Optimal Risk-Taking and Risk Management Decisions of Annuity Insurers under Cumulative Prospect Theory

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Motivations

- DB pensions introduce significant risks (Lin et al., 2015; Cox et al., 2017)
  - Financial market fluctuations
  - Low interest rates
  - New pension accounting standards
  - Improved life expectancy of retirees
- Pension de-risking by DB plans through purchasing buy-in and buy-out annuities from insurers
- Insurers operating in the annuity markets have reported a significant decline in statutory earnings and credit rates.
- In reaction to this, the insurers increase their risk-seeking activities.

Motivations (Cont')

- How an annuity insurer with changing risk preferences should make decisions remains poorly understood.
- Most existing studies view the reference point as exogenous and ignore its determinants.
- There is a lack in our understanding on how risk management changes an insurer's risk preferences and strategies.

Contributions

- We add to the pension de-risking literature by studying an annuity insurer’s optimal risk-taking decisions in the CPT framework.
- We propose a way to endogenously determine a reference point that serves to distinguish different risk preferences of a bulk annuity insurer.
- We study how risk management changes an insurer’s risk preferences and affects the optimal decisions on its annuity business and asset allocation.

Basic Framework

Mortality model

- We apply the Lee and Carter (1992)'s model to describe the mortality dynamics of an annuity insurer:

$$\ln q_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

Annuity contracts

- Suppose an insurer sells buy-out annuities that cover $N_0(x_0)$ male retirees aged x_0 at time 0 in a pension plan.
- The obligations of the bulk annuity insurer to the surviving retired cohort at time *t*:

$$L(t) = N_0(x_0+t) \cdot B \cdot a_{x_0+t} \qquad t = 1, 2, \cdots ,$$

where $a_x = a_{x_0+t} = \sum_{k=1}^{\infty} v_p{}^k \cdot {}_k\, \overline{p}_{x,t}$

Basic Framework (Cont')

Asset allocation

- The bulk annuity insurer invests its funds in *n* asset indices at time 0.
- The dynamic process of asset index *i*, $S_i(t)$, at time *t*, is a geometric Brownian motion:

$$dS_i(t) = S_i(t)[\mu_i(t)dt + \sigma_i(t)dW_i(t)]$$

- The Brownian motions of these asset indices are correlated with covariances equal to

$$\mathrm{Cov}(W_i(t), W_j(t)) = \rho_{ij}\sigma_i\sigma_j, \quad i = 1, 2, \cdots, n; j = 1, 2, \cdots, n,$$

Basic Framework (Cont')

- **Insurance surplus**
  - The insurer's total assets at time t:

$$S(t) = \begin{cases} C_0 + N_0(x_0) \cdot B \cdot a_{x_0} \cdot (1 + l_P) & t = 0 \\ \hat{S}(t) - B \cdot N_0(x_0 + t) & t = 1, 2, \cdots \end{cases}.$$

  - The insurer's total liabilities at time t:

$$L(t) = N_0(x_0 + t) \cdot B \cdot a_{x_0+t} \quad t = 0, 1, 2, \cdots .$$

  - The insurer's total surplus at time t:

$$X(t) = S(t) - L(t)$$

$$= \begin{cases} C_0 + N_0(x_0) \cdot a_{x_0} \cdot (1 + l_P) - N_0(x_0) \cdot a_{x_0} & t = 0 \\ \hat{S}(t) - N_0(x_0 + t) - N_0(x_0 + t) \cdot a_{x_0+t} & t = 1, 2, \cdots \end{cases}.$$

CPT Decision Model

- This study applies CPT to describe a bulk annuity insurer's changing risk preferences.

- **The Reference Point**
  - The reference point based on ROA, $\bar{c}$.
  - The reference surplus level *c(t)* at time *t*:

$$c(t) = \bar{c} \cdot \bar{S}(t-1) + \bar{X}(t-1) \quad t = 1, 2, \cdots$$

- **The Value Function**
  - The two-part power function (Tversky and Kahneman, 1992):

$$v[X(t), c(t)] = \begin{cases} [X(t) - c(t)]^{\alpha} & \text{if } X(t) \geq c(t) \\ -\lambda[c(t) - X(t)]^{\beta} & \text{if } X(t) < c(t) \end{cases} .$$

CPT Decision Model (Cont')

- The cumulative weighting function (Tversky and Kahneman, 1992):

$$w^+(p(t)) = \frac{p(t)^\eta}{[p(t)^\eta + (1-p(t))^\eta]^{1/\eta}} \quad \textbf{if } X(t) \geq c(t)$$

$$w^-(p(t)) = \frac{p(t)^\tau}{[p(t)^\tau + (1-p(t))^\tau]^{1/\tau}} \quad \textbf{if } X(t) < c(t).$$

- The decision weights:

$$\pi_n{}^+(t) = w^+(p_n(t)), \pi_{-m}{}^-(t) = w^-(p_{-m}(t)),$$

$$\pi_i{}^+(t) = w^+(p_i(t) + \ldots + p_n(t)) - w^+(p_{i+1}(t) + \ldots + p_n(t)), 0 \leq i \leq n-1,$$

$$\pi_i{}^-(t) = w^-(p_{-m}(t) + \ldots + p_i(t)) - w^-(p_{-m}(t) + \ldots + p_{i-1}(t)), 1-m \leq i \leq 0,$$

Basic Optimization Problem

- The bulk annuity insurer's total value $V(t)$ at time *t*.

$$V(t) = \sum_{i=1}^{n} [x_i^+(t) - c(t)]^{\alpha} \cdot \pi_i^+(t) + \sum_{i=1-m}^{0} -\lambda [c(t) - x_i^-(t)]^{\beta} \cdot \pi_i^-(t), \quad t = 1, 2, \cdots .$$

- The discounted expected utility $V(t)$ of the insurer at time 0 over *T* periods:

$$\bar{V} = \sum_{t=1}^{T} \nu^t \cdot V(t)$$

Basic Optimization Problem (Cont')

- Our optimization problem is to solve for the optimal annuity business size $N_0(x_0)$, reference point $\bar{c}$, and weights invested in different assets $\omega = [\omega_1, \omega_2, \ldots, \omega_n]$ so as to maximize the discounted expected utility $\bar{V}$ in the CPT framework:

$$\underset{N_0(x_0),\omega,\bar{c}}{\text{Maximize}} \quad \bar{V}$$

subject to the following constraints:

Constraint 1: Risk preference constraint $\bar{V} \geq \bar{U}$

Constraint 2: Overall risk constrain $VaR_{\hat{\alpha}}[X(t)] \geq R, t = 1, 2, \cdots, T,$

Constraint 3: Budget constraint $\omega_1 + \omega_2 + \cdots + \omega_n = 1.$

Constraint 4: Range constraints $N_0(x_0) > 0. \quad 0 \leq \omega_i \leq 1, \quad i = 1, 2, \cdots, n.$

Numerical Illustration

- $\alpha = \beta = 0.88$ and $\lambda = 2.25$ for the value function.
- $\eta = 0.61$ and $\tau = 0.69$ for the weighting function.
- The insurer has an initial capital of $C_0 = 1000$.
- The insurer sells a bulk annuity to insure $N_0(65)$ participants at age 65 in a DB pension plan at time 0.
- The loading of the bulk annuity $l_P = 0.2$.
- Each annuitant will receive an annual survival benefit $B = 1$ as long as he or she survives at the end of each year.

Numerical Illustration (Cont')

- Based on the US male population mortality tables from 1933 to 2010 in the Human Mortality Database, we obtain the parameters $g = -1.46$ and $\sigma_k = 2.44$ for the Lee-Carter model.

- Suppose the annuity insurer invests its funds in $n = 3$ asset indices at time 0: the 3-month T-bill , the Merrill Lynch corporate bond index , and the S&P 500 index with the weights of $\omega = \{\omega_1, \omega_2, \omega_3\}$.

Numerical Illustration (Cont')

- Maximum Likelihood Parameter Estimates of Three Asset Indices.

Asset	Parameter	Estimate	Parameter	Estimate
3-month T-bill	μ_1	0.0348	σ_1	0.0061
Corporate Bond	μ_2	0.0715	σ_2	0.0566
S&P500 Stock	μ_3	0.0964	σ_3	0.1696

- Correlation Coefficients of the Three Asset Indices.

	3-month T-bill	Corporate Bond	S&P500 Stock	IRS Bond
3-month T-bill	1			
Corporate Bond	0.0381	1		
S&P500 Stock	0.0466	0.2534	1	
IRS Bond	0.1398	0.1222	-0.2241	1

Numerical Illustration (Cont')

- We use the Cox-Ingersoll-Ross (CIR) model to describe the dynamics of the pension valuation rate.

$$dr_{p,t} = \vartheta(\theta - r_{p,t})dt + \sigma_r \sqrt{r_{p,t}} dW_{rt}$$

- Maximum Likelihood Parameter Estimates of Pension Valuation Rates.

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
ϑ	0.1821	θ	0.0569	σ_r	0.0035

- Correlation Coefficients of the Three Asset Indices and the Pension Valuation.

	3-month T-bill	Corporate Bond	S&P500 Stock	IRS Bond
3-month T-bill	1			
Corporate Bond	0.0381	1		
S&P500 Stock	0.0466	0.2534	1	
IRS Bond	0.1398	0.1222	-0.2241	1

Numerical Illustration (Cont')

- We maximize the discounted expected utility $\bar{V}$ with respect to the annuity business size $N_0(x_0)$, the weights invested in the three assets $\omega = \{\omega_1, \omega_2, \omega_3\}$ and the reference point $\bar{c}$ over a 10-year time horizon:

$$\underset{N_0(x_0),\omega,\bar{c}}{\text{Maximize}} \; \bar{V} = \sum_{t=1}^{10} \nu^t \cdot \left(\sum_{i=1}^{n} [x_i^+(t) - c(t)]^{0.88} \cdot \pi_i^+(t) + \sum_{i=1-m}^{0} -2.25[c(t) - x_i^-(t)]^{0.88} \cdot \pi_i^-(t) \right)$$

subject to Constraints 1-4.

Numerical Illustration (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

- The bulk annuity insurer should insure around 281 pension participants at the age of 65 at time 0.

Numerical Illustration (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

- The insurer should invest 17.88% of its funds in the 3-month T-bill, 61.58% in the corporate bond index, and 20.54% in the S&P500 index.

Numerical Illustration (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

- Given no dividend payment to its shareholders, the insurer should set the optimal ROA reference point at $\bar{c} = -0.0417$.

Optimal Decision making with Risk Management under CPT

- Suppose the above bulk annuity insurer purchases a reinsurance policy to transfer a proportion η of its bulk annuity business.
- The reinsurer requires a loading of $l_R = \bar{a} + \bar{b}\eta$.
- The bulk annuity insurer pays the following reinsurance premium to its reinsurer:

$$\bar{P}_R = N_0(x_0)B(1 + l_R)\eta a_{x0}$$

$$= N_0(x_0)B\eta a_{x0} + \left(\bar{a}\eta + \bar{b}\eta^2\right) N_0(x_0)Ba_{x0}.$$

Optimal Decision making with Risk Management under CPT (Cont')

- We maximize the insurer's discounted expected utility with reinsurance with respect to the annuity business size $N_0(x_0)$, the asset weights $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$, the reference ROA $\bar{c}$ and the reinsurance ratio η in the CPT framework:

$$\underset{N_0(x_0),\omega,\bar{c},\eta}{\text{Maximize}} \ \bar{V}_R,$$

subject to Constraints 1-4.

Numerical Illustration with Risk Management

TABLE 5. The Optimal Solution with Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	η	$\bar{V}_R$
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424

- The insurer should transfer a proportion $\eta = 0.3142$ of the entire risk to the reinsurer.

Numerical Illustration with Risk Management (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

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	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	η	$\bar{V}_R$
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424

- Reinsurance allows the insurer to underwrite more pension participants.

Numerical Illustration with Risk Management (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

TABLE 5. The Optimal Solution with Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	η	$\bar{V}_R$
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424

- The insurer invests less in risky assets and invests more in safe assets with reinsurance.

Numerical Illustration with Risk Management (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

TABLE 5. The Optimal Solution with Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	η	$\bar{V}_R$
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424

- We observe a decline in the optimal reference point with reinsurance.

Numerical Illustration with Risk Management (Cont')

TABLE 4. The Optimal Solution without Reinsurance

	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	$\bar{V}$
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	547.7639

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	$N_0(65)$	ω_1	ω_2	ω_3	$\bar{c}$	η	$\bar{V}_R$
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424

- With reinsurance, the insurer achieves a 10.44% rise in utility compared to the utility without reinsurance.

Sensitivity Analyses

TABLE 6. Optimal Strategies with Varied Parameter Values

	$N_0(65)$	ω_1	ω_2	ω_3	c	η	V
Baseline - No reinsurance	280.9393	17.88%	61.58%	20.54%	-0.0417	–	547.7639
Baseline - Reinsurance	475.9576	21.79%	58.89%	19.32%	-0.0440	0.3142	604.9424
Panel A: Discount Rate Parameter							
No reinsurance: $\rho = 4\%$	281.8033	17.97%	61.52%	20.50%	-0.0432	–	598.3120
Reinsurance: $\rho = 4\%$	467.3936	21.82%	58.88%	19.30%	-0.0455	0.3009	658.9227
No reinsurance: $\rho = 6\%$	268.7471	16.55%	62.41%	21.03%	-0.0400	–	497.1658
Reinsurance: $\rho = 6\%$	445.3890	20.74%	59.65%	19.61%	-0.0426	0.2873	555.9144
Panel B: Insurance Loading Parameter							
No reinsurance: $l_P = 0.19$	212.9910	14.10%	66.44%	19.46%	-0.0246	–	236.0945
Reinsurance: $l_P = 0.19$	292.4120	18.41%	61.00%	20.59%	-0.0302	0.2023	314.4330
No reinsurance: $l_P = 0.21$	397.4024	20.18%	60.76%	19.06%	-0.0560	–	958.9361
Reinsurance: $l_P = 0.21$	1047.1512	25.17%	61.57%	13.26%	-0.0686	0.2997	1752.8045
Panel C: VaR Probability Parameter							
No reinsurance: $\hat{\alpha} = 0.0225$	249.0838	19.90%	61.40%	18.69%	-0.0246	–	236.0945
Reinsurance: $\hat{\alpha} = 0.0225$	377.8128	21.57%	62.03%	16.39%	-0.0308	0.2127	321.0457
No reinsurance: $\hat{\alpha} = 0.0275$	315.6081	21.63%	56.27%	22.11%	-0.0511	–	741.7373
Reinsurance: $\hat{\alpha} = 0.0275$	958.4938	30.33%	56.63%	13.05%	-0.0598	0.2905	1289.3317
Panel D: ARA Parameter							
No reinsurance: $b = 1.14$	229.5023	7.89%	66.44%	20.84%	-0.0217	–	183.0592
Reinsurance: $b = 1.14$	402.4586	18.59%	61.18%	20.23%	-0.0293	0.2555	272.8478
No reinsurance: $b = 1.16$	564.7472	24.48%	60.76%	9.13%	-0.0601	–	1204.7848
Reinsurance: $b = 1.16$	787.3529	25.13%	64.36%	10.51%	-0.0623	0.2724	1293.0340
Panel E: Reinsurance Loading Parameters							
No reinsurance: $\bar{a} = 0, \bar{b} = 0$	280.9393	17.88%	61.58%	20.54%	-0.0417	–	547.7639
Reinsurance: $\bar{a} = 0.185, \bar{b} = 0.03$	415.8273	18.30%	61.34%	20.36%	-0.0429	0.3009	575.7582
No reinsurance: $\bar{a} = 0, \bar{b} = 0$	280.9393	17.88%	61.58%	20.54%	-0.0417	–	547.7639
Reinsurance: $\bar{a} = 0.195, \bar{b} = 0.01$	476.0095	24.32%	57.63%	18.05%	-0.0444	0.2408	620.47715

Discussion and Practical Implications

- Motivations for corporate risk management:
  - Mitigate financial distress cost (Mayers and Smith Jr, 1982);
  - Reduce agency problems (Myers, 1977; Mayers and Smith Jr, 1987; Mian, 1996; Knopf et al., 2002);
  - Avoid costly external finance (Froot et al., 1993);
  - Increase tax benefits and debt capacity (Smith and Stulz, 1985; Mian, 1996; Nance et al., 1993; Cummins et al., 2001; Graham and Rogers, 2002).

Discussion and Practical Implications (Cont')

- An insurer can control downside risk and improve performance with reinsurance.
  - Mitigate financial distress cost.

- Reinsurance reduces an insurer's risk-taking incentive.
  - Lower risk shifting problem;
  - Lower cost of debt.

Conclusion

- We provide an optimization framework to analyze the optimal decisions of a bulk annuity insurer with changing risk preferences dependent on its performance in the CPT framework.
- We propose a way to endogenously determine a reference point.
- We show that risk management, such as reinsurance, can improve an insurer's value and lessen its risk-taking propensity.

Thank You !

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