Value-Time Curve Psychology

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ABSTRACT

We use value-time charts to explore the psychology underlying the mental simulation of value changes over time. In a series of “thought investments”, an initial value is projected forward to the future and that future value is discounted back to the present. We look at exponential, hyperbolic and hybrid valuation. We explain how the personal characteristics and identities of the temporal selves that project and discount can influence time preference and how, in turn, time preference influences valuation. Present value equals initial value when the valuer has temporal neutrality as, for example, when valuing symmetrically; present value is less than initial value when the valuer has positive time preference (or “myopia”); and present value exceeds initial value when the valuer has negative time preference (or “hyperopia”). £1 “thought invested” today for 30 years might, for example, be worth: £1 today when valued symmetrically; £0.57 to a myopic valuer; and £1.77 to a hyperopic valuer.

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1. INTRODUCTION

How much will £1 be worth in 30 years’ time? What is that future value worth today? In this paper, we examine the psychology underlying the mental simulation of temporal change in value. We illustrate our discussion with some simple value-time curves in which: today’s value is projected into a future value; and that future value is discounted back to a present value.

We exploit the concept of a life as a series of psychologically connected temporal selves. This is a notion developed by philosophers, such as Derek Parfit, and explored by economists and behavioural scientists, such as Robert Strotz and Shane Frederick. The psychological connections at the core of this concept include our memories, personal characteristics and interests all of which contribute to the make-up of our personal identity. Some of us have temporal selves which are well connected psychologically – our personal identity is stable over time. Indeed, according to some philosophies and religions, the temporal selves have the same soul. Some of us, in contrast, experience, or foresee, changes in our personal identity; there is a discontinuity between our temporal selves.

The value-time charts we present, in this paper, record a series of “thought investments” in which a Present Self mentally simulates value forward (“prospection”) and a Future Self (or, to be more exact, the Present Self’s projection of its Future Self) mentally simulates value backwards (“retrospection”). So, the direction of travel, in our value-time charts, is forward in time along a growth curve and back to the present along a discount curve. Prospection requires the Present Self to mentally visit the future; while retrospection requires the Projected Future Self to mentally return to the present (its past). This mental time journey begins and ends at the present time.

Mental time travel theory has traditionally assumed symmetry between prospection and retrospection. Recent research, however, reveals that, for many of us, the ways in which we mentally represent the future and the past are not the same. Symmetry implies temporal neutrality, in other words, no past, present or future time bias or preference. David Brink, the philosopher, traces the concept of temporal neutrality back

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2 Strotz (1955-56)
3 Frederick (2003, 2006)
4 Frederick (2003)
5 Where “thought investment” is used in the same sense as “thought experiment”
6 Kane et al. (2012)
7 Van Boven et al. (2008)
8 Frederick (2003)
to the Epicureans who believed in a symmetry between our pre-natal and post-mortem non-existence\(^1\) – the past time before our birth being a mirror-image of the time to come after our death.\(^1\)

Many of us are not temporally neutral. Some of us have positive time preference – we value the present more than the future. We are “myopic”. Some of us, in contrast, are “hyperopic”. We value the future over the present – we have negative time preference.

The Epicureans’ presumption of non-existence post-mortem (their belief that the soul dies with the body) imposes a finite time horizon on our existence and, as the economists Mancur Olson and Martin Bailey argue, a finite time horizon is consistent with positive time preference\(^1\). Myopic personalities, generally, prefer present pleasures (and future pain) to future pleasures (and present pain) of a similar magnitude.

Negative time preference, on the other hand, has been linked to a desire for improving sequences of outcomes which, in turn, can be attributed partly to anticipation and partly to loss aversion.\(^1\) Pleasure from anticipation is enhanced if the best comes last; while loss aversion induces a preference for increasing values (gains) over time. Negative time preference is consistent with an extended or infinite time horizon (such as that needed to accommodate a family dynasty or a belief in post-mortem existence).

The value-time charts we use to trace our valuers’ mental time travels are but constructs of the mind. These are thought investments only. No money is invested; no real time passes. This is the transformation of value as mentally simulated by a valuer. Because the future is uncertain, projected and present value are subjective. They reflect the valuer’s personal identity – the memory of a past investment, perhaps, or the valuer’s innate optimism or pessimism. Because prospection and retrospection are psychologically distinct, because a valuer’s personal characteristics can have different effects on the growth and discount curves and because the personal identities of the different temporal selves are not always well-connected, present value is not necessarily the same as initial value. Sometimes it is higher, sometimes lower.

Our paper focuses on the psychology of valuing financial assets, but value-time curve psychology has relevance also to inter-temporal costs and benefits, such as those relating to pensions and health-care, and to inter-generational issues, such as those arising from climate change.

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\(^{10}\) Greene and Sullivan (2013)  
\(^{11}\) Brink (2011)  
\(^{12}\) Lucretius, cited by Brink (2011). This “symmetry argument” presumes, of course, non-existence both before and after life.  
\(^{13}\) Olson and Bailey (1981)  
\(^{13}\) Loewenstein and Prelec (1991)
Why do we project and discount? Because we want not only to know today’s worth tomorrow, but also to know tomorrow’s value today.

We look at two methods of mentally simulating value change – exponential and hyperbolic.

Exponential valuation is standard in time value of money calculations (growth and discount rates being postulated not to vary today, tomorrow or a year from now) and economists usually assume that discount functions are exponential. Exponential savers foresee ever-increasing returns, thanks to the benefits of compounding.

Hyperbolic valuation, in contrast, assumes that the short-term rate of value change differs from the long-term rate. According to Mitchell and Utkus (2004), hyperbolic savers expect £1 saved to grow more rapidly in the short-term than in the long-term. So, they perceive decreasing benefits to long-term savings – rewards are expected to accelerate quickly and then taper off.

The growth curves in Chart 1 illustrate these exponential and hyperbolic valuation methods. In the chart, £1 is projected forward 30 years: exponentially to £5.74 and hyperbolically to £3.32. The thought investments that we consider below build on these projections by discounting these future values back to the present time. To keep things simple, we assume a positive expected return on all thought investments and no inflation over the period. Because it is the differences between value-time curves, rather than the curves themselves, that matter, we hold the growth curve fixed, for the most part, and allow the discount curve to vary.

For each of these two methods (exponential and hyperbolic), we consider three expressions of time preference: symmetry (an expression of temporal neutrality) (in section 2); myopia (positive time preference) (in section 3); and hyperopia (negative time preference) (in section 4). We then discuss hybrid mental simulation of value – the use of one method for prospection and a different method for retrospection (in section 5). We conclude by considering whether symmetry, myopia and hyperopia are persistent or flexible. We begin with symmetric valuation.

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15 Angeletos et al. (2001)
17 Berns et al. (2007)
18 Rasmussen (2008) notes that hyperbolic valuation differs additionally from exponential valuation in that it is based on relativistic distance in time from the present, rather than absolute time – the date...
20 Fixed, or standard, exponential and hyperbolic growth curves are apposite given that recent research reveals mental time travel to the future to be more prototypical than that back to the past (Kane et al. (2012)).
2. **SYMMETRIC VALUATION**

The Symmetric Valuer has temporal neutrality. The Symmetric Valuer values the present and the future equally. The Symmetric Valuer looks backwards and forwards in time in the same way (prospection and retrospection are identical, save for the direction of travel). If, for example, the projected growth rate is optimistic so, too, will be the discount rate. Similarly, if the growth rate is pessimistic. To the Symmetric Valuer, discounting is the exact temporal inverse (or reciprocal) of projecting.

*Symmetric exponential valuation*

We can use the compound interest formula \((1+r)^t\) to describe the exponential growth function for a \(t\)-year investment horizon. Here, \(r\) is the projected annual rate of growth. At an annual growth rate of 6\%, for example, £1 saved today will be worth £5.74 in 30 years’ time (\(1 \times (1.06)^{30} = 5.74\)).

What’s the value today of £5.74 in 30 years’ time? The Symmetric Valuer who projects exponentially uses an exponential discount function to answer this question; and that function incorporates a discount rate equal to the growth rate. We can use the inverted compound interest formula \((1+R)^{-t}\) to define the exponential discount function for a \(t\)-year time horizon. Here, \(R\) is the discount rate which, for the Symmetric Valuer, always equals the growth rate \(r\).
Chart 2 illustrates the exponential value-time curves of the Symmetric Valuer, assuming \( r=R=6\% \). In the chart, £1 is projected forward, at a compound annual growth rate of 6\%, to a value of £5.74 and that future value is then exponentially discounted, at an annual rate of 6\%, to a present value of £1. So, £1, invested today at 6\%, is valued at £5.74 in 30 years’ time; and £5.74 in 30 years’ time at £1 today. By implication, the Symmetric Valuer values £1 today at £1.

**CHART 2: SYMMETRIC VALUE-TIME CURVES SHOWING EXPONENTIAL GROWTH AND DISCOUNTING**

Any change in the Symmetric Valuer’s growth rate is reflected in the discount rate. So, a change in the growth rate has no effect on present value; nor does a change in the length of the valuation period. As Chart 3 illustrates, the Symmetric Valuer values at £1 today every £1 invested in any asset for any period of time. £1 is projected forward at a range of compound growth rates over a range of valuation periods and each future value is then discounted back, at a symmetric rate, to a present value of £1. So, the present value of £1 invested in, for example, equities for, say, 35 years is the same today as £1 invested in bonds for, say, 20 years, namely £1, even though equities are riskier, offer a higher expected return and are held for a longer period than bonds. The Symmetric Valuer is risk indifferent.
Symmetric hyperbolic valuation

We can use the formula \((1+\alpha t)^{\gamma/\alpha}\), where \(\gamma\) and \(\alpha\) are constants, to define the shape of the hyperbolic growth function.

We can use the formula \((1+\alpha t)^{\gamma/\alpha}\) to describe the hyperbolic discount function.\(^{21,22}\) A growing body of experimental evidence suggests that hyperbolic discounting – in which a future value declines at a more rapid rate in the short-term than the long-term – better describes how many of us value delayed rewards than does exponential discounting.\(^{23}\)

Chart 4 illustrates the hyperbolic value-time curves for a Symmetric Valuer, assuming that \(\gamma = 1\) and \(\alpha = 4\) for both the growth and discount curves.\(^{24}\) In the chart, £1 today is projected to grow hyperbolically to £3.32 in 30 years’ time; and that future value is then hyperbolically discounted to a present value of £1. So, £1 today is valued at £3.32 in 30 years’ time; and

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\(^{21}\) Loewenstein and Prelec (1992)

\(^{22}\) Laibson (1998)


\(^{24}\) These are the values underlying the hyperbolic curve in Figure 1 of Mitchell and Utkus (2004) and, as reproduced in, Chart 1 of this paper.
£3.32 in 30 years’ time at £1 today. Again, by implication, the Symmetric Valuer values £1 today at £1.

**CHART 4: SYMMETRIC VALUE-TIME CURVES SHOWING HYPERBOLIC GROWTH AND DISCOUNTING**

As when valuing exponentially, any change in the Symmetric Valuer’s hyperbolic growth rate is reflected in the discount rate. So, a change in the growth rate has no effect on present value; nor does a change in the length of the valuation period. The Symmetric Valuer always values £1 today at £1, whether valuing exponentially or hyperbolically.

Symmetric valuation implies that the same personal characteristics influence the growth and discount curves. It also implies symmetric temporal selves, i.e., a stable personal identity over time. The Symmetric Valuer’s temporal selves are well connected psychologically, so the Projected Future Self values its present (the valuer’s future) the same as the valuer’s present (its past). Because the Projected Future Self cares equally for its Present and Future Selves, it does not discount the Present Self’s future, other than for the growth projected over the valuation period.

The symmetric hyperbolic value-time curves are not identical, but they are temporal mirror-images of one another. So, the rates at which the Present Self projects in the short-term and over the long-term are exactly the same as those at which the Projected Future Self discounts. It is because the Symmetric Valuer has a stable personal identity that its temporal selves adjust the hyperbolic growth and discount rates, from high to low, symmetrically. So, £1 today is always valued at £1.
Because the Symmetric Valuer’s growth and discount curves are symmetric, they can tell us little about this valuer’s personal identity. To tease out further clues about the personal characteristics of valuers, we need to explore differences: in the value-time curves of asymmetric valuers; and between the curves of different valuers. It is not a matter, so much, of whether or not, for example, a valuer is optimistic, it is more about whether the Present Self is optimistic and the Projected Future Self is pessimistic.

3. MYOPIC VALUATION

The Myopic Valuer has positive time preference and values the present more than the future.

The Myopic Valuer values the present for one or both of two reasons. First, some valuers suffer from what the economist George Loewenstein describes as inadequacies in their “ability to imagine the future”.25 Such valuers are short-sighted.26 It is as if they have a “faulty telescopic faculty”.27 Secondly, some valuers experience psychological discomfort when asked to defer immediate gratification, even for a short period.28 Such valuers are impatient.29

The Myopic Valuer uses a discount rate that is higher than the growth rate over the valuation period, i.e., the Myopic Valuer discounts more heavily than does the Symmetric Valuer.

Chart 5 illustrates the Myopic Valuer’s value-time curves when projecting and discounting both exponentially and hyperbolically. In the first case, £1 is exponentially projected forward at a growth rate of 6% to a value of £5.74 and that future value is then exponentially discounted, at a rate of 8%, to a present value of £0.57. So, £1 today is valued at £5.74 in 30 years’ time; and £5.74 in 30 years’ time at £0.57 today. In the second case, £1 today is projected to grow hyperbolically to £3.32 in 30 years’ time and that future value is then hyperbolically discounted to a present value of £0.57.30 So, £1 today is valued at £3.32 in 30 years’ time; and £3.32 in 30 years’ time at £0.57 today.

Both of these thought investments produce a present value less than the initial value. By implication, the Myopic Valuer values £1 today, invested for 30 years, at £0.57. We noted, earlier, that a change in the length of the valuation period has no effect on symmetric valuation (£1 invested today in any symmetric thought investment is always worth £1). In contrast, any increase in the valuation period will result in a decrease in the Myopic Valuer’s

25Loewenstein (1992)

26Elster (1979) cited by Bartels and Urminsky (2011)

27Pigou (1920) cited by Frederick et al. (2002)

28Loewenstein (1992)

29Olson and Bailey (1981)

30We set $y=1$ and $\alpha = 4$ for the growth function; and $y=1$ and $\alpha = 2.34$ for the discount function
present value calculation, while a decrease in the period will increase present value. But present value for the Myopic Valuer is always less than initial value.\(^{31}\)

What could cause short-sightedness or impatience? What influences might differences in the personal identities of the Myopic Valuer’s temporal selves exert on the value-time curves? Which personal characteristics, that lie passive when time-adjusting value in one direction, might emerge to influence the rate of change in the other direction? The Myopic Valuer either underestimates the growth rate or uses a discount rate that reflects factors other than, or additional to, those that determine the projected growth rate.

An inability to imagine the future might cause the Present Self to underestimate the rate of value change. If the valuer’s temporal selves are loosely connected psychologically, its Projected Future Self might not share this predisposition. In this case, the discount rate would exceed the growth rate. If the growth rate is not underestimated, however, we must look for other reasons for the mismatch in value-time curves.

CHART 5: MYOPIC VALUE-TIME CURVES SHOWING BOTH EXPONENTIAL AND HYPERBOLIC GROWTH AND DISCOUNTING

If discounting is multi-causal or “multiply-determined”,\(^ {32}\) we should look for a number of explanations for this asymmetric valuation. We explore four key ones: psychological discontinuity; existential uncertainty; risk aversion; and loss aversion.

\(^{32}\) Because the hyperbolic time-curves become flatter in the direction of time travel, increasing the length of the valuation period will affect exponential valuation more than hyperbolic valuation (which is one reason why, for example, environmental campaigners argue for hyperbolic discounting of future costs). See Farmer and Geanokoplos (2009)
Psychological discontinuity

We noted, earlier, that the Symmetric Valuer has a stable personal identity over time. The Myopic Valuer’s temporal selves, in contrast, are loosely connected psychologically. Derek Parfit suggests that where the selves are not fully connected psychologically, it is rational for the one to care less about the other than itself.33 If the Projected Future Self cares less for the Present Self than for the Future Self, it might discount future values heavily. Some research confirms that subjects who anticipate large changes in their psychological connections over time tend to use higher discount rates than those who are told they have stable identities.34

Existential uncertainty

We also noted earlier that a finite time horizon is consistent with positive time preference;35 as is uncertainty as to the length of the horizon. A finite time horizon of uncertain length is, of course, a defining feature of the life of a human being.

The “brevity and uncertainty of human life”36 means that the Present Self cannot be sure that the Future Self will exist. So, the Projected Future Self cannot be sure that the Future Self exists. If the Projected Future Self believes that the Future Self might not exist to enjoy future values, it might discount them.37

Risk aversion

Research suggests that many of us are risk averse.38 Consider the choice between:

- a gamble that offers an 85% chance of winning £1000 (with a 15% chance of nothing); and
- the alternative of winning £800 for sure.

34 Bartels and Urmsky (2011), Bartels and Rips (2010), although Frederick (2003) found no such relationship
35 Olson and Bailey (1981)
36 Loewenstein (1992)
37 Indeed, it might discount them hyperbolically – heavily at first and then at a reducing rate as the present approaches and confidence in the continued existence of the self increases.
38 Andersen et. al (2008)
According to the laws of probability, the expected value of an uncertain outcome is equal to the sum of the values of each possible outcome weighted by that outcome’s expected probability. So, the gamble has an expected value of £850 \([0.85 \times 1000] + (0.15 \times 0)\] which exceeds that of the sure thing. Yet, research by Daniel Kahneman and Amos Tversky reveals that many of us prefer the sure thing – we’re risk averse.\(^{39}\) We discount the expected value of uncertain gains. It might be, then, that a valuer’s discount rate exceeds the growth rate because of risk aversion.

If risk aversion increases with the riskiness of an investment, so, too, will the difference between the growth and discount curves of a Myopic Valuer with risk aversion. So, £1 invested in equities might be valued at less than £1 invested in bonds (and both at less than £1).

Risk aversion is, however, unlikely to explain myopic valuation of riskless investments, unless, of course, even a riskless investment is considered to be uncertain, psychologically. Myopic valuation of a riskless investment is more likely to be a characteristic of a risk-seeking valuer. This explains, too, the use of high discount rates by gamblers and those with risk-seeking behaviours such as alcohol and drug abuse.\(^{40}\) These are people whose temporal selves are loosely connected – the Projected Future Self caring little for the Present Self’s well-being.

**Loss aversion**

Loss aversion is a stronger psychological trait than risk aversion. Consider the choice between:

- an 85% chance of losing £1000 (with a 15% chance of losing nothing); and

- a sure loss of £800.

Now, many (if not most) of us prefer the gamble over the sure loss – we discount the expected value of uncertain losses.

Kahneman and Tversky call this tendency, to be risk averse for sure gains and risk seeking to avoid sure losses, the “certainty effect”: we underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty.\(^{41}\) The certainty effect is encapsulated in “Prospect Theory”.\(^{42}\)

Prospect Theory proposes that value (see Chart 6) is defined by gains and losses in relation to a reference level of wealth and is:

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\(^{39}\)Kahneman and Tversky (1984)

\(^{40}\)Chabris et. al. (2008)

\(^{41}\)Kahneman and Tversky (1979)

\(^{42}\)Kahneman and Tversky (1979)
• concave in the domain of gains, due to risk aversion;
• convex in the domain of losses, as a result of risk seeking;
• sharply kinked at the reference point; and
• loss-averse – steeper for losses than for gains by a factor of about 2-2.5.

CHART 6: PROSPECT THEORY’S VALUE FUNCTION

How can Prospect Theory explain the use of a discount rate higher than the growth rate? Well, the projection of value by the Present Self (as represented by the growth curve) feels like a loss to the Present Self and a gain for the Future Self. If the growth curve represents a gain for the Future Self, the discount curve must represent a loss of value for the Future Self. We feel losses more than gains so, if it suffers from loss aversion, the Projected Future Self will discount at a rate higher than the growth rate.

There are parallels here with the hyperbolic growth curve, in Chart 5, which is concave, like the Prospect Theory gains curve, and the hyperbolic discount curve, also in Chart 5, which is convex, like the Prospect Theory losses curve. Further, the discount curve is steeper than the growth curve, just as the Prospect Theory losses curve is steeper than its gains curve. These parallels are so compelling that, the behavioral scientist, Eugene Caruso and colleagues recommend that hyperbolic valuation be unified in a three-dimensional value function together with Prospect Theory’s gain-loss asymmetry and what they term “temporal value asymmetry” – the tendency, of many of us, to value future events more highly than equivalent events in the equidistant past.43

43 Caruso et al. (2008), D’Argembeau and van der Linden (2004)
4. HYPEROPIC VALUATION

The Hyperopic Valuer has negative time preference and values the future over the present.

The Hyperopic Valuer values the future for, one or both of, two reasons. First, some valuers have a strong desire for accumulation.\(^44\) Such valuers are far-sighted,\(^45\) have self-control and can resist, or abstain from, immediate gratification. Secondly, some valuers obtain pleasure from anticipating the future and its rewards, including a “bequest motive”.\(^46\) In short, some valuers enjoy wealth for its own sake.

The Hyperopic Valuer, uses a discount rate lower than the growth rate, i.e., the Hyperopic Valuer discounts less heavily than does the Symmetric Valuer.

CHART 7: HYPEROPIC VALUE-TIME CURVES SHOWING BOTH EXPONENTIAL AND HYPERBOLIC GROWTH AND DISCOUNTING

Chart 7 illustrates the Hyperopic Valuer’s value-time curves, both when valuing exponentially and hyperbolically. In the first case, £1 is projected forward exponentially at a growth rate of 6% to a value of £5.74 and then that future value is exponentially discounted, at a rate of 4%, to a present value of £1.77. So, £1 today is valued at £5.74 in 30 years’ time; and £5.74 in 30 years’ time at £1.77 today. In the second case, £1 today is projected to grow hyperbolically to £3.32 in 30 years’ time and that future value is then hyperbolically discounted.

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\(^{44}\) Loewenstein (1992)

\(^{45}\) Keinan and Kivetz (2008)

\(^{46}\) Loewenstein (1992)
discounted to a present value of £1.77.\textsuperscript{47} So, £1 today is valued at £3.32 in 30 years’ time; and £3.32 in 30 years’ time at £1.77 today.

Both of these thought investments produce a present value greater than the initial value. By implication, the Hyperopic Valuer values £1 today, invested for 30 years, at £1.77.

As noted, earlier, the Symmetric Valuer, when conducting thought investments, always values £1 today at £1. In contrast, any increase in the valuation period will result in an increase in the Hyperopic Valuer’s present value calculation; and similarly, any decrease in valuation period will cause a decrease in present value. But present value for the Hyperopic Valuer is always more than initial value.

If a valuer discounts heavily because its temporal selves are loosely connected psychologically, we might expect that a valuer who discounts at a rate lower than the growth rate does so because its temporal selves are strongly connected. Indeed, research confirms that a strong psychological connection between the temporal selves facilitates saving – individuals who anticipate that their future personal identity will overlap considerably with their current identity tend to accumulate more financial assets than do those who sense little such overlap.\textsuperscript{48} It is because the Projected Future Self cares for the Present Self that it discounts lightly the Present Self’s future.

We might expect, too, that the Hyperopic Valuer’s Projected Future Self would not suffer risk aversion, loss aversion and existential uncertainty but, instead, might be more of a risk seeker, more optimistic, over-confident or altruistic than its Present Self. If risk aversion causes the Myopic Valuer to discount risky investments more than safe ones, an appetite for risk might cause a Hyperopic Valuer to discount risky investments less than safe ones. In which case, the Hyperopic Valuer will value £1 invested in equities at more than £1 invested in bonds (and both at more than £1). But an appetite for risk cannot, of course, by itself explain hyperopic valuation of riskless investments.

Uncertainty about the future can be “double-edged”: it can cause the future to be valued more than the present, as well as less.\textsuperscript{49} Risk aversion, for example, can encourage “saving for a rainy day”;\textsuperscript{50} and existential uncertainty can incentivise financial provision for an unexpectedly long life.

A spirit of altruism means enjoying giving for its own sake.\textsuperscript{51} Such altruism can be personally motivated, as by the desire to build a family dynasty, or socially motivated, as by a concern for inter-generational welfare and for the protection of the environment.

\textsuperscript{47} We set $\gamma = 1$ and $\alpha = 4$ for the hyperbolic growth function; and $\gamma = 1$ and $\alpha = 8.9$ for the hyperbolic discount function

\textsuperscript{48} Hershfield et al. (2009)

\textsuperscript{49} Olson and Bailey (1981)

\textsuperscript{50} Olson and Bailey (1981)

\textsuperscript{51} Cowen and Parfit (1992)
5. HYBRID VALUATION

Hybrid valuation – the use of one method to project value and a different method to discount – is, like myopic and hyperopic valuation, asymmetric. Hybrid valuation is flexible - the Hybrid Valuer sometimes values the present and the future equally, sometimes values the present over the future and sometimes values it less than the future. So, the Hybrid Valuer has temporal neutrality sometimes, positive time preference sometimes and negative time preference at other times.

Chart 8 provides one example of hybrid valuation: exponential growth (which is standard in time value of money calculations) combined with hyperbolic discounting (which many of us use to value delayed rewards). The chart includes three discount curves: one resulting in a present value less than initial value; a second illustrating a present value greater than initial value; and a third whereby £1 today is valued at £1. The length of the investment horizon (and not just the growth and discount rates) can influence which of these outcomes arises.

The middle discount curve, in chart 8, produces a present value of £1, but this is not a symmetric valuation. Any reduction in the length of the investment horizon will result in a reduction in present value, as in the case of the lower discount curve (myopic valuation: £1 today will be valued at less than £1). In contrast, any increase in the period will cause the value-time curves to cross over, as in the case of the upper discount curve (hyperopic valuation: £1 today will be valued at more than £1). Similarly, absent a compensating adjustment in the discount rate, any increase (or decrease) in the growth rate will result in an uplift (or reduction) in present value.\(^5\)

The use of one method to project value and an entirely different method to discount value implies either that the Hybrid Valuer’s temporal selves are fundamentally different from one another (an unstable personal identity over time) or that mental simulation of temporal change in value differs radically depending on the direction of time travel. Research confirms that there are systematic differences in the ways in which we mentally simulate the future and the past.\(^5\) Events in prospection, for example, seem more similar to one another than do past events.\(^5\) Further, because we have experienced the past, retrospection is subject to more “reality checks” than prospection.\(^5\)

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\(^{5}\) An alternate form of hybrid valuation would be hyperbolic projection combined with exponential discounting. With this later combination, present value decreases as the length of the valuation period increases.

\(^{53}\) Van Boven et. al (2008)

\(^{54}\) Kane et al. (2012)

\(^{55}\) Kane et al. (2012)
The mental time journey traced by the growth curve in Chart 8 is shorter than that traced by the middle discount curve, even though the latter starts and ends where the former ends and starts. This implies that the future feels closer to the Present Self than does the equidistant past to the Projected Future Self. Recent research confirms a systematic asymmetry (a “temporal Doppler effect”), whereby future events are psychologically closer than past events of equivalent objective distance. As time passes, the Present Self draws ever closer to the Future Self, while the Past Self recedes. Diminishing temporal separation makes the future seem closer to the Present Self, while increasing temporal separation makes past events seem ever more distant.

When we considered, earlier, myopic and hyperopic valuation, we discovered that myopia and hyperopia persist – the Myopic Valuer always values £1 today, thought invested for 30 years, at less than £1; the Hyperopic Valuer always values £1 today at more than £1; and, of course, the Symmetric Valuer always values £1 today at £1. In contrast, we can see from Chart 8 that, for the Hybrid Valuer, the relationship of present value to initial value is flexible depending on the length of the valuation period (and the discount rate) – sometimes present value might be more than initial value; sometimes less; and occasionally it is the same.

The Hybrid Valuer’s temporal selves use different methods to mentally simulate value change – in Chart 8, for example, the Present Self projects exponentially and the Projected Future Self discounts hyperbolically. The exponential per-period growth rate is constant, but

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56 Caruso et. al (2012)
the hyperbolic discount rate declines over time. However, it is the hyperbolic curve that determines time preference. This valuer is myopic over a short time period but hyperopic in the long-term. Taking the uppermost discount curve, for illustration, a thought invested £1 is valued at less than £1 over a 15 year period, but at more than £1 over a 30 year horizon.

In this paper, we suggest that myopia is symptomatic of loosely-connected temporal selves, whereas hyperopia indicates a stable personal identity. How can the temporal selves be loosely-connected in the short-term but well-connected long-term? The answer might be “hyperbolic valuation” – short-term short-sightedness, indicating an obsession with the present (a preference for one apple today over two tomorrow), combined with longer-term far-sightedness, suggesting a concern for the future (a preference for two apples in 51 days over one apple in 50 days).

6. CONCLUSION

In this paper, we have undertaken a series of thought investments in which we consider how an individual’s assessment of the value of an investment depends on how that individual projects and discounts over time.

Our first two examples show how £1 thought invested by a Symmetric Valuer is valued at £1. Neither the projected growth rate nor the valuation period affects present value in these cases (£1 invested today in equities for 10 years has the same present value as £1 invested in bonds for 5 years). Nor does it matter whether the projected rate is exponential, hyperbolic or that of any other valuation method. The Symmetric Valuer has temporal neutrality and is risk indifferent. Symmetric valuation, we suggest, is indicative of a valuer with a stable personal identity over time whose personal characteristics exercise similar influences over the growth and discount rates.

In our second two thought investments, we introduce myopic valuation. Here, £1 kept in cash is valued (both exponentially and hyperbolically) more highly than £1 thought invested in any other asset class for any period of time. The Myopic Valuer has positive time preference (£1 thought invested in any asset class is attributed a higher present value the shorter the investment period) and can be either risk averse (£1 invested in bonds is valued at more than £1 invested in equities for the same duration) or risk seeking (the equity investment is the more highly valued). We attribute myopic valuation (asymmetric valuation with the growth rate lower than the discount rate) to psychological unease when asked to imagine the future or to defer immediate gratification. Such unease may, in turn, be due to risk or loss aversion, existential uncertainty or loose psychological connections between the temporal selves (which can be a direct cause of myopia, when one temporal self cares less for the other than itself, or an indirect cause, when the personal characteristics of the temporal selves exercise different influences on the growth and discount curves).

Our next two thought investments feature hyperopic valuation. Here, an invested £1 is valued (both exponentially and hyperbolically) more highly than £1 in cash. The Hyperopic

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57 Thaler (1981)
Valuer has negative time preference (£1 thought invested in any asset class is attributed a higher present value the longer its investment period) and can be either risk seeking or risk averse. As with the Myopic Valuer, the relative valuation of equities and bonds depends on the interaction of time and risk preferences. The Hyperopic Valuer is far-sighted and possessed of self-control. We associate hyperopic valuation (asymmetric discounting with the growth rate higher than the discount rate) with a valuer whose Projected Future Self is optimistic, over-confident or altruistic.

Finally, we consider hybrid valuation – the use of one valuation method (for example, exponential or hyperbolic) to project and another to discount. Hybrid valuation is asymmetric. Asymmetric valuation, in general, and hybrid valuation, in particular, highlights the distinction between prospection and retrospection. Hybrid valuation is indicative of either a valuer who recognises this distinction or a valuer whose temporal selves regard temporal value change in fundamentally different ways. Hybrid valuation is flexible – sometimes present value is more than initial value (as with hyperopic valuation), sometimes it is less (as with myopic valuation) and, occasionally, it is the same.

Flexible valuation – fluctuating from, for example, myopia to hyperopia – need not be restricted to hybrid valuation. It will be a feature, too, of purely exponential or hyperbolic valuation provided the growth and discount curves are determined independently of one another. If, for example, during a period of market exuberance, the projected growth curve shifts steeply upwards without a compensating move in the discount curve, myopic valuers will become progressively more hyperopic. Indeed, myopia and hyperopia might co-exist within the same investor – a valuer being myopic when considering one thought investment but hyperopic when valuing an alternative. A valuer might, for example, have difficulty resisting sweets and cigarettes (myopia) but also have a tendency to overwork and perpetually postpone vacations (hyperopia).58 Finally, if we can shift from myopia to hyperopia, it might be that we can shift, too, between exponential, hyperbolic and hybrid valuation methods.

This paper does not add to the experimental research literature on temporal change in value. Rather, it draws on the extant literature to consider how projecting and discounting (prospection and retrospection) might be combined in practice. The growth curves in Chart 1 of this paper are derived from Mitchell and Utkus (2004) who entitle their chart “Exponential versus Hyperbolic Discounters: Growth of $1 Over Time”. The implications of this heading are that we project exactly as we discount and that some of us project and discount exponentially while others do so hyperbolically. Most of the experimental research literature on temporal change in value referenced in our paper is focused on discounting. Much of that literature, like Mitchell and Utkus (2004), implicitly views discounting (retrospection) as identical to projecting (prospection), save for the direction of travel. However, as we have explained, recent research suggests that that there are, sometimes, differences in the ways in which we mentally simulate the future and the past – retrospection and prospection are

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58 Keinan and Kivetz (2008)
not necessarily the same. Our thought investments illustrate some of the ways in which those differences might influence valuation.
REFERENCES


