Rethinking Age-Period-Cohort Mortality Trend Models

Daniel Alai       Michael Sherris

Centre of Excellence in Population Ageing Research

Actuarial Studies, Australian School of Business
University of New South Wales

September 8, 2011
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
Motivation

- Models where age parameters interact with period parameters are well studied in the literature (e.g. Lee-Carter models).

- Rather than building upon such a structure, with its accepted pitfalls, we propose to take a step back.

- By rethinking model fundamentals, our approach is to clearly separate the influence of factors that drive mortality improvement.

- Inspired by models found in non-life insurance, we
  - identify trends with reliable estimation procedures (maximum likelihood), and
  - propose a framework that has the potential to improve forecasting performance.
The Mortality Triangle

- Traditionally, mortality data has been presented with an emphasis on the calendar year of death (period tables).
- By transforming the data as shown below, we
  - shift the emphasis to year of birth, and
  - format the data in a non-life setting (triangle data).

![Mortality Triangle Diagram]

**Figure:** Transforming life insurance data, the mortality triangle.
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
The Non-Life Insurance Model

Model Assumptions

- Increments $X_{i,j}$ are independent (overdispersed) Poisson distributed.
- The regression formula is given by

$$\eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j}, \quad i \in \{0, \ldots, I\}, \ j \in \{0, \ldots, J\},$$

where $\beta_{1,0} = \beta_{2,0} = 0$.
- The link function is given by $g(\mu) = \ln(\mu)$.

This model replicates the expected future liabilities produced by the classic deterministic chain ladder method!
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
Model Assumptions

- **Deaths** \( D_{i,j} \) are independent (overdispersed) Poisson distributed.
- The regression formula is given by
  - **Age-Period:** \( \eta_{i,j} = \beta_0 + \beta_{2,j} + \beta_{3,i+j} \), \((AP)\)
  - **Age-Cohort:** \( \eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j} \), \((AC)\)
  - **Age-Period-Cohort:** \( \eta_{i,j} = \beta_0 + \beta_{1,i} + \beta_{2,j} + \beta_{3,i+j} \), \((APC)\)

  where \( \beta_{1,0} = \beta_{2,0} = \beta_{3,0} = 0 \).

- The link function is given by \( g(\mu) = \ln(\mu) \).

- Age, period, and cohort effects are modelled with distinct parameters for each age, calendar year of death, and year of birth.
- Note that the age-cohort model is exactly the non-life insurance model.
Fitting the Models to Norwegian Mortality Data

- Norwegian period mortality data dating back from 1846 to 2008 *(source: the Human Mortality Database).*

- In the paper, we contrast this with Australian data. The models can be fit to any country data with relative ease.

- We study the regression parameter trends:
  - age trend,
  - calendar year (period) trend,
  - birth year (cohort) trend,

- To gauge the fit, we study the errors with respect to the omitted trend (either period or cohort).
The Age-Period Model: Age Trend

AP Model: Norwegian Age Trend

Age Regression Coefficient Values

D. H. Alai (CEPAR, UNSW)  Rethinking APC Mortality Trend Models  September 8, 2011  8 / 21
The Age-Period Model: Period Trend

AP Model: Norwegian Calendar Year Trend

Calendar Year Regression Coefficient Values

D. H. Alai (CEPAR, UNSW) Rethinking APC Mortality Trend Models September 8, 2011 9 / 21
Residuals of the AP Model Plotted Against Birth Year

AP Model: Norwegian Residuals vs. Birth Year

![Graph showing residuals vs. birth year for the AP Model.](chart.png)
The Age-Period Model

- The age trend follows the typical shape of average log-mortality rates.
  - Relatively high infant mortality.
  - Accident hump present in early adulthood.
  - Decaying mortality for the older ages (typically modelled as linear).

- The age trend retains this pattern for the age-cohort and age-period-cohort models, where it exhibits slightly more decay for the older ages.

- The model has difficulty fitting the older ages (centenarians).

- The period trend is downward (indicating mortality improvement) and roughly linear.
  - Similar to the mortality index found in the Lee-Carter model.

- The residuals are not well behaved. It appears birth year is a significant covariate and aught to be included in the model!
  - Note: large residuals are result of the model’s difficulties fitting centenarians.
The Age-Cohort Model: Cohort Trend

AC Model: Norwegian Birth Year Trend

Birth Year
Birth Year Regression Coefficient Values

D. H. Alai (CEP AR, UNSW)
Rethinking APC Mortality Trend Models
September 8, 2011 12 / 21
The Cohort Trend - Omitting the First 50 Cohorts

AC Model: Norwegian Birth Year Trend

Birth Year Regression Coefficient Values

Birth Year
Residuals of the AC Model Plotted Against Calendar Year

AC Model: Norwegian Residuals vs. Calendar Year
The Age-Cohort Model

- The cohort trend is downward (indicating mortality improvement) and roughly quadratic.
  - It is a *smooth* trend compared with the period trend found in the AP model.
- It *appears* to have high uncertainty!
  - This uncertainty originates from the earliest cohorts. If we remove, say, the first 50 cohorts, our confidence intervals become much tighter.

- The residuals are much better behaved. With the exception of 1918, the period covariates do not appear needed in the model.
  - Note: large residuals are result of the model’s difficulties fitting centenarians.
In the **full** age-period-cohort model (right), the period trend changes dramatically compared to the age-period model (left).

- The cohort trend remains unchanged when moving from the age-cohort to the age-period-cohort model.
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
The Lee-Carter Model

Mortality improvements vary considerably among different age groups.

Lee and Carter introduce:

- $\kappa_t$, an index representative of the mortality improvement over time, $t \in \{1, 2, \ldots, T\}$.
- $b_x$, a measure of the share of that general improvement by age, $x \in \{0, 1, \ldots, \omega\}$.

They model the log mortality rate as

$$\ln(m_{x,t}) = a_x + b_x \kappa_t + \varepsilon_{x,t},$$

where $\varepsilon_{x,t}$ has mean zero and variance $\sigma_\varepsilon$. 
The Implied Lee-Carter Cohort Effect

An age-period interaction term is really just a stand-alone cohort term.

- Let $\gamma_i = \text{average}\{b_xk_t\}$, where the average is over the available combinations of $x$ and $t$ suitable for cohort $i$.
- We call $\gamma_i$ the implied cohort effect and compare it with the stand-alone cohort effect in our model framework.
Plan

- Introduction
- Generalized Linear Models in Non-life Insurance
- Mortality Trends Models
- Lee-Carter Mortality Models
- Conclusions
The Moral of the Story

We fit mortality rates using three available indices, age, calendar year, and year of birth.

- Age certainly plays an important role.
- We show that year of birth dominates over calendar year of death and should be the second index included in the model!
- If necessary, one off period effects can be included, such as 1918 for the Norwegian data.

Comparison with the Lee-Carter model

- The popular Lee-Carter model uses a bilinear period effect that compares to a stand alone cohort effect.
- It is well known that the Lee-Carter model has difficulties forecasting due to the fact that the share of overall mortality improvement for age \( x \) is constant over time.
- A stand alone cohort effect does not have this problem.
On the Horizon

Problems to consider in the future

- Cohort effects need to be updated after realizing a new year of data.
- Life expectancy, forecasting, and uncertainty.
Thank you!