Best Practice Life Expectancy: An Extreme Value Approach

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Some Facts

- Best Practice Life Expectancy (BPLE) is the maximum life expectancy observed among nations at a given age.

- At birth, has been increasing almost linearly - beginning in Scandinavia c. 1840 - at about 3 months per year (Oeppen and Vaupel, 2002).

- Life expectancy trends may fit better than individual-country trends in age-standardized (log) death rates (White, 2002).
Some Facts

- Nations experience more rapid life expectancy gains when they are farther below BPLE and tend to converge towards BPLE (Torri and Vaupel, 2012).

- It is sensible to consider national mortality trends in a larger international context rather than individual projections (Lee, 2006; Wilmoth, 1998).
Females $e_0$

Female Best Practice $e_0$

- Iceland
- Japan
- Norway
- NZ (non-maori)
- Sweden
Male Best Practice $e_0$

- Australia
- NZ (non-MAori)
- Denmark
- Norway
- Iceland
- Sweden
- Japan
- Switzerland
- Netherlands
Male Best Practice $e_{65}$

- Australia
- Denmark
- Iceland
- Japan
- Norway
- Switzerland
Breakpoints

Figure: Breakpoints in the trend of the highest life expectancies at birth and age 65, males and females separately, from 1900 - 2012.
Empirical motivation

**Figure:** Left panel: raw and detrended data. Right panel: kernel density and fitted GEV distribution.
Theoretical motivation

Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independent, identically distributed random variates all having a common distribution function $F(x)$.

Let $M_n = \max\{X_1, X_2, \ldots, X_n\}$.

The distribution of the maxima, $M_n$, converges (for large $n$) to the Generalized Extreme Value (GEV) Distribution.
The Generalized Extreme Value Distribution

\[ G(z) = \exp \left\{ - \left[ 1 + \xi \left( \frac{z - u}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} \]

- \( u \) is the location parameter
- \( \sigma \) is the scale parameter
- \( \xi \) is the shape parameter, which determines the tail behaviour
  - \( \xi > 0 \): polynomial tail decay and the Fréchet Distribution
  - \( \xi = 0 \): exponential tail decay and the Gumbel Distribution
  - \( \xi < 0 \): bounded upper finite end point and the Weibull Distribution
Quantiles
Inverting the GEV distribution function:

\[ z_p = \mu - \frac{\sigma}{\xi} \left[ 1 - \left\{ -\log(1 - p) \right\}^{-\xi} \right], \]

where \( p \) is the tail probability and \( G(z_p) = 1 - p \)

Return Levels
- Simply a different way of thinking about the quantiles.
- If data are annual the \((1 - p)\)th quantile would be exceeded on average once every \(1/p\) years.
Fitted Model

\[ GEV(u_t = 59.6 + 0.24t, \quad \sigma = 1.31, \quad \xi = -0.48) \]
Projections, Females $e_0$
Projections, Females $e_0$
A probability distribution has been fit so the usual tools are available.

<table>
<thead>
<tr>
<th>Year</th>
<th>$P(e_{0}^{\text{max}} &gt; 90)$</th>
<th>$P(e_{0}^{\text{max}} &gt; 95)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020</td>
<td>35%</td>
<td>&lt; 0.001%</td>
</tr>
<tr>
<td>2050</td>
<td>&gt; 99.99%</td>
<td>91%</td>
</tr>
</tbody>
</table>
Fit model using data up to 1980.
Compare Observed 10 Year Maxima vs 10 Year return Levels.

Mean Absolute Difference (MAD) = 0.67 years
Mean Absolute Percentage Error = 0.8%
Figure: Normality tests for residuals of ARIMA(2,1,1) fitted to female $e_0$ BPLE. Left panel: QQ Plot; Right panel: histogram.
Innovations Process

- Assumption of Gaussian errors is often arbitrary and can be poorly fitting.
- GEV is more flexible and is able to capture the shape of different error distributions - not just symmetric.
- In practice Gaussian often provides a reasonable fit but GEV should be considered as an alternative for the innovations process.
Method can be used similarly to the Torri and Vaupel (2012) approach to forecasting life expectancy:

- Either through projecting BPLE directly, which is preferable
- Or using the GEV as the innovations process in an ARIMA model

EVT can identify in an objective way whether life expectancy is actually at an extreme level rather than just "high"

EVT can be used to obtain probabilities and/or levels of extreme longevity


