Heterogeneous Expectations and Speculative Behavior in Insurance-linked Securities

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Motivation

- **Background:**
  - **Principle of Insurance:** Law of Large Numbers
    - The greater the number of similar exposures, the more predictable the outcome
  - **Life Insurance Pricing Risks**
    - Mortality—when death occurs
    - Persistency—how long the customer keeps the policy
    - Expenses—how efficient the company operates
    - Investment—return of and on invested cashflows.
  - **Risks insured (life insurance)**
    - Premature death
    - Outliving accumulated savings
**Figure:** Life Expectation At Birth (Source: World Bank)
Solutions:

- **Natural Hedge**
  - Natural hedge between life insurance and annuity
  - Ineffective and cost prohibitive

- **Reinsurance**
  - capacity constraint

- **Securitization**
  - Transfer risk to investors in capital market
  - Derivative securities with payoffs link to certain insurance risk
Securitization: ILS

Key elements

- Price
- Demand v.s supply
- Ability to hedge risks
- ...
Securitization: ILS

Pricing

- **No-arbitrage approach**: Cairns, Blake and Dowd (2006), Bauer, Boerger and Russ (2010)
- **Maximum Entropy**: Kogure and Kurachi (2010)
- **Indifference pricing approach**: Cui (2008), Cox, Lin and Pederson (2010)
- **Tâtonnement approach**: Zhou, Li and Tan (2011) and Chen, Sherris, Sun and Zhu (2012)
  - A gradual calibration of supply and demand
Securitization: Tâtonnement approach

Agent I (hedger) v.s Agent A (investor)

Agents are expected utility maximizers.

Two investment choices in Zhou, Li and Tan (2011)
  - the insurance-linked security
  - agents can lend/borrow at the same risk-free interest rate $r$
Securitization

Agent A
Securitization

Insurance-linked Security → Agent A
Securitization

Insurance-linked Security  <-> Agent A  <-> Risky Asset
Securitization

- ILS offers a new class of investment opportunities
- Agent A wants to optimize his whole portfolio.
In this paper

- We follow the Tâtonnement approach and use the BH framework given by Brock and Hommes (1997, 1998).

- Main questions:
  - Whether ILS can help the insurance company to hedge its risk?
  - Whether ILS as a new investment tool can increase the investment profit?
  - What relationship between the risky asset and ILS?

- Three cases:
  - Static model with two agents
  - Dynamic model with two agents
  - Dynamic model with three agents
Set-up: One-period

- $q$: the risk agent $I$ wants to hedge
- Cash flow for agents

- $\Theta_I$, $\Theta_A$: quantities of ILS traded by agents $I$ and $A$
- $\Phi_A$: quantity of the risky asset traded by agent $A$
Set-up: One-period

- Agent A
  - wealth process
    \[ W_A = (\omega_A - \Theta_A P - \Phi_A S)R + \Theta_A g(q) + \Phi_A y, \quad R = 1 + r \]
  - Agent A is a myopic mean-variance maximizer based on his own belief:
    \[ \sup_{\Theta_A, \Phi_A} E_0^A(W_A) - \frac{k_A}{2} V_0^A(W_A), \]
    where \( E_0^h \) and \( V_0^h \) denote the ‘beliefs’ of agent \( h \) about the conditional expectation and conditional variance at time 0 and \( k_h \) denotes the risk aversion of agent \( h \).
Set-up: One-period

- **Agent I**
  - **Hedging term:** Basis risk \((BR)\)
    \[ BR = f(q) - \Theta_I(g(q) - RP). \]
  - If \( BR \leq 0 \), then it means this type of ILS can completely hedge all the risks and even more, the profit can be obtained by ILS.
  - Otherwise, it means there exists the basis risk between this type of ILS and the risk hedge demand.
  - **Objective:**
    \[ \inf_{\Theta_I} E_0^I(BR) + \frac{k_I}{2} V_0^I(BR). \]
Notations:

\[ \mathcal{Y}_h = E_h^0(y), \quad \mathcal{G}_h = E_h^0(g(q)), \quad (h = A, l) \]

\[ \sigma_{h,y}^2 = V_h^0(y), \quad \sigma_{h,g}^2 = V_h^A(g(q)), \quad \sigma_{h,f}^2 = V_h^A(f(q)), \]

\[ \text{cov}_{A,gy} = \text{cov}_0^A(g(q), y), \quad \text{cov}_{l,gf} = \text{cov}_0^l(g(q), f). \]
Equilibrium: Walrasian scenario

\[
\begin{align*}
\Theta_A + \Theta_I &= 0 \\
\Phi_A &= 0
\end{align*}
\] (1)

**Theorem**

*The equilibrium prices of ILS and the risky asset respectively are*

\[
P^* = R^{-1} \left( G_I + k_I cov_{I,gf} \right) k_A \sigma_{A,g}^2 + G_A k_I \sigma_{I,g}^2 \\
S^* = R^{-1} \left( \gamma_A + k_A cov_{A,gy} \frac{G_I + k_I cov_{I,gf} - G_A}{k_A \sigma_{A,g}^2 + k_I \sigma_{I,g}^2} \right)
\] (2) (3)
The relationship between the risky asset and ILS

\[
\frac{\gamma_A - RS^*}{k_A} = \frac{G_A - RP^*}{k_A} + \frac{\sigma_{A,g}^2 - \text{cov}_{A,gy}}{\sigma_{l,g}^2} \frac{G_I + k_I \text{cov}_{l,gf} - RP^*}{k_l}.
\]

(4)
**ILS v.s. Risky Asset**

- The relationship between the risky asset and ILS

\[
\frac{\gamma_A - RS^*}{k_A} = \frac{G_A - RP^*}{k_A} + \frac{\sigma_{A,g}^2 - \text{cov}_{A,gy}}{\sigma_{I,g}^2} \frac{G_I + k_I \text{cov}_{I,gf} - RP^*}{k_I}.
\]

(4)

- "CAPM" format

\[
\gamma_A - RS^* = \frac{\text{cov}_{A,gy}}{\sigma_{A,g}^2} (G_A - RP^*). \tag{5}
\]

(5)
The relationship between the risky asset and ILS

\[
\frac{\gamma_A - RS^*}{k_A} = \frac{G_A - RP^*}{k_A} + \frac{\sigma^2_{A,g} - \text{cov}_{A,gy}}{\sigma^2_{I,g}} \frac{G_I + k_I \text{cov}_{I,gf} - RP^*}{k_I}.
\]

"CAPM" format

\[
\gamma_A - RS^* = \frac{\text{cov}_{A,gy}}{\sigma^2_{A,g}} (G_A - RP^*).
\]

If \(\text{cov}_{A,gy} = 0\), then

\[
S^* = R^{-1}\gamma_A.
\]

If \(\text{cov}_{A,gy} \neq 0\), then

\[
\gamma_A - RS^* = \frac{\text{cov}_{A,gy}}{\sigma^2_{A,g}} (G_A - RP^*) = k_A \text{cov}_{A,gy} \Theta^*_A.
\]
Basis Risk

- $BR = f(q) - \Theta^*_I(g(q) - RP^*)$ measures the level of the risk hedge through ILS.

- If $\sigma_{A,g} = \sigma_{I,g} = \sigma$ and $k_A = k_I = \kappa$, then

\[
BR = f(q) - \Theta^*_I(g(q) - RP^*) \\
= \frac{\kappa}{4\sigma^2} \left( \text{cov}_{I,g} - \frac{g(q) - G_I}{\kappa} \right)^2 + \left( f(q) - \frac{(g(q) - G_A)^2}{4\kappa \sigma^2} \right) \\
= \frac{1}{4\kappa \sigma^2} \left( \kappa \text{cov}_{I,g} - (g(q) - G_I) \right)^2 + \left( f(q) - \frac{(g(q) - G_A)^2}{4\kappa \sigma^2} \right)
\]
\( \kappa \text{cov}_I = g(q) - G_I \)

(a) \( G_A = 1 \)

(b) \( \text{cov}_{I,g} = 1 \) and \( \kappa \text{cov}_{I,g} = g(q) - G_I \)

**Figure:** Basis risk at \( g(q) = 0.5, f(q) = 1, \sigma_{I,g} = \sigma_{A,g} = 1, k_I = k_A = 1 \).
Set-up: Multi-period

- $\Theta_{h,t}$ and $\Phi_{h,t}$ respectively are the net position of the risky asset and ILS held by agent $h$.

- The wealth process of Agent A is

$$W_{A,t+1} = W_{A,t}R + \Theta_{A,t}R_{P,t+1} + \Phi_{A,t}R_{S,t+1}$$

- Similarly, the basis risk of agent $l$ would be

$$BR_{t+1} = f(Q_{t+1}) - \Theta_{l,t}R_{P,t+1}.$$ 

where $R_{P,t+1} = g(Q_{t+1}) + P_{t+1} - RP_{t+1}$ and $R_{S,t+1} = S_{t+1} + y_{t+1} - RS_{t}$.
At every time, the two agents will choose the position of the risky asset and ILS to optimize their utilities based on their own beliefs

\[
\begin{align*}
\text{Agent } A : & \quad \sup_{\Theta_{A,t}, \Phi_{A,t}} E_{t}^{A}(W_{A,t+1}) - \frac{k_A}{2} V_{t}^{A}(W_{A,t+1}), \\
\text{Agent } I : & \quad \inf_{\Theta_{I,t}} E_{t}^{I}(BR_{t+1}) + \frac{k_I}{2} V_{t}^{I}(BR_{t+1}),
\end{align*}
\]

where $E_{t}^{h}$ and $V_{t}^{h}$ respectively are the conditional expectation and variance of agent $h$ ($h = A, I$) at time $t$ based on the publically available information $\mathcal{F}_{t} = \{P_{0}, \cdots, P_{t-1}, S_{0}, \cdots, S_{t-1}\}$. 
Assume that agent $I$ adopts the fundamental analysis to analyze the price trend while agent $A$ prefers using the technical methods to estimate the price value, that is

\[
E_t^A(S_{t+1}) = S_{t-1} + \gamma_A(S_{t-1} - S_{t-1}^\sharp), \quad \gamma_A > 0
\]
\[
E_t^A(P_{t+1}) = P_{t-1} + \alpha_A(P_{t-1} - P_{t-1}^\sharp), \quad \alpha_A > 0
\]
\[
E_t^I(P_{t+1}) = P_{t-1} + \alpha_I(P_{t-1}^* - P_{t-1}), \quad \alpha_I \in [0, 1].
\]

For agent $A$, the reference prices are calculated by the moving average method

\[
S_t^\sharp = \frac{1}{2}(S_{t-1} + S_t),
\]
\[
P_t^\sharp = \frac{1}{2}(P_{t-1} + P_t).
\]
For agent $i$, the fundamental price comes from

$$E^I_t (g(Q_{t+1}) + P^*_{t+1} - R P^*_t) + k_I \text{cov}_{i,Pf} = 0.$$ 

In the case where the coupon process $\{g(Q_t)\}$ is IID, $E^I_t(g(Q_{t+1})) = G_I$ which is a constant and a standard notion of ‘fundamental’ is

$$P^*_t = \frac{G_I + k_I \text{cov}_{i,Pf}}{R - 1}.$$
Proposition

The equilibrium prices of ILS and the risky asset are

\[
\bar{P} = r^{-1} \left( \frac{r}{k_A \sigma^2_{A,P}} G_A + \frac{r + \alpha_I}{k_I \sigma^2_{I,P}} (G_I + k_I \text{cov}_I P_f) \right) + \frac{r + \alpha_I}{k_I \sigma^2_{I,P}},
\]

\[
\bar{S} = r^{-1} \left( \gamma_A + k_{A \text{cov}_{A,PPI}} \frac{r + a_I}{rk_I \sigma^2_{I,P} + (r + \alpha_I)k_A \sigma^2_{A,P}} (G_I + k_I \text{cov}_I P_f - G_A) \right),
\]

which are stable when \( \alpha_A k_I \sigma^2_{I,P} < 2R(k_A \sigma^2_{A,P} + k_I \sigma^2_{I,P}) \) and \( \gamma_A < 2R \).
Figure: \( \alpha_A k_I \sigma_{I,P}^2 = 2R(k_A \sigma_{A,P}^2 + k_I \sigma_{I,P}^2) \) and \( \gamma_A = 2R \).
Set-up: Three agents

- Except agents I and A, there is another agent who is just interested in the risky asset.

- The wealth process of agent $B$ is

$$W_{B,t+1} = W_{B,t} R + \Phi_{A,t} R_{S,t+1}.$$ 

- Equilibrium conditions

$$n_{I,t-1} \Theta_{I,t} + n_{A,t-1} \Theta_{A,t} = 0$$
$$n_{A,t-1} \Phi_{A,t} + n_{B,t-1} \Phi_{B,t} = 0$$

where $n_{h,t}$ is the population fraction of agent $h$ ($h = I, A, B$) and $n_{I,t} + n_{A,t} + n_{B,t} = 1$
The equilibrium price satisfies

\[
\frac{n_{A,t-1}}{k_A} E_t^A(R_{S,t+1}) + \frac{\sigma_{A,S}^2 - \text{cov}_{A,PS}}{\sigma_{S,B}^2} \frac{n_{B,t-1}}{k_B} E_t^B(R_{S,t+1})
\]

\[
= \frac{n_{A,t-1}}{k_A} E_t^A(R_{P,t+1}) + \frac{\sigma_{A,P}^2 - \text{cov}_{A,PS}}{\sigma_{P,I}^2} \frac{n_I}{k_I} (E_t^I(R_{P,t+1}) + k_I \text{cov}_{I,Pf})
\]
The equilibrium price satisfies

\[ \frac{n_{A,t-1}}{k_A} E_t^A(R_{S,t+1}) + \frac{\sigma_{A,S}^2 - \text{COV}_{A,PS}}{\sigma_{B,S}^2} \frac{n_{B,t-1}}{k_B} E_t^B(R_{S,t+1}) \]

\[ = \frac{n_{A,t-1}}{k_A} E_t^A(R_{P,t+1}) + \frac{\sigma_{A,P}^2 - \text{COV}_{A,PS}}{\sigma_{I,P}^2} \frac{n_I}{k_I} (E_t^I(R_{P,t+1}) + k_I \text{COV}_{I,P}) \]

**Special case:** \( n_I = 0 \), then

\[ E_t^A(R_{p,t+1}) = \frac{\text{COV}_{A,PS}}{\sigma_{A,S}^2} E_t^A(R_{S,t+1}). \]
Agents cannot switch to each other, that is to say, \( n_I, t \equiv n_I, n_A, t \equiv n_A, n_B, t \equiv n_B \) and \( n_I + n_A + n_B = 1 \).

Agents \( A \) and \( B \) can switch to each other.

- The population fractions are updated by the well known logit model probabilities

\[
n_{h,t} = \frac{\exp(\beta_h U_{h,t})}{Z_t},
\]

where \( Z_t = \exp(\beta_A U_{A,t}) + \exp(\beta_B U_{B,t}) / (1 - n_I) \).

- \( U_{h,t} \) is the performance measure

\[
U_{h,t} = \pi_{h,t} + \eta_{h,t} U_{h,t-1},
\]

where \( \eta_h \) represents the ‘memory strength’ of agent \( h \) and \( \pi_{h,t} \) is given by realized profits for agent \( h \), that is

\[
\pi_{A,t} = R_{P,t} \cdot \Theta_{A,t-1} + R_{S,t} \cdot \Phi_{A,t-1}
\]

\[
\pi_{B,t} = R_{S,t} \cdot \Phi_{A,t-1}.
\]
NS bifurcation
nA v.s. nB

$t$

$n_A$

$n_B$
The impact of the existence of ILS

- Only if investing in the risky asset (i.e. choosing Strategy B), is the aggregate wealth

\[ M_{B,t+1} = M_{B,t} + \pi_{B,t+1}. \]  

(8)

- If choosing the better one between Strategy A and Strategy B, the aggregate wealth is

\[ M_{AB,t+1} = M_{AB,t} + \pi_{A,t+1} \cdot I_{\pi_{A,t+1} > \pi_{B,t+1}} + \pi_{B,t+1} (1 - I_{\pi_{A,t+1} > \pi_{B,t+1}}). \]  

(9)
(a) Aggregate wealth

(b) Difference between different strategies
Conclusion

- We construct a heterogeneous agent model to link the ILS market and the tradition financial market.

- Through the study of basis risk, we analyze the reason of success or failure about the issuance of ILS.

- We study the reaction of the ILS and the tradition risky asset and the role of the ILS in investment portfolio.

- We can discuss the impact of different parameters.

- Future works:
  - Market-maker scenario
  - Inventory effect
  - Optimizing the VaR of insurance companies
  - ...