Restricted Boltzmann Machine (RBM)

The probability of configuration \((v, h)\) is given by Boltzmann (Gibbs) distribution:

\[
P(v, h) = \frac{1}{Z} e^{-E(v, h)}, \quad Z = \sum_{\{v, h\}} e^{-E(v, h)}
\]

where the energy of a configuration \((v, h)\) is given by expression:

\[
E(v, h) = \sum_{i=1}^{m} a_i v_i + \sum_{j=1}^{n} b_j h_j + \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} v_i h_j
\]
Stochastic activation units

2. Hidden node $j$ firing probability

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

3. Deciding whether node $j$ fires or not:

$$z \sim U[0,1]$$

IF $\sigma(x) \geq z$

THEN $h_j = 1$

ELSE $h_j = 0$

1. Aggregated input for the hidden node $j$

$$x = b_j + \sum_{i=1}^{m} w_{ij} v_i$$
Training algorithm for RBM

Marginal probability of visible state:

\[ P(v) = \frac{1}{Z} \sum_h e^{-E(v,h)} \]

Weight updates with stochastic gradient ascent (maximisation of the likelihood of the observed data):

\[ \Delta w_{ij} = \eta \frac{\partial \log(P(v))}{\partial w_{ij}} \]

The gradient has the form:

\[ \frac{\partial \log(P(v))}{\partial w_{ij}} = \langle v_i h_j \rangle_{\text{data}} - \langle v_i h_j \rangle_{\text{model}} \]

where \( \langle \ldots \rangle \) denotes expectations under the distribution specified by the subscript.
Algorithm 1 k-step contrastive divergence

Result: Weights and biases updates
Input: training minibatch $S$
Input: model parameters $a_i, b_j, w_{ij}, i = 1, \ldots, N, j = 1, \ldots, M$ (before update)
Initialization: $\forall i, j : \Delta w_{ij} = \Delta a_i = \Delta b_j = 0$

for $\mathbf{v} \in S$ do
  $\mathbf{v}^{(0)} \leftarrow \mathbf{v}$
  for $t = 0, \ldots, k - 1$ do
    for $j = 1, \ldots, M$ do
      sample Bernoulli random variable $h_j^{(t)} \sim p(h_j | \mathbf{v}^{(t)})$
    end
    for $i = 1, \ldots, N$ do
      sample Bernoulli random variable $v_i^{(t+1)} \sim p(v_i | h^{(t)})$
    end
  end
  for $i = 1, \ldots, N, j = 1, \ldots, M$ do
    $\Delta w_{ij} \leftarrow \Delta w_{ij} + \eta \left( p(h_j = 1 | \mathbf{v}^{(0)}) v_i^{(0)} - p(h_j = 1 | \mathbf{v}^{(k)}) v_i^{(k)} \right)$
  end
  for $i = 1, \ldots, N$ do
    $\Delta a_i \leftarrow \Delta a_i + \eta \left( v_i^{(0)} - v_i^{(k)} \right)$
  end
  for $j = 1, \ldots, M$ do
    $\Delta b_j \leftarrow \Delta b_j + \eta \left( p(h_j = 1 | \mathbf{v}^{(0)}) - p(h_j = 1 | \mathbf{v}^{(k)}) \right)$
  end
end
Transformation of the real-valued datasets

**Algorithm 2** Real-valued to integer to binary transformation (training phase)

**Result:** Conversion of real-valued data set into binary features

**Input:** $X_{\text{real}}^{(n)}(l)$ — a real-valued data set: $l = 1, \ldots, N_{\text{samples}}$, $n = 1, \ldots, N_{\text{variables}}$

**for** $n = 1, \ldots, N_{\text{variables}}$ **do**

\[
X_{\text{min}}^{(n)} \leftarrow \min(l)(X_{\text{real}}^{(n)}) - \varepsilon_{\text{min}}^{(n)}, \quad \varepsilon_{\text{min}}^{(n)} \geq 0
\]

\[
X_{\text{max}}^{(n)} \leftarrow \max(l)(X_{\text{real}}^{(n)}) + \varepsilon_{\text{max}}^{(n)}, \quad \varepsilon_{\text{max}}^{(n)} \geq 0
\]

**for** $l = 1, \ldots, N_{\text{samples}}$ **do**

\[
X_{\text{integer}}^{(n)}(l) \leftarrow \text{int} \left( 65535 \times \left( X_{\text{real}}^{(n)}(l) - X_{\text{min}}^{(n)} \right) / \left( X_{\text{max}}^{(n)} - X_{\text{min}}^{(n)} \right) \right)
\]

\[
X_{\text{binary}}^{(n)}(l) \leftarrow \text{binarize} \left( X_{\text{integer}}^{(n)}(l) \right)
\]

**end**

Each data sample is represented by a 16-digit binary number ($2^{16} - 1 = 65535$) with every digit becoming a separate feature. The total number of features is $16 \times N_{\text{variables}}$.

---

**Algorithm 3** Binary to integer to real-valued transformation (sampling phase)

**Result:** Conversion of the generated 16-digit binary sample into real-valued sample

**Input:** $\hat{X}^{(n)}[m]$ — generated 16-digit binary sample: $m = 0, \ldots, 15$, $n = 1, \ldots, N_{\text{variables}}$

**for** $n = 1, \ldots, N_{\text{variables}}$ **do**

\[
\hat{X}_{\text{integer}}^{(n)} \leftarrow 0
\]

**for** $m = 0, \ldots, 15$ **do**

\[
\hat{X}_{\text{integer}}^{(n)} \leftarrow \hat{X}_{\text{integer}}^{(n)} + 2^m \times \hat{X}^{(n)}[15 - m]
\]

**end**

\[
\hat{X}_{\text{real}}^{(n)} \leftarrow X_{\text{min}}^{(n)} + \hat{X}_{\text{integer}}^{(n)} \times \left( X_{\text{max}}^{(n)} - X_{\text{min}}^{(n)} \right) / 65535
\]

**end**

Alexei Kondratyev and Christian Schwarz (2019)

*The Market Generator*

Illustrative example – feature extraction

- Open source dataset of wine samples (University of California, Irvine)

- 178 wine samples, 13 real-valued features, 3 grape types

- Our plan:
  1) Standardise and binarise features
  2) Split original dataset into training and validation datasets (70:30)
  3) Train MLP Classifier and make predictions for the validation dataset
  4) Train combined classifier consisting of RBM and MLP and make predictions for the validation dataset
  5) Compare results
Illustrative example – feature extraction

MLP Classifier

88.9% correct predictions

<table>
<thead>
<tr>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>17</td>
</tr>
<tr>
<td>Class 2</td>
<td>3</td>
</tr>
<tr>
<td>Class 3</td>
<td>0</td>
</tr>
<tr>
<td>Class 1</td>
<td>2</td>
</tr>
<tr>
<td>Class 2</td>
<td>18</td>
</tr>
<tr>
<td>Class 3</td>
<td>0</td>
</tr>
<tr>
<td>Class 1</td>
<td>0</td>
</tr>
<tr>
<td>Class 2</td>
<td>1</td>
</tr>
<tr>
<td>Class 3</td>
<td>13</td>
</tr>
</tbody>
</table>

RBM + MLP Classifier

94.4% correct predictions

<table>
<thead>
<tr>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>18</td>
</tr>
<tr>
<td>Class 2</td>
<td>1</td>
</tr>
<tr>
<td>Class 3</td>
<td>0</td>
</tr>
<tr>
<td>Class 1</td>
<td>1</td>
</tr>
<tr>
<td>Class 2</td>
<td>20</td>
</tr>
<tr>
<td>Class 3</td>
<td>0</td>
</tr>
<tr>
<td>Class 1</td>
<td>0</td>
</tr>
<tr>
<td>Class 2</td>
<td>1</td>
</tr>
<tr>
<td>Class 3</td>
<td>13</td>
</tr>
</tbody>
</table>

mlpc = MLPClassifier(hidden_layer_sizes=(7, 10), alpha=0.001, tol=0.0001, max_iter=200, random_state=0)

rbm = BernoulliRBM(n_components=7, n_iter=200, random_state=0)
mlpc = MLPClassifier(hidden_layer_sizes=(10), alpha=0.001, tol=0.0001, max_iter=200, random_state=0)
classifier = Pipeline(steps=[('rbm', rbm), ('mlpc', mlpc)])
Generation of synthetic market data

QQ-plots of USDJPY spot FX daily log-returns (dataset: 1999-2019)

Dataset vs. Normal

RBM-generated samples vs. Normal

RBM-generated samples vs. Dataset

Alexei Kondratyev and Christian Schwarz (2019)
The Market Generator
https://ssrn.com/abstract=3384948
Generation of synthetic market data (continued)

Reconstruction of correlations

<table>
<thead>
<tr>
<th>Currency pairs</th>
<th>Pearson</th>
<th>Spearman</th>
<th>Kendall</th>
<th>Pearson</th>
<th>Spearman</th>
<th>Kendall</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD/GBPUSD</td>
<td>63.7%</td>
<td>62.5%</td>
<td>45.3%</td>
<td>55.7% (±4.4%)</td>
<td>59.1% (±2.2%)</td>
<td>42.7% (±1.8%)</td>
</tr>
<tr>
<td>EURUSD/USDJPY</td>
<td>−28.2%</td>
<td>−31.2%</td>
<td>−21.8%</td>
<td>−26.0% (±2.5%)</td>
<td>−28.7% (±1.4%)</td>
<td>−20.0% (±1.0%)</td>
</tr>
<tr>
<td>EURUSD/USDCAD</td>
<td>−44.3%</td>
<td>−38.6%</td>
<td>−26.8%</td>
<td>−40.1% (±3.8%)</td>
<td>−37.1% (±1.8%)</td>
<td>−25.7% (±1.3%)</td>
</tr>
<tr>
<td>GBPUSD/USDJPY</td>
<td>−13.5%</td>
<td>−21.2%</td>
<td>−14.7%</td>
<td>−15.5% (±5.4%)</td>
<td>−21.2% (±0.9%)</td>
<td>−14.6% (±0.7%)</td>
</tr>
<tr>
<td>GBPUSD/USDCAD</td>
<td>−43.1%</td>
<td>−35.8%</td>
<td>−24.8%</td>
<td>−38.5% (±3.2%)</td>
<td>−34.3% (±2.4%)</td>
<td>−23.7% (±1.7%)</td>
</tr>
<tr>
<td>USDJPY/USDCAD</td>
<td>3.0%</td>
<td>7.4%</td>
<td>5.1%</td>
<td>5.6% (±2.5%)</td>
<td>8.5% (±1.5%)</td>
<td>5.8% (±1.0%)</td>
</tr>
</tbody>
</table>

Reconstruction of tail behaviour

<table>
<thead>
<tr>
<th>Currency pair</th>
<th>data set</th>
<th>RBM-generated samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st percentile</td>
<td>99th percentile</td>
</tr>
<tr>
<td>EURUSD</td>
<td>−1.58%</td>
<td>1.53%</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>−1.48%</td>
<td>1.34%</td>
</tr>
<tr>
<td>USDJPY</td>
<td>−1.73%</td>
<td>1.66%</td>
</tr>
<tr>
<td>USDCAD</td>
<td>−1.42%</td>
<td>1.50%</td>
</tr>
</tbody>
</table>

Reconstruction of annualised volatilities

<table>
<thead>
<tr>
<th>Currency pair</th>
<th>Historical volatility</th>
<th>RBM-generated samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD</td>
<td>9.7%</td>
<td>9.8% (±0.7%)</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>9.4%</td>
<td>9.5% (±0.7%)</td>
</tr>
<tr>
<td>USDJPY</td>
<td>10.2%</td>
<td>10.4% (±0.4%)</td>
</tr>
<tr>
<td>USDCAD</td>
<td>8.9%</td>
<td>8.6% (±0.8%)</td>
</tr>
</tbody>
</table>

Alexei Kondratyev and Christian Schwarz (2019)
The Market Generator
https://ssrn.com/abstract=3384948
RBM can be used to generate either fully independent or autocorrelated samples with desired degree of autocorrelation.

RBM-generated daily log-returns (EURUSD)

Alexei Kondratyev and Christian Schwarz (2019)  
*The Market Generator*  
https://ssrn.com/abstract=3384948
RBM is able to handle non-stationarity through conditional sampling.

### Conditional sampling

<table>
<thead>
<tr>
<th>Currency pair</th>
<th>Low volatility environment</th>
<th>High volatility environment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Historical volatility</td>
<td>Historical volatility</td>
</tr>
<tr>
<td></td>
<td>RBM-generated samples</td>
<td>RBM-generated samples</td>
</tr>
<tr>
<td>EURUSD</td>
<td>6.8%</td>
<td>12.2%</td>
</tr>
<tr>
<td></td>
<td>6.7% (±0.2%)</td>
<td>10.8% (±0.5%)</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>6.9%</td>
<td>13.1%</td>
</tr>
<tr>
<td></td>
<td>6.8% (±0.3%)</td>
<td>13.7% (±0.7%)</td>
</tr>
<tr>
<td>USDJPY</td>
<td>8.0%</td>
<td>12.8%</td>
</tr>
<tr>
<td></td>
<td>8.9% (±0.3%)</td>
<td>12.7% (±0.6%)</td>
</tr>
<tr>
<td>USDCAD</td>
<td>6.2%</td>
<td>13.0%</td>
</tr>
<tr>
<td></td>
<td>6.4% (±0.2%)</td>
<td>11.6% (±0.7%)</td>
</tr>
</tbody>
</table>

Alexei Kondratyev and Christian Schwarz (2019)

*The Market Generator*

Further Applications

- Applications:
  - Data Anonymisation
  - Outlier Detection
  - Fighting Overfitting

- RBMs can be trained on relatively small datasets, which often are too small to train GANs.

- We apply our model on several standard medical and financial datasets.

- We generate synthetic data (anonymised dataset) with the same statistical properties as the original data (i.e. a potentially sensitive dataset that can be used to deduce patients or clients details).

Alexei Kondratyev, Christian Schwarz and Blanka Horvath (2020)
Data Anonymisation, Outlier Detection and Fighting Overfitting with Restricted Boltzmann Machines
https://ssrn.com/abstract=3526436
Example 1: Wisconsin dataset

Figure 1: F1 scores for different classifiers trained on the original and synthetic datasets for Benign (left chart) and Malignant (right chart) classes with performance observed on the same original testing datasets. The dots indicate mean values and error bars indicate 1/2 and 1 standard deviations.
Example 2: Coimbra dataset

Figure 2: F1 scores for different classifiers trained on the original and synthetic datasets for the Healthy Control (left chart) and Patient (right chart) classes with the performance compared to the same original testing datasets. The dots indicate mean values and the error bars indicate 1/2 and 1 standard deviations.
Example 3: Australian credit approval dataset

Figure 3: F1 scores for different classifiers trained on the original and synthetic datasets for Class 1 (left chart) and Class 2 (right chart), comparing the performance on the same original testing datasets. The dots indicate mean values and error bars indicate 1/2 and 1 standard deviations.
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