Health and Mortality Hedging: A Revisit of the Optimal Product Mix for Life Insurers

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Motivation

- Life insurers are facing mortality/longevity risk and health shocks, in addition to the interest rate risk.

- Studies on the natural hedging strategy between an insurer's life insurance and annuity business have been well developed (e.g., Cox and Lin, 2007; Wong et al. 2015). However, little research has been done on how mortality/longevity risk intertwines with health shocks and how this interplay affects a life insurer's immunization and hedging strategies.

- There exists an inherent connection between health shocks and mortality/longevity risk since the death represents the end state in an individual's health state transition process.

Motivation

- Long-term Care Insurance Market
  - In the US, 2/3 of individuals aged 65 + will need some long-term care, either at home or at a long-term care facility (Chapman, 2012).
  - According to the National Health Expenditures Survey compiled by the Centers for Medicare & Medicaid Services (CMS), the U.S. spent $239 billion on LTC expenses in 2014.
  - Thus far, Medicare and Medicaid have largely provided such services. However, given funding deficiency at both the state and federal government levels, there is an increasing need to fund individuals' long-term care costs through either out-of-pocket savings or private insurance plans.
  - After two decades of rapid growth in the mid- to late 1980s and 1990s, the LTC insurance industry in the U.S. has experienced significant contraction, both in terms of sales as well as insurers participating in the market.

Motivation

- It is meaningful to examine how an insurer constructs an hedging/immunization strategy in the context of health and mortality risk.
  - In the U.S., one of the major reasons for most of insurers exiting the market is the high capital requirement to support the LTC insurance products.
  - Most insurers' LTCI policies issued before the mid-2000s have seen adverse experience when compared to their original pricing assumptions.
  - If insurers can hedge mortality risk and health shocks across different lines of business, the capital requirement can be reduced and insurers can provide more affordable LTC insurance products.

Literature

- Cox and Lin (2007) suggest mortality risk in the life insurance business can be partially offset by longevity risk in the annuity business and such a natural hedging strategy can help life insurers manage mortality/longevity risk and remain competitive.

- Tsai et al. (2010) introduce a conditional value-at-risk minimization method to determine an optimal mix of life insurance and annuity products.

- Wang et al. (2010) and Plat (2011) study mortality immunization strategies based on the effective mortality duration by assuming a uniformly proportional change in the force of mortality and one-year death probability, respectively.

Literature

- Tsai and Chung (2013), Lin and Tsai (2013, 2014) define mortality durations and/or mortality convexities under the linear hazard transform and then explore their applications to mortality immunization.

- Luciano et al. (2012) and Liu and Sherris (2015) develop delta-gamma hedging allowing for both stochastic interest rates and stochastic mortality rates.

- Koijen et al. (2016) is the first to define health and mortality delta of life and health insurance products. They find that households choose an optimal portfolio of insurance products to replicate the optimal health and mortality delta predicted by the life-cycle model.

Health Transition Model

- Three health states: depends on the No. of ADLs

$$h_t = \begin{cases} 1, & \text{healthy (at most 1 ADL and no cognitive impairment)} \\ 2, & \text{disabled (2-6 ADLs or cognitive impairment)} \\ 3, & \text{death} \end{cases}$$

- The individual's health evolve from period t to t+1 according to a Markov chain with a transition matrix, where the (i, j)-th element is

$$p_{x,t}^{ij} = \Pr\left(h_{x,t+1} = j \mid h_{x,t} = i\right)$$

Health Transition Model

- We follow Fong et al. (2015) and Shao et al. (2015) to estimate Customers' health transition matrix at age n, $H_E(n)$ (n=45, …, 100):
  - Assume that health dynamics follows a time-continuous, inhomogeneous Markov process with age-dependent transition intensities.
  - Calculate the empirical age-dependent transition intensities across the three health states using the HRS 1998-2010 data. We adopt a Poisson Generalized Linear Model (GLM) to graduate these empirical transition intensities and obtain the graduated age-dependent transition intensities from age 45 to 100.
  - Derive the three-state age-dependent transition probabilities via Kolmogorov differential equations from the corresponding graduated transition intensities.

Calibration of Health Dynamics

- Adjusting $H_E(n)$ to match 2006 HMD life-table mortalities via auxiliary matrix $M_1(n)$, such that the fourth element of $\alpha\Pi_{n=0}^{m}[H_E(n)M_0(n)]$ matches the respective HMD mortalities and the projected mortalities

$$M_1 = \begin{bmatrix} 1+d_1 & 0 & -d_1 \\ 0 & 1+d_1 & -d_1 \\ 0 & 0 & 1 \end{bmatrix}$$

CBD Mortality Model

- We use a two-factor mortality model (Carins et al. 2006) to fit the mortality curve

$$q\left(t,x\right)=\frac{e^{A_1(t+1)+A_2(t+1)(x+t)}}{1+e^{A_1(t+1)+A_2(t+1)(x+t)}}$$

- To forecast the future distribution of $A(t)=\left(A_1(t),A_2(t)\right)'$, we assume

$$A(t+1)=A(t)+\mu+CZ\left(t+1\right)$$

Life Insurance

- Let $\mathbf{I}_t(j)$ denote an indicator function that is equal to one if the insured is in health state *j* in period *t*. Suppose a term life insurance of maturity *n* is issued in period *t*. In any period *s*, it will pay out a death benefit (normalized to one) of $D_{L,t+s}(n-s \mid h_{t+s}) = \mathbf{I}_{t+s}(1)$ per unit upon the death of the insured.
- The pricing of life insurance depends on the insured's age and health at issuance of the policy. Conditional on being in health state in period *t*, the price of *n*-period life insurance per unit of death benefit is

$$P_{L,t}(n \mid h_t) = \sum_{s=1}^{n} \frac{p_t(s \mid h_t)}{R_L^s}$$

where $R_L \le R$ is the discount rate. The pricing of life insurance is actuarially fair when $R_L = R$, while $R_L < R$ implies a markup.

Annuity

- Suppose there is a deferred annuity of maturity *n* issued in period *t.* It will pay out a constant income (normalized to one) of

$$D_{A,t+s}(n-s\mid h_{t+s})=\begin{cases}0, & \text{if } s<n\\ 1-\mathbf{I}_{t+s}(1), & \text{if } s\geq n\end{cases}$$

per unit in each period $s\in\{1,\ldots,T-t\}$ that the insured is alive, where *T* denotes the maximum attainable age.

- Conditional on being in health state *h* in period t, the price of an n-period annuity per unit of income is ,

$$P_{A,t}(n\mid h_t)=\sum_{s=n}^{T-t}\frac{q_t(s\mid h_t)}{R_A^s}$$

where $R_A \leq R$ is the discount rate.

Long-term care insurance

- Denote $M_t(j)$ the out-of-pocket long-term care expenses in period *t*, depending on the realization of health state *j*. Then the long-term care insurance of maturity *n* issued in period *t* covers

$$D_{H,t+s}(n-s \mid h_{t+s}) = \mathbf{I}_{t+s}(2)\left(M_{t+s}(2) - M_{t+s}(3)\right)$$

per unit in each period that the insured is in poor health.

- Conditional on being in health state *h* in period t, the price of n-period long-term care insurance per unit of coverage is

$$P_{H,t}(n \mid h_t) = \sum_{s=1}^{n} \frac{\pi_t^s(h_t, 2)\left(M_{t+s}(2) - M_{t+s}(3)\right)}{R_H^s}$$

where $R_H \le R$ is the discount rate.

Hedging Strategy

- We minimize the CVaR of the insurance portfolio to decide the optimal product mix

$$\underset{w^i}{\text{Min}} \quad E\left[r_p \,\middle|\, r_p \geq r_p(\alpha)\right]$$

$$s.t. \quad \sum_i w^i \cdot \pi^i \geq \bar{\pi},$$

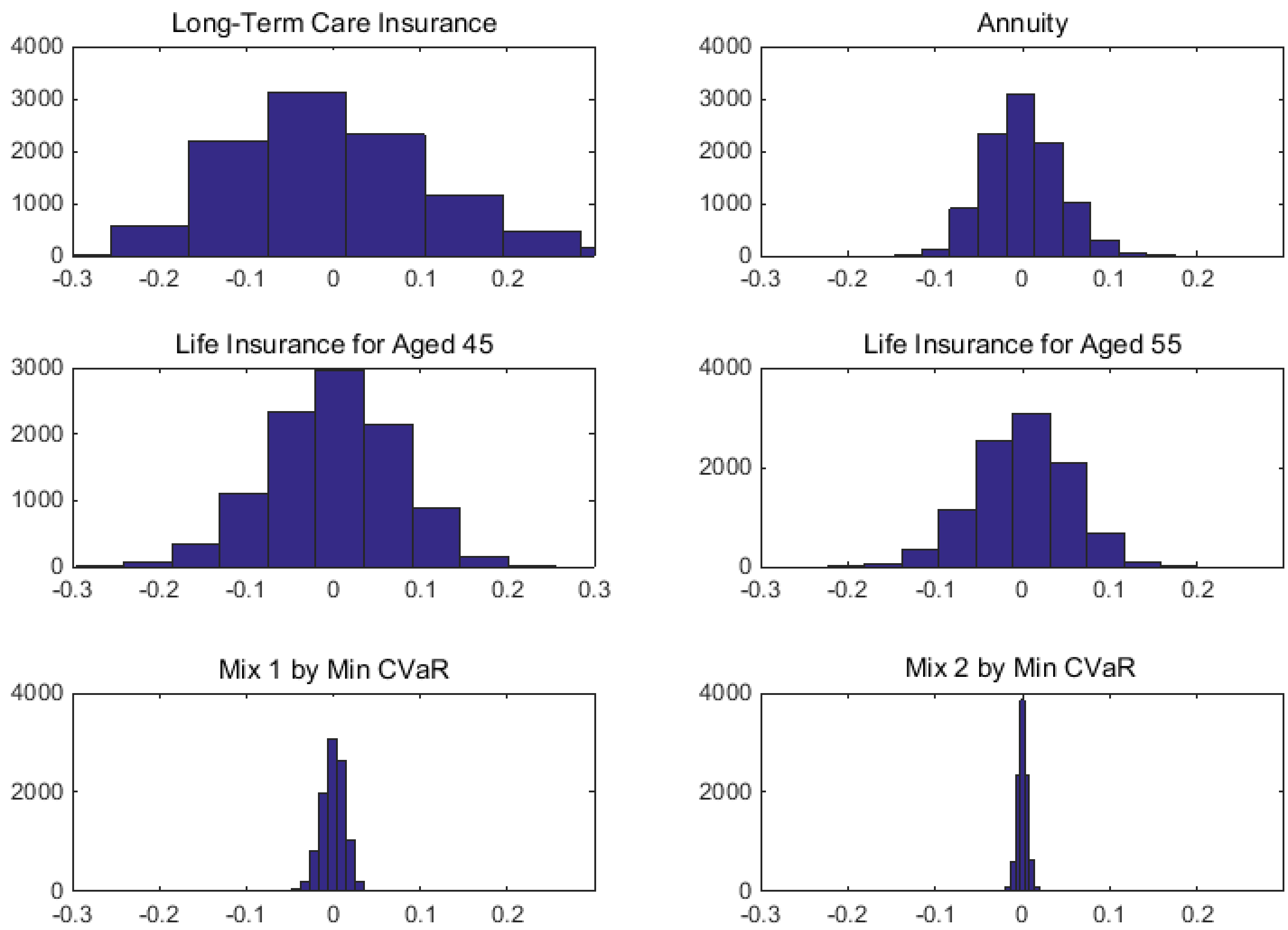
$$\sum_i w = 1, \quad \text{and} \quad 0 \leq w^i \leq 1$$

- We define the loss percentage of the portfolio $r_p = \sum_i w^i r^i$
- w^i is the weight for product *I*
- $r^i = \dfrac{v^i - E\left(v^i\right)}{E\left(v^i\right)}$ is the loss proportion for product *i*
- π^i is the profit loading for product *i* calculated from the Sharpe-Ratio

Basic Assumptions

	Whole-life LTC insurance	Whole-life annuity	Whole-life insurance
Age/Gender	65/men	65/men	45 or 55/men
Payout benefits	$10 per year	$1 per year	$100
Premium type	single	single	single
Interest rate	3%	3%	3%
Deferred period	Immediately	Immediately	Immediately
Health-Mortality Dynamics	Health transition matrix	CBD Model, using 1933-2010 data	CBD Model, using 1933-2010 data
Premiums	$9.3390	$11.5802	$30.56/$40.45

Loss Distributions of Different Products



Loss Distribution for Each Product and CVaR

Model	α=0.99	α=0.95	α=0.90	Mean	Std.	loading
Whole-life LTC	0.3774	0.2800	0.2321	0.0000	0.1182	0.0177
Whole-life annuity	0.1192	0.0904	0.0760	-0.0000	0.0411	0.0062
Whole-life insurance 1 for aged 45	0.1790	0.1420	0.1227	0.0000	0.0728	0.0109
Whole-life insurance 2 for aged 55	0.1305	0.1031	0.0891	0.0000	0.0533	0.0080
The CVaRM 1	0.0339	0.0264	0.0226	0.0000	0.0134	0.0020
The CVaRM 2	0.0171	0.0123	0.0102	0.0000	0.0057	0.0009

Optimal Product Mix

	Product Mix 1, including Whole-life LTC, annuity and life insurance 1 for aged 45			Product Mix 2, including Whole-life LTC, annuity and life insurance 2 for aged 55		
Model	α=0.99	α=0.95	α=0.90	α=0.99	α=0.95	α=0.90
Whole-life LTC	0.138	0.139	0.141	0.232	0.232	0.233
Whole-life annuity	0.389	0.391	0.394	0.134	0.136	0.137
Whole-life insurance 1 for aged 45	0.473	0.470	0.465	---	---	---
Whole-life insurance 2 for aged 55	---	---	---	0.634	0.632	0.630

Conclusions

- We model health transition process and relate health shocks to mortality risk which can help life insurers achieve better hedging effectiveness.

- We calculate an optimal product mix for insurance companies to hedge against mortality and health risk.

- A better hedging/immunization strategy can reduce insurers' cost and the high capital requirement for LTC insurance and thus advance the development of private LTC insurance market.

Future work

- Compare optimal product mix derived from two scenarios, i.e., considering mortality risk only and considering mortality and health risk at the same time, for the portfolio consisting of life insurance and annuity.

- Set up delta-gamma-hedging strategies.

- Optimal asset allocation between multiple assets.

Questions? Comments?

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