On the Valuation of Reverse Mortgages with Surrender Options

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Outline

Reverse mortgage

- The aging population structure and increases in longevity have caused steady retirement income declines from both the public and private pensions.
- To maintain a sustainable replacement ratio, many private and capital market solutions have been proposed.
- Reverse mortgage (RM): one of such longevity risk transfer solutions, which provides seniors access to their home equity without a home sale or monthly mortgage payments until closing.

Non-recourse clause

- Reverse mortgages are sold with a non-recourse clause to protect the borrower from owing more than the proceeds of the collateralized property.
- Lenders of RM can hedge this crossover risk by participating in the Home Equity Conversion Mortgage (HECM) program in US.
- Most RM contracts in the US are under the HECM program.

Pricing and risk analysis

- Weinrobe (1988), Boehm and Ehrhardt (1994), Case and Schnare (1994), Szymanoski (1994).
- Contingent claim framework: Chen et al. (2010), Li et al. (2010), Lee et al.(2012), Wang et al. (2016).
- Securitization of crossover risk: Wang et al. (2008), Huang et al. (2011), Yang (2011).
- Profitability and risk profile: Alai et al. (2014), Cho et al. (2013), Lee and Lo (2016).

Mortgage prepayment

- Borrowers can repay the RM loan early, which could significantly affect the cost and risk profile of a reverse mortgage contract.
  - In a sluggish housing market, a RM borrower would rarely terminate the contract because of the nonrecourse clause.
  - However, the motivation of early repayment could be significantly strengthened when the housing price appreciates.
- Average annual HECM prepayment index has been steadily increasing from 4.12% in Jan 2011 to 16.61% as of Mar 2017 (including assignment to FHA).
- Market share for HECM Refinance loans hovered between 2.3%-8.5% in FY 2005-2011.

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Objective and methodology

- **Objective**: In this project, we aim to fill the gap by exploring the impact of the surrender behaviors on the cost of RM insurance.
  - Prior studies: typically consider the termination by exogenous decrements, i.e., cease of the borrower's life.
  - In our settings: the termination of a RM loan is based on two factors, the surrender and the mortality.
- **Methodology**: Following Milevsky (2001) and Gao and Ulm (2012), we propose a multi-period rational choice model based on a constant relative risk aversion utility function to analyze the early repayment.

Literature review

For traditional life insurance products and variable annuities,

- Empirical drivers of lapse rate:
  - level of interest rate (Kuo et al., 2003)
  - emergency fund hypothesis (Outreville, 1990)
  - product and policyholder characteristics (Eling and Kiesenbauer, 2014; Knoller et al., 2016)
  - macroeconomic variables and company specific determinants (Kim, 2005; Kiesenbauer, 2012)
- Contingent claim framework: Bacinello (2003), Bernard et al. (2014).
- Affine intensity-based framework: Russo et al. (2017).

Reverse mortgage contract

- Consider a lump-sum reverse mortgage with a constant interest rate.
- Maximum insured amount is assumed to equal to the housing value $H(0)$ for simplicity.
- The accrued outstanding balance at t, $BAL(t)$:

$$BAL(t) = (\pi_0 H(0) + BAL(0))\,(1 + \pi_m)^{t-1} e^{(r+\pi_r)t}, \quad t = 1, 2, \ldots$$

  - π_0: upfront premium rate
  - π_m: annual ongoing premium rate
  - r: risk-free rate
  - π_r: mortgage spread

House price process

- House price process follows a geometric Brownian motion under the physical measure $\mathbb{P}$:

$$\frac{dH(t)}{dt} = (\mu_H - \delta)\, dt + \sigma_H dW^{\mathbb{P}}(t)$$

  - δ is the rental rate
  - σ_H denotes the volatility
  - $W^{\mathbb{P}}(t)$ is a standard Brownian motion under $\mathbb{P}$.

- Under the risk-neutral measure $\mathbb{Q}$

$$\frac{dH(t)}{dt} = (r - \delta)\, dt + \sigma_H dW^{\mathbb{Q}}(t)$$

  - $W^{\mathbb{Q}}(t)$ is a standard Brownian motion under $\mathbb{Q}$.

CRRA utility function

- We assume that surrender behaviors follow the intertemporal utility function with a constant relative risk aversion (CRRA) utility:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1, \\ \ln(c), & \gamma = 1, \end{cases}$$

  - $1/\gamma$: intertemporal substitution elasticity between consumption in two different periods

- For a lump-sum reverse mortgage, the lump-sum borrowing amount is converted to annuity payments when considering intertemporal utility.

Total utility with RM payments

- Given a retirement income of p per period, the intertemporal utility of entering a RM contract is

$$U_R(0) = \sum_{t=0}^{\omega-x} \beta^t \, {}_t p_x \cdot u\left(p + c_t\right) + \sum_{t=0}^{\omega-x} \zeta \, \beta^{t+1} \, {}_t p_x \, q_{x+t} \cdot u\left(\left(H(t+1) - BAL(t+1)\right)^+\right)$$

  - c_t: includes the RM tenure payment $BAL(0)/[(1+L)\,\ddot{a}_x]$ (L is loading) and the rental income.
  - β: subjective discount factor.
  - ζ $(0 \le \zeta \le 1)$: relative bequest motive.

- At the end of any period t, the borrower may keep the contract with utility

$$\begin{aligned} U_R(t) &= \sum_{s=0}^{\omega-x-t} \zeta \, \beta^{s+1} \, {}_s p_{x+t} \, q_{x+t+s} \cdot u\left(\left(H(t+s+1) - BAL(t+s+1)\right)^+\right) \\ &+ \sum_{s=0}^{\omega-x-t} \beta^s \, {}_s p_{x+t} \cdot u\left(p + c_{t+s}\right) \end{aligned}$$

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Total utility after surrendering

- We assume that the borrower has to refinance in order to pay off the outstanding balance.

$$BAL(t) < \mathrm{PLF}_{x+t} \cdot H(t)$$

where PLF_{x+t}: the principal limit factor at age $x+t$.

- At t, the borrower may surrender with revised utility

$$\begin{aligned} U_S(t) &= \sum_{s=0}^{\omega-x-t} \zeta\, \beta^{s+1} {}_s p_{x+t}\, q_{x+t+s} \cdot u\left(\left(H(t+s+1) - BAL'(t+s+1)\right)^+\right) \\ &+ \sum_{s=0}^{\omega-x-t} \beta^s {}_s p_{x+t} \cdot u\left(p + c'_{t+s}\right) \end{aligned}$$

where $c'_{t+s} = c_{t+s} + \frac{H(t)\cdot(\mathrm{PLF}_{x+t} - \pi_{or}) - BAL(t)}{(1+L)\,\ddot{a}_{x+t}}$: revised cash flows at $t+s$.

Optimal surrender time

- Based on the CRRA utility, the borrower may surrender at t if

$$\mathbb{E}[U_S(t)\,|H(t)] > \mathbb{E}[U_R(t)\,|H(t)]$$

- The borrower will receive optimal utility with surrender time

$$\tau_S = \inf\left\{t : \mathbb{E}[U_S(t)\,|H(t)] > \mathbb{E}\left[U_R(t)\,|H(t)\right] + \max\left(0, \Delta_t\right)\right\}$$

where

$$\Delta_t = \max_{s \geq 1}\left\{\mathbb{E}\left[\beta^s\, {}_s p_{x+t}(U_S(t+s) - U_R(t+s))\,|H(t)\right]\right\}$$

Parameters

- House price process
  - risk-free rate r: 2.5%
  - rental rate δ: 2%
  - growth rate of housing price $\mu_H - \delta$: 3.43%
  - volatility of housing price σ_H: 10%
- Reverse mortgage
  - mortgage spread π_r: 2%
  - upfront premium rate π_0: 2.5%
  - annual ongoing premium rate π_m: 1.25%
  - origination fee for refinance π_{or}:1.5%
- CRRA utility
  - subjective annual discount factor β: 0.97
  - risk aversion parameter γ: 0.5
  - relative bequest motive ζ: 0.5

Results

- Borrower's characteristics
  - We use U.S. male population mortality data from 1970-2015 to fit the Lee-Carter model(1992).
  - We assume $p = 0$ for simplicity.
- Numerical methods
  - Hull and White's binomial tree (1994, 1996) with monthly time steps.
  - Borrower's surrender decision under $\mathbb{P}$ measure.
  - Fair loan-to-value ratio (PLF) under $\mathbb{Q}$ measure.
- Outcome
  - For a borrower aged 70, its fair PLF is 37.09% (as property value) with surrender option, which is 0.53% lower than the PLF without surrender option.

Premiums comparison

Table 1: Premium Reductions and Underpricing ($\sigma_H = 10\%$)

Age	PLF_s	$PLF_{ns} - PLF_s$	Premium Reduction	Underpricing %
70	0.3709	0.53%	5.46%	2.40%
75	0.4546	0.58%	5.89%	2.52%
80	0.5451	0.59%	6.41%	2.64%
85	0.6380	0.53%	6.72%	2.54%
90	0.7280	0.44%	6.64%	2.40%

- Premium Reduction: premium income decrease from the no surrender option case.
- Underpricing: premium deficit as percentage of the expected insurance costs, if PLF_{ns} is used but surrender is allowed.

Impact of σ_H

Table 2: Premium Reductions and Underpricing ($\sigma_H = 7.5\%$)

Age	PLF_s	$PLF_{ns} - PLF_s$	Premium Reduction	Underpricing %
70	0.4011	0.24%	2.76%	1.16%
75	0.4875	0.26%	3.11%	1.24%
80	0.5795	0.28%	3.58%	1.35%
85	0.6724	0.26%	3.93%	1.36%
90	0.7606	0.21%	3.93%	1.33%

Conclusion

- We analyzed the cost and risk profile of a reverse mortgage contract in the presence of surrender.
- A CRRA utility based choice model is used to characterize borrower's surrender behaviors.
- Numerical evidences are provided to show the importance of surrender option in RM pricing.