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Equilibrium adjustment, basis risk and risk transmission in spot and forward foreign exchange markets

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This study investigates the risk transmission between the spot and forward foreign exchange markets. In particular, the effect of innovation basis and signs of shocks in both markets are assessed. The market is less predictable when the spot and forward markets have experienced shocks of opposite signs. The spot market and the forward market are less predictable when both the spot and forward markets have experienced higher uncertainty in the previous periods, but the forward market is influenced more by the uncertainty of its own.

I. INTRODUCTION

The relationships between spot and derivatives markets have long been extensively studied. While there is a huge body of literature on the study of spot and forward foreign exchange rates, it is mostly centred on the first moment conditions regarding expectations and market efficiency. The recent interest in the time-varying variance in financial time series has prompted studies on volatility spillovers or risk transmission between spot and derivatives markets in the commodities market and the stock market, e.g., Ng and Pirrong (1996) on the oil price, and Crain and Lee (1996) on volatility in wheat spot and futures markets. Although there are plenty of studies on the time-varying volatility in foreign exchange markets, most of them have considered the spot rates, or the spot and forward/futures separately, e.g., Baillie and Bollerslev (1989, 1990), McCurdy and Morgan (1988), Hsieh (1988) and Copeland and Wang (1993, 1994). A limited number of papers on multivariate volatility models includes Diebold and Nerlove (1989) in which an unobserved stochastic volatility model is applied. Some of the studies on the relationships between spot and forward/futures exchange markets are univariate in nature, e.g. Chatrath *et al.* (1996) links the time-varying volatility expressed in the univariate GARCH in the spot market with futures trading volume. Another strand of research on volatility spillovers is in the same market but between different geographical domains, e.g., Koutmos and Booth (1995) and Karolyi (1995) on volatility transmission in international stock markets.

The purposes of this study are to investigate volatility spillovers in the spot and forward foreign exchange markets in a bivariate setting involving the spot and forward rates of the same currencies and, in particular, to assess the effect of signs of shocks in both markets and the markets' response to the basis risk. The latter would have constructive implications in the foreign exchange market. Rather than on model deliberation, the study is more on such empirical issues which, to our knowledge and judgement, are less studied. The study adopts a VAR specification in the mean equation and uses the daily data. The forward exchange rates are the 30-day forward contracts, and therefore a daily frequency in the data set would cause serial correlation due to overlapping. There are two approaches to dealing with this problem. Hansen's GMM as in Hansen and Hodrick (1980) is in a single equation framework. The use of the VAR approach with overlapping data is justified and applied by Hokkio (1981), Baillie *et al.* (1983) and MacDonald and Taylor (1989). Hallwood and MacDonald (1994) argue rightly that the VAR approach is more efficient which this study agrees with. The VAR approach poses no special problems in the estimation

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involving serial correlated disturbances arising from overlapping data. When the residuals are assumed to follow a multivariate ARCH process, the parameters are obtained by estimating the multivariate maximum likelihood function. This approach enables one to utilize as much as possible information embedded in the daily foreign exchange data. Although the use of monthly frequency for a data set consisting of one month forward rates and spot rates would eliminate the overlapping problem, it would result in information loss.[1] This is, to a certain extent, reflected in the well known observation that the time varying volatility is either weak or has disappeared altogether in weekly, fortnightly and monthly data as reported by many researchers, e.g. Baillie and Bollerslev (1989); though surprisingly, Alexakis and Apergis (1996) find strong ARCH phenomenon even in the quarterly data.

The rest of the paper is organized as follows. Section II discusses the models for testing volatility spillovers with attention being paid to the effect of signs of shocks. The property of the data and the preliminary statistics are presented and discussed in section III. Empirical evidence and findings are reported in section IV. Finally section V is a brief summary.

II. MODELS AND TESTING PROCEDURES

General setting up

The system of equations for the spot and forward exchange rates is specified as an extended VAR, which incorporates a forward premium into a simple VAR suggested and used by Hokkio (1981), Baillie *et al.* (1983) and MacDonald and Taylor (1989). In addition, the covariance of the extended VAR is time-varying which allows for and mimics volatility spillovers or transmission between the spot and forward foreign exchange markets. The model with such attributes is as follows:

$$\begin{aligned}\Delta s_t &= c_1 + \gamma_1(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{1i}\Delta s_{t-i} \\ &\quad + \sum_{i=1}^{m} \beta_{1i}\Delta f_{t-i} + \varepsilon_{1t} \\ \Delta f_t &= c_2 + \gamma_2(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{2i}\Delta s_{t-i} \\ &\quad + \sum_{i=1}^{m} \beta_{2i}\Delta f_{t-i} + \varepsilon_{2t} \qquad \varepsilon_t \mid \Omega_{t-1} \sim N(0, H_t)\end{aligned} \tag{1}$$

where s_t is the logarithm of the spot rate, f_t is the logarithm of the forward rate, Δs_t is the percentage change in the spot rate, Δf_t is the percentage change in the forward rate, and H_t is the vector of the time varying variance extended from its univariate counterpart, widely known as GARCH (Generalized AutoRegressive Conditional Heteroscedasticity).

The inclusion of the forward premium is not merely for setting up an ECM model, it keeps information in levels while still meeting the requirements for stationarity. Although there are arguments about the property of the forward premium, its inclusion makes the system informationally and economically complete by reserving information in levels (original variable) and reflecting expectations in the market. In Section III, the property of the forward premium will be checked before it is empirically included in estimation.

In this study, the positive definite parameterization, known as BEKK as suggested by Baba *et al.* (1990), is used to model the conditional time-varying variance:

$$H_t = C^{*\prime}C^* + A^{*\prime}\varepsilon_{t-1}\varepsilon_{t-1}'A^* + B^{*\prime}H_{t-1}B^* \tag{2}$$

where C* is a symmetric $(N \times N)$ parameter matrix, and A^* and B^* are unrestricted $(N \times N)$ parameters. The important feature of this specification is that it builds in sufficient generality, allowing the conditional variances and covariances of the two time series to influence each other, and at the same time, does not require to estimate a large number of parameters. For $p = q = 1$, Equation 2 will only have 11 parameters compared to 21 parameters of the vech representation.[2] Even more importantly, the BEKK pro-

[1] According to the Nyquist sampling theorem, if a time series is sampled at the frequency of $2f_h$, then all frequency components lower than f_h would be reserved and can be recovered. In other words, any frequency components higher than f_h would be lost and unrecoverable. If the monthly data are ever to achieve the same results as the daily data, one should assume there are no fluctuations which have a frequency higher than half a month in exchange rates. This assumption, however, is highly unlikely to hold. Refer to one of the books on signals and systems, e.g., Ziemer, R. E., Tranter, W. H. and Fannin, D. R. (1993) *Signals and Systems: Continuous and Discrete*, 3rd edn, Macmillan Publishing Company, NY.

[2] The full parameterization, or the vech representation is as follows:

$$\text{vech}\,(H_t) = \text{vech}\,(C) + \sum_{i=1}^{q} A_i \text{vech}\,(\varepsilon_{t-i}\varepsilon_{t-i}') + \sum_{i=1}^{p} B_i \text{vech}\,(H_{t-i})$$

For $p = q = 1$, the above equation becomes

$$\text{vech}\,(H_t) = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

So, the simplest bivariate model has 21 parameters.

cess guarantees that the covariance matrices are positive definite under very weak conditions; and it can be shown that under certain nonlinear restrictions on A^* and B^*, Equation 2 and the vech representation are equivalent (Engle and Kroner, 1995). In the bivariate system with $p = q = 1$, Equation 2 now takes the form:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \times \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{12,t-1} & h_{22,t-1} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{3}$$

We will extend this matrix expression to show how the signs of shocks can be reflected in the next subsection so that their effect can be exploited and investigated.

The effect of signs of shocks

The extension of Equation 3 yields the following expression:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 & a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{12}a_{21} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 \\ a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{12}a_{21} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 & a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11}^2h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{21}^2h_{22,t-1} & b_{11}b_{21}h_{11,t-1} + (b_{12}b_{21} + b_{11}b_{22})h_{12,t-1} + b_{21}b_{22}h_{22,t-1} \\ b_{11}b_{21}h_{11,t-1} + (b_{12}b_{21} + b_{11}b_{22})h_{12,t-1} + b_{21}b_{22}h_{22,t-1} & b_{12}^2h_{11,t-1} + 2b_{12}b_{22}h_{12,t-1} + b_{22}^2h_{22,t-1} \end{bmatrix}$$

i.e.:

$$\begin{aligned} h_{11,t} &= c_{11} + (a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2) \\ &\quad + (b_{11}^2h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1} + b_{21}^2h_{22,t-1}) \\ h_{12,t} &= h_{21,t} \\ &= c_{12} + [a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{12}a_{21} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ &\quad + a_{21}a_{22}\varepsilon_{2,t-1}^2] \\ &\quad + [b_{11}b_{21}h_{11,t-1} + (b_{12}b_{21} + b_{11}b_{22})h_{12,t-1} \\ &\quad + b_{21}b_{22}h_{22,t-1}] \\ h_{22,t} &= c_{22} + (a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2) \\ &\quad + (b_{12}^2h_{11,t-1} + 2b_{12}b_{22}h_{12,t-1} + b_{22}^2h_{22,t-1}) \end{aligned} \tag{4}$$

Looking at the diagonal elements in the above matrix, i.e. $h_{11,t}$ and $h_{22,t}$, we can assess the impact of the shock in one series on the uncertainty or volatility of the other, and the impact could well be asymmetric or even be one way effective only. In particular, one might also be interested in assessing the effect of the signs of shocks in the spot and forward markets. To this end the diagonal elements representing the previous shocks can be rearranged as follows:

$$\begin{aligned} a_{11}^2\varepsilon_{1t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2t-1}^2 &= (a_{11}\varepsilon_{1,t-1} + a_{21}\varepsilon_{2,t-1})^2 \\ a_{12}^2\varepsilon_{1t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2t-1}^2 &= (a_{12}\varepsilon_{1,t-1} + a_{22}\varepsilon_{2,t-1})^2 \end{aligned} \tag{5}$$

It is clear that a_{11} and a_{22} represent the effect of the shock on the future uncertainty in the same market, and a_{21} and a_{12} represent the cross effect, i.e., the effect of the shock in the forward rate on the future uncertainty in the spot rate, and vice versa. The interesting point is that, if a_{11} and a_{21} have different signs, then the shocks with different signs in the spot and forward rates tend to increase the future uncertainty in the spot rate. Similarly, if a_{12} and a_{22} have different signs, the future uncertainty in the forward market might increase if the two shocks have different signs. Regarding the impact of the variance and covariance on the future uncertainty or volatility, the cross effect is the same, except that there would be no negative sign in the variance or standard deviation.

The basis risk and the adjustment in the first and second moments

Let the basis be expressed in the logarithm, i.e., $f_t - s_t = \log(F_t/S_t)$, as against the traditional form. Then, the basis is in fact the forward premium, and exchange rate changes in response to the forward premium, or the error correction between the spot and forward rates, are in fact the adjustment to changes in the basis. Further express the basis or the forward premium as:

$$f_t - s_t = f_{t-1} - s_{t-1} + \varepsilon_{2,t} - \varepsilon_{1,t} \tag{6}$$

i.e., the basis in the current period is the basis in the previous period plus the difference in innovations in the two market. From Equation 6, it is clear that innovations themselves are not the basis risk. It is the difference in innovations that contributes to the basis risk. The difference is the source of the basis risk and is called innovation basis in the rest of the paper.

Now the relationship between the spot and forward rates in the first and second moments can be further exploited. If changes in spot and forward rates can adjust to the full impact of the forward premium or the error correction in the first moment, then there would be no role for the innovation basis to play in the second moment, i.e., any basis change will not cause future uncertainty in the two markets. If, however, changes in the spot and forward rates are unable to totally correctly adjust to the basis change, uncertainty is raised and the two markets become more volatile. In other words, the basis risk is not eliminated by making adjustment to basis changes in the first moment.

III. DATA[3] AND THEIR PROPERTY

In this study, the daily spot and forward foreign exchange rates of the British pound, German mark, French franc and Canadian dollar against the US dollar are used. All of the data sets start from 02/01/76 and end on 31/12/90. So there are 3758 observations in each series. These long period high frequency time series data enable one to observe a very evident GARCH phenomenon in a bivariate system.

Summary statistics on changes in the spot and forward rates and the forward premium for the four currencies are presented in Table 1. One of the purposes is to rule out the unit root in these variables, especially in the forward premium which displays controversies on the existence of a unit root, or perhaps has a root very close to the unit circle, as reported by a few researchers, e.g., Kwiatkowski *et al.* (1992), Crowder (1994) and Baillie and Bollerslev (1993, 1994). In this research, the ADF unit root test is used and the lag length in the ADF test is decided by the AIC. The ADF test clearly rejects a unit root in the changes in spot and forward rates in all of the four currencies with the ADF statistic being around -18. The ADF statistic is much smaller (in absolute value) for the forward premium ranging from -6.7561 for the French franc to -3.4500 for the British pound, but is still able to reject the null of a unit root. Notice that most of the studies which report high persistence in the forward premium use monthly data. In monthly data, most of the high frequency components in daily premium data are lost. Because the lower frequency components correspond to persistence, monthly premia are therefore more persistent than daily premia. With these results, it is felt to be acceptable to include the forward premium without causing the non-stationarity problem.

During this period, only the German mark has experienced appreciation against the US dollar; the other three currencies have undergone depreciation. This can be observed in the negative figures for the mean value of the change in the German spot rate, forward rate and forward premium; and the positive figures for the other three currencies. Forward exchange rates have anticipated appreciation or depreciation which is reflected in the mean value of forward premia: they have the same sign as that of changes in the spot rate or forward rate.

The diagnostic checking is provided in Table 2. For the daily data, the VAR model includes five lags of spot and forward rate changes to cover one cycle of season. The statistics in the table show the VAR model fits well, with no serial correlation in the mean of the residuals measured by the Ljung–Box Q* statistic. However, there does appear remarkable serial correlation in the squared residuals. After adjusting for GARCH in the squared residuals, the Q^* statistic is not significant for all of the four currencies. The characteristics revealed by the statistics in Tables 1 and 2 justify the specification of a bivariate VAR with a forward premium and time-varying covariance.

IV. RESULTS AND DISCUSSION

Although attention is on the time-varying patterns in the second moment, the first moment properties are discussed first. As mentioned earlier, depreciation or appreciation of a currency against the US dollar is reflected in the mean value of spot rate changes and forward rate changes, which are negative for the German mark and positive for the rest of the currencies. The forward premium, as it is the difference between the one month forward rate contracted on the same day and the spot rate, also displays this tendency. However, no matter whether a currency experiences depreciation or appreciation in the long-run, a negative coefficient is always assigned to the forward premium, and assigned to both spot and forward equations, as reported in Table 3. Therefore, departure of the forward rate from the spot rate would be reversed only in the forward rate equation. It is the forward rate that adjusts to the errors in the basis, and the long-run equilibrium is maintained through this channel or mechanism. This may differ from some of the previous studies using the monthly data. With the daily data, last period's (yesterday's) forward rate is not able to be compared with the corresponding spot rate in this period (today). From the information flow point of view, today's forward rate or spot rate does not necessarily take the match or mis-match of yesterday's spot rate and the matured forward rate, which was contracted and determined a month ago, seriously. By contrast, today's forward rate and spot rate would consider newer information which includes yesterday's forward rate, which will mature in a month's time, and any new information, more seriously. In this sense, interaction between the spot and forward rates and markets is intensive, as (a) they share a common information set, (b) the unbiasedness of the forward rate cannot be checked when the new forward contract has to be made, and (c) any anticipated mis-match of the forward rate and spot rate, due to the arrival of new information, cannot be corrected by revising the previous forward contracts, it is today's forward rate that is to respond and make adjustment. This is evident as the forward premium is significant

[3] Data sources: The foreign exchange rate data used in this study are retrieved from Data Stream.

Table 1. *Summary statistics*

	BP			DM			FF			CD		
	Δs_t	Δf_t	$f_t - s_t$	Δs_t	Δf_t	$f_t - s_t$	Δs_t	Δf_t	$f_t - s_t$	Δs_t	Δf_t	$f_t - s_t$
Mean[a]	0.0473	0.0476	1.9987	−0.1194	−0.1192	−2.3127	0.0615	0.0616	2.2846	0.0399	0.0402	1.4492
Median[a]	0.0000	0.0000	2.1299	0.0000	0.0000	−2.3576	0.0000	0.0000	1.7425	0.0000	0.0000	1.4013
Max[a]	38.6558	37.8143	14.8432	55.2631	55.9874	25.3591	58.6254	51.4550	28.5769	39.5516	38.3118	21.9599
Min[a]	−46.6738	−46.9582	−7.4618	−40.1228	−39.8732	−30.7550	−39.0833	−38.6851	−9.2792	−37.6435	−36.5034	−8.0629
σ[a]	6.3719	6.3880	2.9398	6.5041	6.4718	2.1000	6.4260	6.3840	3.6093	3.1470	3.1983	1.6328
ADF[b]	−18.0794	−18.0943	−3.4500	−17.1435	−17.1908	−4.6321	−17.0237	−17.1717	−6.7561	−18.0780	−18.1808	−4.5681
Skewness	−0.0384	−0.0166	0.1103	−0.0400	−0.0454	−0.5161*	0.2167*	0.1643*	1.4629*	0.0182	0.0957†	0.2340*
Kurtosis	3.9229*	4.0483*	1.3543*	3.8891*	4.0356*	25.1797*	6.5168*	5.5009*	6.2224*	17.6573*	15.6249*	7.6240*

Note: a. Being timed by 1000.
b. The lag length in the ADF test is decided by the AIC.
* Significant at 1% level, † significant at 5% level.

Table 2. *Statistics of diagnostic checking*

$$\Delta s_t = c_1 + \gamma_1(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{1i}\Delta s_{t-i} + \sum_{i=1}^{m} \beta_{1i}\Delta f_{t-i} + \varepsilon_{1t}$$

$$\Delta f_t = c_2 + \gamma_2(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{2i}\Delta s_{t-i} + \sum_{i=1}^{m} \beta_{2i}\Delta f_{t-i} + \varepsilon_{2t} \qquad \varepsilon_t \,|\, \Omega_{t-1} \sim N(0,\ H_t)$$

	BP		DM		FF		CD	
	ε_{1t}	ε_{2t}	ε_{1t}	ε_{2t}	ε_{1t}	ε_{2t}	ε_{1t}	ε_{2t}
$Q^*(20)$	18.7899	18.7090	28.8294	21.8077	27.4759	26.5192	22.8862	23.0074
$Q^{*2}(20)$	555.5257*	539.0187*	368.4184*	324.4452*	184.9602*	220.9753*	418.9194*	382.6533*
$Q_h^{*2}(20)$	31.5052[n1]	26.3293	19.4011	22.8247	18.5053	29.5267	9.9707	10.0772

$Q^*(20)$ is the Ljung–Box test for serial correlation (of order 20) in the residuals; ε_{1t} is the residual from the spot equation, ε_{2t} is the residual from the forward equation.
$Q^{*2}(20)$ is the Ljung–Box test for serial correlation in the squared residuals.
$Q_h^{*2}(20)$ is the Ljung–Box test for serial correlation in the squared residuals, being adjusted for GARCH.
n1. marginally significant at 5% level (p-value = 0.0489), but $Q_h^{*2}(15)$ is 13.8209 with a p-value of 0.5392.
* significant at 1% level, † significant at 5% level.

in both the spot and forward equations for all currencies except for the spot equation of French franc, and the coefficients (not reported as they are not of interest in this study and there are 20 such coefficients for one currency, and can be obtained upon request) of these lagged variable are jointly significant in both equations.

Now let us turn to the focus of this study. Table 3 presents the results of the major second moment statistics. The time-varying variance is, in general, evident in both the spot and forward markets. The variance transmission displays clear asymmetry, that is, volatility spillovers from the forward market to the spot market is stronger than that from the spot market to the forward market. In addition, shocks with opposite signs in the two markets would be inclined to raise future uncertainty or volatility. Consider the British pound first. a_{12} and a_{21} are both significant at 1% level, but the magnitude of the former is about half the size of the latter, implying that the effect of the shock in the forward market on the spot market volatility is larger than that on the forward market induced by the shock in the spot market. Turning to the B matrix representing the long-run effect or the previous variance, the volatility spillovers are one directional from the forward market to the spot market, as b_{21} is significant but b_{12} is not significant at all. Notice, b_{22} is also insignificant, which means there is only ARCH in the forward exchange rate. Further scrutiny on the signs of a_{12} and a_{22} suggests that the future volatility in the forward market would be higher if the two shocks have different signs. Therefore, the innovation basis would raise forward market uncertainty but not spot market uncertainty. In the modelling term, even if the model performs well for error correction with a significant γ_2, it is not able to explain all the disequilibrium in the first moment. From the market point of view, although the forward market responded to the basis change, the forward market is not able to remove all the basis risk in its equilibrium adjustment process, and it is this bit of remaining basis risk that further contributes to the time-varying volatility. In the case of the German mark, the asymmetry is more apparent, where a_{12} is not significant at all but a_{21} is significant at 1% level. As such, the shock in the forward market would affect the future volatility in the spot market, but the shock in the spot market has no influence on the future volatility in the forward market. In addition, a_{11} and a_{21} have different signs, so the shock with opposite signs in these two markets would tend to increase the future volatility in the spot market, but the source of the basis risk is more from the forward market than from the spot market as a_{21} is much larger that a_{11}. Nevertheless, the forward market seems to be immune from the basis risk. As far as the previous variance is concerned, b_{12} and b_{21} are both significant, but the size of the former is much smaller than that of the latter, so asymmetry exists in this respect too. Again, b_{22} is not significant; the forward rate would only have the ARCH effect if it were not considered in a bivariate system. The strongest asymmetry occurs in the exchange rates of the Canadian dollar. The volatility spillovers are absolutely one directional from the forward rate to the spot rate. That is: a_{12} and b_{12} are not significant at any conventional levels, whereas a_{21} and b_{21} are both significant at 1% level. In the case of the French franc, the influence of the previous variance is clearly one directional from the forward to the spot measured by b_{12} and b_{21}. The GARCH effect is strong in the forward rate as well as in the spot rate. Regarding the previous shocks, the influence is also more from the forward market to the spot market; both a_{12} and a_{21} are significant but a_{21} is much larger than a_{12}. Furthermore, a_{11} and a_{21} have different signs and a_{21} is relatively small, so the spot market suffers from the basis risk and the source of the basis risk is mostly in the spot market.

Therefore, the four currencies have similar patterns in the time-varying volatility. Most results regarding the time-varying volatility and asymmetry are not new and the study has merely confirmed them, though the revelation about asymmetry and the explanations are slightly different. The contribution of this study, however, lies in the findings on the effect of the innovation basis or the sign effect, which links the basis risk to the markets' response to a basis change and the ability of the model to adjust to the innovation basis in the first moment. The sign effect is most profound in the British pound and the German mark. In the case of the British pound, the shock in the spot rate has the largest influence (but is still relatively small) on the forward uncertainty amongst the four currencies, and the sign effect takes place in the forward market. In the German mark, the spot shock has no influence on the forward uncertainty but the forward shock has the largest influence on the spot uncertainty, and the sign effect is found in the spot market. Comparing these two currencies, the German mark market is much more asymmetric, i.e., the forward market predominates over the spot market, and the spot market seems to passively adjust to the events in the forward market. Although there exists asymmetry in the British pound market, the spot market has some influence on the forward market, and to a certain extent, causes the forward market to make corresponding adjustments. In addition, the British spot market is more efficient in a sense that it is able to adjust to the innovation basis and eliminate the basis risk. The French franc is similar to the German mark, but the sign effect in the spot market is much less substantial; and there is not such effect in the Canadian dollar. The trading activities in the French franc and Canadian dollar are relatively modest which may explain the less significant volatility spillovers in these two currencies.

Before ending this section, it is necessary to check that the parameters obtained so far satisfy the covariance stationarity. In Table 4, all four eigenvalues for each currency are reported. Details of derivation and calculation

Table 3. *Equilibrium adjustment and risk transmission between spot and forward foreign exchange markets*

$$\Delta s_t = c_1 + \gamma_1(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{1i}\Delta s_{t-i} + \sum_{i=1}^{m} \beta_{1i}\Delta f_{t-i} + \varepsilon_{1t}$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0,\ H_t)$$

$$\Delta f_t = c_2 + \gamma_2(f_{t-1} - s_{t-1}) + \sum_{i=1}^{m} \alpha_{2i}\Delta s_{t-i} + \sum_{i=1}^{m} \beta_{2i}\Delta f_{t-i} + \varepsilon_{2t}$$

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{12,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

	BP	DM	FF	CD
c_1	0.00025†	−0.00058*	0.00012	0.00013†
	(2.3740)	(4.3591)	(1.2106)	(2.2491)
γ_1	−0.12034*	−0.23731*	−0.03214	−0.05255
	(3.9301)	(5.5473)	(1.3884)	(1.8081)‡
c_2	0.00027†	−0.00057*	0.00014	0.00015†
	(2.5077)	(4.0781)	(1.4735)	(2.5582)
γ_2	−0.12536*	−0.23203*	−0.05197†	−0.05436‡
	(4.0293)	(5.1272)	(2.2460)	(1.8156)
a_{11}	0.51775*	−0.20555*	1.00020*	0.53282*
	(13.3029)	(3.8590)	(148.5928)	(8.4157)
a_{12}	−0.24576*	0.00539	0.03776*	−0.05055
	(6.8053)	(0.1062)	(5.4583)	(0.8144)
a_{21}	0.45452*	1.17328*	−0.06149*	0.40725*
	(11.5872)	(22.2677)	(8.7868)	(6.5068)
a_{22}	1.21688*	0.96138*	0.89990*	0.98706*
	(33.3811)	(19.1147)	(124.5143)	(16.0892)
b_{11}	0.43475*	1.00966*	0.24888*	0.52226*
	(4.8987)	(16.8606)	(8.0209)	(8.0757)
b_{12}	−0.10742	−0.18033*	−0.04565	0.01389
	(1.2389)	(3.5195)	(1.4180)	(0.2278)
b_{21}	−0.66683*	−1.16582*	0.10881*	−0.22376*
	(7.5985)	(19.1740)	(3.5734)	(3.5935)
b_{22}	−0.13151	−0.05456	0.40644*	0.28611*
	(1.5358)	(1.0563)	(12.7895)	(4.7730)

Note: * Significant at 1% level, † significant at 5% level, ‡ significant at 10% level. t-statistics in brackets. Constant terms in the second moment are not reported.

of eigenvalues can be found in Judge *et al.* (1988) or Baba *et al.* (1990). The positioning of four eigenvalues on the complex plane is displayed in Appendix 1. On the diagrams, the horizontal axis is for the real part, and the vertical axis is for the imaginary part of the eigenvalue, with the reference circle being the unit circle. The root or eigenvalue of the unconditional covariance equation is not always real. The mod, which is $(r^2 + i^2)^{1/2}$ (r – real part, i – imaginary part), determines stationarity and persistence. The imaginary part of the eigenvalue also plays a role. If the mod is smaller than unity or inside the unit circle, and if the imaginary part is nonzero, then the conditional covariance converges to the unconditional covariance with oscillation. If the mod is smaller than unity and if the imaginary part is zero, then the conditional covariance converges to the unconditional covariance gradually without oscillation.

It can been seen that the largest module of the eigenvalues for the British pound, German mark and Canadian dollar is around 0.96 in modules, so the time varying volatility is highly persistent. In the French franc case, the largest module of eigenvalue is just above unity, suggesting that the unconditional covariance does not exist. There is one explanation for this. According to Nelson (1990), Lumsdaine (1991) and Bougerol and Picard (1992), even if a GARCH (IGARCH) model is not covariance stationary, it is strictly stationary or ergodic, and the standard asymptotically based inference

Table 4. *Verifying covariance stationarity: the eigenvalues. Unconditional covariance:*

$$E(\sigma_t^2) = [I - (A^* \otimes A^*)' - (B^* \otimes B^*)']^{-1} vec\,(C_0^{*\prime} C_0^*)$$

$(A^* \otimes A^*)' + (B^* \otimes B^*)'$	BP	DM	FF	CD
λ_1 (real, imaginary)	0.963, 0.000	0.969, 0.000	1.003, 0.000	0.969, 0.000
λ_1 (mod)	0.963	0.969	1.003	0.969
λ_2 (real, imaginary)	0.852, 0.000	0.570, 0.000	0.995, −0.022	0.699, 0.000
λ_2 (mod)	0.852	0.570	0.996	0.699
λ_3 (real, imaginary)	0.628, 0.000	0.022, 0.000	0.995, 0.022	0.698, 0.000
λ_3 (mod)	0.628	0.022	0.996	0.698
λ_4 (real, imaginary)	0.608, 0.000	0.017, 0.000	0.988, 0.000	0.600, 0.000
λ_4 (mod)	0.608	0.017	0.988	0.600

Note: In a situation that all eigenvalues are smaller than one in modules, the covariance is confirmed stationary.

procedures are generally valid. The analysis on the eigenvalues of the Kroneker product of the covariance matrices reveals that the time varying volatility is also highly persistent in a bivariate setting for foreign exchange rate data. In addition, though the BEKK specification has proved a helpful analytical technique for volatility transmissions, especially the impact of the signs of the shocks in different markets, in empirical research, the covariance stationarity is not so easy to satisfy and is not always guaranteed.

V. CONCLUSIONS

In this study, the spot and forward exchange rates of the British pound, German mark, French Franc and Canadian dollar against the US dollar are used to study the risk transmission between the spot and forward foreign exchange markets. In particular, the study has assessed the effect of innovation basis in both markets and the markets' response to the basis risk from the market perspective; and the equilibrium adjustments in both the first and second moment from the modelling standpoint. The adoption of the VAR approach enables the use of high frequency data without serious concern about overlapping. In addition, in a bivariate setting, one is able to investigate risk transmission or volatility spillovers in the two markets with the BEKK-GARCH model.

The research has obtained a number of findings regarding the market and modelling. First, the basis risk contributes to the volatility in foreign exchange markets, which is reflected in the effect of innovation basis or the sign effect of shocks: the market becomes less predictable when the two markets experienced shocks of opposite signs. This finding is profound as market participants would be more used to the same predicting errors in these two related exchange rates than the errors with different signs, e.g., a situation in which the spot rate ended up higher than anticipated while the forward rate turned out to be too low, would confuse the market and consequently increase the volatility.

Second, the effect of innovation basis is due to the failure of the market in responding to a basis change; that is, the market is not able to remove all the basis risk in its equilibrium adjustment process and immune itself from the basis risk, and it is this bit of remaining basis risk that further contributes to the time-varying volatility.

Third, the effect of innovation basis is due to the failure of the model to correct all the disequilibrium movement to the full in the first moment. Although the correction in the first moment works and works well, it fails to perform the adjustment to the extent which would have left no role for the innovation basis to play in the second moment.

Fourth, although there is asymmetry in risk transmission between the spot and forward markets, the spot market is not totally passive only to receive and process information disseminated from the forward market. Instead, the spot market also influenced, though to a lesser extent, the movements in the forward market in both the first moment and second moment. That is, there is price discovery in the spot market too, and volatility in the spot market can also spillover to the forward market. This is more evident in this study than in some of the previous studies, and the difference is attributable to the use of data with different sampling frequencies. Since the daily data sets are more efficient and precise in processing information flows, the results obtained are profound. Nevertheless, the forward exchange rates bear some information and, in particular, the expectations of the future exchange rate movement, which seem to have considerable influence on the spot market. The other results regarding the time-varying volatility are in line with most previous studies.

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APPENDIX: EIGENVALUES ON THE COMPLEX PLANE

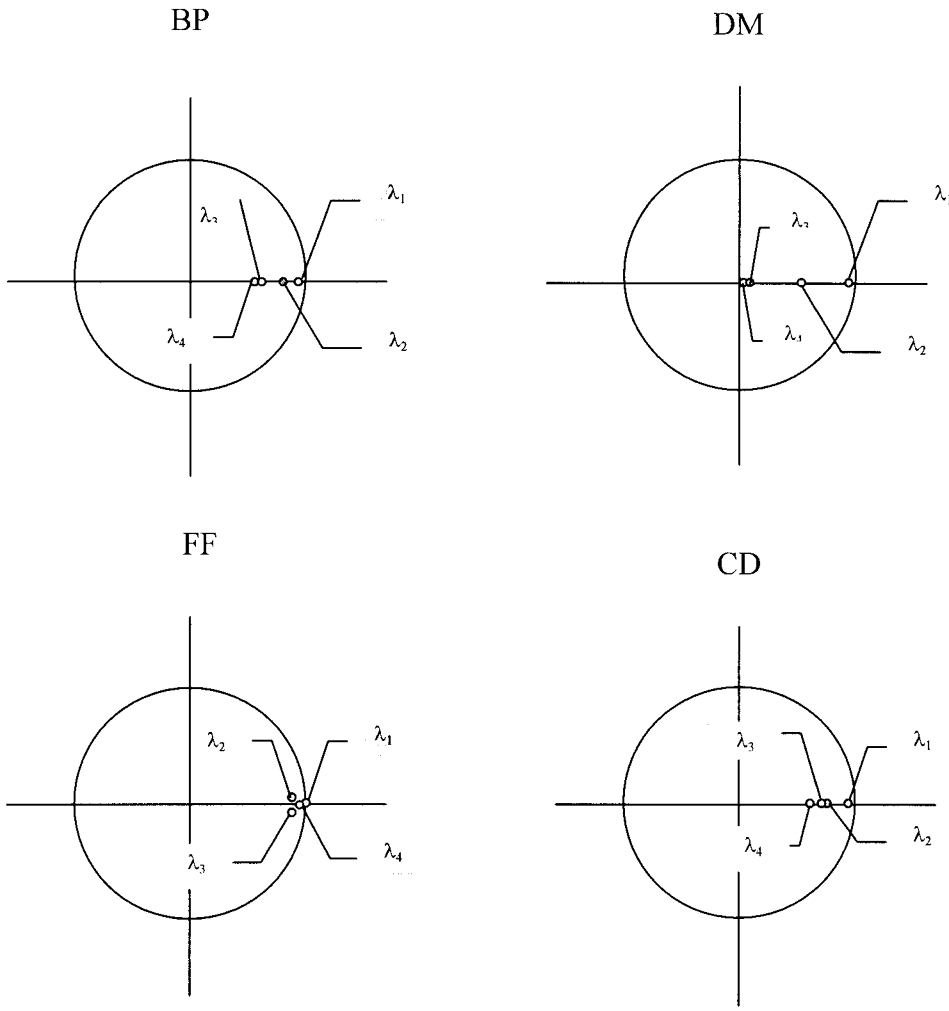

Fig. A1. *Eigenvalues of covariance matrices on the complex plane* (*the horizontal axis is for the real part, and the vertical axis is for the imaginary part of the eigenvalue the reference circle is the unit circle*)