

Testing for efficiency and rationality in foreign exchange markets—a review of the literature and research on foreign exchange market efficiency and rationality with comments

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Abstract

This paper specifies two VAR models for testing efficiency and expectations in foreign exchange markets. The sufficient conditions for efficiency and rational expectations, by imposing restrictions on the VAR parameters, are derived. Based on these models, issues on testing efficiency and rationality are discussed with reference to previous empirical studies in the area. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

This paper presents a framework to test for foreign exchange market efficiency and expectations which encompasses two main approaches: single equation regression and the VAR (Vector Auto Regression). Two VAR models are specified for the purpose. The primary motivation is to inquire into the appropriateness of the standard hypothesis being tested in this area of research. The hypothesis is not subject to the choice of estimation methods, for it is assumed that the parameters obtained

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are, at least unbiased and, may be efficient, in statistical terms. Appropriateness means that even if one has obtained consistent and efficient estimators for the parameters, whether these parameters will provide appropriate guidelines for accepting or rejecting the hypothesis regarding market efficiency and rational expectations. This concerns us, because the majority of the studies have rejected the hypothesis of market efficiency and rationality to such an extent that the parameters have not only the wrong magnitudes but also the wrong signs, which do not appear to be justified by theory or practice in foreign exchange markets. Therefore, it is important to ask whether such studies are testing the right things; and whether too much effort has been invested to obtain good estimators for the parameters which do not really rule anything in or rule anything out.

This paper is organized as follows. Section 2 presents the specifications of single equation regression and the VAR model, which are commonly used in hypothesis testing. In Section 3, a minor amendment is made to the basic VAR, to develop model A, which can be reduced to one kind of single equation regression specifications and the basic VAR. We then propose a second model, called model B, which can be reduced to any kind of single equation regression specifications, and would be the same as model A under certain circumstances. Some special cases of both models A and B are illustrated in Section 4 to demonstrate that the standard hypothesis being tested is not appropriately laid out. Discussions of the “news” model as a solution to the problem and the explanatory power of “news” is explored in Section 5. Section 6 provides a summary.

2. Review of the literature and the basic testing equations

2.1. Single equations

The single equation regression method tests whether the hypothesis $H_0: \lambda_0 = 0, \lambda_1 = 1$ holds in the following equation:

$$s_{t+\tau} - s_t = \lambda_0 + \lambda_1(f_t - s_t) + \mu_{t+\tau}, \quad (1)$$

where s_t is the logarithm of the spot exchange rate S_t , f_t is the logarithm of the forward rate F_t contracted at t and matures at $t + \tau$. This method has been used by Bilson (1981), Fama (1984), Gregory and McCurdy (1984), Gweke and Feige (1979), Hansen and Hodrick (1980), Hsieh (1984), and Longworth (1981), who all reported the rejection of the null hypothesis. Although these studies were mainly conducted in the 1980s, no recent research has disproved their findings; in fact, support was offered by Cavaglia et al. (1994) after taking into account the respective effects of irrationality and time-varying risk premia. Other related work includes Peel and Pope (1995) who use ARCH (Auto Regressive Conditional Heteroscedasticity)/ GARCH (Generalised ARCH) and risk premia, but whose conclusions regarding the mean parameters are similar.

Under the umbrella of single equation regression, there are two types of tests,

one using non-overlapping data and the other using overlapping data. The former is represented by Fama (1984), who shows that the OLS method can perform well. The latter is represented by methods such as the GMM (Generalized Method of Moments), popularized by Hansen (1982), which are used to correct the biases resulting from overlapping in data sets. The difference is in the way of obtaining correct estimates for the parameters involved and not in the hypothesis itself. The hypothesis H_0 is equivalent to $E_t\{s_{t+\tau}\} = f_t$, where $E_t\{\}$ is expectations formed in t . Table 1 summarizes the main findings of the research using single equation methods with both overlapping and non-overlapping data sets. These include the three most influential studies in the 1980s by Bilson (1981), Fama (1984) and Gregory and McCurdy (1984), and some more recent ones. It can be seen that the rejection of the hypothesis is overwhelming.

Foreign exchange markets may or may not be efficient and some investors may or may not be rational. Either situation is not unacceptable to most academics and practitioners. The problem, however, is the severity of rejection of the hypothesis. The degree of departure of the estimated parameters from the null hypothesis values is to such an extent in some studies that even the researchers themselves suspect the results, yet leave it alone to convince others of their value. In fact, many attempts have been made to investigate how the parameters are biased or distorted, instead of examining whether the hypothesis holds or not—a departure from the initial issue itself. Puzzled by the obvious failure in the majority of the empirical studies, there have emerged some recent studies adopting new approaches to dealing with the old issue, for example, Huisman et al. (1998) and Schotman et al. (1997), in the spirit of panel data analysis. The panel approach differs from single equation regression in that the former estimates several single equations “cross-sections” at the same time, allowing for possible cross-sectional correlation in their residuals. In addition, pooling across currencies technically increases the number of observations and reduces the effect of the small sample problem. Therefore, the difference is in the estimation techniques, and not in the underlying methodology, i.e. they all test whether the hypothesis $H_0: \lambda_0 = 0, \lambda_1 = 1$ holds in Eq. (1), or in a group of cross-sectional equations in the form of Eq. (1). In this sense, we put the panel approach in the category of single equation analysis.

What the panel approach, when accompanied by other assumptions, has achieved and their results can be summarised as follows. Schotman et al. (1997) apply the multiplicative news component, i.e., the forward premium is multiplied by a random variable, instead of a constant coefficient alone. This, in fact, raises the variability of the regressor in Eq. (1), i.e. the forward premium which is highly persistent. Nevertheless, whether this multiplicative news component is fundamental to the determination of changes in the spot exchange rate is unknown. As argued later in this paper, any variables which add more changeable components (higher frequency components) to the right hand side of the equation may “improve” estimation, even if they are simply random variables not fundamentally related to the exchange rate mechanism. Huisman et al. (1998) suggest that there may be co-existing factors which bias the estimates. They include a random time effect in the panel estimation procedure similar to that of Schotman et al. (1997), and claim that the efficiency of

Table 1

Test of Eq. (1): forward premium as an unbiased predictor, US\$ vis-à-vis the listed currencies

Author/country	λ_0	λ_1	R_2	Overlapping/non-overlpg	Period
Bilson:				Non-overlpg	01/74–01/80
Belgium	5.270	0.027	0.027		
Canada	−4.010	−0.804	0.050		
France	0.407	−0.849*	0.061		
Germany	6.737	−0.208	0.007		
Italy	−7.428	−0.372	0.178		
Japan	3.917	−0.665	0.049		
Netherlands	7.285*	−1.741*	0.086		
Switzerland	11.32	−0.184	0.010		
UK	1.928	0.628	0.002		
Fama:				Non-overlpg	08/73–10/82
Belgium	−0.50	−1.58*	0.04		
Canada	−0.25*	−0.87	0.01		
France	−0.64*	−0.87	0.01		
Germany	0.36	−1.32	0.00		
Italy	−1.14*	−0.51	0.01		
Japan	0.12	−0.29	0.00		
Netherlands	0.21	−1.43	0.01		
Switzerland	0.81	−1.14	0.00		
UK	−0.57*	−0.90	0.01		
Gregory/McCurdy ^a :				Non-overlpg	01/73–01/81
Canada	0.002	−0.756	0.01		
Cavaglia et al. ^b :				Overlapping	01/86–12/90
Belgium		−4.2005			
Canada		0.9094			
France		−3.4259			
Germany		−6.1755*			
Italy		−5.1734*			
Japan		−9.6632*			
Netherlands		−7.0037*			
Switzerland		−5.5380*			
UK		−1.1202			

The decimal points are kept as in the original papers.

The asterisk indicates statistical significance; necessary calculations and adjustments have been made.

^a Regression equation is $\frac{S_{t+1} - S_t}{S_t} = \alpha + \beta \left(\frac{F_t - S_t}{S_t} \right) + \mu_{t+1}$, instead of Eq. (1) exactly.

^b Forward rates are the three-month contracts; data are sampled at the monthly interval.

estimation is increased by using a system of multiple exchange rates. Both studies have obtained slope estimates, i.e. λ_1 in Eq. (1), in the range of 0.45–0.55, which are both significantly different from zero and from unity. Thus, the null hypothesis remains rejected but the severity of rejection is reduced.

2.2. The VAR approach

The VAR approach is claimed by Hallwood and MacDonald (1994) to be more efficient than single equation regression in the study of spot and forward rate relationships. They argue that since this approach exploits the time series properties of the data, the tests should be more efficient. This paper views the advantages of a VAR model rather differently. First, it does not treat the forward rate as being exogenous as single equation models do. Dynamic feedback between changes in the spot rate and in the forward rate is allowed, which is close to reality. Second, lagged changes in both spot and forward rates are explicitly incorporated to provide a fuller and clearer picture. Serially correlated residuals in single equation models may reflect or partly reflect these lagged variables, but it is not clear what this serial correlation is about—serial correlation in single equations is a matter to be corrected to obtain right estimates of parameters, rather than useful elements in an information set. Finally, our version of VAR, incorporating exchange rate variables in levels, utilises more information which would otherwise be lost in difference operations. In mathematics, this loss of information means the boundary condition is not established or recoverable. While in finance and economics, the loss of information in levels means that rational bubbles could exist.

The VAR approach to modelling foreign exchange rate behaviour is proposed by Hakkio (1981) Hakkio, 1981 and Baillie et al. (1983). Baillie (1989), Baillie and McMahon (1989), MacDonald and Taylor (1990a,b) are among this category. A typical bivariate model including the spot and forward exchange rates in differences would be:¹

$$\begin{aligned}\Delta s_t &= \sum_{i=1}^m \alpha_{1i} \Delta s_{t-i} + \sum_{i=1}^m \beta_{1i} \Delta f_{t-i} + \varepsilon_{1t}, \\ \Delta f_t &= \sum_{i=1}^m \alpha_{2i} \Delta s_{t-i} + \sum_{i=1}^m \beta_{2i} \Delta f_{t-i} + \varepsilon_{2t},\end{aligned}\tag{2}$$

where $\Delta s_t = s_t - s_{t-1}$ is the one period difference which is to be one day instead of the τ period difference in Eq. (1). As a result, the data series are not overlapping, though serial correlation may exist. The hypothesis H_0 , i.e. that the forward rate is the unbiased predictor of the future spot rate, is tested by imposing constraints on the VAR coefficients matrix.

While empirical studies on foreign exchange market efficiency and rational expectations have the same objectives, they have their own distinctive features. In this

¹ Notice what Hakkio (1981) and Baillie et al. (1983) have tested are the restrictions in difference, i.e. $E\{\ln S_{t+\tau} - \ln S_{t+\tau-1} | I_{t-1}\} = E\{\ln F_t - \ln F_{t-1} | I_{t-1}\}$, not $E\{\ln S_{t+\tau} | I_{t-1}\} = E\{\ln F_t | I_{t-1}\}$. However, the former is only a necessary condition for the latter, not the sufficient condition. Baillie et al. have used the same VAR in levels as well, but their analysis and conclusions are based on that in first differences as in Hakkio. Nevertheless, they have all rejected the null hypothesis, so the empirical results and conclusions are entirely justified.

paper, we would like to emphasise not only the sampling frequencies, but also the intervals over which the difference operation is taken. In general, the difference operation in Eq. (1) is over τ , the forward exchange contract period; and in a VAR specification, the difference operation is usually daily. Therefore, the parameters in a single equation specification and those in a VAR model are not directly comparable. To resolve whether the hypothesis H_0 with Eq. (1) has been appropriately specified, we need a VAR model which encompasses the single equation regression. To set up models with such characteristics, two amendments are required. The first is to include a forward premium, and the second is to change the difference period from one day to τ , the contract period. The model specifications for these purposes are presented in the next section.

3. Conditions for market efficiency and rationality in VAR models

3.1. Model A: Differenced at thinner interval k ($\tau \geq k \geq 1$), sampled at the same interval

$$\Delta s_t = \gamma_1 p_{t-k} + \sum_{i=1}^m \alpha_{1i} \Delta s_{t-i,k} + \sum_{i=1}^m \beta_{1i} \Delta f_{t-i,k} + \varepsilon_t, \quad (3a)$$

$$\Delta f_t = \gamma_2 p_{t-k} + \sum_{i=1}^m \alpha_{2i} \Delta s_{t-i,k} + \sum_{i=1}^m \beta_{2i} \Delta f_{t-i,k} + v_t, \quad (3b)$$

with

$$E(\varepsilon_t) = 0, E(\varepsilon_t \varepsilon_{t-l}) = \begin{cases} \sigma_\varepsilon^2 & l = 0, \\ 0 & l \neq 0, \end{cases}$$

$$E(v_t) = 0, E(v_t v_{t-l}) = \begin{cases} \sigma_v^2 & l = 0, \\ 0 & l \neq 0, \end{cases}$$

and

$$E(\varepsilon_t v_{t-l}) = \begin{cases} \sigma_{\varepsilon v} & l = 0, \\ 0 & l \neq 0, \end{cases}$$

where $\Delta s_t = s_t - s_{t-k}$, $\Delta f_t = f_t - f_{t-k}$, $p_t = f_t - s_t$, $s_t = \ln S_t$, $f_t = \ln F_t$, S_t is the spot exchange rate and F_t is the τ day forward exchange rate. $k = \tau/\kappa$, where κ is the number of observations in one contract period. When $k = 1$ ($\kappa = \tau$), the time series are of daily frequency, while when $k = \tau$ ($\kappa = 1$), only one observation is used in each contract period. The difference in operation is over k periods with $\tau \geq k \geq 1$, time series are also sampled at the same interval, and therefore there is no overlapping.

Only when a τ period difference is used in exchange rates in Eqs. (3a) and (3b), an estimated result of $\gamma_1 = 1$ and $\alpha_i = \beta_i = 0$ is equivalent and comparable to $H_0: \lambda_0 = 0, \lambda_1 = 1$ in Eq. (1). This is because H_0 means f_{t-k} is the optimal predictor of the τ period ahead spot rate $s_{t+\tau-k}$. $E_{t-k}\{s_t\} = f_{t-k}$ is a valid proposition only when $k = \tau$; for $k < \tau$, γ_1 is not bound to be unity. To test the hypothesis of

efficiency and expectations, the procedure would be similar to those used in the basic VAR model (Eq. (2) with the differenced spot and forward exchange rates only). That is, γ_1 and γ_2 , together with $\alpha_{i,j}$, $\beta_{i,j}$ ($i = 1,2$) should jointly satisfy the restrictions imposed on them. In Eqs. (3a) and (3b), the role of the forward premium has a different economic interpretation from that in Eq. (1). When daily data are available, yesterday's premium obviously contains more useful information than that in the premium 30 days earlier.

The hypothesis to be tested is that the forward exchange rate is an unbiased predictor of the future spot exchange rate. In this specification, the necessary and sufficient condition for the null of H_0 to hold, i.e. $E_t\{(s_{t+\tau} - s_t) | I_t\} = f_t - s_t$, is:

$$E_t \left\{ \sum_{i=1}^{r/k} \Delta s_{t+i-k} \right\} = p_t \tag{4}$$

The forward premium p_t can be expressed as:

$$p_t = f_t - s_t = \Delta f_t - \Delta s_t + f_{t-k} - s_{t-k} = (1 + \gamma)p_{t-k} + \sum_{i=1}^m \alpha_i \Delta s_{t-i-k} + \sum_{i=1}^m \beta_i \Delta f_{t-i-k} + \xi_t \tag{3c}$$

where $\gamma = \gamma_2 - \gamma_1$, $\alpha_i = \alpha_{2i} - \alpha_{1i}$, $\beta_i = \beta_{2i} - \beta_{1i}$ and ξ_t is a zero mean white noise residual. This is supplementary to the bivariate VAR (Eqs. 3(a) and 3(b)) and will be useful in the following illustration, and therefore, is numbered as Eq. (3c).

Eqs. (3a), (3b) and (3c) imply that: if the forward premium is relatively more persistent (or close to the unit circle), i.e. the effect of a shock lasts relatively longer, than the first difference of spot or forward exchange rates, then γ would be very close to zero, or the value of γ_1 would be very close to that of γ_2 , which has been found in a few empirical studies.

To express Eqs. (3a), (3b) and (3c) in a first order autoregressive system, we define the following matrices and vectors.² Let \mathbf{A} be a $(2m+1) \times (2m+1)$ matrix:

$$\begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,m} & \beta_{1,1} & \beta_{1,2} & \dots & \beta_{1,m} & \gamma_1 \\ 1 & & \dots & 0 & 0 & & \dots & 0 & 0 \\ 0 & 1 & & 0 & 0 & & \dots & 0 & \\ \dots & & & & & & & & \\ \alpha_{2,1} & \alpha_{2,2} & & \alpha_{2,m} & \beta_{2,1} & \beta_{2,2} & \dots & \beta_{2,m} & \gamma_2 \\ 0 & & & & 1 & 0 & \dots & & 0 \\ 0 & & & & 0 & 1 & & & 0 \\ \dots & \dots & & & & & & & \\ \alpha_1 & \alpha_2 & & \alpha_m & \beta_1 & \beta_2 & \dots & \beta_m & 1 + \gamma \end{bmatrix}$$

² The only difference with Ballie et al. (1983) is the addition of the last column and last row.

The last row in \mathbf{A} is some combination of the parameters in the first and $(m+1)^{\text{th}}$ rows, except for its last element which is $(1 + \gamma_2 - \gamma_1)$, therefore matrix \mathbf{A} is of full rank. In fact, all of the parameters are estimated with the bivariate equation system (3a) and (3b).

Further define the $(2m+1)$ dimension vectors:

$$x_t' = [\Delta s_t \Delta s_{t-k} \dots \Delta s_{t-(m-1)k} \Delta f_t \Delta f_{t-k} \dots \Delta f_{t-(m-1)k} p_t],$$

$$\omega_t' = [\varepsilon_t 0 \dots 0 v_t 0 \dots 0 \xi_t],$$

$$d = [10 \dots 000 \dots 0], \text{ First element is } 1,$$

$$e = [00 \dots 010 \dots 0], \text{ } (m + 1)^{\text{th}} \text{ element is } 1,$$

$$g = [00 \dots 000 \dots 1], \text{ } (2m + 1)^{\text{th}} \text{ element is } 1.$$

Then Eqs. (3a), (3b) and (3c) can be written as:

$$x_t = \mathbf{A}x_{t-k} + \omega_t, \tag{5}$$

and

$$\Delta s_t = d\mathbf{A}x_{t-k} + \varepsilon_t, \tag{6a}$$

$$\Delta f_t = e\mathbf{A}x_{t-k} + v_t, \tag{6b}$$

$$p_t = gx_t. \tag{6c}$$

The LHS of Eq. (4) can be expressed as:

$$E_t \left\{ \sum_{i=1}^{\tau/k} \Delta s_{t+i \cdot k} \right\} = d(\mathbf{A}^{\tau/k} + \mathbf{A}^{\tau/k-1} + \dots + \mathbf{A})x_t, \tag{7}$$

and the RHS of Eq. (4) is:

$$p_t = gx_t. \tag{8}$$

Accordingly we have the restrictions on the VAR parameters for the null of H_0 to hold:

$$d(\mathbf{A}^{\tau/k} + \mathbf{A}^{\tau/k-1} + \dots + \mathbf{A}) = g. \tag{9}$$

3.2. Model B: Difference interval equals to the forward contract period τ , sampled at the same or thinner intervals k ($\tau \geq k \geq 1$)

The difference operation is over τ period, i.e. $\Delta s_t = s_t - s_{t-\tau}$, $\Delta f_t = f_t - f_{t-\tau}$. There is overlapping as long as $k < \tau$. For example, there is information overlapping in Δs_t and Δs_{t-k} ; as s_{t-k} , part of Δs_t , precedes $s_{t-\tau}$ in Δs_{t-k} .

Regarding information content, a one period difference and τ period difference are the same, except that the first $\tau-1$ observations are not available for use in the latter. This is almost totally unimportant when there are several thousand daily data points. There are disadvantages associated with this specification, however; one of the most obvious being that it causes overlapping in data series. Nevertheless, it has

the advantage of encompassing Eq. (1), and one of the purposes of presenting this specification is to compare it with Eq. (1) to reveal some important implications for empirical work.

The VAR system has a similar appearance to that in Section 3.1:

$$\Delta s_t = \gamma_1 p_{t-\tau} + \sum_{i=1}^m \alpha_{1i} \Delta s_{t-i:k} + \sum_{i=1}^m \beta_{1i} \Delta f_{t-i:k} + \varepsilon_t, \tag{3a'}$$

$$\Delta f_t = \gamma_2 p_{t-\tau} + \sum_{i=1}^m \alpha_{2i} \Delta s_{t-i:k} + \sum_{i=1}^m \beta_{2i} \Delta f_{t-i:k} + v_t, \tag{3b'}$$

But $\Delta s_t = s_t - s_{t-\tau}$, $\Delta f_t = f_t - f_{t-\tau}$, and the lagged forward premium is $p_{t-\tau}$. Eq. (4) becomes:

$$E_t\{\Delta s_{t+\tau}\} = p_t \tag{4'}$$

No summation is needed, as the difference is over τ , the contract period. The forward premium p_t can be expressed as:

$$p_t = f_t - s_t = \Delta f_t - \Delta s_t + f_{t-\tau} - s_{t-\tau} = (1 + \gamma)p_{t-\tau} + \sum_{i=1}^m \alpha_i \Delta s_{t-i:k} + \sum_{i=1}^m \beta_i \Delta f_{t-i:k} + \xi_t. \tag{3c'}$$

Let **A** be a $(2m + \kappa) \times (2m + \kappa)$ matrix:

$$\left[\begin{array}{cccccccccccc} \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,m} & \beta_{1,1} & \beta_{1,2} & \dots & \beta_{1,m} & 0 & \dots & 0 & \gamma_1 \\ 1 & 0 & \dots & & & & & & & & & 0 \\ 0 & 1 & 0 & \dots & & & & & & & & 0 \\ \dots & & & & & & & & & & & \\ \alpha_{2,1} & \alpha_{2,2} & \dots & \alpha_{2,m} & \beta_{2,1} & \beta_{2,2} & \dots & \beta_{2,m} & 0 & \dots & 0 & \gamma_1 \\ 0 & \dots & & & 1 & 0 & \dots & & & & & 0 \\ 0 & \dots & & & 0 & 1 & 0 & \dots & & & & 0 \\ \dots & & & & & & & & & & & \\ \alpha_1 & \alpha_2 & \dots & \alpha_m & \beta_1 & \beta_2 & \dots & \beta_m & 0 & \dots & 0 & 1 + \gamma \\ 0 & \dots & & & & & & & 1 & 0 & 0 & \\ \dots & & & & & & & & & & & \\ 0 & \dots & 0 & & & & & & 0 & 1 & 0 & \end{array} \right] \left. \begin{array}{l} (\kappa-1) \text{ 0s before } \gamma_1 \\ \\ \\ \\ \\ \\ (\kappa-1) \text{ rows} \end{array} \right\}$$

$$\begin{aligned}
 x_t' &= [\Delta s_t \Delta s_{t-k} \dots \Delta s_{t-(m-1)k} \Delta f_t \Delta f_{t-k} \dots \Delta f_{t-(m-1)k} p_t \dots p_{t-\tau}], \\
 \omega_t' &= [\varepsilon_t 0 \dots 0 v_t 0 \dots 0 \xi_t], \\
 d &= [10 \dots 000 \dots 0], \text{ First element is } 1, \\
 e &= [00 \dots 010 \dots 0], \text{ } (m + 1)\text{th element is } 1, \\
 g &= [0, 0 \dots 00010 \dots 0], \text{ } (2m + 1)\text{th element is } 1, \text{ } (\kappa - 1) \text{ 0s after } 1,
 \end{aligned}$$

Eq. (7) becomes:

$$E_t\{\Delta s_{t+\tau}\} = d\mathbf{A}^{\tau/k}x_t. \tag{7'}$$

Eq. (8) is the same. Eq. (9), the conditions for the null of H_0 to hold, becomes:

$$d\mathbf{A}^{\tau/k} = g. \tag{9'}$$

4. Some special cases

4.1. $k = \tau (\kappa = 1)$ in Eqs. (3a), (3b) and (3c)

The difference operation is over the period τ , i.e. there is only one observation in one contract period. The necessary and sufficient condition is $d\mathbf{A} = g$. By analyzing the matrix and vectors involved, the following equations can be seen to hold:

$$\begin{aligned}
 \gamma_1 &= 1, \\
 \alpha_{1,i} &= 0, \quad (i = 1, \tau \dots m), \\
 \beta_{1,i} &= 0, \quad (i = 1, \tau \dots m).
 \end{aligned} \tag{10}$$

There are no restrictions on $\gamma_2, \alpha_{2,i}$ and $\beta_{2,i} (i = 1, \tau \dots m)$. Under such circumstances, lagged Δs_t and Δf_t do not have a role in Eq. (3a), and Eq. (3b) is redundant. Therefore, Eqs. (3a), (3b) and (3c) and their necessary and sufficient conditions effectively reduce to that of Eq. (1).

4.2. $k = \tau (\kappa = 1)$ in Eqs. (3a') and (3b')

This case is exactly the same as in Section 4.1. It would be most interesting to examine thinner sampling intervals with Eqs. (3a') and (3b'), as in the following sub-section.

4.3. $k = \tau/2 (\kappa = 2)$ in Eqs. (3a') and (3b')

In this case, the sampling is at an interval of $\tau/2$, or there are two observations in one contract period. Most research using fortnightly data would fall in this category, and it can be easily extended to the case of weekly data. Therefore, the results from analysing this case are applicable to many empirical studies. Referring to Eq.

(9'), the necessary and sufficient condition is $d\mathbf{A}^{\tau/k} = d\mathbf{A}^2 = g$. Let us concentrate on γ_1 and γ_2 , the parameters for the forward premium. The conditions are:

$$\begin{aligned}\gamma_1 &= 1, \\ \alpha_{1,1}\gamma_1 + \beta_{1,1}\gamma_2 &= 0, \\ \alpha_{1,1}^2 + \alpha_{2,1}\beta_{1,1} &= 0, \\ \alpha_{1,1}\beta_{1,1} + \beta_{1,1}\beta_{2,1} &= 0.\end{aligned}\tag{11}$$

for one lag in the VAR model; and,

$$\begin{aligned}\gamma_1 &= 1, \\ \alpha_{1,1}\gamma_1 + \beta_{1,1}\gamma_2 &= 0, \\ \alpha_{1,1}^2 + \alpha_{1,2} + \alpha_{2,1}\beta_{1,1} &= 0, \\ \alpha_{1,1}\alpha_{1,2} + \alpha_{2,2}\beta_{1,1} &= 0, \\ \alpha_{1,1}\beta_{1,1} + \beta_{1,1}\beta_{2,1} &= 0, \\ \alpha_{1,1}\beta_{1,2} + \beta_{1,1}\beta_{2,2} &= 0,\end{aligned}\tag{12}$$

for two lags in the VAR model, respectively. When $\kappa > 2$ (i.e. the sampling interval is thinner than fortnightly), and the conditions are similar; the first condition in Eqs. (11) and (12) is the same, but there are more restrictions on other parameters, as stated by the above equalities. Therefore, on the one hand, confirmation of $\gamma_1 = 1$ alone cannot rule in market efficiency. On the other hand, the specification of Eq. (1) effectively imposes $\alpha_{i,j} = 0$ and $\beta_{i,j} = 0$ ($i,j = 1,2,\dots$), in Eqs. (3a') and (3b') (although serial correlation in $\mu_{t+\tau}$ implies it may include lagged Δs_t and Δf_t). These untested restrictions are likely to affect the value of γ_1 , and due to this distortion in the value of γ_1 , market efficiency cannot be ruled out, simply because the estimate of $\gamma_1 \neq 1$.

4.4. Discussions

One may notice that $\alpha_{i,j} = 0$ and $\beta_{i,j} = 0$ is one of the instances which make the conditions in Eqs. (11) and (12) hold. But this instance and the associated conditions are unlikely to materialize in practice. We can discuss this issue in three ways. First, let us explain it intuitively. It is a well observed fact in empirical studies that the forward premium is highly persistent and close to having a unit root. Therefore, the covariance function of the forward premium is rather different from the covariance function of Δs_t , and the difference between the two variables cannot be a white noise residual. In Eq. (1), the serially correlated residual $\mu_{t+\tau}$ is in fact a combination of lagged Δs_t and Δf_t . Although the GMM, like certain other estimation procedures may be able to obtain a correct estimate for γ_1 , it does not test for the other conditions in Eqs. (11) and (12). Second, the overlapping data series Δs_t can always be expressed as $\Delta s_t = s_t + s_{t-k}$, where s_t is a random variable and may or may not have serial correlation. The first lag term is $\Delta s_{t-k} = s_{t-k} + s_{t-\tau}$. As both Δs_t and Δs_{t-k} contain s_{t-k} , they should be correlated. Therefore, $\alpha_{1,1} \neq 0$ leads to $\beta_{1,1} \neq 0$

and $\gamma_2 \neq 0$. The condition $\gamma_2 \neq 0$ also suggests that the forward rate will also adjust to the forward premium, which is overlooked by the single equation approach. Lastly, we compare the spectra of Δs_t and p_t , as plotted in Fig. 1. Without lagged Δs_t and Δf_t , the graphed “equation” certainly does not hold.

For non-overlapping data, one might argue that there should be no problem for

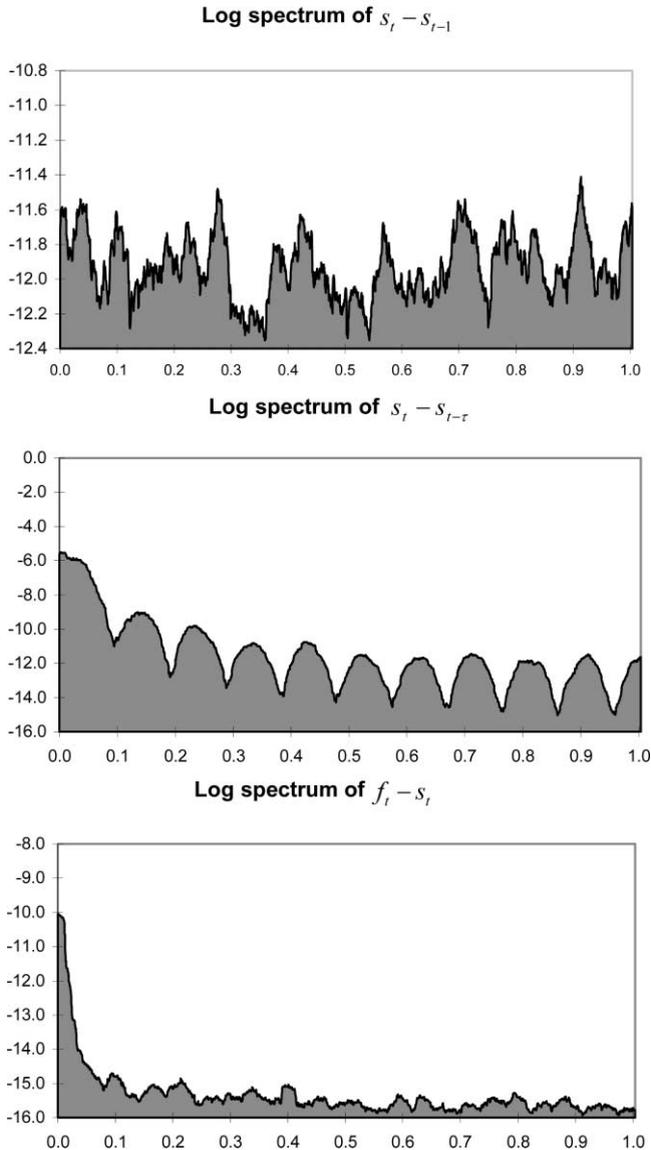


Fig. 1. Spectra of changes in the spot rate and the forward premium (German mark vis-à-vis US\$ 02/01/76–31/12/90).

$\lambda_0 = 0$ and $\lambda_1 = 1$ to be the necessary and sufficient conditions. But do we lose some critical information by using data of such a low frequency?³ Simply observing the graphs of the spectra for changes in the spot and forward rates and the forward premium in Fig. 1 provides an answer to this question. One would be lucky if information is lost proportionally across all frequencies for changes in exchange rates and for the forward premium. But unfortunately, changes in exchange rates and the forward premium have quite different covariance functions and, therefore, disproportionate components across frequencies. From Fig. 1, it can be seen that virtually no information has been lost in the forward premium when the sampling interval moves from the daily frequency to the monthly frequency or to even lower frequencies. But a large portion of information is lost in the spot (or forward) rate changes if one uses monthly data. After arguing against the use of monthly or quarterly data in empirical work, a new question then arises: is the daily frequency high enough not to lose important information? We can probably justify the use of the daily frequency as the appropriate working frequency given the way in which forward exchange contracts are made.

In summary, an estimate of unity for λ_1 in Eq. (1) cannot guarantee market efficiency which is also subject to other conditions in Eqs. (11) and (12), and $\gamma_1 \neq 1$ cannot rule out market efficiency because the estimate may be distorted by imposing inappropriate restrictions on $\alpha_{i,j}$ and $\beta_{i,j}$.

The VAR methodology as adopted in this paper differs from the single equation approach in the way it handles the issues raised above. It treats the forward exchange rate as being endogenous, in a (dynamic) system, which is close to reality and to agents' behaviour. It proves that there is a role for lagged changes in exchange rates, which bring more changeable components to the right hand side of the equations. Unlike other studies including news components, either multiplicative or additive, on the right hand side of the regression equation to increase the variability in a random fashion, our approach suggests that the variability is systematically built in, and regulated by exchange rate dynamics. News, in the sense of important economic and monetary announcements and policy changes, does not happen frequently; and news, in the sense of daily innovations, should be close to random walks. Therefore news does not bias (down) the estimates systematically, unless there are deficiencies in estimation techniques. Single equation regression approaches, whether using overlapping or non-overlapping data, are static, ignoring exchange rate dynamics. Parameters in the forward equation, λ_1 , α_{2i} and β_{2i} , are completely ignored. As shown above, $\lambda_0 = 0$ and $\lambda_1 = 1$ is one of the conditions for market efficiency which cannot be tested directly. In fact, $E_t\{s_{t+\tau}\} = f_t$ is a fair game which allows for serial corre-

³ According to the Nyquist sampling theorem, if a time series is sampled at the frequency of $2f_h$, then all frequency components lower than f_h would be reserved and can be recovered. In other words, any frequency components higher than f_h would be lost and unrecoverable. If the monthly data are ever to achieve the same results as the daily data, one should assume there are no fluctuations which have a frequency higher than half a month in exchange rates. This assumption, however, is highly unlikely to hold. Refer to one of the books on signals and systems, e.g., Ziemer, R.E., Tranter, W.H. and Fannin, D.R. (1993), *Signals and Systems: Continuous and Discrete*, 3rd ed., Macmillan, New York.

lation, while the left hand side of Eq. (1), $\Delta s_{t+\tau}$ is close to a random walk with non-overlapping data. From an economic point of view, the forward premium, $f_t - s_t$, catches the long-run fundamentals of the trend movement, whereas $\Delta s_{t+\tau}$ contains a great deal of trading noise. These are reflected by their statistical properties: $f_t - s_t$ is persistent and $\Delta s_{t+\tau}$ is much more variable. So, while the fair game $E_t\{s_{t+\tau}\} = f_t$ can be inferred by the VAR methodology, it does not necessarily turn into Eq. (1).

5. The “news” model as a solution?

The results of two approaches of utilizing “news” as an explanatory variable are to be found in Table 2. The first, used by MacDonald (1983), regresses changes in the spot exchange rate on the forward premium and “news” variables. i.e., the procedure adds “news” to the variables in Eq. (1)⁴. The second, used by MacDonald

Table 2
Selected “news” model results

Author/country	“news” 1		“news” 2		Forward premium	Period
MacDonald (1983) ^a					with	01/72–12/79
France–US	–0.291	(0.46)	–1.155	(1.02)		
Germany–US	–2.017*	(3.27)	3.173*	(2.66)		
UK–US	–0.156	(1.06)	0.172	(0.14)		
MacDonald and Torrance (1988) ^b					without	10/81–08/85
France–UK	–0.008	(0.07)	–0.123	(0.72)		
Germany–UK	–0.132	(1.21)	–0.131	(0.77)		
Japan–UK	–0.325*	(2.49)	–0.111	(0.56)		
Swaziland–UK	–0.018	(0.16)	0.164	(0.95)		
US–UK	–0.222	(1.26)	0.120	(0.45)		
Copeland (1989) ^c					without	01/77–01/84
Germany–US	–0.294	(1.64)	–0.019*	(8.24)		
UK–US	0.069	(0.29)	–0.003*	(1.71)		

*statistically significant, *t*-statistic in brackets.

^a “news” 1 and “news” 2 are shocks in money growth in the domestic and foreign countries, respectively.

^b “news” 1 and “news” 2 are unexpected and expected percentage changes in monetary aggregates.

^c “news” 1 is money shock, “news” 2 is relative interest rate. Industrial production is also included in the “news” model, and is insignificant in both cases. In another regression equation, industrial production is replaced by share index with similar results.

⁴ The actual regression equation is $s_{t+1} - f_t = \gamma_0 + b_1 \text{news}1 + b_2 \text{news}2 + \mu_{t+1}$. It is in fact obtained by subtracting the forward premium from both sides of Eq. (1), plus “news” variables.

and Torrance (1988) and Copeland (1989), regresses changes in the spot rate on “news” variables without the forward premium. The findings of these studies show, in general, the “news” variables are significant only in a small number of cases. The results suggest that the “news” variables do explain some, but not much, of the change in foreign exchange rates.

The spectra of foreign exchange rates presented in Fig. 1 shows that changes in the spot rate and the forward premium differ substantially. The daily changes in the spot rate are close to white noise, and as such, can rarely be explained by other variables. The spectrum of the monthly changes in the spot rate with daily sampling, is an aggregation of the spectrum of the daily changes with a phase-shift, which introduces “cycles” in the spectrum. Though the latter is not so white as the former, it still has substantial higher frequency components. In contrast, the spectrum of the forward premium consists only of very low frequency components. Therefore, it is possible to make Eq. (1) a better fit by adding variables with some higher frequency components to compensate for the lack of such components in the forward premium. It is a well recognized fact that the forward premium is much more persistent than changes in spot or forward rates; and the variance of the forward premium is much smaller than that of changes in spot or forward rates. In other words, the forward premium is arguably close to having a unit root, or is fractionally integrated with long memory; while changes in spot and forward rates are quite certainly stationary.

If we ignore the argument for the existence of a unit root in the forward premium, how can we make the left hand side of Eqs. (3a') and (3b'), which is not persistent, equal to a highly persistent forward premium on the right hand side? One solution, which people do not pursue intentionally, is to forcibly introduce a constant term (which by definition is very persistent) into the estimation procedure to counterbalance the persistence in the forward premium, since reducing the persistent component will increase the relative importance of the less persistent components. This has been well observed in most empirical work. Another solution, which researchers do pursue intentionally, is to include a risk premium or “news” into the equation and effectively introduce less persistent components on the right hand side. Both methods would make the models “fit better”. Therefore, the explanatory power of the “news” model appears superficial rather than fundamental.

6. Summary

In this paper we have specified two VAR models for testing efficiency and expectations in foreign exchange markets. By incorporating a forward premium, these two models encompass the two main approaches used in empirical investigations: single equation regression and the VAR. The forward premium provides vital information embedded only in the level variables to test the sufficient conditions while retaining stationarity in the statistical equations. This would solve the dilemma of either using the variables in levels which violate the requirement for estimation in the VAR model; or using the variables in differences with the sufficient conditions being untested due to information loss.

This study has the following results and conclusions. First, the sufficient conditions can only be met with the exchange rate variables in levels being included. The basic VAR, as used in the previous studies, is only able to test whether the necessary conditions hold; and as such, efficiency and rationality are not guaranteed, although a rejection of the hypothesis will be enough to reach a verdict.

Second, the single equation regression method ignores exchange rate dynamics and could distort the true parameters even if the estimated parameters are statistically sound, whether overlapping data or non-overlapping data are used in research. In the former, where the sampling period is thinner than the forward contract period, although serial correlation in the residual is recognized and measures have been taken to achieve an unbiased estimate of parameter λ_1 , the pattern of the residual's serial correlation has never been analysed. In fact, conditions $\lambda_0 = 0$, $\lambda_1 = 1$ in Eq. (1) alone cannot guarantee efficiency and rationality. The restrictions imposed on the parameters for the variables in the difference, as discussed in Section 4, imply that the residual has to obey a certain pattern to justify efficiency and rationality, in addition to the restrictions on the long run parameters. In the latter, the serial correlation problem is avoided by adopting non-overlapping data, i.e. the sampling period is the same as the forward contract period. However, this approach completely ignores the dynamic behaviour of foreign exchange market operations; and the agent, by assumption, will not adjust his position in a period shorter than the forward contract period. Even if λ_0 and λ_1 in Eq. (1) were unbiased in statistical terms, they would be distorted in economic terms. Indeed, much critical information reflecting the market and agent behaviour, especially that of the higher frequency components, has been lost in estimation, leaving the results obtained with little economic relevance and appeal.

Third and finally, the “news” model, which introduces “news” or a risk premium into Eq. (1), effectively adds less persistent components to the right hand side of the equation. Eq. (1) is unlikely to hold properly, due to substantially different characteristics reflected in their spectral patterns on the two sides of the equation. Therefore, variables with higher frequency components would compensate for the relative lack of such components on the right hand side and make the model a “better fit”. To a certain extent, the “news” variables have played such a role, which does not appear to be fundamental to exchange rate determination.

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