Too Little of A Good Thing?
A Reexamination of Market Failures in Endogenous Growth Models.

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August 12, 2004

Abstract

This paper improves on previous attempts to quantify the size of market failures in models of endogenous technological change. Although it is well known that the private firms have incentive to allocate too much or too little research, previous attempts have had difficulties in estimating the size of market failures due to elusive key parameters. Estimating a model of cost reduction using current dynamic panel data improves upon the previous attempts. A model of cost reducing research and development more clearly states the channels for the market failures by showing the effects of research on prices and market size. Dynamic panel data methods offer a more reliable estimate of the aforementioned elusive parameters. The result of the estimation shows that the socially optimal level of research may only be 151% of the private allocation. The result is much lower than previously believed. Previous studies have placed the social optimal level of research as high as 400% of the private allocation.
It is a widely held belief that too little research and development is undertaken by private firms. In the United States, such policies as R&D tax credits and research subsidies have been created to address this problem. However, few studies have been undertaken to estimate the optimal levels of research. The question still remains as to how much research is optimal. Without a reliable estimate of optimal research levels the appropriateness of policies such as the R&D tax credit is unclear.

Currently, models of economic growth through endogenous technological change have presented different answers to the question of how much research is optimal. Models of endogenous technological change often provide reasons to believe there are incentives for firms to invest in too little research. However, the theory does not conclusively state that too little research is undertaken. These models also include reasons to believe there may exist too much research.

There are two major reasons that private firms may undertake too little research. One reason is due to spillovers from research. Previous research by firms may lower the cost of future research. Firms in the future can copy or reverse engineer the innovation at a lower cost than the initial innovation. Research may also solve some problems that are prerequisite for future innovations. The innovating firm is not compensated for the positive benefits to future researchers and therefore has incentive to produce too little research.

Another reason firms may produce too little research is due to the consumer surplus appropriation or monopoly distortion effect. The problem lies in imperfect competition. Although imperfect competition is needed for firms to invest in research and development, it results in problems with excess capacity. From society’s viewpoint, the value of innovations provided by private firms will be greater than the social cost. The consumer sur-
plus gains from increased innovation will not be realized due to the presence of market power among firms. This will lead to too little research being undertaken.

These models also present two major reasons why firms may produce too much research. One reason is due to the creative destruction or business stealing effect. New innovations decrease market shares of existing firms. This can occur from new technologies making older models obsolete or simply due to a cost reduction by one firm. This gives firms incentives to invest in too much research. From society’s viewpoint the transfer of profits presents a zero net gain. Therefore, a social cost is incurred that offers no social benefit.

Firms may also have incentive to produce too much research when patent races are present. Firms have incentives to research similar products. This duplication of research results in wasted resources in that only one firm will obtain the right to produce the good. The losing firm spends real resources that give no benefit to the firm or to society.

The summation of each of the previous mentioned effects imply that there can exist too much or too little research. Theory provides some answers to the questions. In some cases such as models of cost reduction or variety expansion models, the results are often clear. Although several market failures may be present, the presence of constant elasticity utility functions or the imposition of symmetry among firms results in market failures offsetting each other. For instance, the consumer surplus appropriation effect may be equal to and offset by the business stealing effect. In a case like this, if there exist no duplication of research then, only the spillover effect is left. In models that involve destruction of technologies such as quality ladder models, the answer is often ambiguous. There will exist incentives to
underinvest in research along with incentives to overinvest in research. In these models, the theory gives no clear indication of which incentive will be stronger.

Given the many market failures and their importance, it is somewhat surprising little empirical work has been done to estimate their size. The reason may be largely due to that in the past key parameters have been very difficult to obtain. In papers such as Stokey(1995), Jones and Williams(1998), and Jones and Williams(2000) key parameters remain elusive. These studies calibrate models by using previous microeconomic estimates aimed at measuring returns to research and development. The approach is lacking somewhat in that the estimates were obtained by using quite different assumptions than those made by models of endogenous technological change.

One instance of the microeconomic models using different assumptions in their estimation occurs with the creation of knowledge capital. The problem is that the traditional approach to estimating knowledge capital is the linear approach explained in Griliches(1993). Here, knowledge capital is seen as the summation of prior research expenditures minus depreciation. This method may be appropriate for estimating the returns to R&D yet, it does not tell the entire story. This measurement includes a mixture of current returns and past spillovers that need to be separated in order to accurately estimate a model of endogenous technological change. When these effects are unobserved the results can vary widely and be very sensitive to assumptions about the unobserved variables. For instance, Jones and Williams(2000) find that the socially optimal allocation of research is sensitive to the size of spillovers and the size of the congestion parameter. Different calibrations for these parameters cause the optimal allocation of R&D to vary from 98% to 255% of the private level of research.
The approach of a single representative firm has also caused some problems in estimating the channels of externalities. For instance, Comin (2002) points to the inconsistency that it is believed that large R&D externalities exist and yet, a small percentage of GDP is devoted to research. Large externalities would imply firms would want to invest large amounts in future research due to high returns yet, data show that research is a small percentage of GDP. This seems inconsistent only in the framework of a representative firm who draws only on public knowledge capital. In this case, the representative firm framework does not show how a rival firm’s ability to use public capital can dilute the returns from innovation.

The seeming inconsistency can be explained by a model such as the one used in this paper. The presence of many firms coupled with a distinction between private and public knowledge capital provides the answer. The large externalities come through channels of both private and public stocks of knowledge. A firm can not fully block its nonrival technology. Increasing investment in research today will make a firm more productive tomorrow but the spillover effect indicates the firm’s rivals will also be more productive. Rivals can copy or improve upon the innovation and therefore, decrease the gain in in market share and profits of the initial innovator. When the benefits of research are not fully appropriable firms will have not have an incentive to make large R&D investments. This explains why there can exist large externalities and there still exist a small allocation of research.

The previous techniques can be improved upon using a model of cost reduction and recent advances in dynamic panel data analysis. A model of Cost Reduction similar to Peretto (1998b) is used to guide the empirical section and show the private and social optimal allocations of research. This model differs some from Peretto (1998b) in that to obtain constant growth
rates in the steady state the evolution of knowledge exhibits diminishing returns to the stock of knowledge rather than constant returns. This eliminates the need for entry and allows the knowledge spillover effect to be estimated from the data.

The cost reduction approach is also beneficial in that it is easier to identify effects such as profit stealing and consumer surplus appropriation through the effect of research on prices and market share. The easy identification of channels is an improvement on the representative firm approach. This allows the model to show how returns on research are diluted due to spillovers and therefore, why large spillovers can exist alongside small allocations of R&D.

The model also gives insight into appropriate regressions to estimate the previously elusive spillover effect. The evolution of knowledge is used to give insight into an appropriate knowledge production function. Recent advances in dynamic panel data methods are used to estimate the previously elusive parameters efficiently. The use of firm and industry level data coupled with current dynamic panel data techniques allow separation of the effects of spillovers from the return to R&D and use an reliable econometric approach to calibrating the model.

The results are similar to Comin(2002) in that research intensity provides little or no contribution to productivity. Comin(2002) uses this argument to show that the social and private levels of research are very close if not the same. The findings in this paper disagree in that it is found there is a significant difference between the socially optimal level of research and the private allocation. This difference is estimated as smaller than previously believed but at 151% the estimate is significant.

The paper is organized as follows. Section one presents a model of endogenous technological change and solves for the private allocation of re-
search. Section two presents the social planner’s problem and solves for the socially optimal level of research. The socially optimal level of research is then compared to the private level and market failures are identified. Section three presents an approach to estimating the key parameters identified in sections one and two. Section four concludes the paper.

1 The Model

In order to find how much research and development is optimal, a model must be constructed. The model gives theoretical insight to the factors that will determine the optimal size of research allocations. The model will also give insight into appropriate regressions that will yield estimates of the size of optimal levels of research. This section begins with the Consumer’s allocation problem which provides insights into how market shares are obtained and is also necessary to solve the social planner’s problem. Once the Consumer’s allocation problem is solved, the firm’s allocation problem is then solved and it is shown how the private firm will allocate research. The firm’s allocation problem will also give insights to proper measurement of variables and regressions to estimate.

1.1 The Consumer’s Allocation Problem

The consumer’s problem can be separated into two stages. In the first stage, consumers choose the static level of consumption to maximize utility at time \( t \). Consumers have symmetric preferences over a continuum of differentiated goods. At any given point in time consumers will choose consumption to maximize

\[
D_t = \left[ \int_0^N \frac{x_i}{x_i + 1} \, di \right]^{rac{1}{\epsilon}}
\]
subject to

\[ E - \int_0^N P_i x_i \, di = 0 \],

where \( x_i \) is consumption of good \( i \), \( \epsilon \) is the elasticity of substitution between products, \( N \) is the number of goods existing at time \( t \), \( E \) represents consumption expenditures, and \( P_i \) is the price of good \( i \). It is assumed that each firm makes one good that is differentiated from other firms’ goods. Solving the allocation problem produces the standard demand function produced by Dixit-Stiglitz preferences,

\[ x_i = \frac{EP_i^{-\epsilon}}{\int_0^N P_j^{1-\epsilon} \, dj} \],

where \( P_j \) is the price of good \( j \).

It is important to note for future use that market share is given by

\[ S_i = \frac{P_i^{1-\epsilon}}{\int_0^N P_j^{1-\epsilon} \, dj} \].

This is easily seen by multiplying equation (2) by the firm’s price and then dividing by expenditures. The firm’s market share is then expressed as a share of consumer’s expenditures. As the firm’s price become smaller than rival firms’ prices market share increases.

In the second stage of the problem, consumers choose a time path of expenditures independent of the static consumption decision. Consider a closed economy with infinitely-lived dynastic households that supply labor services. Each household is endowed with one unit of labor, which it supplies inelastically. Population grows at the rate \( n \). The intertemporal maximization problem for the household is the standard model discussed in Barro and Sala-i-Martin(1995 ch. 2). The representative agent chooses expenditures to maximizes lifetime utility,

\[ U_t = \int_t^\infty e^{(\rho-n)(\tau-t)} \frac{x_i^{1-\sigma}}{1-\sigma} \, d\tau \]
subject to the budget constraint

\[ \dot{\omega} = \omega(r - n) + W - E, \]

where \( x_{i\tau} \) is the optimal level of static consumption derived in equation 2, \( \rho \) represents the discount factor, \( r \) is the interest rate, \( \omega \) is the endowment of assets consumers possess and \( W \) represents wages. Applying standard optimal control techniques it is shown that the optimal time path of consumer expenditure is

\[ \frac{\sigma \dot{E}}{E} = r - \rho + (1 - \sigma) \frac{\dot{P}_i}{P_i}. \]

The solution implies that in steady state, where expenditures do not change, the interest rate will be

\[ r = \rho - (1 - \sigma) \frac{\dot{P}_i}{P_i}. \]

1.2 The Firm’s Allocation Problem

1.2.1 Technology

The technology available to a firm can be seen as a simple Ricardian production function. Innovations cause labor augmenting technological change and therefore reduce cost. New innovations are obtained by research and increase the existing stock of private knowledge capital. The current stock of knowledge capital provides a reduction in labor needed to produce the good. Each firm produces a differentiated consumption good with technology,

\[ x_{it} = A_{it}^\theta L_{it}, \]

where \( x_{it} \) is the differentiated good produced by firm \( i \) at time \( t \), \( A_{it} \) is the stock of knowledge available to firm \( i \) at time \( t \), and \( L_{it} \) is the amount of labor used to produce good \( x \). It can be seen form the previous equation that
the current stock of knowledge capital provides reduction in labor needed to produce the good. The firm’s stock of knowledge can be seen as a measure of productivity.

Research over time will increase the stock of knowledge. In addition to research there are two other factors that affect the growth of knowledge. Microeconomic literature has identified spillovers exist from both public and private stocks of knowledge. There exist a private pool of knowledge in which the firm can access. This is given by the variable $A_{it}$. This represents the sum of past productivity advances by the firm.

The firm and its rivals can also draw upon a public source of knowledge. Productivity advances by firms will become public knowledge from which all firms can copy or improve upon. One way of writing this idea is that to assume that firms who have higher productivity and therefore larger market shares contribute more to the public stock of knowledge. The stock of knowledge can then be written as

\begin{equation}
T_t = \int_0^N S_{it} A_{it} di, \tag{9}
\end{equation}

where $T_t$ is the stock of public knowledge, and $N$ is the number of firms. This can be interpreted as an aggregated measure of productivity.

Growth of productivity over time will be affected by a firm’s research as well as its current knowledge and the knowledge it obtains from other firms. There will exist spillovers from private and public knowledge that make innovations less expensive. This idea can be captured by showing the evolution of the stock of knowledge as

\begin{equation}
\dot{A} = A_{it}^\phi T_t^\gamma R_{it}, \tag{10}
\end{equation}
where $R$ represents labor devoted to research and development. The current stocks of private and public knowledge decrease the cost of innovation making research more productive.

This formulation is the beginning of the explanation of why large externalities to research can exist while firms invest only in a small amount of research. It is important to note that this formulation breaks from previous models in that it differentiates two types of knowledge a firm can draw from. Firms can draw from public and private sources. Private knowledge stocks benefit only the individual firm while all firms can draw from public capital. The distinction is necessary due to the ability of firms to imitate current technologies and problems with upholding secrecy of cost-reducing research. Firms can not fully block the diffusion of their own non-rival technology. This allows for spillovers to be attained from public knowledge. This creates an externality that will make research too small.

At this stage, it is appropriate to give some preliminary interpretations to the parameters in equation 10. The parameter $\phi$, when estimated to be greater than zero, represents a spillover effect similar to the standing on shoulders effect presented by Caballero and Jaffe(1993). Previous innovations solve problems that will lower the cost of future innovations. It will be shown in the following section of the paper that this parameter represents an externality.

On the surface it would seem that the spillover from private knowledge would be completely internalized. This does not occur due to the two sources of knowledge. As Arrow(1962) points out, leakages of technology are inevitable given the embodiment of knowledge in products and the mobility of personnel among firms. Therefore, returns from fully taking advantage

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1Levin(1998), pg. 424
of the spillover from private knowledge will be diluted due to the fact that the firms private knowledge also contributes to public knowledge. This results in firms not taking full advantage of the spillover effect. This explains why there can exist small levels of research in the presence of large externalities to knowledge. The firm does not receive full compensation for its innovations because the resulting increase in public knowledge decreases the gain in market share.

When the parameter $\gamma$ is greater than zero, it represents the degree of knowledge spillovers from other firms. Jaffe(1986) shows that spillovers between firms tend to be localized. Firms who produce products that are more similar to the innovating firm tend to give a larger spillover. This leads to the interpretation that the public stock of knowledge is best seen by the firm as an industry level stock. Firms outside of the industry will provide only small spillovers to the firm’s research. This intra-industry spillover effect represents the degree to which rival firm’s technologies will lower the cost of research to the innovating firm.

1.2.2 Optimization by the firm

The firm’s decision making process can be viewed as a two part problem. In the short run, the stock of private knowledge is given and firms determine a level of prices and output that maximize profits. The presence of imperfect competition gives firm market power and the ability to set price. Firms will follow the familiar inverse elasticity rule to determine the markup above marginal cost. From equation 2 it is easy to prove that the elasticity of demand is $\epsilon$. Solving equation 8 for $L_{it}$ shows that the firm’s marginal cost
is given by $A^{-\theta}_{it}$. Therefore, the markup will be

$$(11) \quad \frac{P_i}{A^{-\theta}_{it}} = \frac{\epsilon}{\epsilon - 1}.$$  

In the long run, firms can change their stock of private knowledge by investing in research. Firms will want to maximize market value in the long run and therefore maximize cash flow. Assuming that, there exist perfect foresight and firms are valued only by fundamentals, the value of the firm equals the discounted flow of profits net of research. These assumptions give the following equation for a firm’s value,

$$(12) \quad V_{it} = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi_{i\tau} - W_R R_{i\tau} d\tau,$$

where $V_{it}$ is the value of the firm at time $t$, $r$ is the interest rate, $\pi$ is the flow of profits, $W_R$ is a constant wage paid to labor devoted to research and $R_{it}$ is the labor devoted to research by firm $i$ at time $t$.

The flow of profits can be obtained through equation $(8)$. Normalizing wages yields

$$(13) \quad \pi_{it} = (P_i - A^{-\theta}_{it}) X_{it}.$$  

The long run decision making process maximizes the value of the firm subject to the evolution of private knowledge. Substituting equation $(13)$ into equation $(12)$ yields,

$$(14) \quad \text{Max}_{R_{it}} V_{it} = \int_t^\infty e^{-\int_t^\tau r(s)ds} (P_i - A^{-\theta}_{i\tau}) X_{i\tau} - R_{i\tau} d\tau$$  

subject to $\dot{A} = A^{\phi T^\tau}_{it} R_{it}$.

Applying Standard optimal control techniques and rearranging the solution yields,

$$(15) \quad R_{it} = \frac{\theta(\epsilon - 1)\pi_{it}g_A}{r + \gamma g_T}.$$
where \( g_x \) is used to denote the growth rate of variable \( x \).

In order to make comparisons to the social planner's problem it is necessary to impose symmetry among firms. All firms being identical is a simplifying assumption that will make the social planner's problem easier to solve. In the symmetrical case, the public stock of knowledge becomes

\[
T = NS_i \bar{A}_t = \bar{A}_t, \tag{16}
\]

where \( \bar{A} \) is the average stock of knowledge capital. The public stock of knowledge is the average of each firm's stock of knowledge. Since all firms are now identical, the stock of public knowledge is equal to the stock of private knowledge for each firm.

At this point the solution for the private allocation of research is implicit. In order to obtain estimates of the desired effects it is necessary to solve for the growth rate of knowledge. Taking advantage of the fact that in steady state, growth rates are constant after several manipulations the growth rate of the average firm's stock of knowledge can be written as

\[
g_A = \frac{n}{1 - \phi - \gamma}, \tag{17}
\]

where \( n \) represents the growth rate of the labor force. The solution to the growth rate of knowledge now allows an explicit solution to be found. Imposing symmetry on equation (15) and substituting equations (16) and (17) yields,

\[
R_P = \frac{\theta(\epsilon - 1) \pi \frac{n}{1 - \phi - \gamma}}{r + \gamma \frac{n}{1 - \phi - \gamma}}, \tag{18}
\]

where \( R_P \) represents labor allocated to research under the private solution. Comparing this expression to the social planner’s allocation of research will show the divergence of the private allocations of research from socially optimal allocations and the market failures present in the model.
2 Optimal Allocation of Research and Development

2.1 The social planner’s problem

In the previous section, firms chose levels of research to maximize profits over time. Maximization of social welfare would imply a different solution. A social planner would be concerned with maximizing per capita consumption over time rather than profits. The social planner’s problem can be presented as

\[ \max_{R_t} U_t = \int_t^\infty e^{-(\rho-n)(\tau-t)} \left[ \left( \int_0^\infty \frac{x_{it}^{\epsilon-1}}{\hat{L}_t} \right)^{1-\sigma} \right], \]

subject to

\[ \hat{L}_t = L_t + R_t \]

\[ \dot{A}_t = A_t^{\phi+\gamma} R_t , \]

where \( \hat{L} \) represents the labor force. Substituting equations 8 and 20 into equation 19 and applying optimal control techniques gives

\[ R_S = \frac{\theta(\epsilon-1)\pi g_{\hat{A}}}{\rho + \theta(\sigma-1)g_{\hat{A}}}, \]

as the socially optimal level of research.

2.2 Market Failures

The socially optimal and private allocations of labor devoted to research have now been solved. Creating a ratio of the socially optimal allocation to the private allocation shows if there is too much or too little research and by how much the socially optimal allocation differs from the private allocation. Dividing equation 21 by equation 18 yields,

\[ \frac{R_S}{R_P} = 1 + \frac{\gamma n}{1-\phi-\gamma}, \]
Due to the symmetry imposed on the model the model predicts too little research will be undertaken by the private firm. This assumes that $\phi + \gamma < 1$. This assumption is necessary to insure constant growth rates in the steady state. If $\phi + \gamma \geq 1$ then accelerating growth rates will occur and the solution for $g_A$ is not valid because it depends on growth rates being constant in the steady state. This provides a check for the validity of the model.

The ratio shows the market failures involved in the model. The presence of the parameter $\gamma$ in the numerator shows that private firms will allocate too little research. $\gamma$ represents the effect of the intra-industry spillover. Firms will not be compensated for the addition to public knowledge capital. Therefore, they will not take into account the decrease in cost to innovations by other firms caused by their research. The Social planner takes into account the benefit to all firms and allocates a greater level of research.

The ratio of socially optimal research to private levels is also increasing in the parameter $\phi$. This shows the standing on shoulders effect. The private stock of knowledge will lower cost of future innovations yet firms will not take full advantage of this spillover. To understand why this happens an expression that shows how private and social research changes as $\phi$ changes is needed. Differentiating the social and private allocations with respect to $\phi$ yields,

$$\frac{\delta R_P}{\delta \phi} = \frac{rn\theta(\epsilon - 1)\pi}{(r(1 - \phi - \gamma) + \gamma n)^2}$$

and

$$\frac{\delta R_S}{\delta \phi} = \frac{rn\theta(\epsilon - 1)\pi}{(r(1 - \phi - \gamma))^2}.$$

Note the presence of the term $\gamma n$ in the private allocation. This term shows how the addition to public knowledge capital dilutes returns to private re-
search. This will cause the firm to not take into account the full value of present research and produce too little research.

It seems that only positive market failures occur. This is due to the symmetry of the model. The solution actually contains two market failures that cancel each other. If the terms are not cancelled the ratio is written as

\[
\frac{R_S}{R_P} = 1 + \frac{\gamma g \tilde{A}}{r + \sigma \theta \epsilon g \tilde{A} - \sigma \theta \epsilon g \tilde{A}}.
\]

Although the terms \(\sigma \theta \epsilon g \tilde{A}\) and \(-\sigma \theta \epsilon g \tilde{A}\) are equal and therefore, cancel each other they represent important effects.

The term in which, \(\sigma \theta \epsilon g \tilde{A}\) is positive represents the consumer surplus appropriation effect. This is due to the inefficiency caused by imperfect competition. As \(\epsilon\) becomes larger the industry approaches perfect competition and more consumer surplus is gained. This also brings the ratio of socially optimal research to private research closer to 1. The social planner takes into account how people will discount the future advances and the cost reduction provided. The elasticity of cost reduction also plays a role in that the larger this parameter is, the more consumer surplus can be gained from innovation.

The term in which, \(\sigma \theta \epsilon g \tilde{A}\) is negative represents the business stealing effect. As the elasticity of cost reduction and the elasticity of demand rise the firms can take profits from other firms more easily. The rise in the benefit from the cost reduction means research will have a bigger impact on prices while the rise in elasticity of demand means the products are easier to substitute. As the growth rate of knowledge increases more business stealing will occur. This allows the firm to take business away from rivals. The private firm does not take into account the social cost imposed on its rivals. The social planner does take this effect into account. Again the social
planner will discount this cost according to the intertemporal elasticity of substitution.

The result show a clear theoretical solution. Three market failures are present. Two of the market failures give incentive too little research while one gives incentive for too much research. Due to symmetry, the consumer surplus appropriation effect is equal to the business stealing effect. This leaves only the spillover effect which gives incentive for firms to produce too little research. In order to find the degree of underinvestment, it is necessary to calibrate the model. The unknown parameters $n, \gamma, \phi, \sigma, \theta, \epsilon$ and $r$ need to be calibrated to quantify the size of the market failures involved.

3 Estimating the Size of Market Failures

3.1 The Spillover Equation

In order to estimate the parameters, a knowledge production function needs to be created. The model does not have a direct production function for knowledge. However, the theory does show how knowledge will change over time. A equation that would lead to an evolution of knowledge such as equation [10] would show current knowledge as being a function of past and present research levels, past and present levels of private knowledge, and past and present levels of public knowledge. Such equation could look like this,

$$A_{it} = \int_{t_0}^{t} A_{\phi, \gamma} T^{\gamma} R_{\tau} d\tau$$

(24) $A_{it} = \int_{t_0}^{t} A_{\phi, \gamma} T^{\gamma} R_{\tau} d\tau$

This formulation could represent a knowledge production function. This function has its drawbacks. From previous microeconomic research it is clear lags to research exist. It takes time for the research expenditures to be become a design that can be produced and marketed. It will also take
time to copy other firm’s technologies or improve upon them. Therefore the regression to be estimated may be better seen in discreet time where it is possible to account for lags.

Microeconomic evidence gives some insight into proper lag times. Pakes and Shankerman(1987) cite previous studies that the total lag time from the idea to be researched to application can be from 1.17 to 2.62 years. This would imply research should have a lag time of 2 to 3 years before it adds to the stock of private knowledge. Mansfield(1987) finds that when firms immitate other firms’ technology a lag time also exist. Mansfield(1987) finds that firms can copy research in as little as 70% of the time it takes to develop an innovation on its own. This would imply that public knowledge capital should have a lag of at least one year. Taking this information into account allows estimation of the equation,

\[ \ln A_{it} = \phi \ln A_{i(t-1)} + \lambda \ln R_{i(t-2)} + \sum_{j=1}^{N} \gamma \ln T_{i-j} + (\mu_i + \epsilon_{it}). \]  

(25)

In this equation \( \lambda \) represent possible increasing or decreasing returns to research, \( \mu_i \) represents an unobserved firm specific effect, and \( \epsilon_{it} \) represents a remainder disturbance term. Estimation of this equation will yield estimates of the parameters \( \phi \) and \( \gamma \).

3.2 data and measurement issues

3.2.1 data sources and measurements

In order to estimate the equation given in the previous section, the data must draw upon two sources. As previously stated, private and public knowledge can be seen as firm level and industry level productivity respectively. The approach measures total factor productivity as proxies for knowledge capital. Although the model exhibits a Ricardian production function, labor
productivity is not the appropriate measurement. Firms will employ capital as well as labor. Therefore, total factor productivity can be seen as a better measurement.

The data need to encompass both firm level data to construct firm level TFP measures which, proxy for private stocks of knowledge, and industry level TFP measures which, proxy for public stocks of knowledge. The data used are available through the NBER. The firm level data is drawn from Bronwyn Hall’s firm level productivity panel and the industry data is drawn from the NBER productivity panel. The Hall data set is an unbalanced panel that contains 1253 firms that have data spanning 2 to 31 years. The panel covers the years from 1960 to 1990. The data used is a reduction of the Hall data set. Industry data had to be matched to firm level data so, older firms that were classified under 2 or 3 digit SIC codes could not be matched because the NBER data provides only 4 digit SIC codes.

Industry and firm level TFP measures are constructed in similar fashion. These are calculated by subtracting the share weighted logs of real inputs from the log of real sales. The inputs included were plant and equipment as a proxy for capital, the number of workers employed as a proxy for labor, and material cost. The TFP measures were not indexed as is standard in analysis. The panel was unbalanced therefore, finding a base year would have caused significant data loss. The TFP measurements can be seen as an imperfect proxy. The method used implies perfect competition and constant returns to scale neither of which are assured. However, other measures have similar drawbacks. For instance, calculating a Solow residual to measure TFP assumes that knowledge enters into the production function in a linear fashion which is not assumed by the model. The Solow residual method can account for imperfect competition as in Norbin(1993) yet, it can be difficult
to separate the markup from the TFP measure.

Some inputs for TFP measures had to be constructed. The Hall data set did not provide all variables needed to calculate TFP. Factor shares, material cost, and deflators were missing. Deflators were provided by the NBER productivity panel. Material Cost and factor shares were constructed using industry and firm level data. Material cost were assumed to be similar among firms. This allowed the industry level material share of sales to be used to construct material costs for the firm.

To construct factor shares for the firm industry data was used. For labor shares, industry payroll was divided by sales to construct the share for the first year. Subsequent years were constructed by using the average of the current year’s factor share and the previous year’s factor share. Material shares were constructed using the same scheme. Constant returns to scale were assumed and capital shares were computed as a residual.

The Hall data set provided nominal research and development expenditures. Using this measure could cause some bias. If correlations between firm size and research exist, than an estimate for research may reflect the fact larger firms do more research rather than the returns sought. To correct for this, nominal research and development expenditures divided by real sales is used as a measure of research intensity.

### 3.2.2 econometric issues

Equation 25 has some special issues regarding estimation. The lagged term of firm TFP, $A_{i(t-1)}$ will cause the estimation to be biased. The bias can come from two sources. First by design $A_{i(t-1)}$ will be highly correlated with the firm specific effect included in the error term. This will cause estimation of the parameter $\phi$ to be inconsistent.
The other source of bias comes from the remainder disturbance term. If errors are serially correlated the estimate of $\phi$ will be biased due to high correlation of $A_{i(t-1)}$ and the remainder disturbance term. If there exist no serial correlation, attempts to remove the firm specific effect will also result in correlation between $A_{i(t-1)}$ and the disturbance term. If a within transformation is used, the mean used in the transformation will be correlated with the remainder disturbance because $A_{it}$ will be correlated with the remainder disturbance. If first differences are used then $A_{i(t-2)} - A_{i(t-1)}$ will be correlated with the difference of the remainder term $e_{t-1} - e_t$. This make efficient estimation of the parameter $\phi$ difficult. Standard panel techniques will cause the estimate of $\phi$ to be biased downward.\(^2\)

There have been several approaches that have been suggested to correct for the bias in estimation. Recently, General Method of Moments estimation has been suggested. The identification of moment conditions allows for the construction of valid instruments. One approach is presented in Arellano and Bond(1991), hereafter AB(91). The authors suggest using first differences and instruments for the differences of $A_{i(t-1)}$. The authors show moment conditions allow for past lags of levels of the variable $A_{it}$ to be used as instruments for instruments for $\Delta A_{i(t-1)}$. All past lags up to two periods back would be valid instruments. These lags should be correlated with $\Delta A_{i(t-1)}$ and uncorrelated with the error term provided the error term does not follow an AR(2) pattern.

AB(91) show that the levels variables for research and industry level TFP can also be used as instruments. How these are treated depends on assumptions made about the variables. If these variables are assumed to be exogenous then all previous lags and the current values can be used.

\(^2\)Bond(2002)
as additional instruments. If these variables are consider predetermined, all previous lags up to one period back are valid instruments. If the variables are endogenous all previous lags up to two periods back are valid instruments. The AB(91) regressions are run under all three assumptions for the industry TFP variable. Research intensity is always consider endogenous because it is constructed using firm level sales as is firm level TFP.

Blundell and Bond(1998) point out that there exist a problem with AB(91). If the variable A follows a unit root or approaches a unit root then the instruments in AB(91) become weak. The weak instrument present another source of bias in estimation. To correct for this the Blundell and Bond method, hereafter BB(98), suggest incorporating the Arrellano and Bond method into a system of equations. In this system level and first difference equations are used and the parameter $\phi$ is common to all equations. BB(98) show that the first differences one period back can be used for instruments for the level equation. This increases the efficiency of the estimates.

3.2.3 Results from and calibration of the spillover equation

In the preliminary stage of estimation, equation 25 is estimated using OLS and then a within transformation. Both of these methods provide biased estimates yet, they provide useful information as well. Bond(2002) shows that the OLS estimates are biased upward while the within estimates are biased downward. The OLS estimate therefore gives an upper bound for $\phi$ while the within estimate gives a lower bound. A valid set of instruments will provide an estimate that is significantly lower than the OLS estimate yet significantly higher than the within estimate. The results from this stage are given in table 1. The upper bound for $\phi$ is estimated at .868 and the
lower bound is estimated to be .639. Any estimates outside this range show invalid instruments are used.

In stage two, the equation,

\[ \Delta \ln A_{it} = \phi \Delta \ln A_{i(t-1)} + \sum_{j=1}^{3} \gamma_j \Delta \ln T_{i-j} + \lambda \Delta \ln R_{t-2} \]  

is estimated using the AB(91) method and then the results are compared to the BB(98) method. The lag lengths for the variables T and R were found using a recursive t method. The evidence previously discussed was used as a guide for the first lag to include. The results are presented in tables 2, 3 and 4. These regressions are run under all three assumptions for the treatment of industry TFP.

This leaves a question of which regression was the best regression. It was clear that using regressions treating industry TFP as exogenous gave invalid instruments for both the BB(98) method. The leaves a choice between assuming T is endogenous or predetermined under either BB(98) or AB(91). The Sargan statistic reported indicates if the instruments are valid. The method that gives the lowest Sargan statistic is considered the preferred method. It was found that BB(98) yielded the second lowest Sargan statistic. However, the differenced Sargan statistic which indicates if the extra instruments are valid did not show that BB(98) where T is treated as endogenous was better than AB(91) where T is treated as endogenous. However, there is the possibility that firm level TFP is highly persistent therefore, the BB(98) method is chosen as the best method. The results used to calculate the size of failures are the BB(98) results due to yet, the AB(91) results may be valid if there is reason to believe TFP is not highly persistent.

The estimates for \( \phi \) and \( \gamma \) can now be used to find the ratio of socially optimal research to private research. A couple of notes are in order. With
regards to the parameter $\gamma$, in this estimation the total spillover for industry TFP is sought. This implies that the effect over time needs to be seen. This can be found by summing the $\gamma$ estimates. There are also two important variables that can not be found using the regression approach. These are the growth of the labor force and the interest rate. These variables are calibrated from previous literature. The growth of the labor force is measured in Jones and Williams (2000) by using BLS data from 1948 to 1997. They find $n = .0144$. The interest rate $r$ is calibrated to the long run trend on returns to the stock market which is estimated at seven percent. 

Given the estimates and the calibrations, the ratio of socially optimal research to private research is found to be 151%. This is found using the estimates $\phi = .804$ and $\gamma = .136$ This implies $\gamma + \phi = .94$. The findings for the total spillover $\phi + \gamma$ are different than those found in Jones and Williams (2000). Their calibrations place the total spillover to be between $.45$ to $.86$. The findings show a larger spillover than the one calibrated by Jones and Williams.

Ironically, the size of the ratio of socially optimal research to private research is with the bounds estimated by Jones and Williams but much smaller than the upper bound. Jones and Williams (2000) estimated the ratio to be 98% to 255%, my findings indicate 151%. This seems ironic in that the spillovers are large however, the degree of underinvest ment is perhaps smaller. This is caused by the division of two types of knowledge capital. A large portion of the spillover from private capital may be internalized by firms. This would mean that the size of private research would be closer to optimal.

The other interesting finding is that research intensity has little effect

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3Mehara and Prescott (1985)
on TFP growth as Comin(2002) shows. Using the AB(91), the contribution of research intensity to TFP growth is small but significant. Using BB(98), the contribution is not significant. This agrees with Comin(2002) in that research intensity contributes little to TFP growth. The spillovers seem to account for a larger portion of TFP growth.

A robustness check was run using current industry TFP in the regression. This assumption was that copying of technology could be done very quickly. The results were not robust to this specification. The total spillover was estimated to be close to or perhaps greater than 1. Giving doubts about the appropriateness of taking advantage of constant growth rates to solve the model. In regressions that supported constant growth rates, the standard errors were extremely high. It was found that a ratio of socially optimal research to private research of 1300% was not significantly different from 100%! The result is largely due to the use of the delta method. As the spillovers approach 1 the standard errors become large. It is possible that the current industry TFP may have added some bias in that it is shown this is an endogenous variable and it was not instrumented for.

3.3 Estimating Profit stealing and Consumer Surplus Appropriation Effects

Another point of interest is the size of consumer surplus appropriation and Profit Stealing effects. Although the symmetry of the model makes these equal, it is still an interesting question as to their size. Equation 23 showed that the two effects would be equal to the term $\sigma \theta g_{\tilde{A}}$. The estimates needed to calibrate $g_{\tilde{A}}$ were obtained when the size of spillovers was estimated. This leaves the variables $\sigma, \theta$ and $\epsilon$ to be calibrated. The data do not allow for estimation of $\sigma$ and $\epsilon$. The parameter $\theta$ can be estimated however.
3.3.1 Values calibrated from other sources

The data and equations provide no direct way to estimate the intertemporal elasticity of substitution, $\sigma$. The parameter has been difficult to estimate in the past. Hall(1988) concluded that $\sigma$ may be very small if not zero while, other works have estimated $\sigma$ close to 1.

The estimate of the intertemporal elasticity of substitution used comes from hVissing-Jorgensen(2002). Vissing-Jorgensen uses data from households who hold assets and compares the data to households who do not hold assets. She finds the intertemporal elasticity of substitution is significantly different from zero for asset holders. She finds that stock holders tend to change consumption less as their returns rise and bond holders change consumption more. However Vissing-Jorgensen does find that non-asset holders seem to have a very low $\sigma$. The lack of evidence from Hall and others may be due to a large number of consumers who hold neither of these assets. The Vissing-Jorgensen study finds that $\sigma$ ranges from .3 to .4 for stock holders and .8 to 1 for bond holders. The estimate of .3 to .4 is also consistent with one technique of estimation used in Patterson and Pesaran(1992).

The model provides a method for calculating the elasticity of substitution between products. Equation 11 shows that firms have a constant markup over marginal cost. Firms will use a markup of $\frac{1}{\epsilon-1}$. An estimate of the markup over marginal cost therefore, gives an estimate of the elasticity of substitution between products. Nadiri and Kim(1993) use a panel of Industries in the United States to estimate a cost function and a demand curve. They use these estimates to calculate the markup over marginal cost. They estimate the markup to be 1.16 times marginal cost. Using this estimate in the markup equation implies an $\epsilon$ of 7.25.
3.4 Estimating $\theta$

Equation 8 provides a straightforward way to estimate the parameter $\theta$. Blundell and Bond (2000) have shown that dynamic panel data techniques can improve the efficiency of production function estimates. This method was not valid due to the high persistence of sales. An AR(3) structure for sales could not be rejected. Therefore even instruments three periods back would be invalid. The production function was estimated using a within transformation and is likely biased downward. This yielded an estimate of $\theta$ of .67. This value along with the previous calibrated values are used to find that the business stealing and Consumer surplus effect measurements range for .44%, assuming a small $\sigma$, to 1.45% when $\sigma$ is close to 1. The result shows a smaller estimate than that of Caballero and Jaffe (1993). Caballero and Jaffe (1993) used patent data to estimate the rate of creative destruction at 4%.

4 Concluding Remarks

This paper has used current dynamic panel data methods along with firm and industry level data to obtain reliable estimates of spillovers that were previously elusive. It has been shown that spillovers may be greater than previously believed. However, these spillovers are possibly somewhat internalized in that the divergence of private allocations of research from socially optimal levels may be smaller than previously believed. The reconciliation of the facts comes from the theory. Firms do internalize some of the spillovers due to the fact that they will reap some of the benefit from knowledge increases. The private allocation is not optimal though simply because as knowledge for one firm grows the public stock of knowledge grows allow-
ing other firms to catch up at less cost. This creates some disincentive to undertake research.

The evidence supports Comin (2002) in that it is found research intensity has little effect on TFP growth. The reason for this is different however. Comin (2002) shows that firms are small in the aggregate and contribute little to aggregate productivity so externalities do not matter. This paper shows that when spillovers are looked upon at levels at which they are most likely to occur the firm can no longer be seen as small. Externalities will therefore, have an impact on the firm’s contribution to aggregated TFP.

The findings of these paper call into question the ideal of private allocations of research diverging from socially optimal levels in an extremely large manner. The paper does show that spillovers may be larger than believed and significant. This model explains how the presence of large spillovers may still lead to small levels of research.
Table 1: Preliminary Results

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<td>R Square</td>
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Notes

- Results obtained using DPD 1.21 for OX
- ** denotes significant at the 1% level
- * denotes significant at the 5% level
- † denotes significant at the 10% level
- Standard errors are not robust to heteroscedasticity
Table 2: GMM industry TFP treated as exogenous

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Notes:

- Results obtained using DPD 1.21 for OX
- t-test for $\phi + \gamma$ and $RS_{RP}$ test if term is significantly different from 1
- The Sargan is calculated using the two step method. Point estimates and standard errors are calculated using the one step method.
- ** denotes significant at the 1% level
- Standard errors are listed in parenthesis below the estimate.
- time dummies and a constant were used in the regression but not reported.
- For the Sargan statistic (x) denotes x degrees of freedom
Table 3: GMM Estimation Industry TFP Treated as Predetermined

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**Notes**

- Results obtained using DPD 1.21 for OX
- t-test for $\phi + \gamma$ and $\frac{R_S}{R_P}$ test if term is significantly different from 1
- The Sargan is calculated using the two step method. Point estimates and standard errors are calculated using the one step method.
- **denotes significant at the 1% level
- Standard errors are listed in parenthesis below the estimate.
- time dummies and a constant were used in the regression but not reported.
- For the Sargan statistic (x) denotes x degrees of freedom
Table 4: GMM Estimation Industry TFP Treated as Endogenous

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Notes:

- Results obtained using DPD 1.21 for OX
- $t$-test for $\phi + \gamma$ and $\frac{R_S}{R_P}$ test if term is significantly different from 1
- The Sargan is calculated using the two step method. Point estimates and standard errors are calculated using the one step method.
- **denotes significant at the 1% level
- Standard errors are listed in parenthesis below the estimate.
- time dummies and a constant were used in the regression but not reported.
- For the Sargan statistic ($x$) denotes $x$ degrees of freedom
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