A Synthesis Mortality Model for the Elderly Effect

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Outline

- Introduction
- Mortality models – brief review
- Synthesis models – process and estimation results
- Extending over 100
- Pricing life annuities
- Conclusion

Introduction

- Longevity risk: the importance and need for modeling mortality rates for the elderly
- Obstacles to better modeling
  - Quality and quantity required
  - Data period
  - Improvement pattern not homogeneous

Introduction

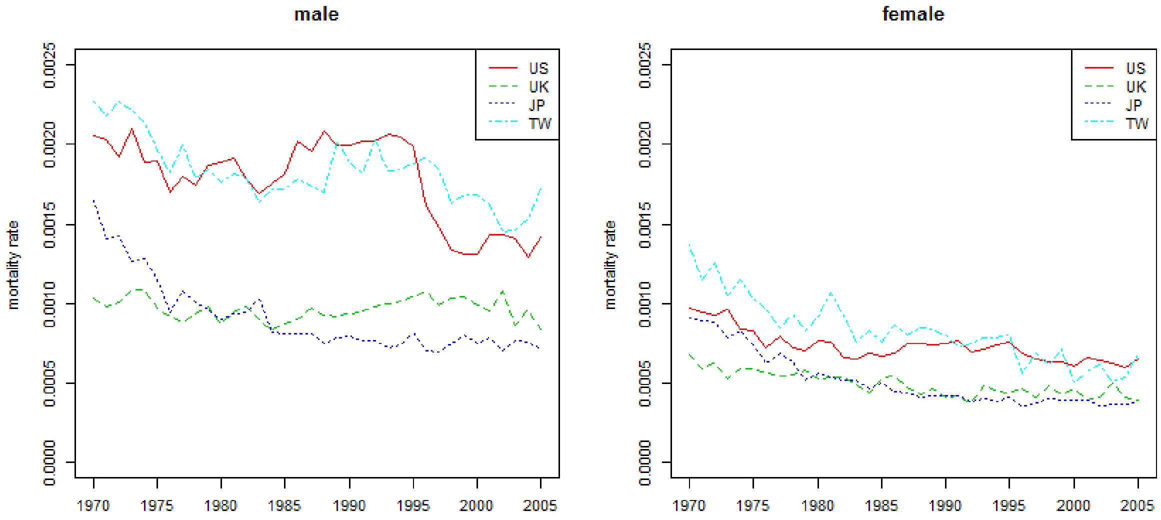

Figure 1. The mortality trend of populations age 30 from 1970 to 2005

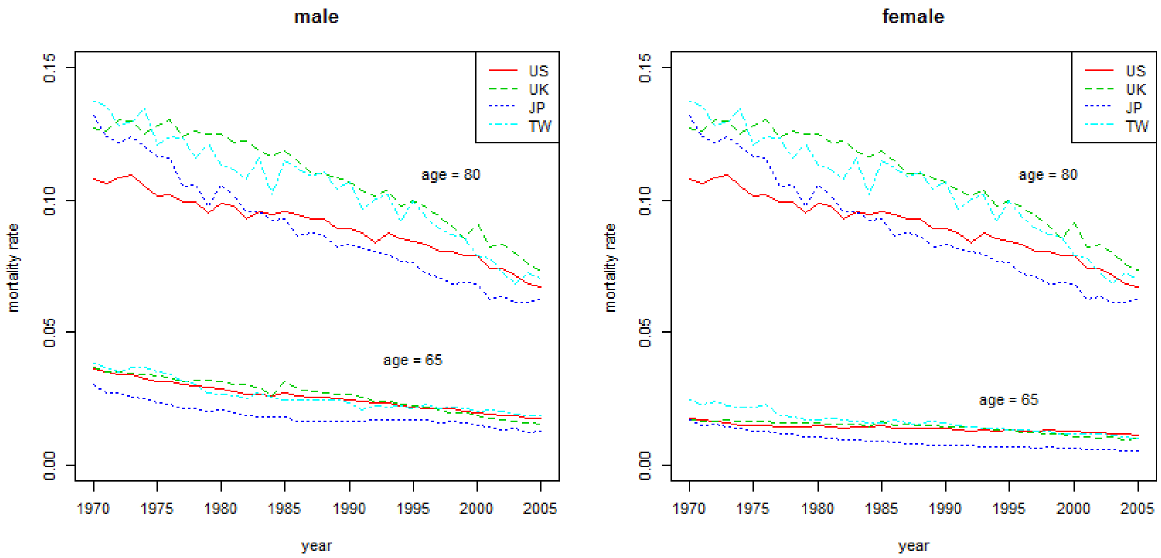

Figure 2. The mortality trends of populations age 65 and 80 from 1970 to 2005

Introduction

Relational models	Stochastic models
◆Fit certain type of function ◆Provide good estimation for age ranges	◆Able to model across age, time and cohort ◆Provide good prediction
◆Inability to model across time ◆Unstable prediction	◆Have difficulty in extrapolating mortality rates for age groups without data

Introduction

- This paper…
  - Propose a synthesis model combining models from both relational and stochastic group
  - Focus on elderly population
  - Extrapolate mortality rates for ages beyond sample age range
  - Application: pricing life annuities

Mortality Models

Model	Object	
Lee-Carter	Central death rate	$\ln m_{x,t} = \alpha_x + \beta_x k_t + \varepsilon_{x,t}$
CBD	Mortality rate	$\text{logit}(q_{x,t}) = \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_{C1,t} + k_{C2,t}(x-\bar{x}) + \varepsilon_{x,t}$
Gompertz	Force of mortality	$\mu_x = BC^x$
Coale-Kisker	Central death rate	$\min_{\alpha_K, s_K} \sum_x w_x \left[\ln m_x - \alpha_K - k_{K,85}(x-84) + \frac{(x-84)(x-85)}{2} s_K\right]^2$
Logistic	Mortality rate	$\kappa_x = -\ln p_x = \frac{\exp\left[\beta_{L,0} + \beta_{L,1}\cdot(x+0.5)\right]}{1+\exp\left[\beta_{L,0} + \beta_{L,1}\cdot(x+0.5)\right]}$

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Synthesis Models – process

- Data
  - Human Mortality Database
  - Countries: U.S., U.K., Japan, Taiwan
  - Year range: 1970~2009
  - Age range: 65~99

Synthesis Models – process

- Back-cast with 10-yr training period and 5-yr testing period

Training period	Testing period
1970 ~ 1979	1980 ~ 1984
1975 ~ 1984	1985 ~ 1989
1980 ~ 1989	1990 ~ 1994
1985 ~ 1994	1995 ~ 1999
1990 ~ 1999	2000 ~ 2004
1995 ~ 2004	2005 ~ 2009

Synthesis Models – process

- Synthesis models considered
  - Lee-Carter + Logistic
  - Lee-Carter + Coale-Kisker
  - Lee-Carter + Gompertz
  - CBD + Gompertz
- Single models: Lee-Carter and CBD

Synthesis Models – process

- Lee-Carter + Gompertz
  - Assume: at year *t*, the force of mortality follows the Gomperz's assumption:

$$\mu_{x,t} = B_t C_t^x \Rightarrow p_{x,t} = \exp\left[-B_t C_t^x (C_t - 1) / \ln C_t\right]$$

  - Further assume that:

$$L_{x,t} = \frac{l_{x,t} + l_{x+1,t+1}}{2} = \frac{l_{x,t} \cdot (1 + p_{x,t})}{2}$$

Synthesis Models – process

- Lee-Carter + Gompertz

$$m_{x,t} = \frac{d_{x,t}}{L_{x,t}} = \frac{l_{x,t} \cdot (1 - p_{x,t})}{L_{x,t}}$$

$$\ln\left(-\ln\left(\frac{1 - \frac{m_{x,t}}{2}}{1 + \frac{m_{x,t}}{2}}\right)\right) = \ln\left(B_t(C_t - 1)/\ln C_t\right) + x \ln C_t$$

Synthesis Models – process

- The Synthesis modeling process

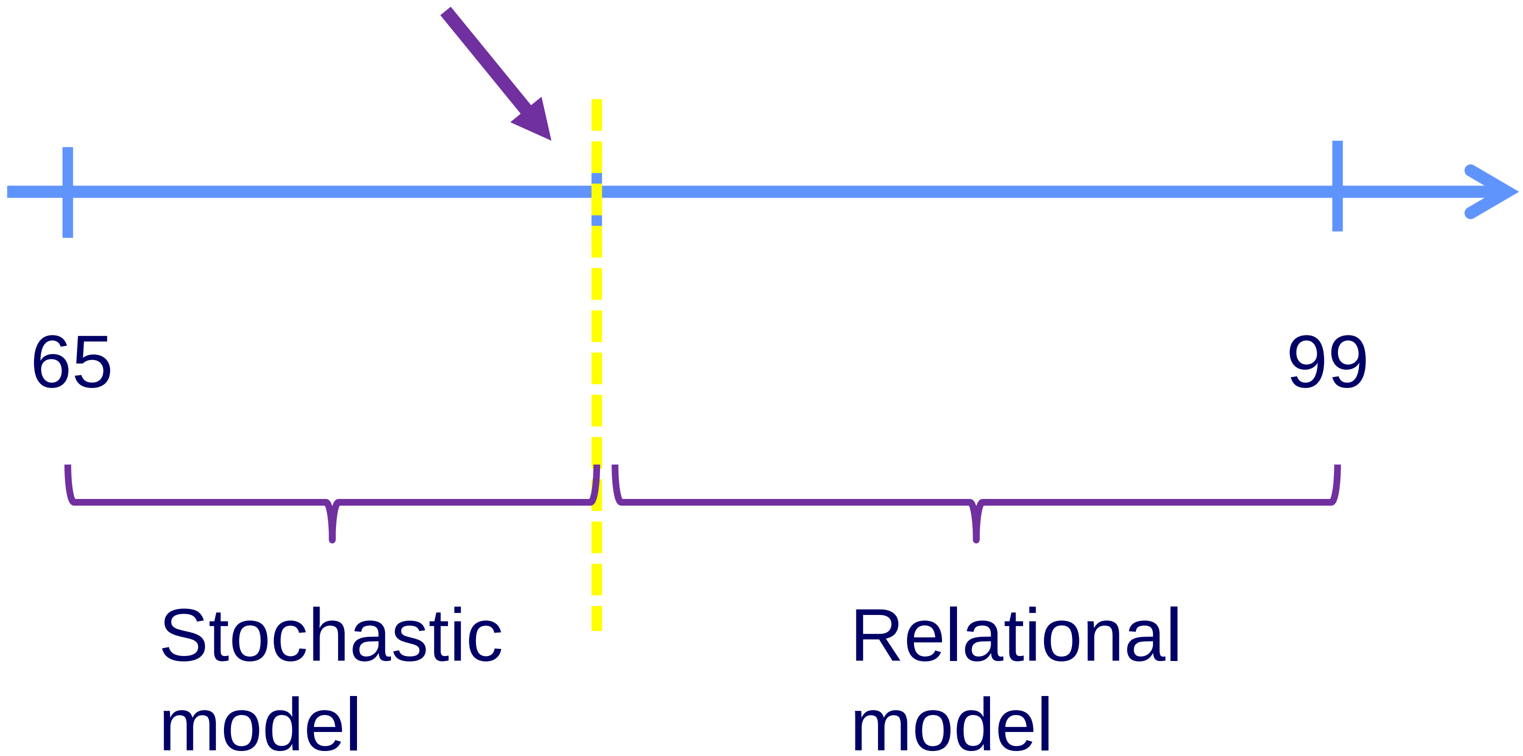
Step 1	Estimate the stochastic parameters from original mortality data
Step 2	Forecast future mortality rates from stochastic parameters
Step 3	Find the transition age using data from each training period
Step 4	For each year, estimate the relational parameters by weighted least square
Step 5	Recalculate mortality rates of ages after transition age with the relational model

Synthesis Models – process

- Transition age
  - Synthesis process not applicable for all age range
  - Mortality pattern changes with time
  - Transition age set for each country and time period by minimizing RMSE

Synthesis Models – process

- Transition age



Synthesis Models – process

Lee-Carter + Coale-Kisker

Data period	U.S.	U.K	Japan	Taiwan
1970-1979	77	77	67	85
1975-1984	77	77	73	96
1980-1989	77	69	76	96
1985-1994	75	68	74	71
1990-1999	75	68	75	77
1995-2004	66	66	75	74

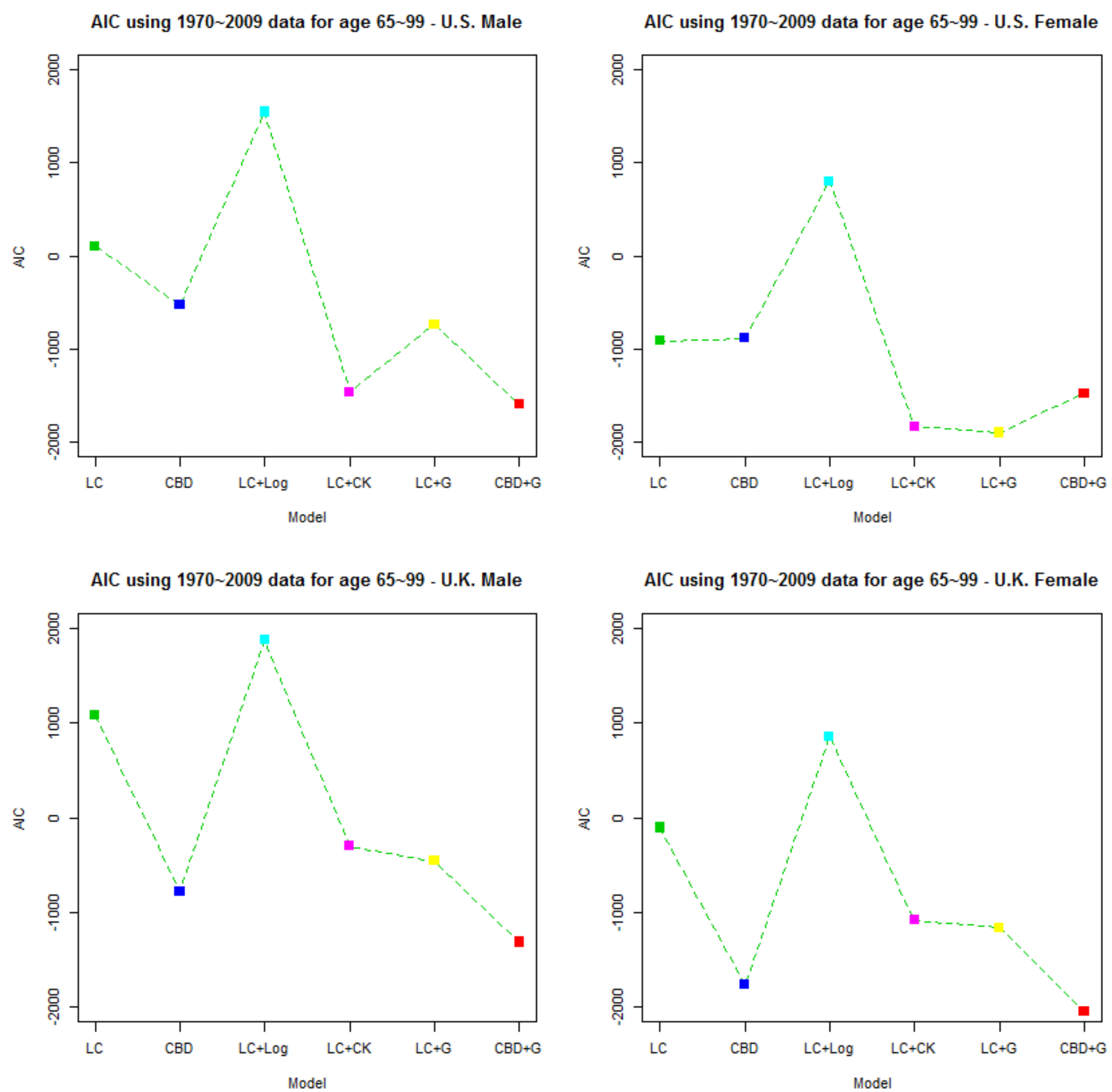
Lee-Carter + Gompertz

Data period	U.S.	U.K	Japan	Taiwan
1970-1979	78	65	75	70
1975-1984	77	65	75	71
1980-1989	77	68	76	75
1985-1994	77	75	77	74
1990-1999	77	65	75	66
1995-2004	78	66	76	79

CBD + Gompertz

Data period	U.S.	U.K	Japan	Taiwan
1970-1979	65	86	98	98
1975-1984	65	81	97	98
1980-1989	65	77	98	98
1985-1994	65	67	98	98
1990-1999	65	69	68	98
1995-2004	65	69	65	98

Synthesis Models – results



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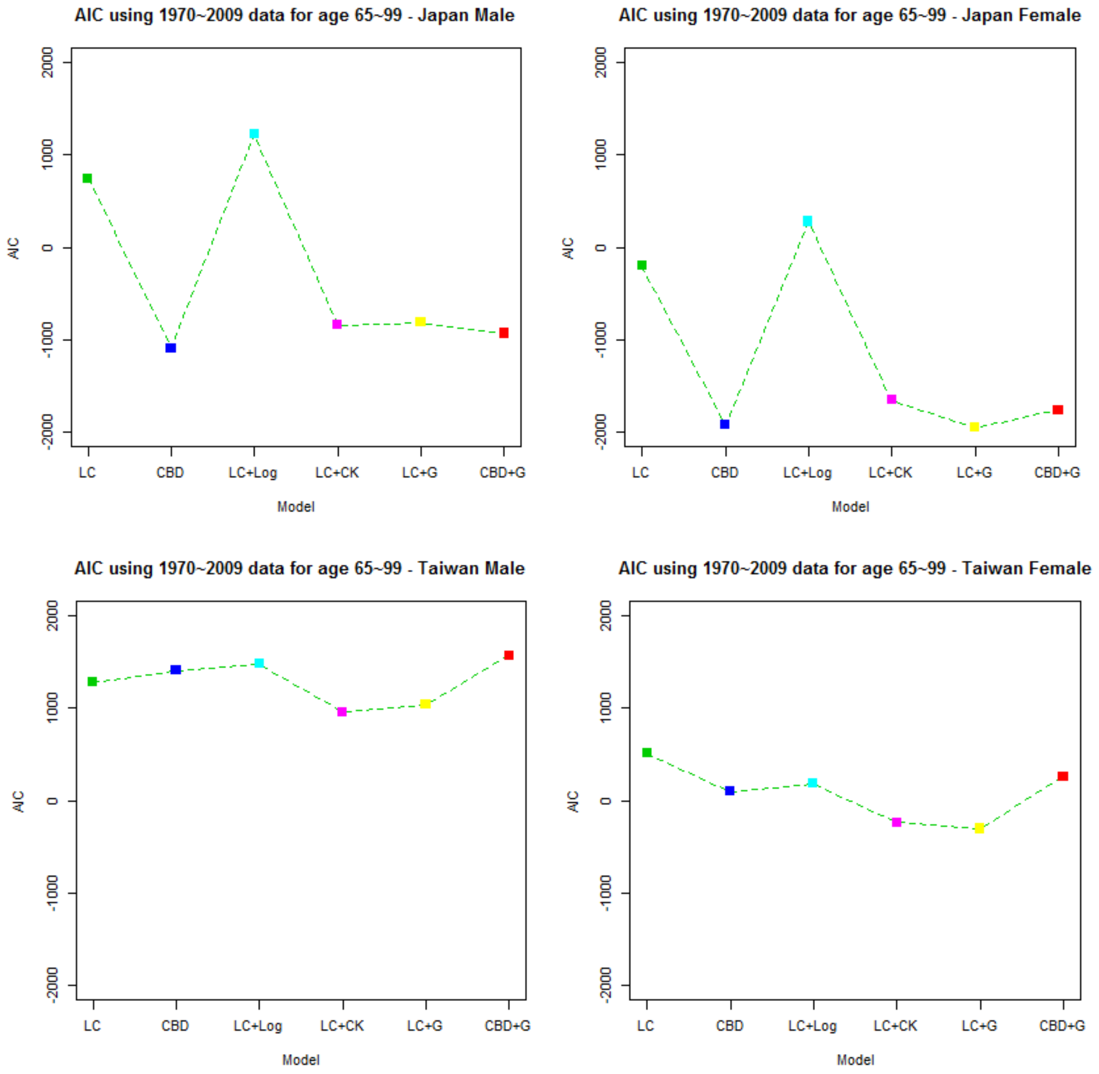

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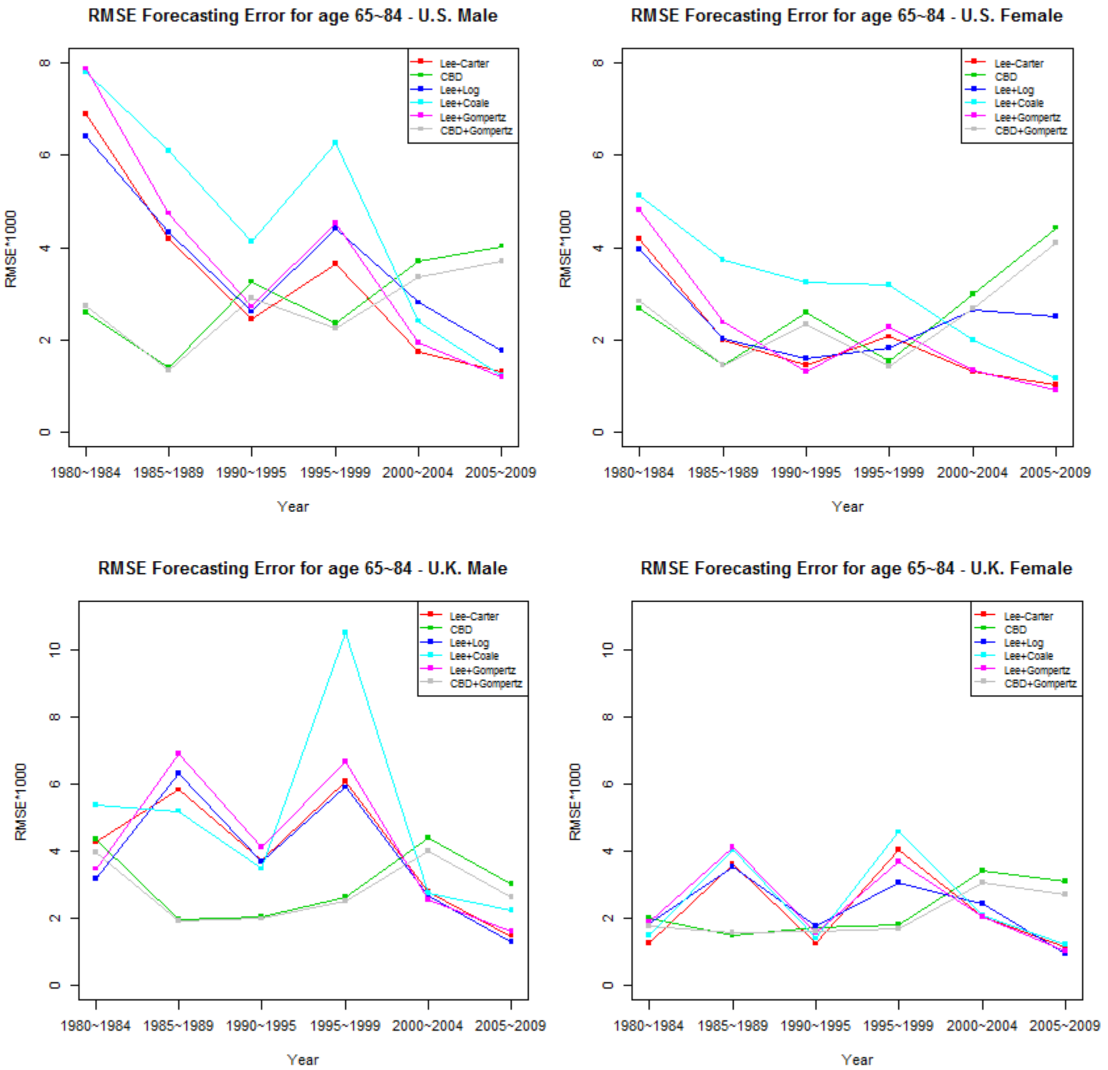

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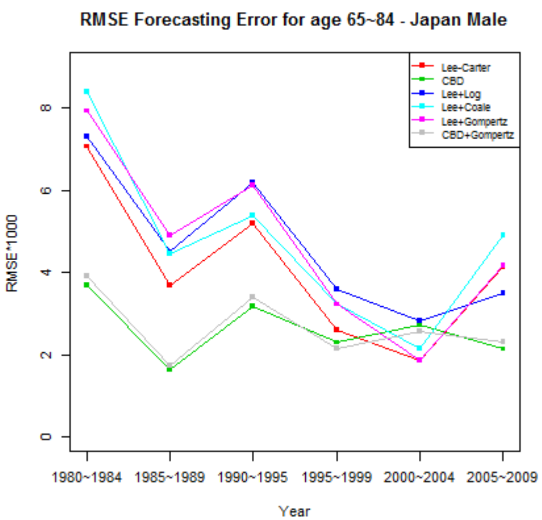

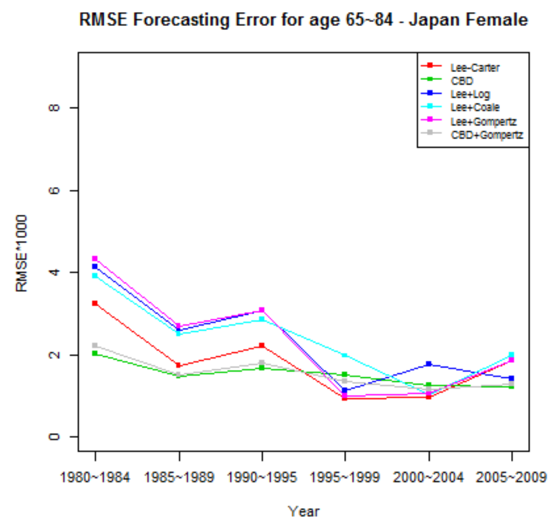

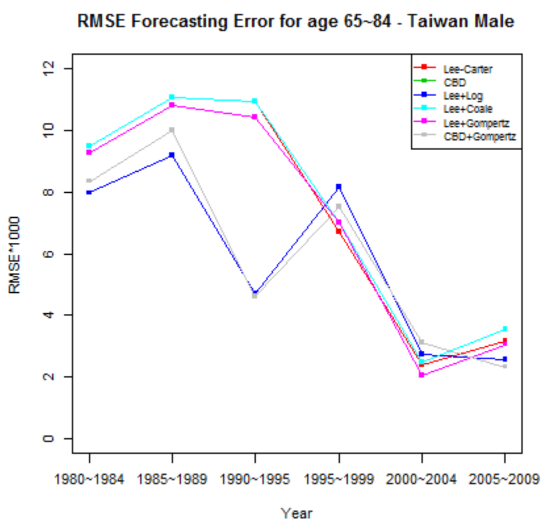

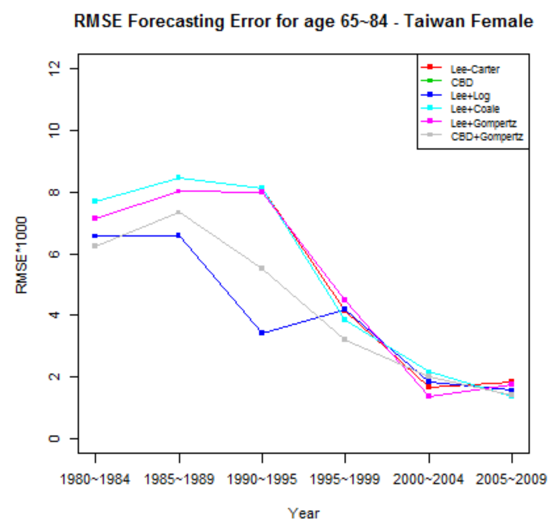

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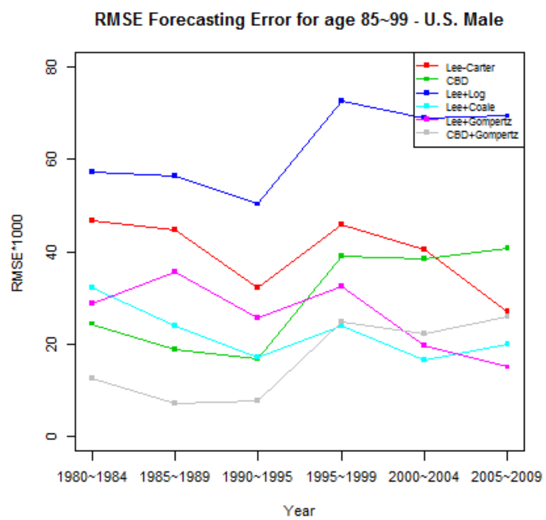
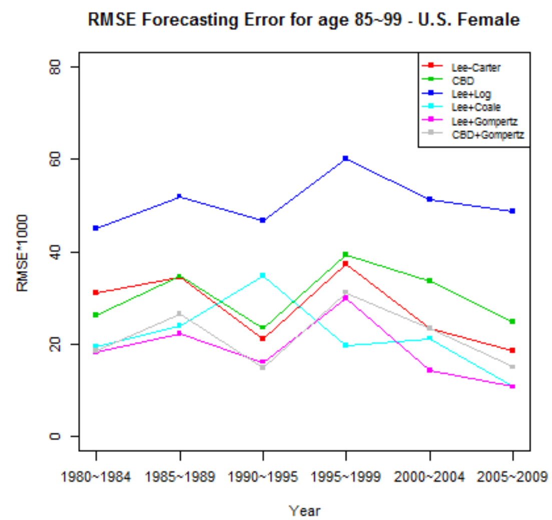
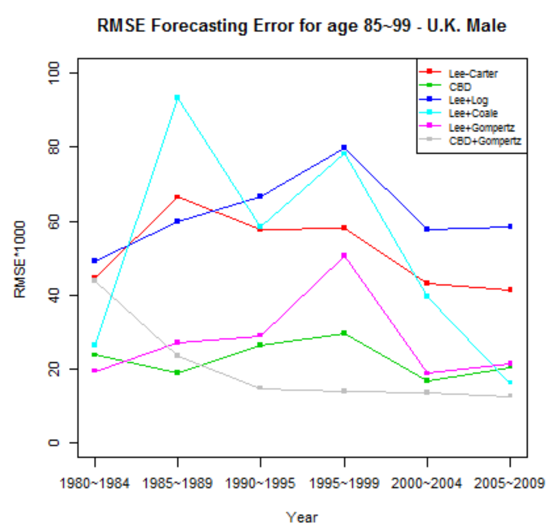
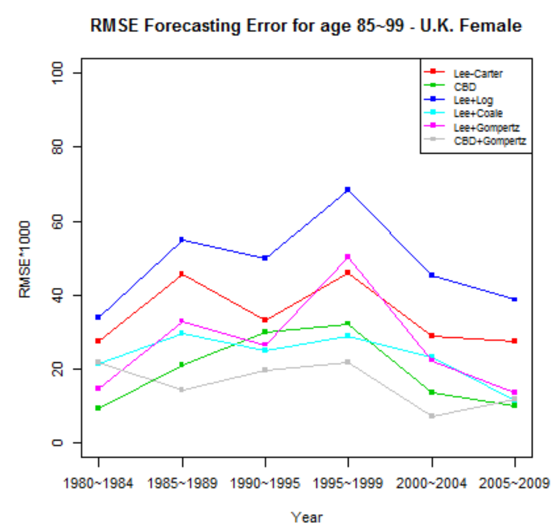
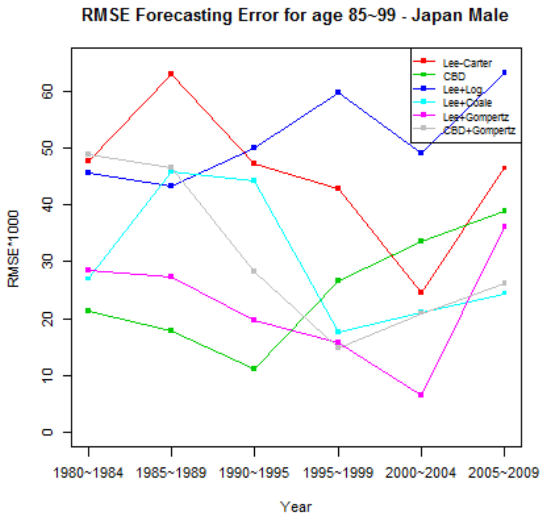


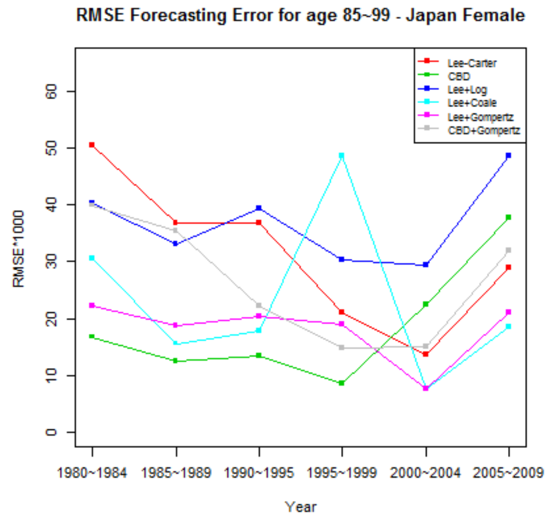


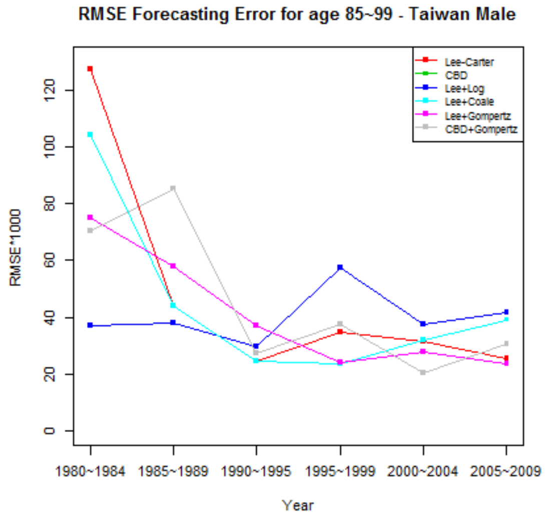


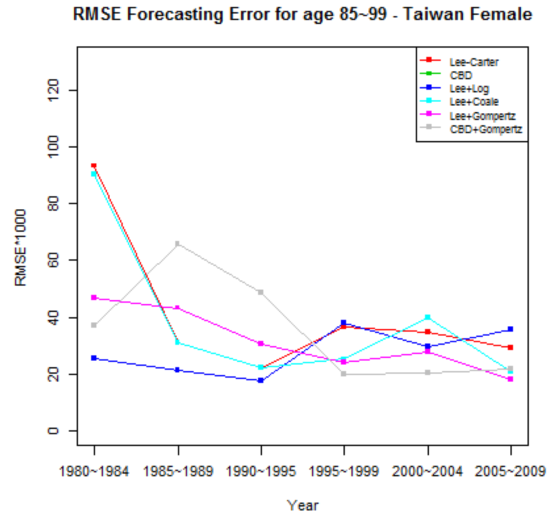


Synthesis Models – results









Synthesis Models – results

- The total number of out-performances

Model \ Age	Fitting		Forecasting	
	65-84	85-99	65-84	85-99
Lee-Carter	13	2	9	0
CBD	17	16	16	13
LC+Logistic	3	2	7	5
LC+Coale-Kisker	0	9	1	8
LC+Gompertz	1	5	7	9
CBD+Gompertz	14	14	8	13

Extending over 100

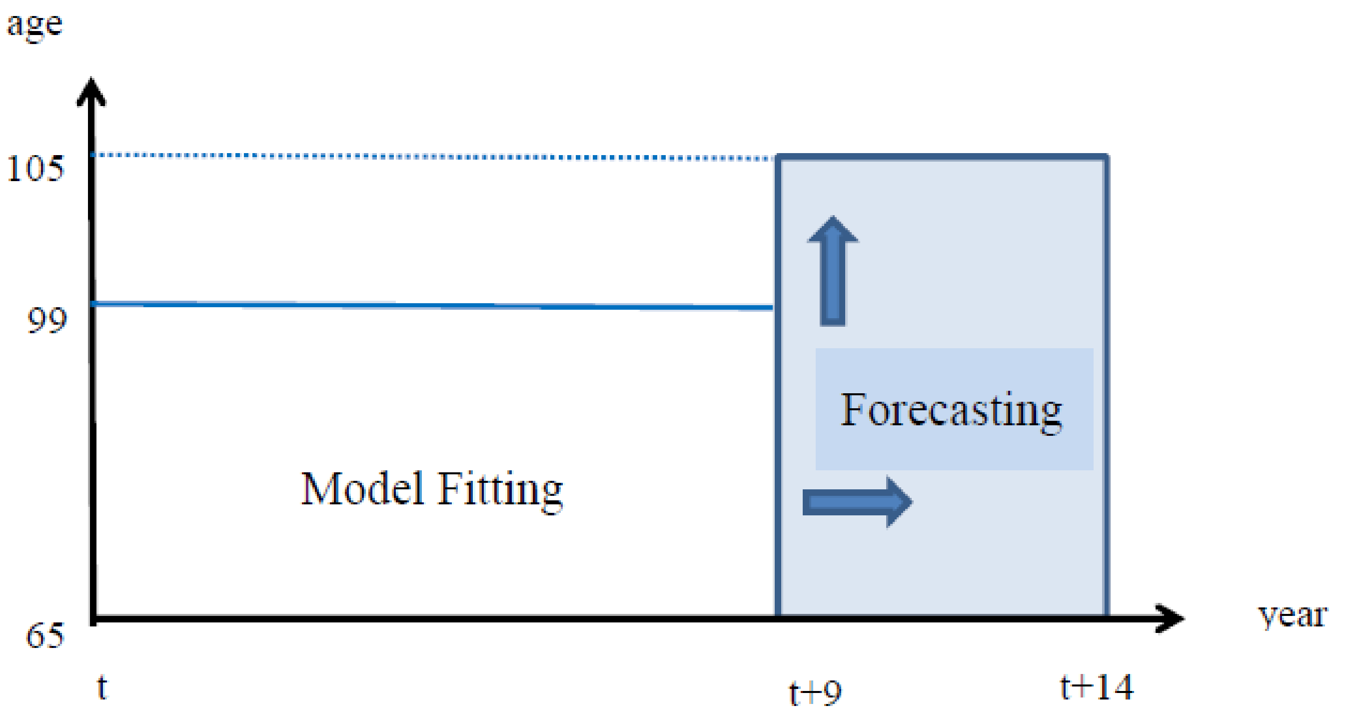


Figure 3. The methodology for future mortality estimation in this study

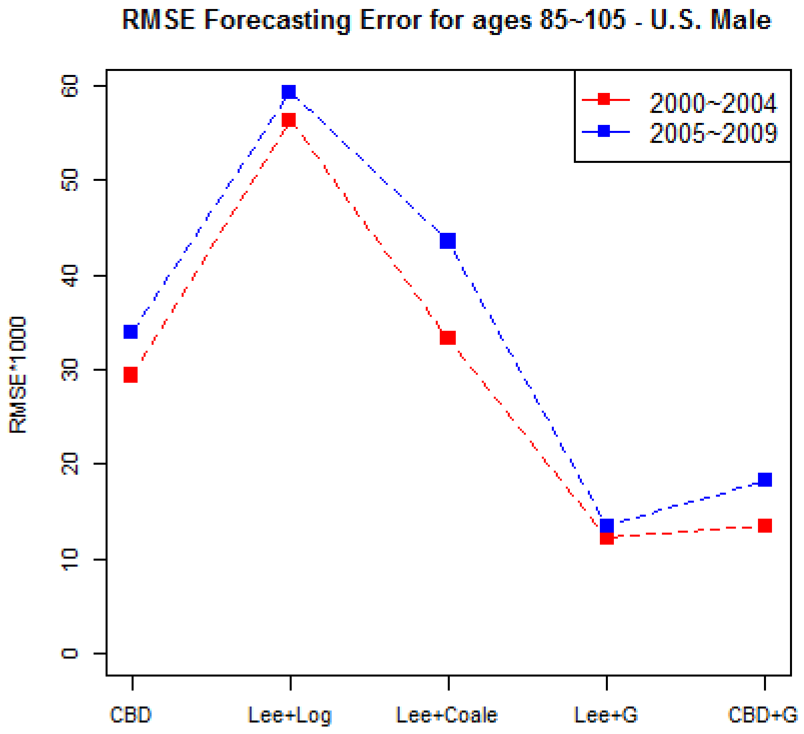
Extending over 100

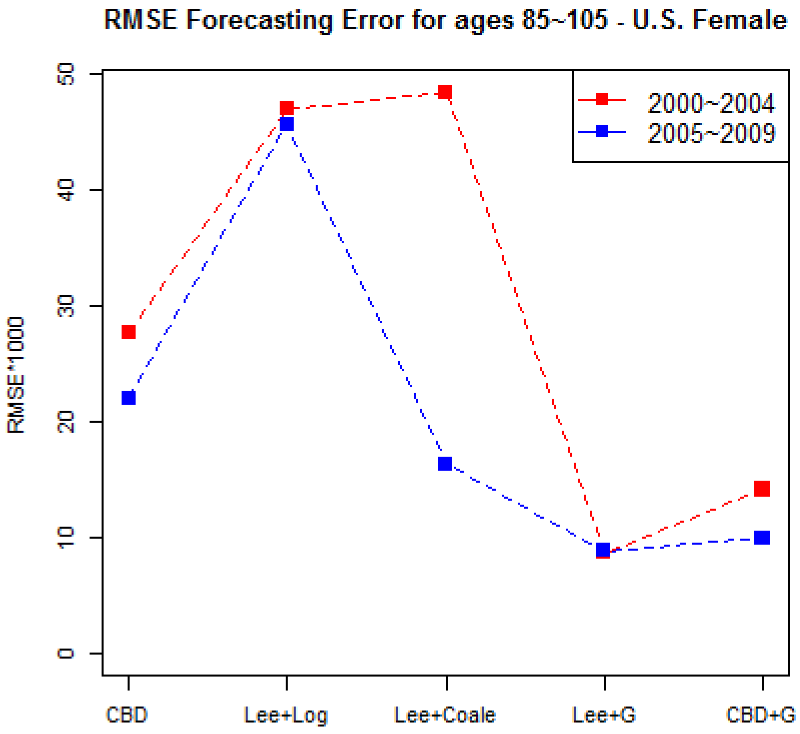
- Data
  - Countries: U.S., Japan
  - Training Testing

    [1990, 1999] => [2000, 2004]

    [1995, 2004] => [2005, 2009]
  - Age range: 65~99 => 105

Extending over 100





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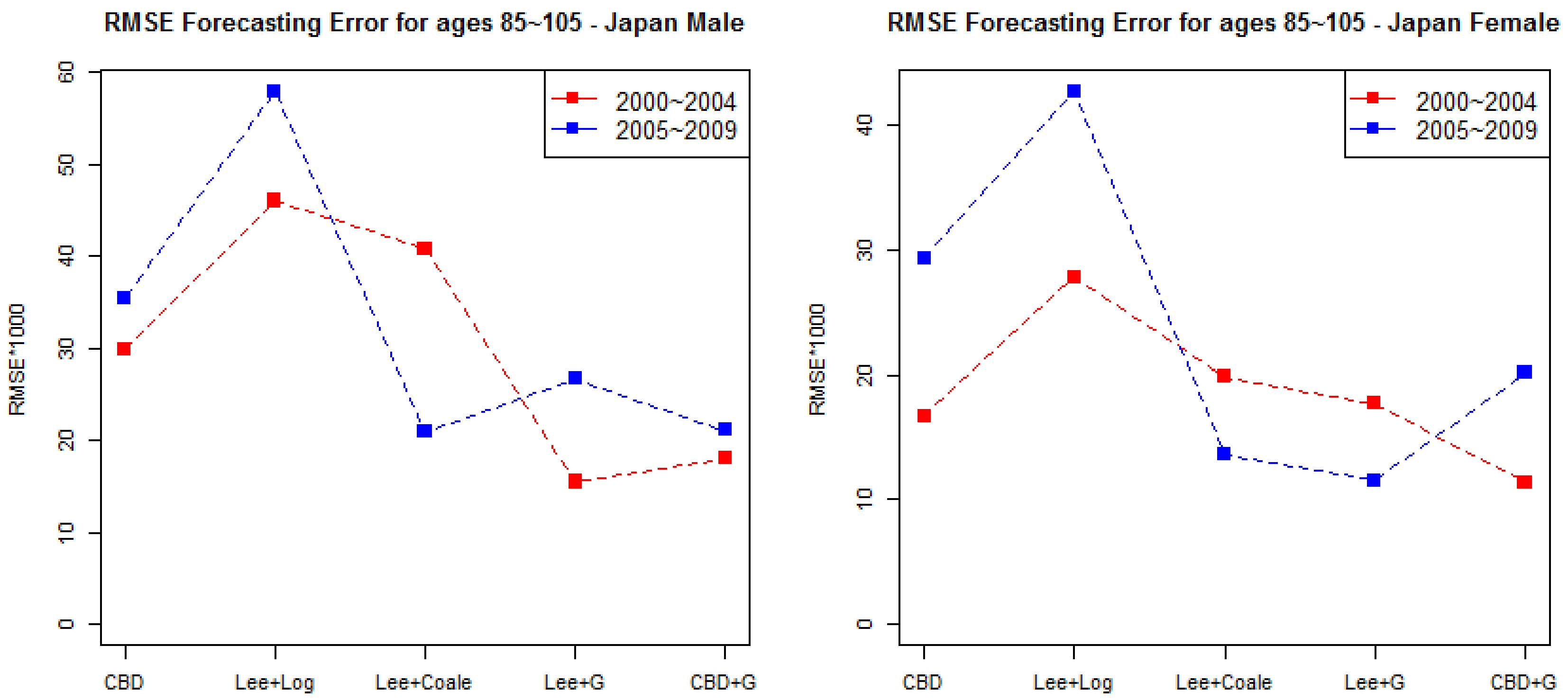

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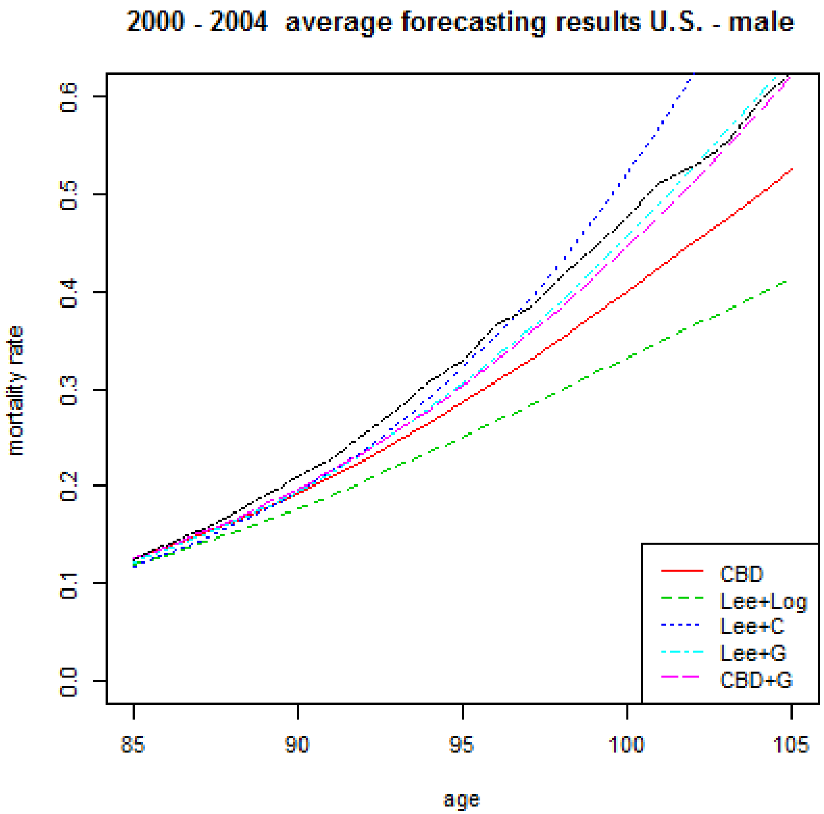

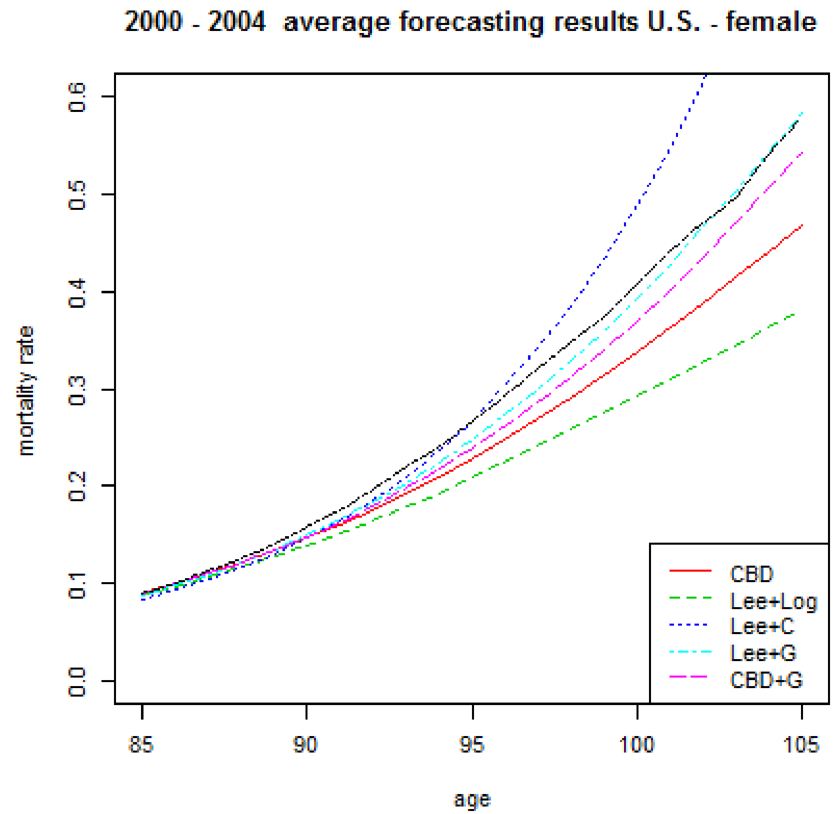

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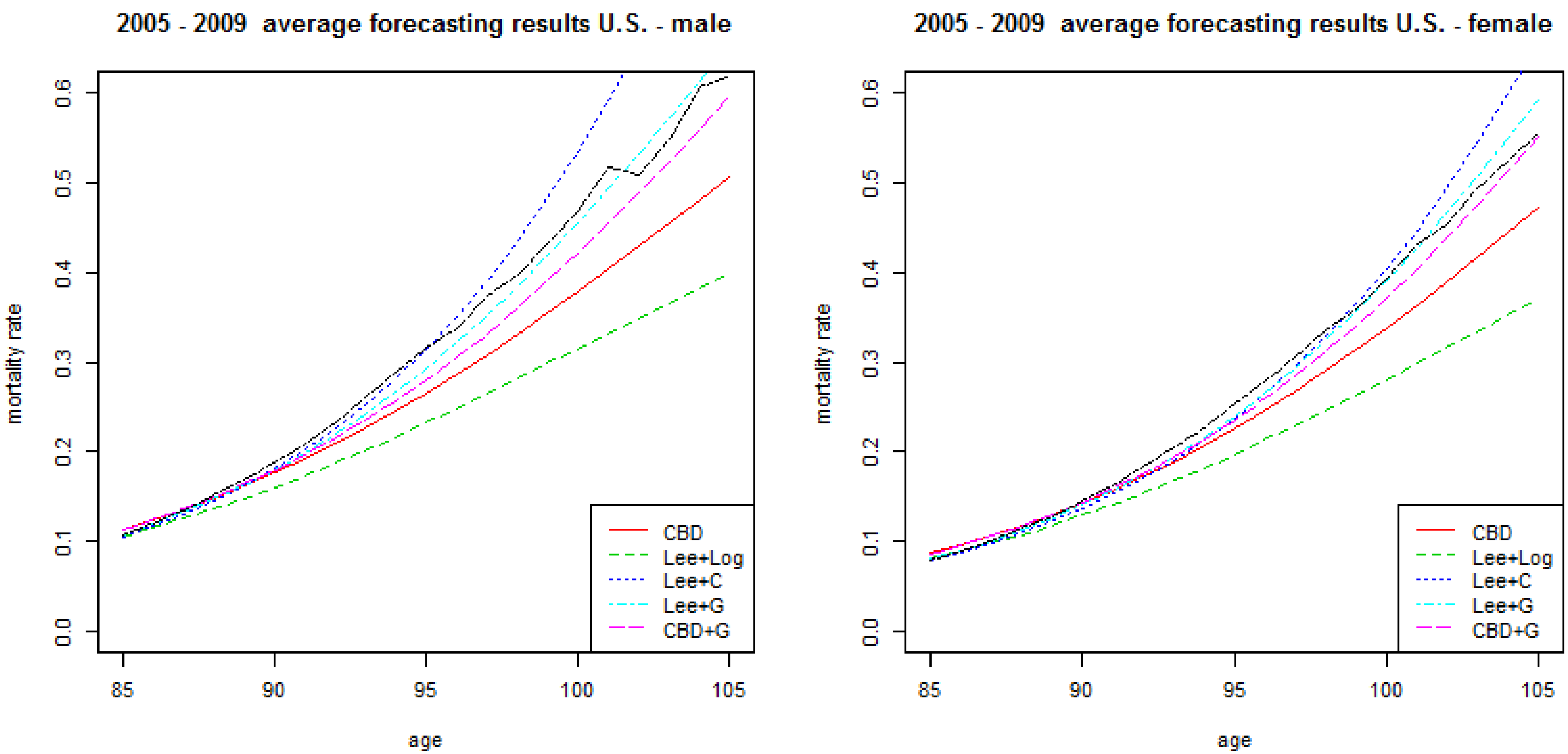

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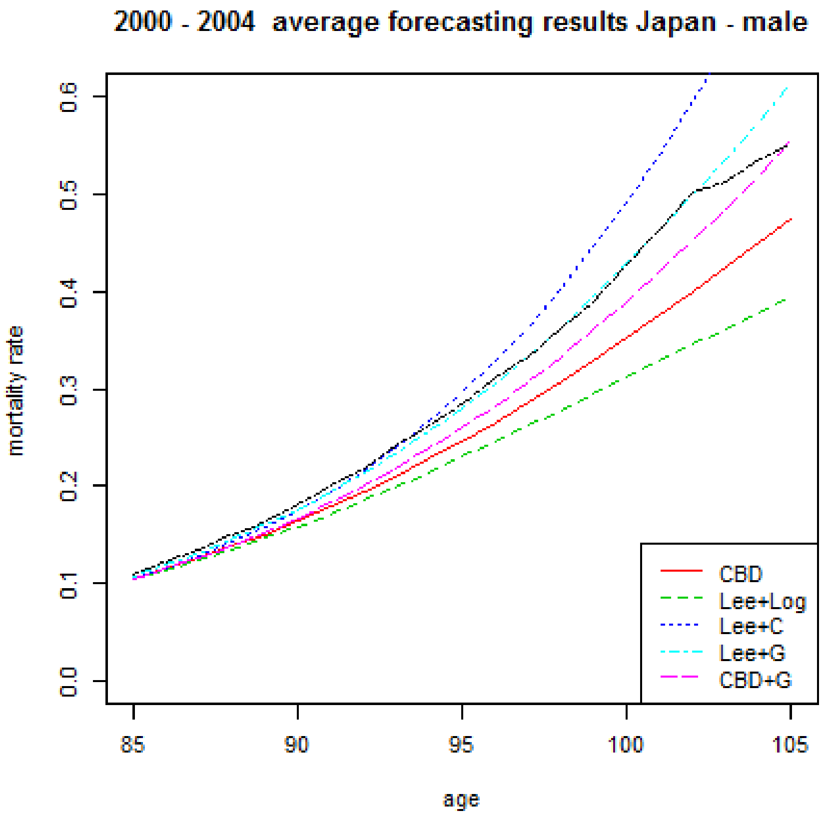

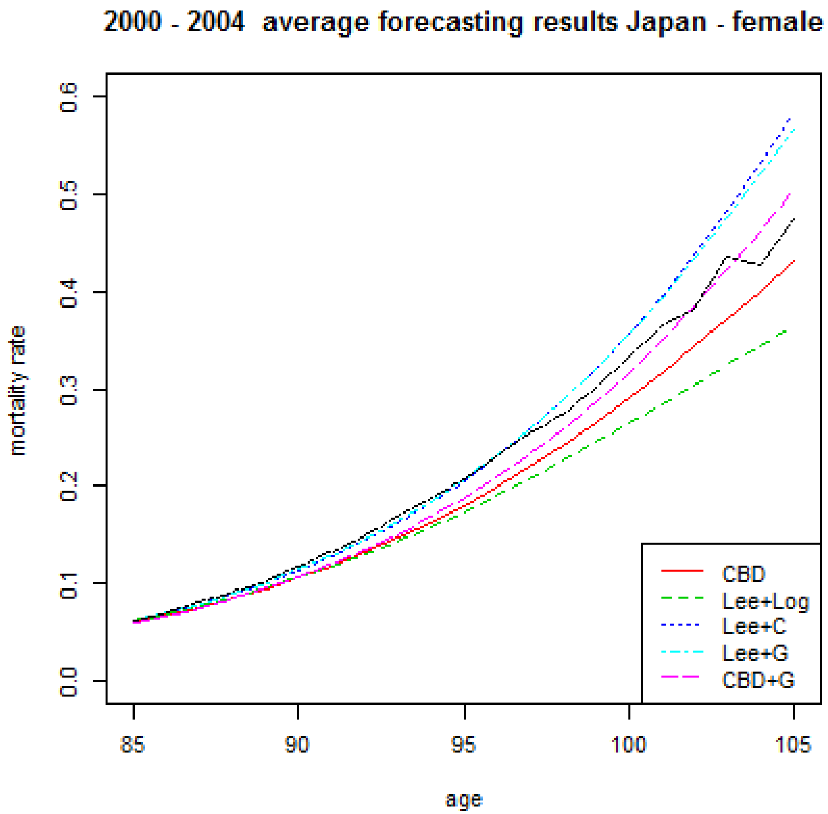

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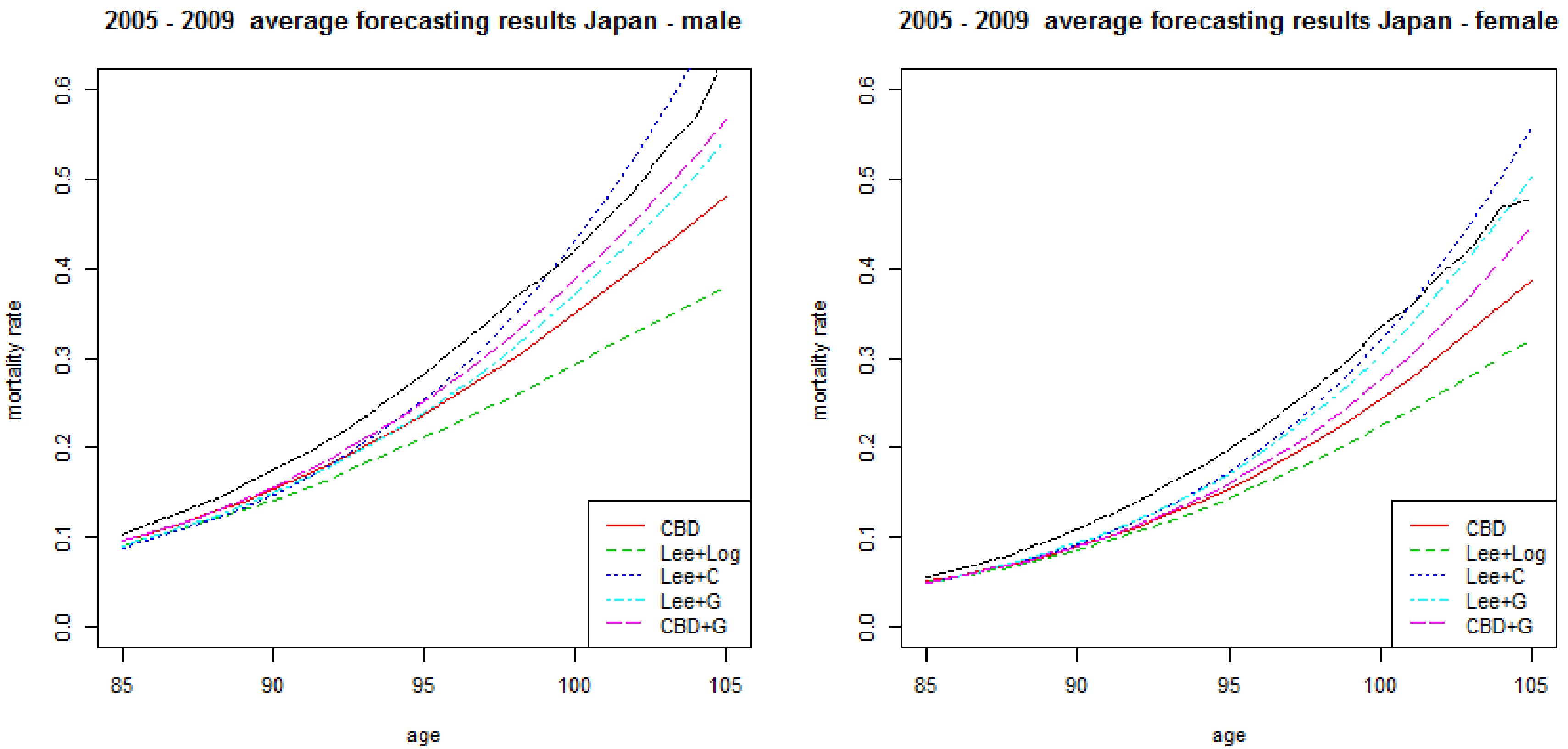
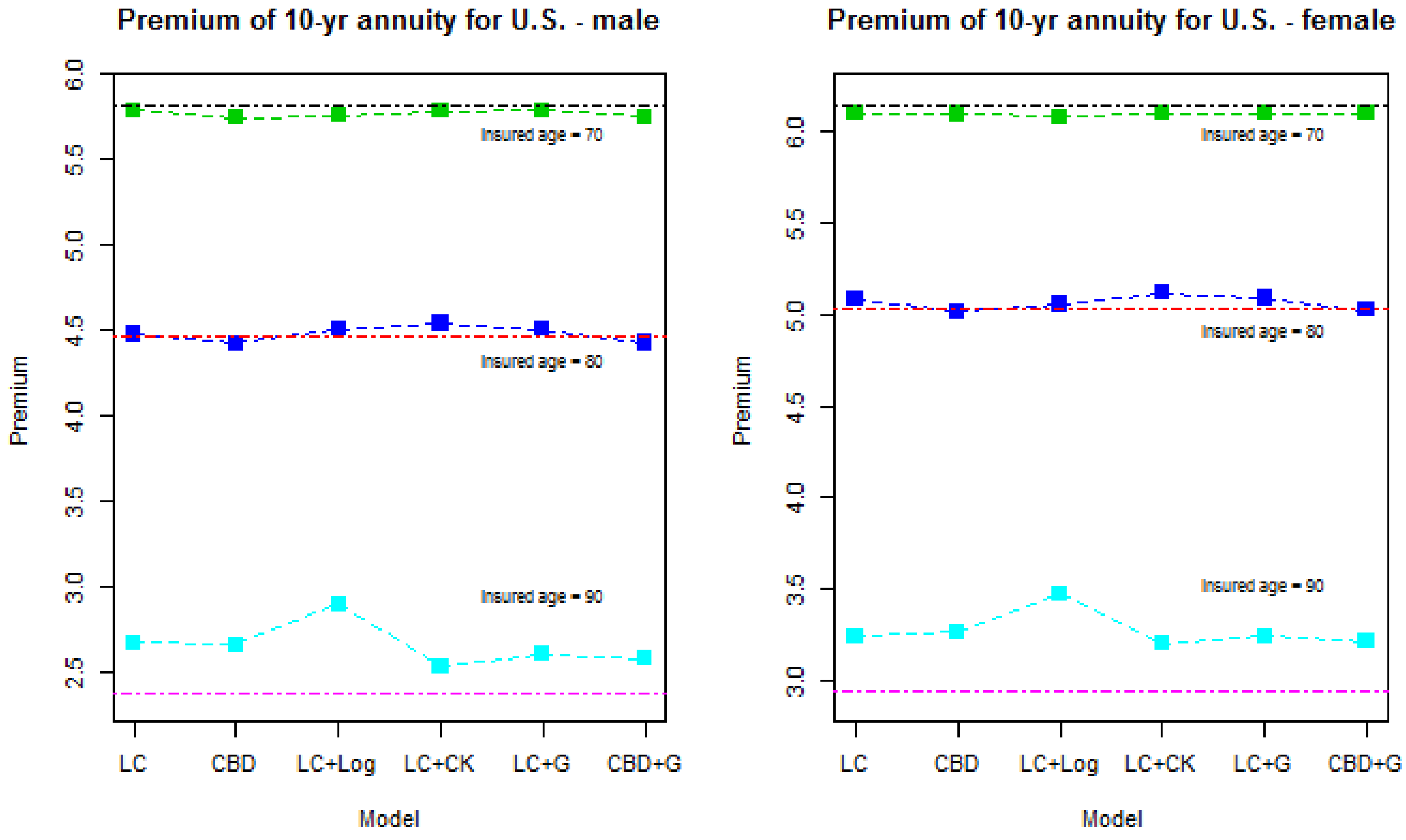


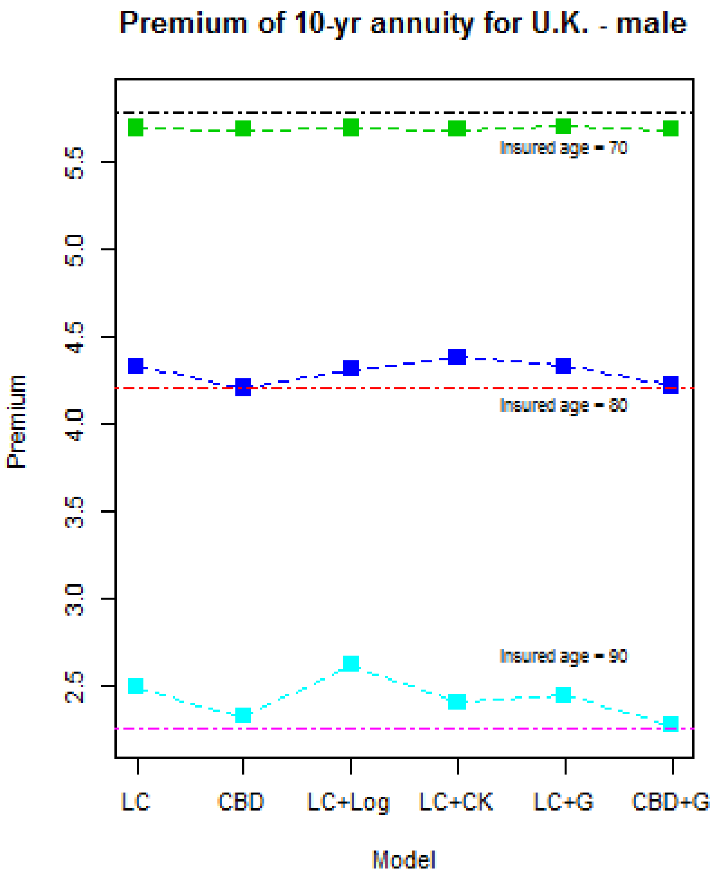
Pricing life annuities

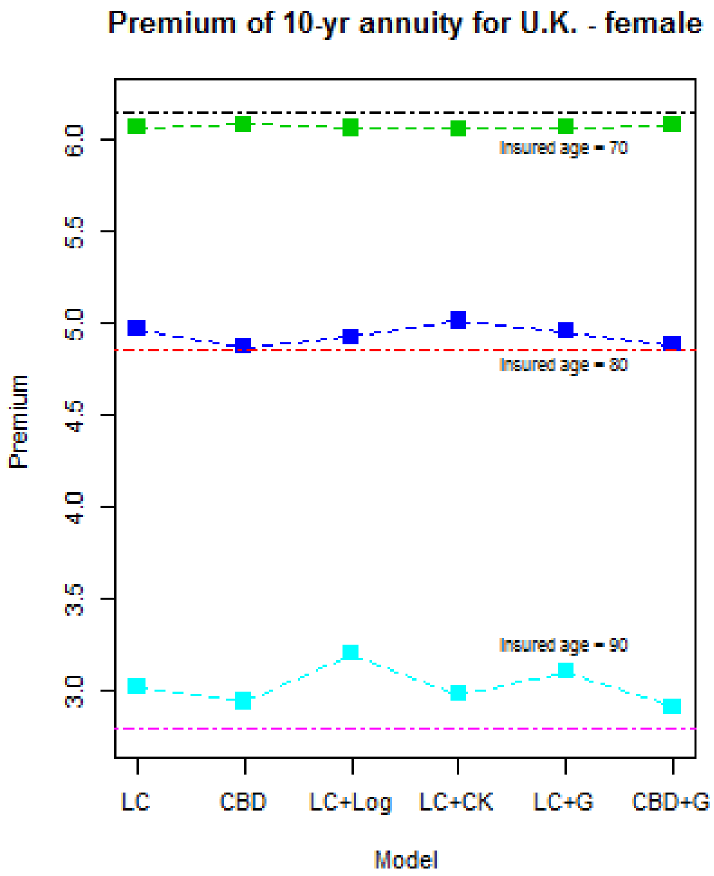
- Product
  - 10-year annuity
  - Single premium
  - Insured age: 70, 80, 90
  - Insured period: [2000, 2009]
  - [1980, 1999] mortality data used

Pricing life annuities



Pricing life annuities





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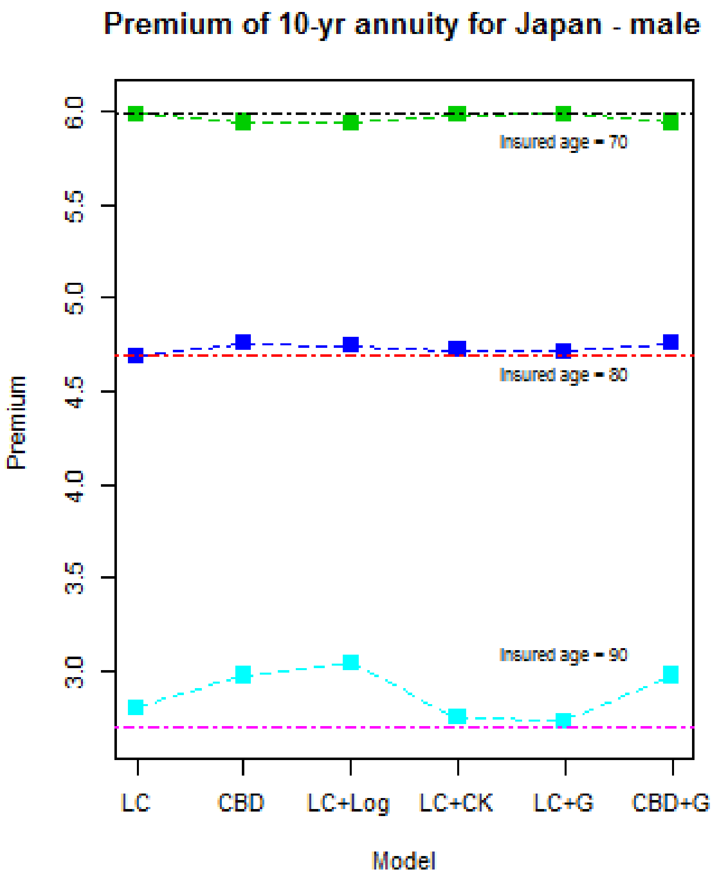
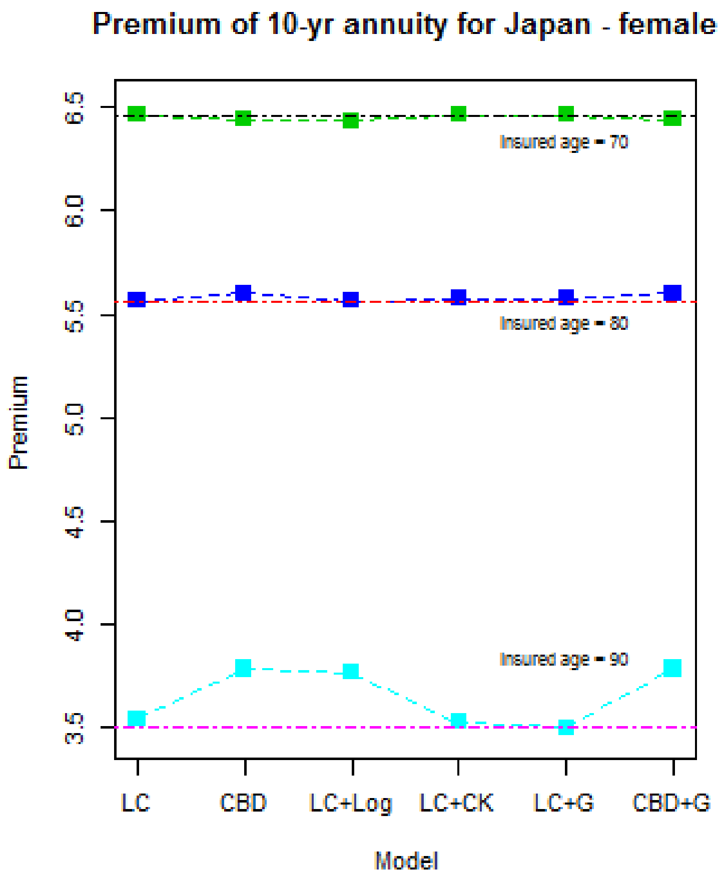
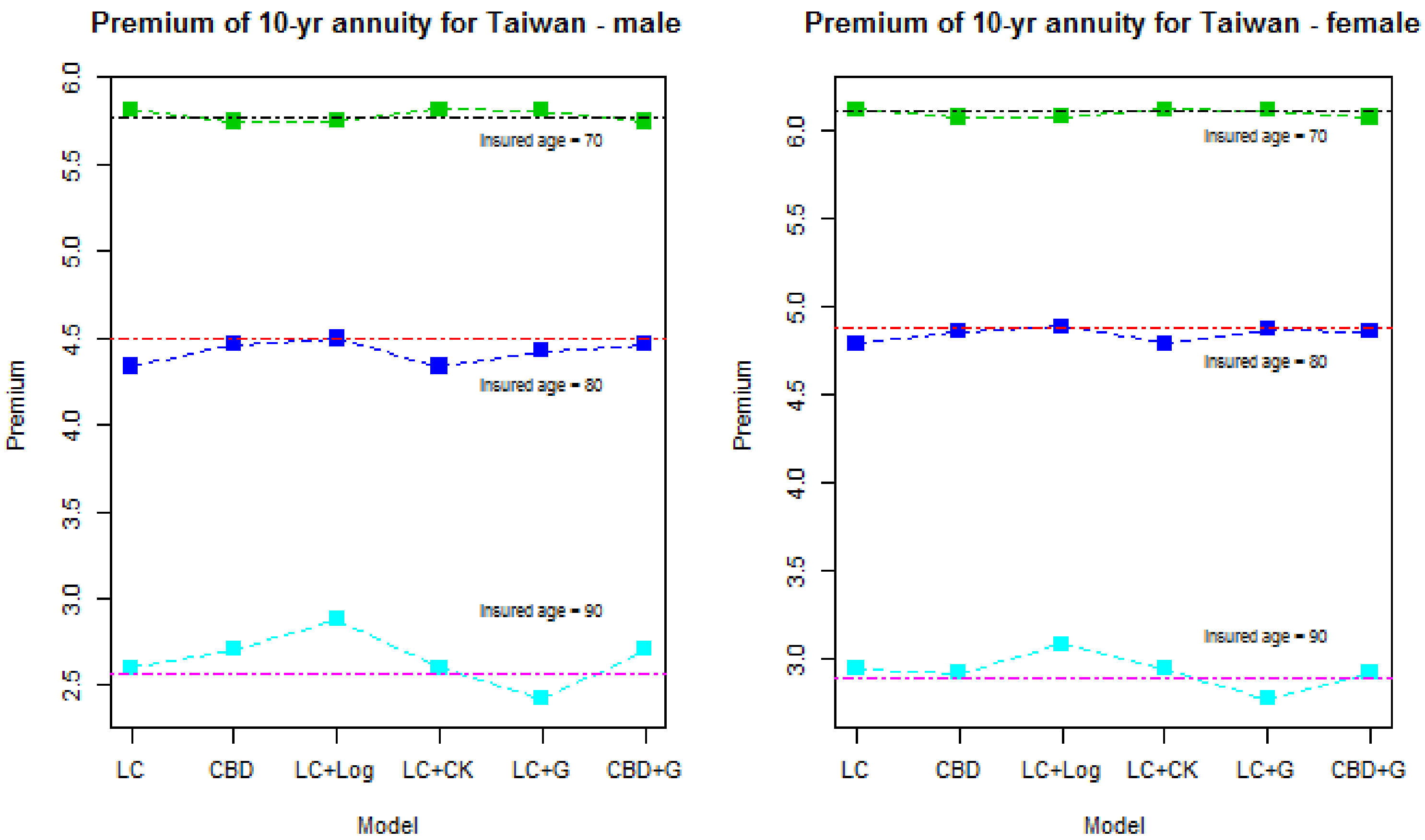




Pricing life annuities



Conclusion

- Post-retirement life has received a lot of attention and the need for modeling mortality rates for the elderly is essential
- We propose a synthesis model, selecting and combining models from both relational and stochastic group

Conclusion

- Our proposed model performs well, especially for the elderly, is a possible choice for the future
- Able to make proper estimation for the oldest-old
- Different models can be used together to decrease longevity risk that insurers face when selling annuity products

THANK YOU!

Mortality Models

- Stochastic models
  - The Lee-Carter model (Lee & Carter, 1992)

$$\ln m_{x,t} = \alpha_x + \beta_x k_t + \varepsilon_{x,t}$$

  - The CBD model (Cairns et al., 2006)

$$\text{logit}(q_{x,t}) = \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_{C1,t} + k_{C2,t}(x-\bar{x}) + \varepsilon_{x,t},$$

Mortality Models

- Relational models
  - The Gompertz model

Force of mortality: $\mu_x = BC^x$

$$p_x = \exp\left[-\int_x^{x+1} \mu_y dy\right]$$

$$= \exp\left[-\int_x^{x+1} BC^y dy\right]$$

$$= \exp\left[-BC^x (C-1)/\ln C\right]$$

Mortality Models

- Relational models
  - The Coale-Kisker model (Coale and Kisker, 1990)

$$m_x = m_{x-1} \cdot \exp\left[k_{K,85} + (x-85) \cdot s \right], \text{ for } x \geq 85$$

$$\min_{\alpha_K, s_K} \sum_x w_x \left[\ln m_x - \alpha_K - k_{K,85}(x-84) + \frac{(x-84)(x-85)}{2} s_K \right]^2$$

Mortality Models

- Relational models
  - The Logistic model

$$\kappa_x = -\ln p_x = \frac{\exp\left[\beta_{L,0} + \beta_{L,1} \cdot (x + 0.5)\right]}{1 + \exp\left[\beta_{L,0} + \beta_{L,1} \cdot (x + 0.5)\right]}$$