility of preferencial execution is sufficient to change the behavior of the arriving trader. Second, we remove the assumption that there is a crowd/dealer. This has an implication in the trading rates, and in particular to the traders that face no competition since they do not have a counterparty to trade, and thus they need to alter their decision problem. Relaxing this assumption, helps verify the compositional changes between traders discussed in Section 3. In both of these cases the main insights about trading rates and investor welfare derived from our model, still hold.

7.1 Trading under opacity

Traders' decisions under limited information for the state of the market have been studied in the past (see, e.g., Degryse et al. 2009; Hendershott and Mendelson 2000). In this section, we assume that the market operates under PBT, but the arriving agents, while they can observe their position on the queue, they do not have knowledge on its composition. In particular, traders have knowledge on the number of orders resting on the book but not the broker affiliation of orders already in the queue. Traders thus have to make their optimal decisions under an opaque environment. Agents then adjust their decision problem by forming expectations on the traders' affiliation that are in the book. We denote by PBT-O, a LOB that operates under PBT priority rules and opacity. We keep the assumption that the two brokers have equal sizes. As with our model presented in Section 2, we construct a system of indifference equations which depict the trade-off between a market and a limit order.[30] We compute the cut-off values of agents for which they are indifferent in submitting a market or limit order and a limit order or refrain from trading. The agent will form her decision based on her private valuation and on her place in the queue regardless its composition, i.e., she has only two relevant states. Since a trader cannot distinguish affiliation in this set up, we expect that her decisions will be less affected by the implementation of PBT. However, the trading protocol would still influence her actions. The first two parts of Proposition 9, depict the relation between PBT-O and PT, while the remaining two between PBT-O and PBT.

[30]The system is defined following the same methodology that we used in Section 2, and for the sake of brevity is not reported.

Proposition 9 *For any tick size the following holds:*

i) A seller facing 'no competition' under PBT-O is more likely to submit a MO, rather than under PT.

ii) A seller facing 'competition' under PBT-O is less likely to submit a MO, rather than under PT.

iii) There is lower likelihood for a seller facing 'no competition' to submit a MO under PBT-O rather than under PBT

iv) There is higher likelihood for a seller facing 'competition' to submit a MO under PBT-O rather than under PBT

Our results are intuitive and in accordance to PBT under transparency. PBT-O is more beneficial for traders that face competition upon arrival. Thus an agent arriving to an empty book does not get any benefit but still she may lose her place in the queue. Thus she accounts for that and becomes more aggressive than in PT. The opposite argument holds for a trader facing competition who becomes less aggressive under PBT-O as compared to PT.

7.2 Priority Rules without the Existence of Crowd/Dealer

In this section, we present the main results from a variant of our model in which we do not assume the existence of a crowd/dealer that is ready to take the opposite side of the trade. We modify our main model presented in Section 2 to accommodate for that difference. A trader who arrives at the market and finds herself in a state in which there is no limit order standing at the book on the opposite side, has to decide between exiting the market or posting a LO. In particular the possibility of trading via MO depends on the existence of LOs in the book. However, the insights that we obtained from our model are still valid. Figure 8 graphs the difference in the trading rates (Panel A) and welfare (Panel B) between PBT and PT for all tick sizes.

*** **Please Insert Figure 8 about here*****

In tight ticks, as in our main set up, no-competition traders dominate and queues are not easily formed. As in our main set up, the likelihood that a first-in-line trader will find herself in an empty (or irrelevant to empty) state for her, is higher in PBT. This is due to the anticipation effect, i.e. the higher probability that a first-in-line trader will lose her position in the queue by a future arriving trader, which makes her more aggressive in order to compensate for that. However, these traders have no other alternative but to submit a LO and as a result, trading volume under PBT is lower. As ticks increase and queues start to form, no-competition traders, who still remain more aggressive under PBT, have a counterparty to trade against, and that leads to a higher trading volume in PBT relative to PT. We remark that under a no crowd/dealer assumption, trading volume and welfare co-move. The rational lies to the fact that under this set up, welfare is always generated by counterparties that both contribute to the overall result. Thus under this modification of our main model the generated welfare findings are in accordance with our main results obtained in Section 3.3. This further strengthens our argument related to the compositional change that we observe in PT and PBT in our model.

8 Conclusion

Priority rules determine how investors reveal their trading intentions in financial markets. In this paper, we compare the impact on market quality and investor welfare of two different allocation rules that are commonly observed in financial markets; price-time priority (PT) and order preferencing as modeled through price-broker-time priority (PBT). PBT is included in the trading protocol in a number of markets including the Canadian and Nordic markets. While in the US price-time priority applies at individual trading venues, order preferencing may happen implying another priority layer in between price and time.

We develop a dynamic microstructure model where dealers or a crowd provide liquidity at the minimum tick, and where investors can trade either by submitting a limit order in the public limit order book or a market order against standing limit orders or against the dealer or crowd. Similar to Parlour (1998), the central intuition of our paper is that each trader knows that her order affects the order placement strategies of those who follow. We add to this by showing that (i) priority rules determine the actions of subsequent investors, and (ii) rational investors anticipate subsequent investors' behavior and therefore may behave differently depending upon priority rules.

Our model generates interesting insights for systematic patterns in order flow, market quality and investor welfare. Systematic patterns in order flow differ between PT and PBT. With PBT, it is more likely to see two consecutive limit orders or two consecutive market orders at the same side of the book than with PT. Consecutive limit orders at the same side of the book through the same (different) broker are less (more) likely under PBT than under PT. Priority rules also affect market quality. In general we find that the average depth of the limit order book is shallower with PBT. This general effect hides an interesting heterogeneity. While it is more likely to have depth in the order book with PT, it is more likely to have a very deep book with PBT. Other market quality statistics depend on the minimum tick. With small ticks, trading rates and fill rates of limit orders are higher with PT than with PBT. The opposite holds for wide ticks. Finally, investor welfare is maximized with PT for small ticks and with PBT for large ticks. We further show that an individual broker when having the choice between PT and PBT has a dominant strategy to implement PBT when maximizing their traders' welfare. When the tick is small, brokers end up in a prisonner's dilemma as they would be better off if they could commit to PT. Investor welfare is higher with PT when the tick is small but higher with PBT with large ticks.

Our model further shows that priority rules are an important driver of how markets may fragment. Our model predicts more off-exchange trading under PT to the extend brokers can circumvent this trading protocol by reporting trades off-exchange when they can execute an arriving market order against a standing limit order deeper in the book. This is a novel result because it identifies one of the driving forces which push traders away from lit markets.

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Table 1: Indifference Cut-off Values

This table contains the cut-off values obtained as a solution of System 2 for an asset with normalized fundamental value $V = 1$. The numbers reported correspond to a seller having affiliation to broker X. The ask and the bid prices are set to 1.1 and 0.9 respectively, which corresponds to a tick $\Delta = 0.2$. By a symmetrical argument a y-seller has identical cut-off values. The first column identifies the irrelevant states, the second refers to PT and the third to PBT. $b_x^S(0,0)$ represent the cut-off value for an x-seller arriving to the market, facing no competition, i.e., an empty or irrelevant to an empty book, while $b_x^S(0,-1^y)$ and $b_x^S(0,-1^x)$ correspond to different intensities of competition i.e. intermediate for PT and soft - tough for PBT.

	PT	PBT
$b_x^S(0,0)$	0.789051	0.790536
$b_x^S(0,-1^y)$	0.850481	0.824353
$b_x^S(0,-1^x)$	0.850481	0.851507

Table 2: Markov Stationary Consolidated Steady State Probabilities

This table reports the consolidated Markov state probabilities derived as the solution of the matrix equation

$$\rho_i = \rho_i \mathcal{M}_i,$$

where $i \in \{PT, PBT\}$ and $\mathcal{M}_i$ denotes the transition matrix of the Markov chain under the protocol i, i.e. the probability that a trader transits from one state of the book to another under PT or PBT and ρ_i represents the vector of all states. The fundamental value V of the asset has been normalized to one and the tick size Δ is set to 0.2, implying bid and ask prices of 0.9 and 1.1 respectively. Columns one and two report the two main categories of no-competition and competition states which are further decomposed to empty and irrelevant states, and columns three and four, correspond to PT and PBT respectively. Exact definitions of the categories are provided in Table C2 in Appendix C.

	Irrelevant States	PT	PBT
No Competition Steady State Probability	Empty book state	74.3531	74.4773
	Irrelevant states to an empty book state	17.9908	17.8767
Competition Steady State Probability	Irrelevant states to Second in Line after y-type	3.8281	3.8229
	Irrelevant states to Second in Line after x-type	3.8281	3.8229

Table 3: Trading Rates in PT and PBT

The table reports the likelihood of observing a market order by an arriving seller under both PT and PBT. The traded asset has a fundamental value V normalized to one and the bid and ask prices are set to 0.9 and 1.1 respectively which correspond to a tick $\Delta = 0.2$. For the calculations we follow formulas analyzed in Section 3.1 i.e.,

$$TR_F = \sum_{i \in B_F} P(MO| \text{ state } s_i)P(s_i) \text{ and } TR_S = \sum_{i \in B_S} P(MO| \text{ state } s_i)P(s_i),$$

for the unconditional trading rate and

$$P(MO|k \text{ in line}) = P(k \text{ in line})^{-1} \sum_{i \in B_j} P(MO| \text{ state } s_i)P(s_i),$$

for the conditional one. Panel A presents the total trading rate and the trading rates for a seller arriving to the market facing no competition and competition. Panel B reports the likelihood of trading given her position in line i.e. the probability that a seller submits a market order, conditional that she arrives in the market in a no competition/competition state. Panel C reports the probability that the trader will submit a market order to sell condition on being first or second in line and the type of competition she faces. Results by symmetry correspond to both x- and y-sellers.

Panel A		
	PT	PBT
Trading Rate	39.6877	39.7080
Trading Rate for First in Line	36.4320	36.5045
Trading Rate for Second in Line	3.2557	3.2034
Panel B		
	PT	PBT
Trading Rate on Condition Trader is First in Line	39.4525	39.5268
Trading Rate on Condition Trader is Second in Line	42.5240	41.8965
Panel C		
	PT	PBT
Trading Rate for Second in Line, after same type	42.5240	42.5753
Trading Rate for Second in Line, after opposite type	42.5240	41.2176

Table 4: Investor Welfare under PT and PBT

This table reports the investor welfare generated under PT and PBT following Section 3.3, that is

$$W \equiv E(\pi(\phi_t(\beta, s))).$$

The traded asset has a fundamental value V normalized to one and the bid and ask prices are set to 0.9 and 1.1 respectively which correspond to a tick $\Delta = 0.2$. Column one reports the overall generated welfare, the welfare under various levels of competition and by different groups of traders, and columns two and three correspond to PT and PBT respectively.

	PT	PBT
Investor Welfare	0.2078	0.2077
Welfare Generated by First in Line	0.1921	0.1920
Welfare Generated by Second in Line	0.0156	0.0157
Welfare Generated by x- seller	0.1039	0.1038
Welfare Generated by y- seller	0.1039	0.1038

Table 5: Fill Rate and Depth

The table reports the fill rate of a sell limit order, i.e., the probability that a submitted limit order gets executed and is computed as:

$$FR = \frac{\sum_{i\in I} P(s_i)P_S(\text{submission of } LO_i)P_S(\text{execution of } LO_i)}{\sum_{i\in I} P(s_i)P_S(\text{submission of } LO_i)},$$

where $P(s_i)$ corresponds to the steady state probability of i state, and $P_S(\text{submission of } LO_i)$, [$P_S(\text{execution of } LO_i)$] is the probability of submission [execution] of a LO when the state of the book is i. The depth is calculated following Section 3.4. We also report the frequency of a 'depth of 1' and 'depth of 2'. The traded asset has a fundamental value V normalized to one and the bid and ask prices are set to 0.9 and 1.1 respectively which correspond to a tick $\Delta = 0.2$. Columns two and three correspond to PT and PBT respectively.

	PT	PBT
Fill Rate	34.6929	34.6063
Average Depth	0.2779	0.2770
Likelihood of 'depth of 1'	23.50	23.33
Likelihood of 'depth of 2'	2.14	2.18

Table 6: Brokers' decision to adopt PT or PBT

This table summarizes the pay-off matrix, for brokers' strategies of implementing PT or PBT, under the assumption that they maximize their investor welfare. Generated welfare when one broker offers PT and the other PBT, is calculated by a modification of our main system of indifference equations presented in Section 2.1. The unique Nash equilibrium is that both brokers offer PBT. The traded asset has a fundamental value V normalized to one and the bid and ask prices are set to 0.9 and 1.1 respectively which correspond to a tick $\Delta = 0.2$.

		Broker Y			
		PT		PBT	
Broker X	PT	0.103906	0.103906	0.103892	0.103934
	PBT	0.103934	0.103892	0.103896	0.103896

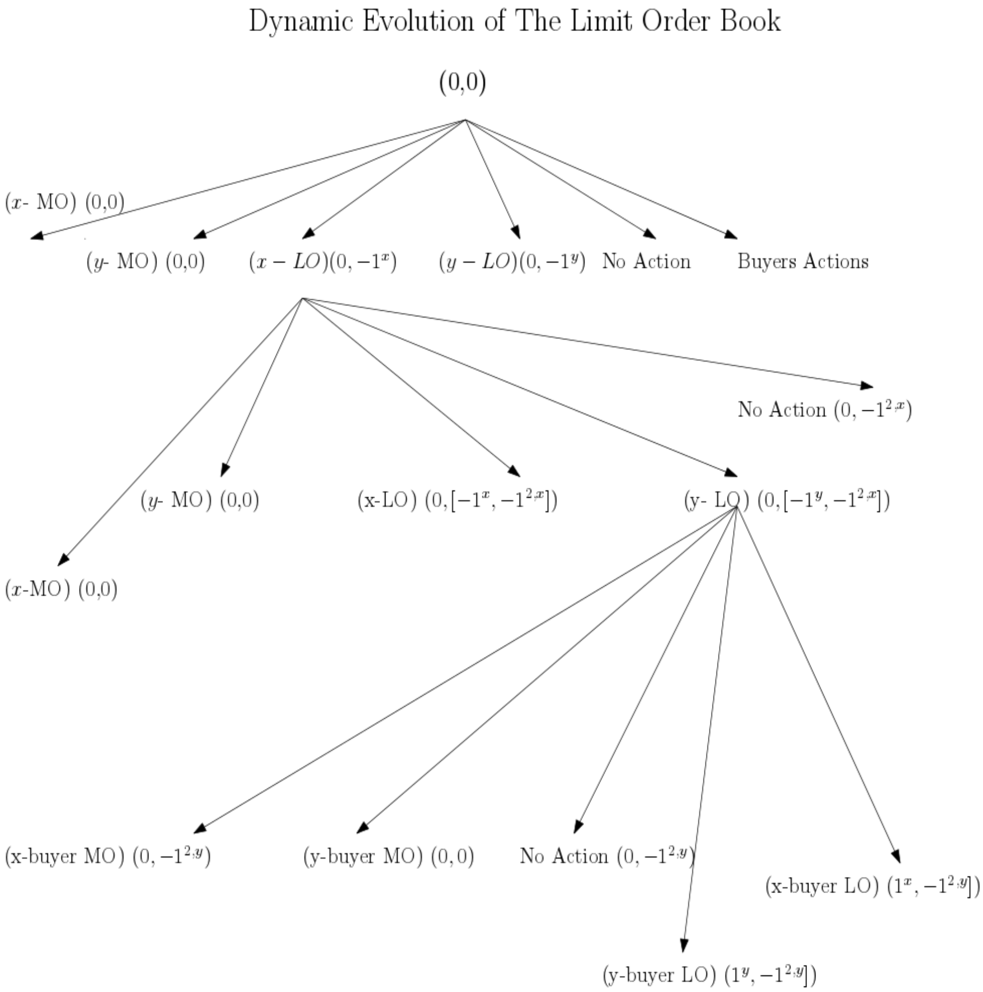

Figure 1: Graphical representation of the Dynamics of Limit Order Book.

This figure illustrates a potential evolution of the book starting from an empty state under PBT. At the first two periods, sellers' actions are depicted. k-MO, (k-LO), $k \in \{x, y\}$, denote a k-seller submitting a MO, (LO). The parenthesis report the action of the arriving seller and the resulting book. To demonstrate a preferential execution we report the potential actions that a buyer can perform the third period, following a specific evolution of the book. The second tree branch on the third level illustrates a queue jump with simultaneous order cancellation.

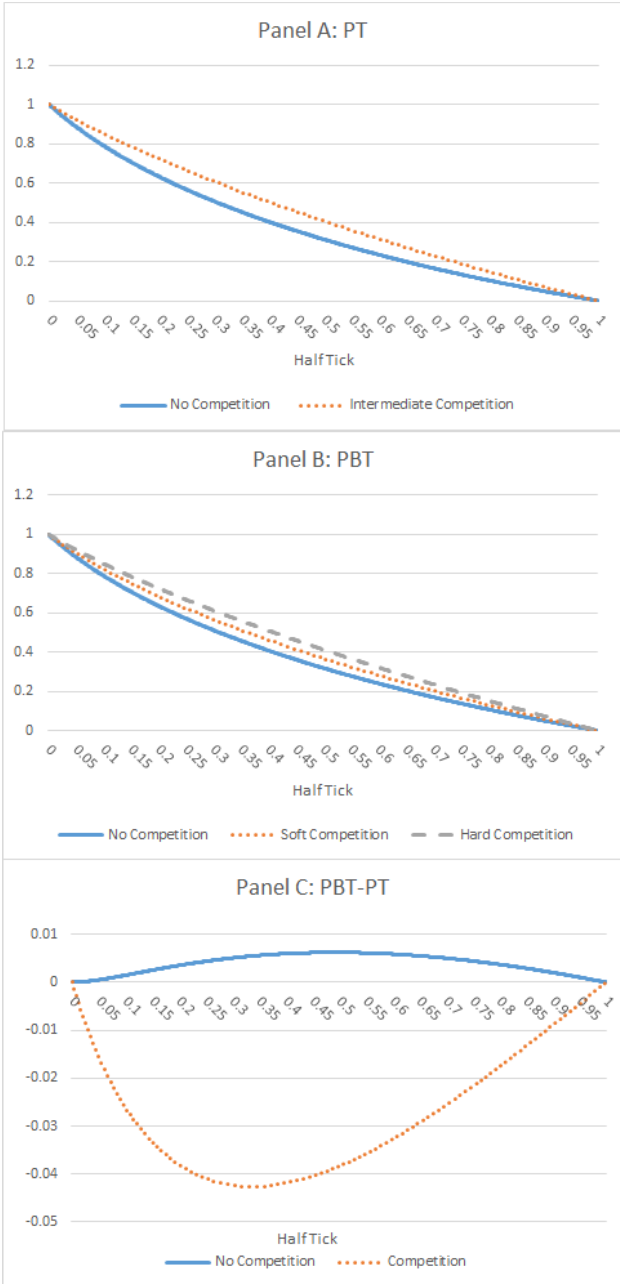

Figure 2: Cut-off values for PT and PBT for all tick sizes.

This figure illustrates the cut-off values of a seller facing deferent levels of competition for all tick sizes for a traded asset with fundamental value V normalized to one. Panel A (B), identifies the cut-off values under PT (PBT) for different levels of competition, i.e., no competition and intermediate competition for PT and no-competition and soft and tough competition for PBT. In Panel C, their difference is depicted. For the difference under competition, in PBT we consider the average of the cut-off values between 'soft' and 'tough' competition.

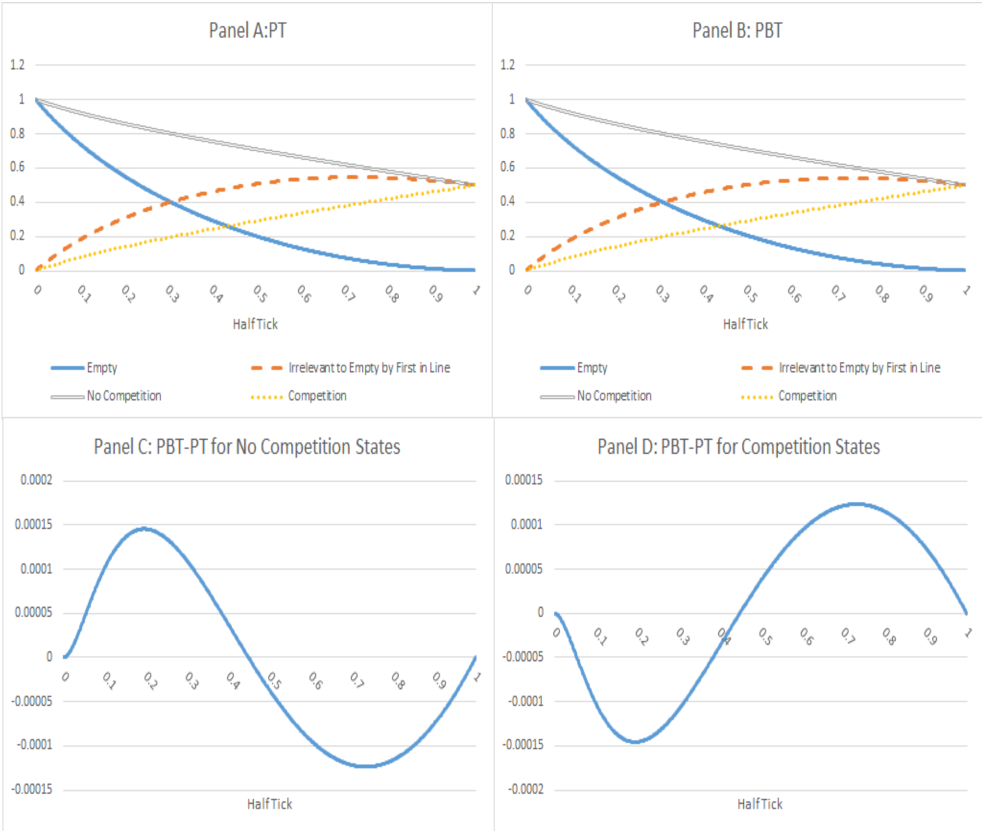

Figure 3: Consolidated Markov Steady States Probabilities for PT and PBT for all tick sizes.

This figure illustrates the consolidated Markov state probabilities derived as the solution of the matrix equation

$$\rho_i = \rho_i \mathcal{M}_i,$$

for all tick sizes and various levels of competition, for both PT and PBT(Panels A and B respectively), as well as their difference PBT-PT for no-competition and competition states (Panels C and D). We denote by i the trading protocol, PT or PBT and $\mathcal{M}_i$ denotes the transition matrix of the Markov chain under the protocol i and ρ_i represents the vector of all states. *Empty* stands for the steady state probability of observing a completely empty book and *Irrelevant to Empty by First in Line* graphs the probability that an arriving seller will face an irrelevant to an empty state. *No competition* refers to their sum, and *Competition* illustrates the consolidated probabilities that would place a trader upon arrival, to a state in which she would face competition, intermediate for PT (Panel A) and combined soft and tough for PBT (Panel B). Panel C (D), shows the difference in probability between PBT and PT for the states that put the arriving trader under no competition (competition). The traded asset has fundamental value V normalized to one.

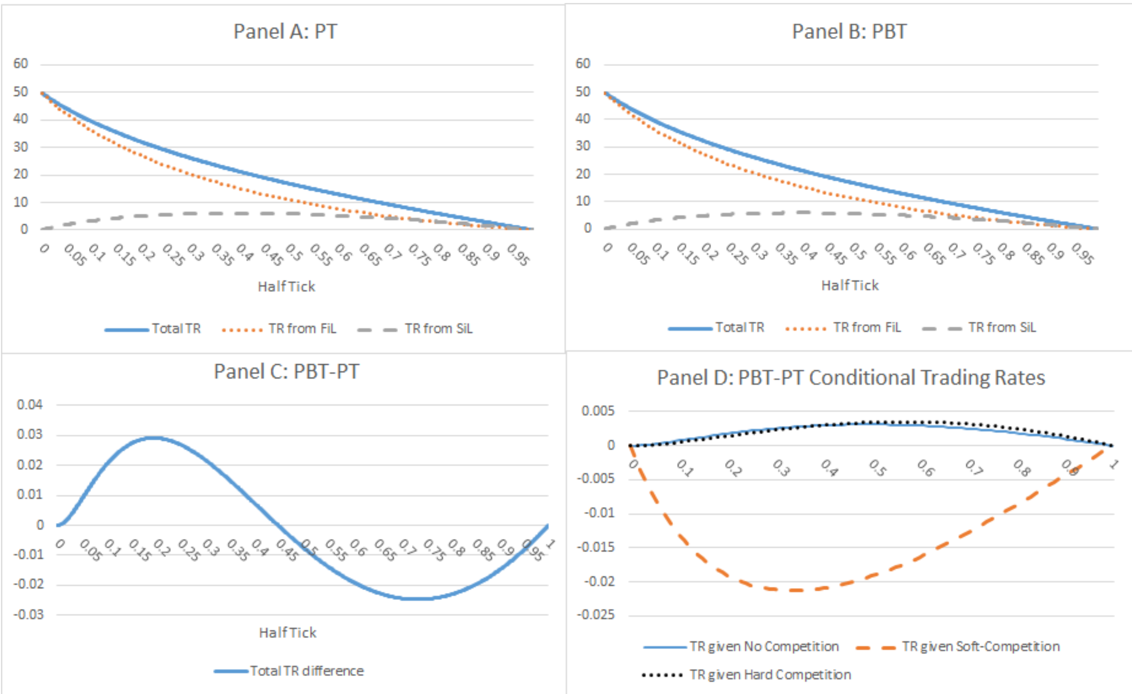

Figure 4: Trading Rates in PT and PBT for all the tick sizes.

This figure illustrates the evolution of total trading rates in PT and PBT under all tick sizes following the calculations of Section 3.1. In Panel A (B), the total trading rate, and the trading rate by a seller that faces no-competition and competition in PT (PBT), are graphed. Panel C illustrates the difference in the total TR between PBT and PT for all tick sizes. In Panel D we plot the difference in the conditional trading rates in PBT and PT regarding the competition that the arriving trader faces. For the computations under competition in Panel D for PT, intermediate competition has been used. The traded asset has fundamental value V normalized to one.

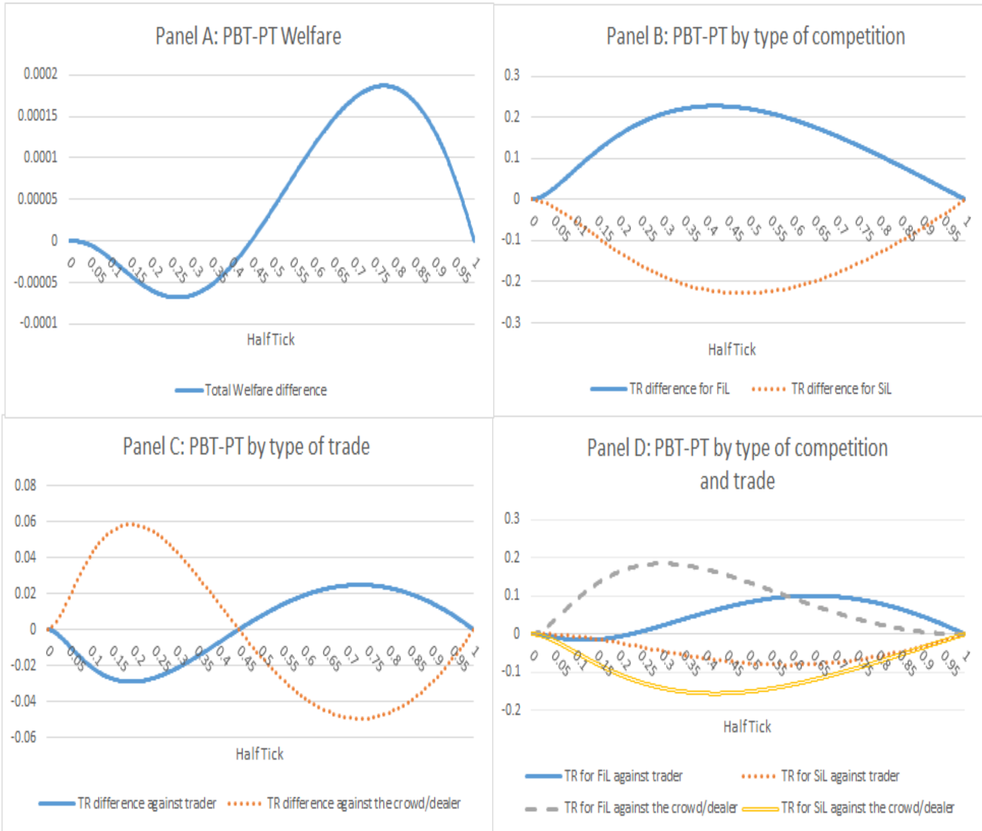

Figure 5: Investor Welfare and trade composition for all tick sizes

This figure illustrates the difference in investor welfare between PT and PBT, as well as differences in trading rates for various groups of traders, for all tick sizes. For the computations we follow the discussion presented in Section 3.3. Panel A graphs the difference in overall welfare between PBT and PT. Panel B illustrates the difference in trading rates by traders facing no-competition and competition. In Panel C, we plot the difference in likelihood that a trader will trade against a counterparty or a crowd/dealer, between PBT and PT. Panel D, further decomposes trades against a counterparty or a crowd/dealer to trades initiated by a first or second in line arriving agent, i.e. by the degree of competition. The traded asset has fundamental value V normalized to one.

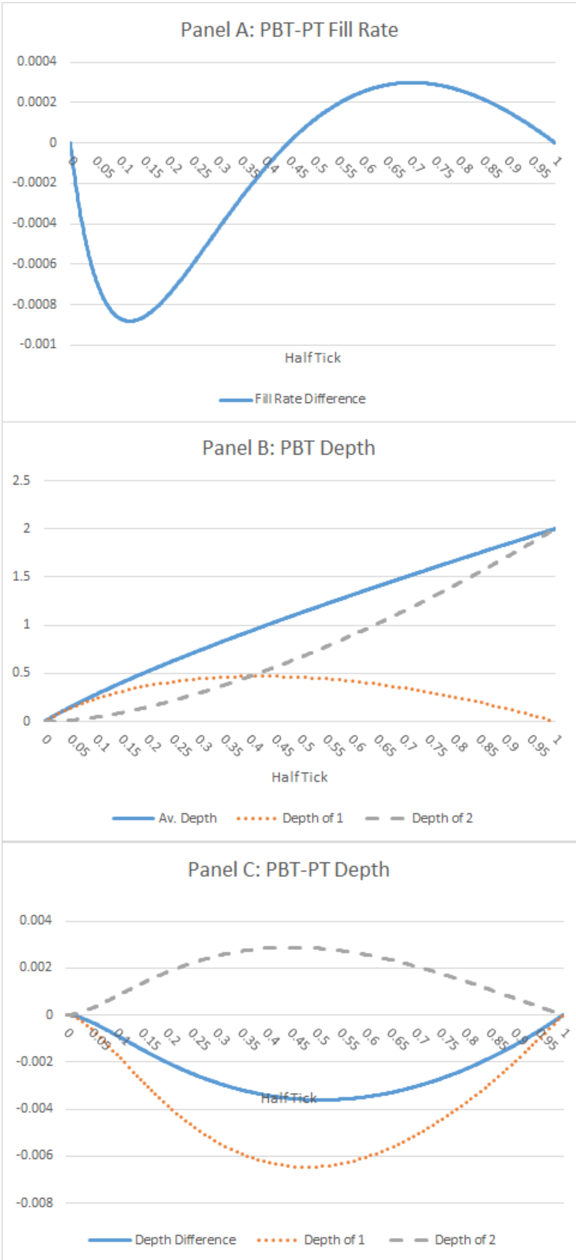

Figure 6: Depth and Fill Rate under PBT and PT for all tick sizes.

The figure illustrates how fill rate and depth evolves for all tick sizes for a traded asset with fundamental value V normalized to one. In Panel A we chart the difference in the fill rate of a sell LO between the two trading protocols. Panel B graphs the average depth, the (unconditional) 'depth of 1' and 'depth of 2' in PBT as a function of tick size. Panel C, plots the difference between PBT and PT for the same variables as in Panel B.

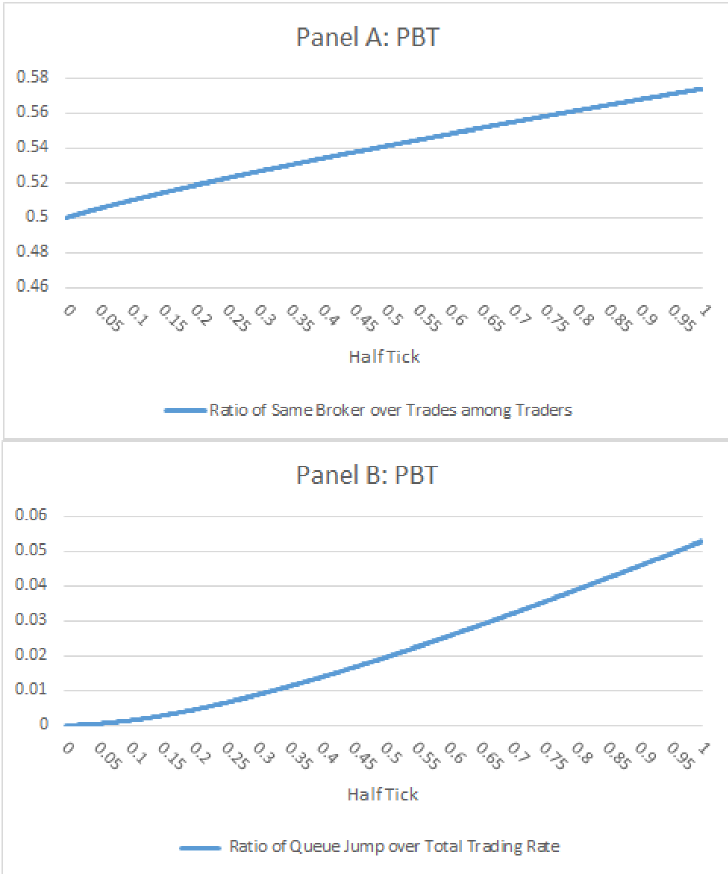

Figure 7: 'Same Broker' matching and off-exchange reporting under PBT for all tick sizes

The figure illustrates how PBT affects 'same-broker' trades and off-exchange reporting for all tick sizes for a traded asset with fundamental value V normalized to one. Panel A reports the ratio of matched investor-investor trades between traders having the same broker over the total amount of trades as a function of the tick size. In Panel B we plot how the ratio of preferential executed orders i.e. queue jump, over the total trading volume, evolves as the tick changes.

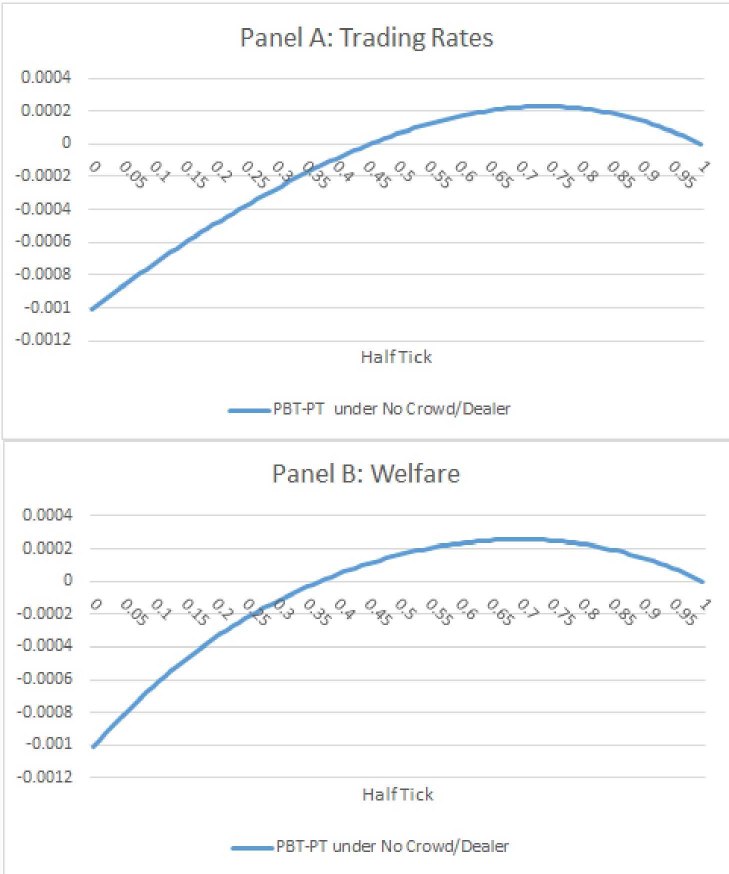

Figure 8: Trading rates and Investor Welfare under No Crowd/Dealer for all tick sizes

The figure plots trading rates and investor welfare for all tick sizes for a traded asset with fundamental value V normalized to one, under the no-crowd/dealer assumption i.e. under the assumption that neither crowd or dealers exist to take the opposite side of the trade. Trading rates and welfare are obtain by a suitable modification of our main model presented in Section 2.1. Panel A charts the difference in trading rates between PBT and PT while in Panel B we plot the difference in welfare as a function of tick size.

Appendix

A Definitions and Proofs

As discussed in Section 2, we define G_b as the execution probability of a submitted limit order, when the following trader is buyer and submits a limit order, or declines trading. G_s is the execution probability of a submitted order, when the next trader is seller. G_s accounts for all possible actions of the arriving trader, i.e. submits a market order, limit order or refrain from trade. G_b and G_s are defined as follows:

$$
\begin{aligned}
G_b =& \Big[\frac{x}{2}P_{t+1}(B/V \leq b \leq b_x^B(0,-1^x))\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(1^x,0))+ \\
&\frac{y}{2}P_{t+2}(b \geq b_y^B(1^x,0))\big) + \frac{y}{2}P_{t+1}(B/V \leq b \leq b_y^B(0,-1^x)) \\
&\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(1^y,0)) + \frac{y}{2}P_{t+2}(b \geq b_y^B(1^y,0))\big) + \frac{1}{2}P_{t+1}(b \leq B/V)) \\
&\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) + \frac{y}{2}P_{t+2}(b \leq b_y^B(0,0))\big)\Big]
\end{aligned}
\tag{3}
$$

and equivalently

$$
\begin{aligned}
G_s =& \frac{x}{2}P_{t+1}(b \leq b_x^S(0,-1^x)) \\
&\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) + \frac{y}{2}P_{t+2}(b \geq b_y^B(0,0))\big)+ \\
&\frac{y}{2}P_{t+1}(b \leq b_y^S(0,-1^x))\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) + \frac{y}{2}P_{t+2}(b \geq b_y^B(0,0))\big)+ \\
&\frac{x}{2}P_{t+1}(b_x^S(0,-1^x) \leq b \leq A/V)\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,-1^x))+ \\
&\frac{y}{2}P_{t+2}(b \geq b_y^B(0,-1^x))\big) + \frac{y}{2}P_{t+1}(b_x^S(0,-1^x) \leq b \leq A/V) \\
&\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,-1^x)) + 0\big) + \frac{1}{2}P_{t+1}(b \geq A/V)\big(\frac{x}{2}P_{t+2}(b \geq b_x^B(0,0)) \\
&+ \frac{y}{2}P_{t+2}(b \geq b_y^B(0,0))\big)\Big]
\end{aligned}
\tag{4}
$$

In the equations above, both x and y are equal to 0.5, but are stated as such in order to identify the desired broker affiliation. In the remaining of the section, we report the proofs of the propositions stated in the main body of the text.

Proof of Proposition 1: The first part of the claim is obvious. We will show that the trader's decision does not depend on the state of the book on the opposite side. Assume an x-seller arriving to the market. We need to show that her cut-off value depends on the state of the book on her side only. Let us analyze only one potential case, since all the rest follow applying similar arguments. We compare the following states of the book, $(0, 0)$ and $(1^x, 0)$, i.e., a completely empty book and a book that has one limit order standing at the bid. Let $b_x^S(0, 0)$ and $b_x^S(1^x, 0)$ be the two corresponding cut-off values. We need to show that $b_x^S(0, 0) = b_x^S(1^x, 0)$. We denote $P(LO(s_i))$ the execution probability of the limit order when the state of the book is s_i. We notice

$$P(LO(0, 0)) = P(LO(1^x, 0)).$$

To see that, assume that the x-seller at t, submits a limit order and let the arriving trader $t + 1$ be seller. If the book at t was $b_x^S(0, 0)$ then this trader faces the state $b_x^S(0, -1^x)$. If the book was $b_x^S(1^x, 0)$ then the trader faces $b_x^S(1^{2,x}, -1^x)$ which is equivalent to $b_x^S(0, -1^x)$ concerning the actions of the arriving trader. The same reasoning holds if at $t+1$ a buyer arrives. Thus the decision of the trader at $t+1$ does not depend on the state of the book on the opposite side that the x- seller faces at t. A similar reasoning holds for the subsequent period and hence, the execution probability of the submitted limit order at t, in both cases is equal, i.e., $P(LO(0, 0)) = P(LO(1^x, 0))$. Since the outside option of a trader on the two states of the book is the same, it results that her behavior, will be identical i.e. $b_x^S(0, 0) = b_x^S(1^x, 0)$. If she had different cut-off values facing these two different states of the book, that would be reflected in a difference in the execution probability of the limit orders. ■

The proof of **Proposition 2** is immediate from the definition of the indifference equation between a MO and a LO.

Proof of Proposition 3: For (i), it is enough to observe that traders, buyers and sellers, exist with same percentages in the population. Moreover, the willingness to trade b, is a draw from a uniform distribution with support over $(0, 2)$ independent to the type of a trader.

For reasons of completeness we will sketch the proof for one state of the book and the remaining follow with similar arguments. Assume that the book is empty. We first notice that a buyer or a seller if they place a LO they have the same probability of execution when both facing an empty book. So we have the following system of indifference equations:

$$\begin{cases} (B - b_x^S(0,0)V) = P(A - b_x^S(0,0)V) \\ (b_x^B(0,0)V - A) = P(b_x^B(0,0)V - B) \end{cases}$$

Solving the system above and substituting one solution to the other we obtain $b_x^S(0,0)$ as a function of $b_x^B(0,0)$, and in particular $b_x^B(0,0) = 2 - b_x^S(0,0)$.

We have that (ii) results from (i) and the properties of cumulative distribution functions. ■

The proof of **Propositions 4** and **5** follow directly by the solution of System 2.

The proofs for **Propositions 6, 7 and 8** are provided via mathematical exhaustion, replicating our results for a wide range of fundamental values and granular tick sizes much finer than the 1 cent tick size found in most markets. Proof of **Proposition 9**, follows directly from the corresponding system of indifference equations that a trader under opacity solves.

B Markov Steady State Equilibrium and Transition Matrix

As explained in the main body of the text, the evolution of the limit order book, defines a Markov chain for which the steady state equilibrium is the stationary distribution ρ that satisfies the following matrix equation

$$\rho\mathcal{M} = \rho,$$

where $\mathcal{M}$ is the transition matrix of the chain. Notice that the transition matrix remains a function of the market shares of the two brokers. The solution provides the steady state distribution, i.e., the likelihood that the arriving trader faces a specific state of the book after a sufficient number of traders have arrived to the market. The steady state distribution exists and is unique. The existence and uniqueness is ensured because the chain is irreducible and a-periodic. Irreducibility means that there is always a positive probability that from a particular state of the book, to be able to return on that state in finite period of time. A-periodicity, is usually more difficult to verify. A state j is periodic with period k, if starting from j we need a multiple of k steps to return to that state. If $k = 1$ then the state is a-periodic. However, given that our Markov chain is irreducible, in order to be a-periodic, it only needs one a-periodic state, which is given when the state of the book is empty. Thus there exists a unique vector representing the steady state probability distribution. Equivalent, this stationary distribution, is the left eigen vector of the transition matrix (Daroch and Senenta, 1965).

C Complementary Tables

Table C1: Steady States Probabilities

The table summarizes the Markov steady state probabilities for the PT and PBT trading protocols when the ask and bid prices are set to 1.1 and 0.9 respectively which corresponds to a tick $\Delta = 0.2$. Panels A and B correspond to PT and Panels C and D to PBT. In both cases, states are divided to those that place the arriving trader to a non-competition state, and to those that put her under competition.

PT					
Panel A: No Competition States with Corresponding Probabilities					
$(0, 0)$	74.3531	$(2^x, 0)$	2.5837	$([1^{2,y}, 1^y], 0)$	0.1194
$(1^x, 0)$	3.2917	$(2^y, 0)$	2.5837	$(1^x, -1^{2,x})$	0.1488
$(1^y, 0)$	3.2917	$([1^{2,x}, 1^x], 0)$	0.1194	$(1^x, -1^{2,y})$	0.1488
$(0, -2^x)$	2.5837	$([1^{2,y}, 1^x], 0)$	0.1194	$(1^y, -1^{2,x})$	0.1488
$(0, -2^y)$	2.5837	$([1^{2,x}, 1^y], 0)$	0.1194	$(1^y, -1^{2,y})$	0.1488
Panel B: Competition States with Corresponding Probabilities					
$(0, -1^x)$	3.2917	$(0, [-1^y, -1^{2,y}])$	0.1194		
$(0, -1^y)$	3.2917	$(1^{2,x}, -1^x)$	0.1488		
$(0, [-1^x, -1^{2,x}])$	0.1194	$(1^{2,y}, -1^x)$	0.1488		
$(0, [-1^x, -1^{2,y}])$	0.1194	$(1^{2,x}, -1^y)$	0.1488		
$(0, [-1^y, -1^{2,x}])$	0.1194	$(1^{2,y}, -1^y)$	0.1488		
PBT					
Panel C: No Competition States with Corresponding Probabilities					
$(0, 0)$	74.4773	$(2^x, 0)$	2.5576	$([1^{2,y}, 1^y], 0)$	0.1187
$(1^x, 0)$	3.2767	$(2^y, 0)$	2.5576	$(1^x, -1^{2,x})$	0.1478
$(1^y, 0)$	3.2767	$([1^{2,x}, 1^x], 0)$	0.1187	$(1^x, -1^{2,y})$	0.1478
$(0, -2^x)$	2.5576	$([1^{2,y}, 1^x], 0)$	0.1317	$(1^y, -1^{2,x})$	0.1478
$(0, -2^y)$	2.5576	$([1^{2,x}, 1^y], 0)$	0.1317	$(1^y, -1^{2,y})$	0.1478
Panel D: Competition States with Corresponding Probabilities					
$(0, -1^x)$	3.2767	$(0, [-1^y, -1^{2,y}])$	0.1187		
$(0, -1^y)$	3.2767	$(1^{2,x}, -1^x)$	0.1478		
$(0, [-1^x, -1^{2,x}])$	0.1187	$(1^{2,y}, -1^x)$	0.1478		
$(0, [-1^x, -1^{2,y}])$	0.1317	$(1^{2,x}, -1^y)$	0.1478		
$(0, [-1^y, -1^{2,x}])$	0.1317	$(1^{2,y}, -1^y)$	0.1478		

Table C2: Irrelevant States For a Seller

This table depicts the irrelevant states for a seller i.e. the states that even though different, lead to the same action for the arriving seller. Column 1, provides the three distinct cases from which the arriving seller determines her action. Column 2 correspond to PT and PBT. Under PT a trader does not distinguishes broker affiliations, thus combines rows two and three.

	PT and PBT
Irrelevant states to $(0,0)$, for a seller	$(0,-1^{2,x}),(1^{2,x},0),(0,-1^{2,y}),$ $(1^{2,y},0),(1^{x},0),(1^{y},0),$ $([1^{2,x},1^{x}],0),([1^{2,y},1^{y}],0),([1^{2,y},1^{x}],0),$ $([1^{2,x},1^{y}],0),(1^{x},-1^{2,x}),(1^{y},-1^{2,y}),$ $(1^{x},-1^{2,y}),(1^{y},-1^{2,x})$
Irrelevant states to $(0,-1^{x})$, for a seller	$(0,[-1^{x},-1^{2,x}]),(0,[-1^{x},-1^{2,y}]),$ $(1^{2,x},-1^{x}),(1^{2,y},-1^{x})$
Irrelevant states to $(0,-1^{y})$, for a seller	$(0,[-1^{y},-1^{2,y}]),(0,[-1^{y},-1^{2,x}]),$ $(1^{2,x},-1^{y}),(1^{2,y},-1^{y})$

Table C3: Indifference Cut-off Values When One Broker Offers PBT

This table contains the equilibrium cut-off values obtained from the extension of our main model in which only broker X offers preferencing services. The ask and bid prices are set to 1.1 and 0.9 respectively which corresponds to a tick $\Delta = 0.2$. In the first column with we have the three states of interest for a trader and column 2 (3) reports the indifference values for an $x(y)$-trader respectively.

	x-seller	y-seller
Empty	0.788974	0.790616
Join Behind Different Type	0.823125	0.851525
Join Behind same Type	0.850466	0.851525

Table C4: Trading Rates When One Broker Offers Preferencing

The table reports the likelihood of observing a market order by the arriving trader for the extension of our model where only broker X operates under PBT. The ask and bid prices are set to 1.1 and 0.9 respectively which corresponds to a tick $\Delta = 0.2$. Column 2 (3) reports trading rates for an $x(y)$-trader respectively. Panel A presents the total trading rate and the trading rates for the first and second in line, given that the arriving trader is $x(y)$-seller. Panel B report the likelihood that a trader will submit a market order given her broker affiliation and her position in line.

Panel A		
	x- seller	y- seller
Trading Rate given the type of the arriving seller	39.6318	39.7638
Trading Rate for First in Line given the type of the arriving seller	36.4304	36.5063
Trading Rate for Second in Line given the type of the arriving seller	3.2014	3.2575
Panel B		
	x- Seller	y- Seller
Trading Rate given trader's type and that she First in Line	39.4487	39.5308
Trading Rate given trader's type and that she is Second in Line	41.8427	42.5762

Table C5: Welfare Results When One Broker Offers Preferencing

The table reports the social welfare generated by traders who are affiliated to X or Y brokers for the extension of our model in which only X offers preferencing services. The ask and bid prices are set to 1.1 and 0.9 respectively which corresponds to a tick $\Delta = 0.2$. Column 2 (3) reports the generated welfare for an $x(y)$-seller respectively. In our computations we have calculated the welfare generated from the trading counterparts, without including brokers and crowd/dealer. Total welfare denotes welfare generated by a seller, and the welfare generated by a first (second) in line denotes the welfare generated by a first (second) in line trader,given that she is affiliated to X or Y broker.

	x- seller	y- seller
Total Welfare Given Trade's Type	0.207868	0.207783
Welfare Generated by First in Line Given Trade's Type	0.192133	0.192057
Welfare Generated by Second in Line Given Trade's Type	0.015734	0.015725