On the Valuation of Reverse Mortgages with Surrender Options

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Introduction

Pricing Model with Surrender Option

Numerical Illustration

Outline

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2. Pricing Model with Surrender Option
   - Basic framework
   - When to surrender

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Reverse mortgage

- The aging population structure and increases in longevity have caused steady retirement income declines from both the public and private pensions.

- To maintain a sustainable replacement ratio, many private and capital market solutions have been proposed.

- Reverse mortgage (RM): one of such longevity risk transfer solutions, which provides seniors access to their home equity without a home sale or monthly mortgage payments until closing.
Non-recourse clause

- Reverse mortgages are sold with a non-recourse clause to protect the borrower from owing more than the proceeds of the collateralized property.

- Lenders of RM can hedge this crossover risk by participating in the Home Equity Conversion Mortgage (HECM) program in US.

- Most RM contracts in the US are under the HECM program.
Pricing and risk analysis


- Contingent claim framework: Chen et al. (2010), Li et al. (2010), Lee et al. (2012), Wang et al. (2016).


Mortgage prepayment

- Borrowers can repay the RM loan early, which could significantly affect the cost and risk profile of a reverse mortgage contract.
  - In a sluggish housing market, a RM borrower would rarely terminate the contract because of the nonrecourse clause.
  - However, the motivation of early repayment could be significantly strengthened when the housing price appreciates.
- Average annual HECM prepayment index has been steadily increasing from 4.12% in Jan 2011 to 16.61% as of Mar 2017 (including assignment to FHA).
- Market share for HECM Refinance loans hovered between 2.3%-8.5% in FY 2005-2011.
Objective and methodology

- **Objective**: In this project, we aim to fill the gap by exploring the impact of the surrender behaviors on the cost of RM insurance.
  
  - Prior studies: typically consider the termination by exogenous decrements, i.e., cease of the borrower’s life.
  
  - In our settings: the termination of a RM loan is based on two factors, the surrender and the mortality.

- **Methodology**: Following Milevsky (2001) and Gao and Ulm (2012), we propose a multi-period rational choice model based on a constant relative risk aversion utility function to analyze the early repayment.
Literature review

For traditional life insurance products and variable annuities,

- **Empirical drivers of lapse rate:**
  - level of interest rate (Kuo et al., 2003)
  - emergency fund hypothesis (Outreville, 1990)
  - product and policyholder characteristics (Eling and Kiesenbauer, 2014; Knoller et al., 2016)
  - macroeconomic variables and company specific determinants (Kim, 2005; Kiesenbauer, 2012)

- **Contingent claim framework:** Bacinello (2003), Bernard et al. (2014).

- **Affine intensity-based framework:** Russo et al. (2017).
Reverse mortgage contract

- Consider a lump-sum reverse mortgage with a constant interest rate.

- Maximum insured amount is assumed to equal to the housing value $H(0)$ for simplicity.

- The accrued outstanding balance at $t$, $BAL(t)$:

$$BAL(t) = (\pi_0 H(0) + BAL(0))(1 + \pi_m)^{t-1}e^{(r+\pi_r)t}, \quad t = 1, 2, ...$$

- $\pi_0$: upfront premium rate
- $\pi_m$: annual ongoing premium rate
- $r$: risk-free rate
- $\pi_r$: mortgage spread
House price process

- House price process follows a geometric Brownian motion under the physical measure \( \mathbb{P} \):
  \[
  \frac{dH(t)}{dt} = (\mu_H - \delta)\,dt + \sigma_H\,dW^\mathbb{P}(t)
  \]

- \( \delta \) is the rental rate
- \( \sigma_H \) denotes the volatility
- \( W^\mathbb{P}(t) \) is a standard Brownian motion under \( \mathbb{P} \).

- Under the risk-neutral measure \( \mathbb{Q} \)
  \[
  \frac{dH(t)}{dt} = (r - \delta)\,dt + \sigma_H\,dW^\mathbb{Q}(t)
  \]

- \( W^\mathbb{Q}(t) \) is a standard Brownian motion under \( \mathbb{Q} \).
We assume that surrender behaviors follow the intertemporal utility function with a constant relative risk aversion (CRRA) utility:

\[
u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1, \\ \ln(c), & \gamma = 1, \end{cases}
\]

- $1/\gamma$: intertemporal substitution elasticity between consumption in two different periods

For a lump-sum reverse mortgage, the lump-sum borrowing amount is converted to annuity payments when considering intertemporal utility.
Total utility with RM payments

- Given a retirement income of $p$ per period, the intertemporal utility of entering a RM contract is

$$U_R(0) = \sum_{t=0}^{\omega-x} \beta^t p_x \cdot u(p + c_t) + \sum_{t=0}^{\omega-x} \zeta \beta^{t+1} p_x q_{x+t} \cdot u((H(t+1) - BAL(t+1))^+)$$

- $c_t$: includes the RM tenure payment $BAL(0)/[(1 + L) \bar{a}_x]$ ($L$ is loading) and the rental income.
- $\beta$: subjective discount factor.
- $\zeta (0 \leq \zeta \leq 1)$: relative bequest motive.

- At the end of any period $t$, the borrower may keep the contract with utility

$$U_R(t) = \sum_{s=0}^{\omega-x-t} \zeta \beta^{s+1} p_x q_{x+t+s} \cdot u((H(t + s + 1) - BAL(t + s + 1))^+)$$

$$+ \sum_{s=0}^{\omega-x-t} \beta^s p_x q_{x+t+s} \cdot u(p + c_{t+s})$$
We assume that the borrower has to refinance in order to pay off the outstanding balance.

\[ BAL(t) < PLF_{x+t} \cdot H(t) \]

where \( PLF_{x+t} \): the principal limit factor at age \( x + t \).

At \( t \), the borrower may surrender with revised utility

\[
U_S(t) = \sum_{s=0}^{\omega-x-t} \zeta \beta^{s+1} s p_{x+t} q_{x+t+s} \cdot u \left( (H(t + s + 1) - BAL'(t + s + 1))^+ \right) \\
+ \sum_{s=0}^{\omega-x-t} \beta^s s p_{x+t} \cdot u \left( p + c'_{t+s} \right)
\]

where \( c'_{t+s} = c_{t+s} + \frac{H(t) \cdot (PLF_{x+t} - \pi_{or}) - BAL(t)}{(1+L) \bar{a}_{x+t}} \): revised cash flows at \( t + s \).
Optimal surrender time

- Based on the CRRA utility, the borrower may surrender at \( t \) if

\[
\mathbb{E}[U_S(t) | H(t)] > \mathbb{E}[U_R(t) | H(t)]
\]

- The borrower will receive optimal utility with surrender time

\[
\tau_S = \inf \{ t : \mathbb{E}[U_S(t) | H(t)] > \mathbb{E}[U_R(t) | H(t)] + \max(0, \Delta t) \}
\]

where

\[
\Delta t = \max_{s \geq 1} \{ \mathbb{E} [\beta^s s p_{x+t} (U_S(t + s) - U_R(t + s)) | H(t)] \}
\]
Parameters

- **House price process**
  - risk-free rate $r$: 2.5%
  - rental rate $\delta$: 2%
  - growth rate of housing price $\mu_H - \delta$: 3.43%
  - volatility of housing price $\sigma_H$: 10%

- **Reverse mortgage**
  - mortgage spread $\pi_r$: 2%
  - upfront premium rate $\pi_0$: 2.5%
  - annual ongoing premium rate $\pi_m$: 1.25%
  - origination fee for refinance $\pi_{or}$: 1.5%

- **CRRA utility**
  - subjective annual discount factor $\beta$: 0.97
  - risk aversion parameter $\gamma$: 0.5
  - relative bequest motive $\zeta$: 0.5
Results

- **Borrower’s characteristics**
  - We assume $p = 0$ for simplicity.

- **Numerical methods**
  - Borrower’s surrender decision under $\mathbb{P}$ measure.
  - Fair loan-to-value ratio (PLF) under $\mathbb{Q}$ measure.

- **Outcome**
  - For a borrower aged 70, its fair PLF is 37.09% (as property value) with surrender option, which is 0.53% lower than the PLF without surrender option.
Premiums comparison

**Table 1:** Premium Reductions and Underpricing ($\sigma_H = 10\%$)

<table>
<thead>
<tr>
<th>Age</th>
<th>$PLF_s$</th>
<th>$PLF_{ns} - PLF_s$</th>
<th>Premium Reduction</th>
<th>Underpricing %</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.3709</td>
<td>0.53%</td>
<td>5.46%</td>
<td>2.40%</td>
</tr>
<tr>
<td>75</td>
<td>0.4546</td>
<td>0.58%</td>
<td>5.89%</td>
<td>2.52%</td>
</tr>
<tr>
<td>80</td>
<td>0.5451</td>
<td>0.59%</td>
<td>6.41%</td>
<td>2.64%</td>
</tr>
<tr>
<td>85</td>
<td>0.6380</td>
<td>0.53%</td>
<td>6.72%</td>
<td>2.54%</td>
</tr>
<tr>
<td>90</td>
<td>0.7280</td>
<td>0.44%</td>
<td>6.64%</td>
<td>2.40%</td>
</tr>
</tbody>
</table>

- **Premium Reduction:** premium income decrease from the no surrender option case.
- **Underpricing:** premium deficit as percentage of the expected insurance costs, if $PLF_{ns}$ is used but surrender is allowed.
Impact of $\sigma_H$

Table 2: Premium Reductions and Underpricing ($\sigma_H = 7.5\%$)

<table>
<thead>
<tr>
<th>Age</th>
<th>$PLF_s$</th>
<th>$PLF_{ns} - PLF_s$</th>
<th>Premium Reduction</th>
<th>Underpricing %</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.4011</td>
<td>0.24%</td>
<td>2.76%</td>
<td>1.16%</td>
</tr>
<tr>
<td>75</td>
<td>0.4875</td>
<td>0.26%</td>
<td>3.11%</td>
<td>1.24%</td>
</tr>
<tr>
<td>80</td>
<td>0.5795</td>
<td>0.28%</td>
<td>3.58%</td>
<td>1.35%</td>
</tr>
<tr>
<td>85</td>
<td>0.6724</td>
<td>0.26%</td>
<td>3.93%</td>
<td>1.36%</td>
</tr>
<tr>
<td>90</td>
<td>0.7606</td>
<td>0.21%</td>
<td>3.93%</td>
<td>1.33%</td>
</tr>
</tbody>
</table>
Conclusion

- We analyzed the cost and risk profile of a reverse mortgage contract in the presence of surrender.

- A CRRA utility based choice model is used to characterize borrower’s surrender behaviors.

- Numerical evidences are provided to show the importance of surrender option in RM pricing.