Optimal Purchase of Life and Longevity Risk Insurance Products for Retired Couples

Andreas Hubener, Raimond Maurer, and Ralph Rogalla

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Motivation and Research Question

Uncertainty of lifetimes is major risk for retired couples:
• risk of outliving their assets and leaving too little bequest
• risk of losing the income of the deceased spouse

How can this risk be hedged by the dynamic allocation using
• term life insurance
• single annuities
• joint and survivor annuities
• stocks and bonds
Literature and Contributions

Brown and Poterba (2000, JRI): welfare gains of full annuitization in joint and survivor annuities for couples in retirement

Horneff et al. (2008, JRI): dynamic annuitization and portfolio choice in retirement framework

Love (2010, RFS): dynamic life cycle portfolio choice for families - investment universe: stocks, bonds, life insurance

Our contribution: discrete time portfolio choice model for a couple in retirement with dynamic annuitization and life insurance purchases
Our Model – Family State

**Family State**

- “family” is Markov chain with four states:
  - couple
  - widow
  - widower
  - both deceased

- transitions are only mortality driven (→ no divorce etc.) and given by the individual one-year survival probabilities.

\[
\begin{align*}
\text{Couple} & \quad p^f (1 - p^m) \\
\text{Widow} & \quad (1 - p^f) \\
\text{Widower} & \quad (1 - p^f) \cdot (1 - p^m) \\
\text{both deceased} & \quad (1 - p^m)
\end{align*}
\]
Our Model – Preferences

Family Preferences

• utility is gained from consumption and bequest in CRRA framework (RRA $\gamma = 5$; time pref. $\beta = 0.96$)

• consumption is normalized by consumption scaling factor $\phi_s$ (“effective family size”)

$\rightarrow$ couple: $\phi_s = 1.3$  singles: $\phi_s = 1$

• bequest Parameter $B = 2$ gives the strength of the bequest motive

\[
J_t = \max\{u(C, s) + \beta E_t [J_{t+1}]\}
\]

Markov chain

\[
u(C, s) = \frac{1}{1 - \gamma} \left(\frac{C}{\phi_s}\right)^{1-\gamma}
\]

\[
\text{Bequest} = \frac{1}{1 - \gamma} \left(\frac{W_t}{B}\right)^{1-\gamma}
\]
Our Model – Financial & Insurance Products

Financial and Insurance Products

- liquid wealth can be invested in
  - riskless bonds (interest rate: \( R_f - 1 = 2\% \))
  - risky stocks (risk premium 4%, volatility 15.7%)

- renewable one-year term life insurance for each spouse

- single annuities for each spouse

- joint annuities – constant payments till the last spouse dies

\[
LP_t = L \cdot \frac{1 - p_t}{R_f}
\]

\[
AP_t = A \cdot \sum_{\tau=t+1}^{T} \frac{p_{\tau,t}}{(R_f)^{\tau-t} \bar{a}_t}
\]

\[
p_{\tau,t}^j = p_{\tau,t}^f + p_{\tau,t}^m - p_{\tau,t}^f p_{\tau,t}^m
\]
Our Model – Financial & Insurance Products

Joint and Survivor Annuities: Survivor Benefit Ratio

• upon first death payments are reduced to survivor benefit ratio $K$

• Annuity pricing factor: $\ddot{a}_t^K = (1 - K)(\ddot{a}_t^f + \ddot{a}_t^m) + (2K - 1)\ddot{a}_t^j$

• single and joint annuities ($K=1$) allow for any survivor benefit ratio $K$

• the overall annuity holdings of the family can be seen as a combination of:
  - j&s annuity with a specific survivor benefit ratio, and
  - an additional single annuity for one spouse

• example:

  **model**
  - wife $A^f = 4$
  - husband $A^m = 3$
  - joint ($K=1$) $A^j = 4$

  **interpretation**
  - j&s ($K=0.7$) $A^{K=0.7} = 10$
  - wife $A^f = 1$
Our Model – Policies

Decision Variables in each Period:

• consumption

• expenditures on life insurances
  o wife
  o husband

• expenditures on annuities
  o wife
  o husband
  o joint
  (availability is restricted to maximum age)

• allocation of (remaining) liquid wealth to stocks and bonds

Solution for optimal decisions found by value function iteration.
Life Cycle Profile without Pre-Annuitized Wealth

Wealth

- liquid wealth
- annuity PV

Life insurance (face values)

- LI husb
- LI wife

Annuity payments

- annu husb
- annu wife
- annu joint
- annu total

Survivor benefit ratio $K$

0,4 0,5 0,6 0,7 0,8 0,9 1
Life Cycle Profile with Pre-Annuitized Wealth (husband)

Wealth

- liquid wealth
- annuity PV

Life insurance (face values)

- LI husb
- LI wife

Annuity payments

- annu husb
- annu wife
- annu joint
- annu total

Survivor benefit ratio $K$

- $K_{husb}$
- $K_{wife}$
Welfare Analysis

- certainty equivalent at age 65:

\[ CE_{65} = \left( (1 - \gamma) \cdot J_{65} \right)^{\frac{1}{1-\gamma}} \]

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Conclusion

• Joint & survivor annuities are useful products to hedge both kinds of mortality risk.

• Liquid wealth (invested mainly in stocks) is preferred over life insurance for bequest.

• Life insurance is used to insure pre-annuitized retirement wealth (e.g. DB pensions) of one spouse. Then they yield high welfare gains.
Thank you!
Backup
• Household may purchase annuities only at the beginning of retirement

• What is the survivor benefit factor of the optimal j&s annuity?

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<tbody>
<tr>
<td>1.00</td>
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</tr>
<tr>
<td>1.10</td>
<td>0.924</td>
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<tr>
<td>1.20</td>
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<td>1.50</td>
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<tr>
<td>1.60</td>
<td>0.639</td>
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<tr>
<td>1.70</td>
<td>0.602</td>
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<tr>
<td>1.80</td>
<td>0.570</td>
</tr>
<tr>
<td>1.90</td>
<td>0.540</td>
</tr>
<tr>
<td>2.00</td>
<td>0.516</td>
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![Graph showing the effect of consumption scaling on survivor benefit ratio.](image)