Forecasting mortality using statistical moments

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Which indicators have been used to forecast mortality?

Life table indicators:

Age	mx	qx	lx	dx	Lx	Tx	ex
0	0.00196	0.00196	1e+05	196	99816	8683802	86.84
1	0.00033	0.00033	99804	33	99788	8583987	86.01
2	2e-04	2e-04	99771	19	99762	8484199	85.04
...	...	...	...	...	...	...	...
108	0.6213	0.47404	171	81	130	264	1.55
109	0.65565	0.49378	90	44	68	134	1.49
110	0.68845	1.00000	46	46	66	66	1.45

Data source: www.mortality.org: Japan, females population, 2014

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Lee and Carter (1992), Carter and Lee (1992), Lee (2000), Lee and Miller (2001), Booth et al. (2002), Currie et al. (2004), Li and Lee (2005), Renshaw and Haberman (2006), Hyndman and Ullah (2007), Russolillo et al. (2011), ...

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Cairns et al. (2006), Debon et al. (2008), Cairns et al. (2009), King and Soneji (2011),...

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Brass (1971), Scherbov and Ediev (2016), ...

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Oeppen (2008), Bergeron-Boucher et al. (2017), Basellini and Camarda (forthcoming), ...

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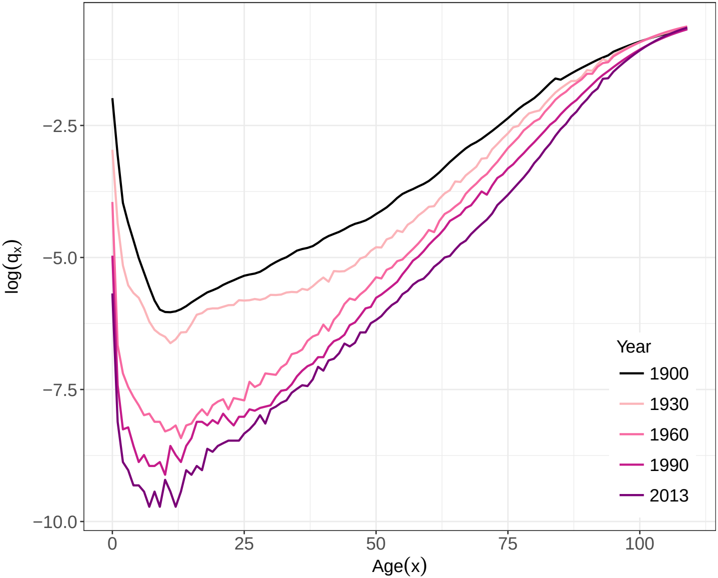
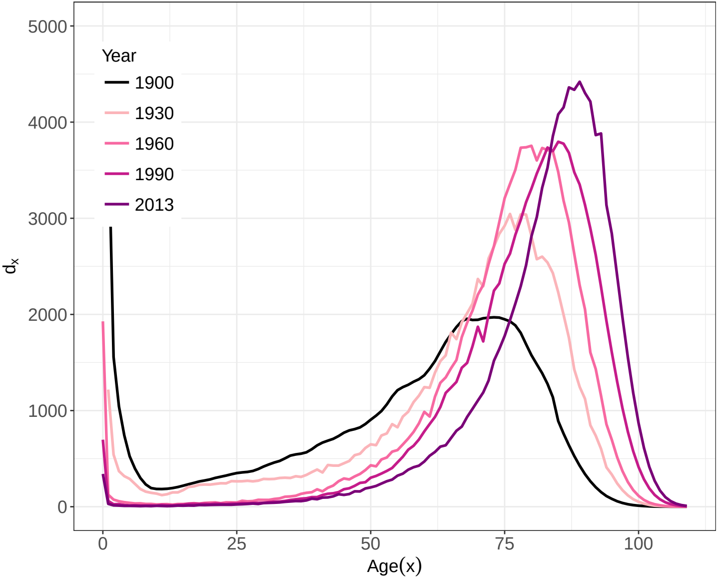
Torri and Vaupel (2012), Raftery et al. (2013), Pascariu et al. (forthcoming 2018), ...

Mortality evolution: q_x



Mortality evolution: d_x



What defines a distribution?

A distribution is defined by the totality of its moments.

Moment: Quantitative measure of the shape of a set of points or probability density → the expected value of the n-th power of a random variable.

$$M(n) = \int_a^\omega (x - c)^n f(x) dx, \quad \text{where} \quad n = 0, 1, 2 \ldots$$

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for example:

$$M(0) = 1 \quad M(1) = Mean \quad M(2) = Variance \quad M(3) = Skewness$$
$$M(4) = Kurtosis \quad \ldots$$

Problem!

All the moments up to infinity are required in order to uniquely identify a density function.

The problem of reconstructing a *pdf* from a limited number of moments is known as **the finite moment problem** (Hausdorff 1921).

Maximum entropy method (*MaxEnt*)

Objective: reconstruct a distribution given a number of known moments that satisfy the maximum entropy condition (Mead & Papanicolaou, 1984).

Construct specific sequences of functions $f_N(x)$ which eventually converge to the true distribution f(x) as N approaches infinity

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Strategy for finding the local maxima - the method of Lagrange multipliers:

$$\mathcal{L} = H + \sum_{n=0}^{N} \lambda_n \left[\hat{M}(n) - M(n) \right].$$

Information entropy

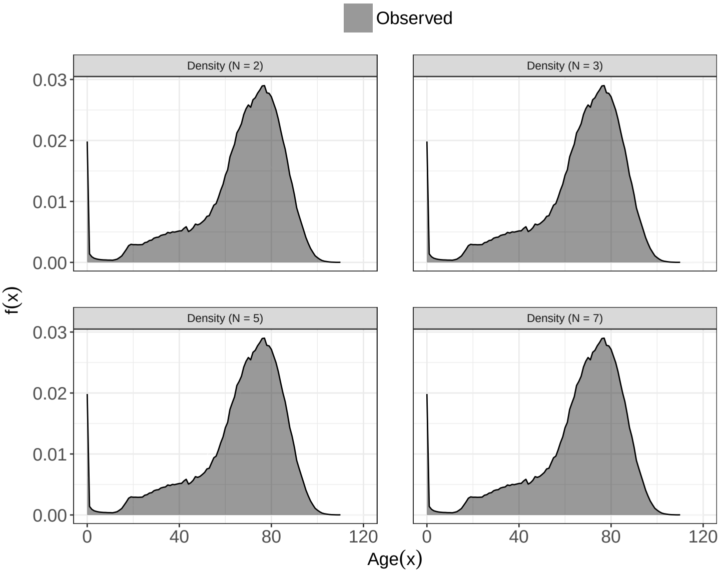
Entropy: the amount of information, $I(x)$, produced by a probabilistic stochastic source of data, x (Shannon, 1951):

$$H = E[I(x)] = E[-\log_b f(x)]$$
$$= -\int_a^\omega f(x) \log_b f(x) dx$$

The entropy is measured in *bits*, *nats* and *bann*.

Zero entropy $\Longrightarrow$ Certain event
Maximum entropy $\Longrightarrow$ Uncertain event. We have no way of learning about the outcome unless we see it!

MaxEnt density reconstruction (USA, 1990, Male population)



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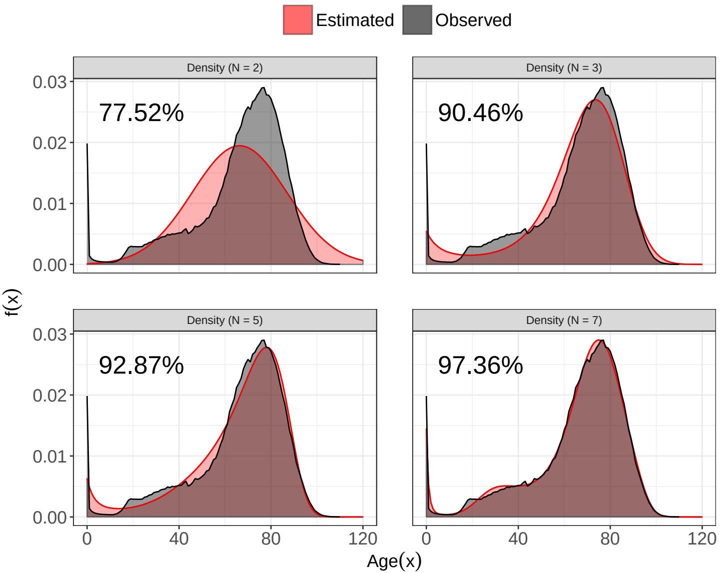
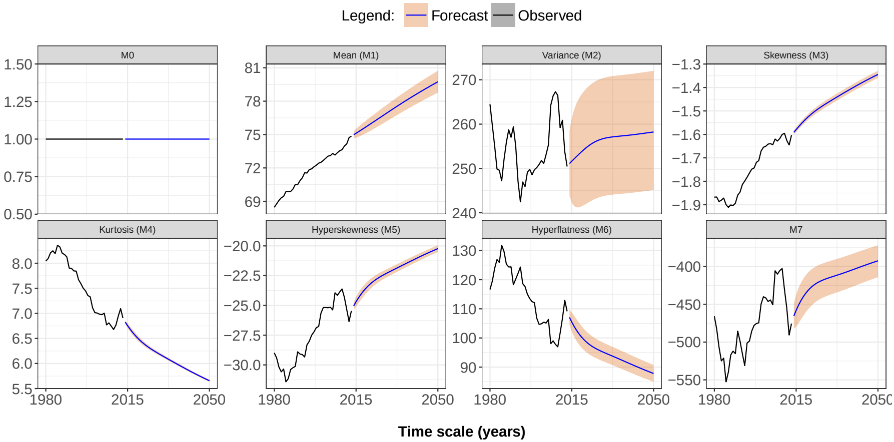


New approach to mortality forecasting

1. Extrapolate the observed raw/central moments of the observed distribution of deaths (D_x or life table d_x) using multivariate time-series models: VAR, VAR-X, VARMA ...
2. Estimate the future distribution using *MaxEnt* method;
3. Convert the estimated f_x into death probabilities q_x either by working a life table starting from d_x or by employing a *Gauss-Newton algorithm* (for higher accuracy).

Forecasting statistical moments

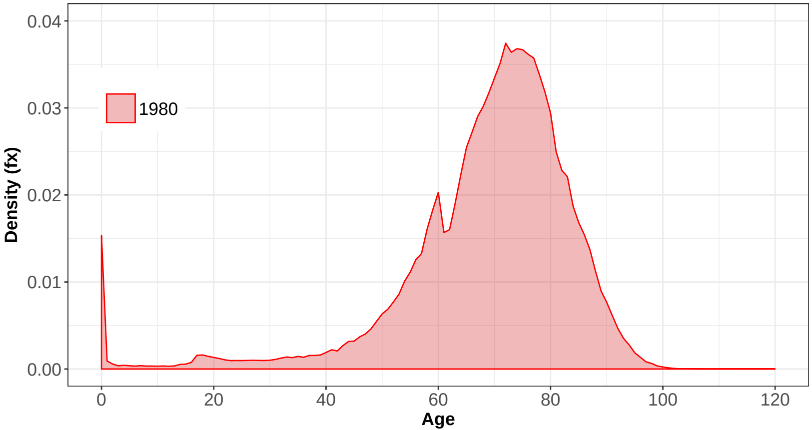
Forecast of central moments of the distribution of deaths together with 95% prediction intervals, using a VAR(1) model



Data source: www.mortality.org: England & Wales, 1980 - 2050, Male population

Distributions of deaths forecast (D_x)

Observed empirical distribution of deaths



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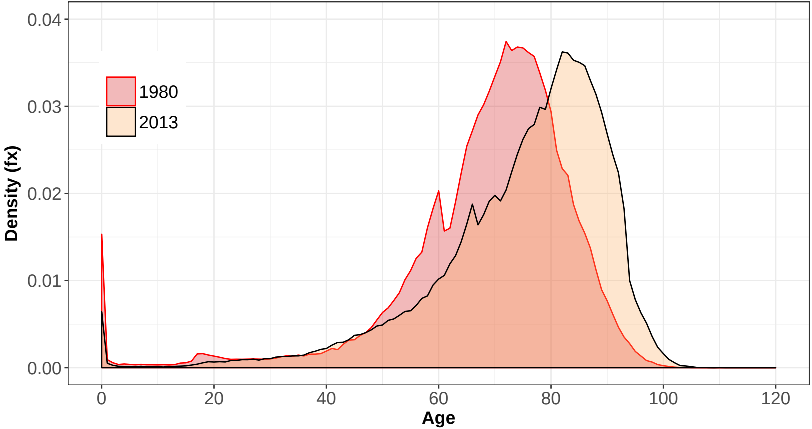

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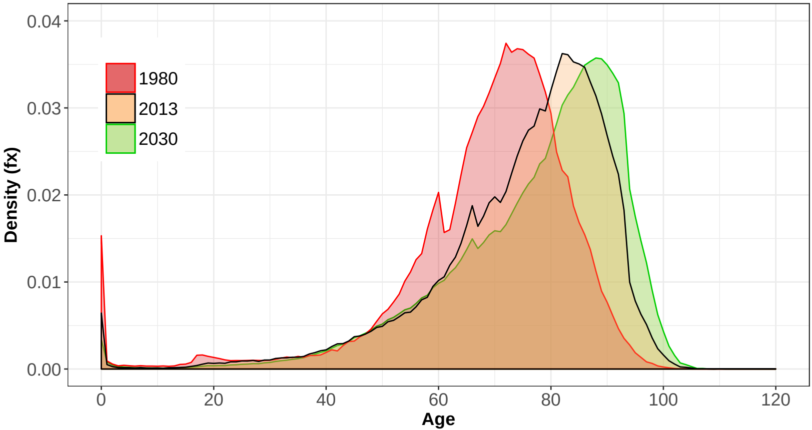
Distributions of deaths forecast (D_x)

Observed and forecast of the distribution using the first 7 moments



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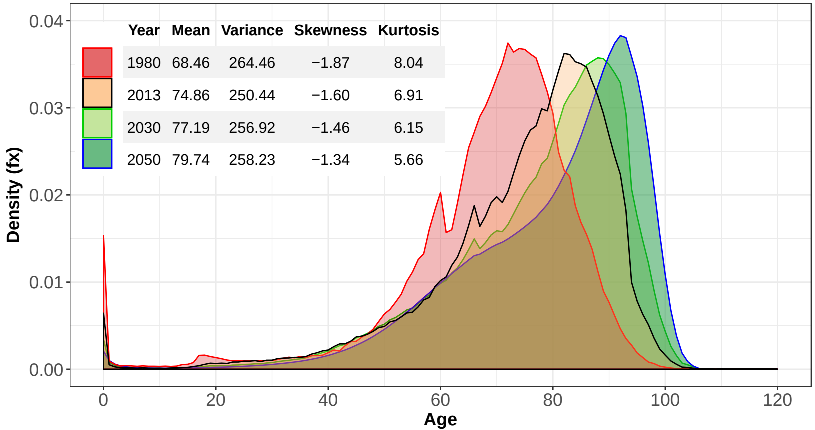

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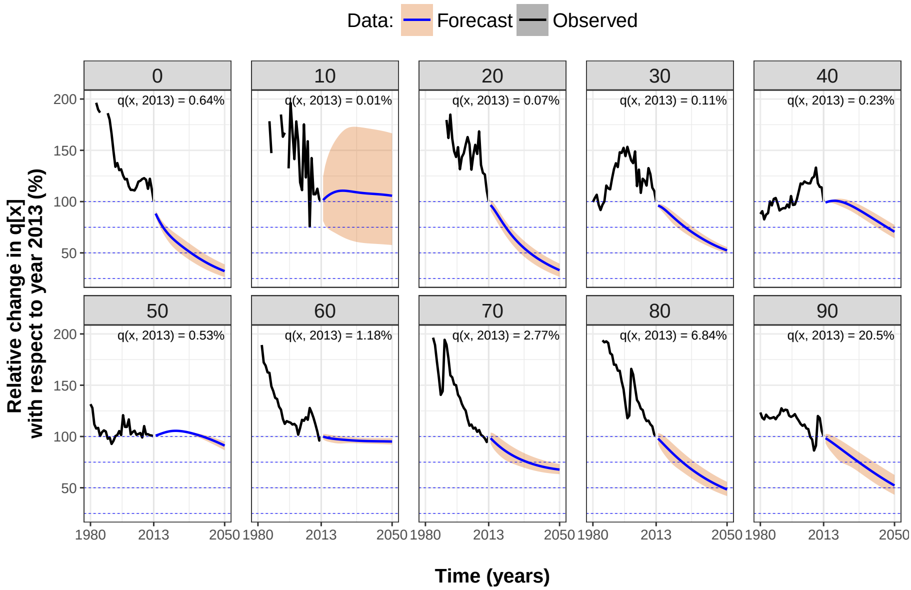
Mortality rates forecast (q_x)



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- No direct/indirect assumption of constant changes in mortality;
- Ability to included exogenous variables in the time series analysis. (information on smoking or obesity, trends in life expectancy, modal age at death, etc.)

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