Longevity 13 Conference

Modeling and Forecasting Age-at-death Distributions

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Taipei, 21\textsuperscript{st} September 2017
Motivation

- **Background:**
  - mortality modeling and forecasting are generally based on mortality rates
  - age-at-death distributions are very informative, yet neglected for modeling and forecasting
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  - mortality modeling and forecasting are generally based on mortality rates
  - age-at-death distributions are very informative, yet neglected for modeling and forecasting

- **Research question:** model and forecast mortality by studying changes in age-at-death distributions
**Age-at-death Distributions**

Smooth Distributions – Japan, Females LT

Summary measures:

- **Longevity** = Modal age at death
- **Lifespan Inequality** = (Relative) variability of death distribution
Age-at-death Distributions

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Age-at-death Distributions

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- **Longevity** = Modal age at death
- **Lifespan Inequality** = (Relative) variability of death distribution
Longevity & Lifespan Inequality

Joint trends studied extensively, but hard to disentangle age-specific contributions

Modal age at death – Females

Gini coefficient (%) – Females
The STAD Model

Notation:

- $x$: age
- $f(x)$: standard distribution
- $g(x)$: observed distribution
- $t(x)$: transformation function

Aim:

Look for a $t(x)$ such that:

- $g(x)$ conforms to $f(x)$ on the warped axis, i.e. $g(x) = f(t(x))$
- $t(\cdot)$ is a segmented function of the difference in modal ages and the change in the variability before and after $M$.

$t(x; s, b_L, b_U) = \begin{cases} M f + b_L (x - s - M f) & \text{if } x \leq M g \\ M f + b_U (x - s - M f) & \text{if } x > M g \end{cases}$
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$$
t(x; s, b_L, b_U) = \begin{cases} 
M_f^f + b_L (x - s - M_f^f) & \text{if } x \leq M_g^f \\
M_f^f + b_U (x - s - M_f^f) & \text{if } x > M_g^f
\end{cases}
$$

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The STAD Model

\[ S = M_g - M_f \]

is the difference between the \( M \) of \( g(x) \) and \( f(x) \) (shifting dynamic of mortality).

**Transformation functions**

- Actual ages, \( x \)
- Transformed ages, \( t(x) \)

**Standard and transformed distributions**

- Ages: \( f(x), f(t(x)) \)
- Transformed ages, \( t(x) \)

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The STAD Model

\[ s = M^g - M^f \] is the difference between the \( M \) of \( g(x) \) and \( f(x) \) 
(shifting dynamic of mortality)
The STAD Model

\( b_L \) and \( b_U \) measure the change in lifespan variability of \( f(x - s) \) before and after \( M^g \) (compression dynamic of mortality)
The Standard Distribution

- **Relational models**: theoretical framework, transformed $f(x)$ captures mortality developments *over time*
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- **Landmark registration**: alignment of observed densities to the mode of the first distribution
The Standard Distribution

![Graph showing Standard - no alignment distributions](image)

- `g_1(x)`
- `g_2(x)`

Distributions vs. Ages
The Standard Distribution

![Graph showing standard distributions with ages ranging from 40 to 100 and distributions labeled as $g_1(x)$, $g_2(x)$, and $f(x)$]
The Standard Distribution

Standard – no alignment

Standard – with alignment

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The Standard Distribution

The distribution for ages 40, 60, 80, and 100 is shown in the graphs. The Standard distribution without alignment and with alignment are compared. The graphs illustrate the changes in the distribution with and without alignment.
Application to observed data

- **Smoothing:**
  - apply continuous model to discrete data
  - avoid rigid parametric mortality structure
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- **Estimation:** for each year, $b_L$ and $b_U$ estimated by maximum likelihood from the assumption:

\[
D_x \sim \text{Poisson} \left( E_x \mu_x \right)
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Application to observed data

- **Smoothing:**
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- **Estimation:** for each year, $b_L$ and $b_U$ estimated by maximum likelihood from the assumption:

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- **Data:** observed death counts and exposure times for females aged 30+ during 1980-2014 in Sweden and France (retrieved from the Human Mortality Database)
Estimation

Shifting parameter $s$:

*Sweden*

*France*
Estimation

Compression/expansion parameters $b_L$ and $b_U$:

**Sweden**

- $b_L$ and $b_U$ from 1980 to 2040.

**France**

- $b_L$ and $b_U$ from 1980 to 2040.
Observed vs Fitted Data

Good performance in terms of goodness-of-fit:

**Sweden – Observed and Fitted LE**

- Observed
Observed vs Fitted Data

Good performance in terms of goodness-of-fit:

Sweden – Observed and Fitted LE

- Observed
- Fitted
Observed vs Fitted Data

Good performance in terms of goodness-of-fit:

France – Observed and Fitted LE

Years

Observed
Observed vs Fitted Data

Good performance in terms of goodness-of-fit:

France – Observed and Fitted LE

Years


Observed
Fitted
Forecasting with univariate ARIMA model

Shifting parameter $s$ forecast with 80% C.I.:
Forecasting with univariate ARIMA model

Shifting parameter $s$ forecast with 80% C.I.:
Forecasting with multivariate VAR model

Compression/expansion $b_L$ and $b_U$ forecast with 80% C.I.:
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Compression/expansion $b_L$ and $b_U$ forecast with 80% C.I.:
Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - Sweden:

**Observed and Forecast LE**

- **Actual**

![Graph showing observed and forecast life expectancy](image)
Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - Sweden:

![Graph showing observed and forecast LE]
Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - Sweden:

**Observed and Forecast LE**

- **Actual**
- **Lee–Carter**
- **STAD**


$e_{30}$

50 52 54 56 58
Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - France:
Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - France:

![Observed and Forecast LE](image)

- **Actual**
- **Lee–Carter**

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Forecasting - $e_{30}$

Remaining female life expectancy with 80% CI - France:

![Graph showing observed and forecast LE](image-url)
Forecasting - $m_x$

Mortality Rates – Sweden

- 1980
- 2014
- STAD 2040
- LC 2040

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Forecasting - $m_x$

Mortality Rates – France

- - - 1980
- - - 2014
- - - STAD 2040
- - - LC 2040

Log($m(x)$) vs Age
Forecasting - $d_x$

Life-table Deaths – Sweden

- 1980
- 2014
- STAD 2040
- LC 2040
Forecasting - $d_x$

Life-table Deaths – France

- - - 1980
- 2014
- STAD 2040
- LC 2040

Age

$\text{Birth cohorts 1980, 2014, STAD2040, LC2040}$

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Modeling and Forecasting Age-at-death Distributions
Summary

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- Forecast remaining life expectancy reflects well the past linear increase and it is more optimistic than the Lee-Carter model
Future work

- Extension to the entire age range
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- Application to longevity risk products & pricing comparison against other models
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- Application to longevity risk products & pricing comparison against other models

- Application to cause-specific mortality
Thanks for your attention.

Comments and/or questions?
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