Pricing Liquidity Risk with Heterogeneous Investment Horizons*

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Abstract

We develop a liquidity-based asset pricing model featuring investors with heterogeneous investment horizons and stochastic transaction costs. In an equilibrium where all investors invest in all assets (integration), we find that the existence of investors with heterogeneous horizons, as opposed to homogeneous horizons, reduces the importance of liquidity risk relative to the standard CAPM market risk and generates a more complex effect of expected liquidity. In an equilibrium where short-term investors do not invest in some more illiquid assets (partial segmentation), our model generates an additional segmentation premium for these assets. We estimate the model for the cross-section of U.S. stocks using GMM and find that our heterogeneous-horizon asset pricing model fares better than a standard liquidity-adjusted CAPM. The segmented version of our model delivers the best cross-sectional fit and generates a substantial effect of expected liquidity on expected returns.

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1 Introduction

The investment horizon is a key feature distinguishing different categories of investors, with high-frequency traders and pension funds at the two extremes of the investment horizon spectrum. Most of the literature on multi-period portfolio choice and asset pricing builds on the theoretical insight of Merton's (1971) hedging demands and demonstrates that multi-period decisions differ substantially from single-period decisions in different model specifications (e.g., Campbell and Viceira, 1999; Balduzzi and Lynch, 1997; Brandt, 1999).

Surprisingly, the interaction between investment horizons and liquidity has attracted much less attention. Even in the absence of hedging demands, heterogeneous investment horizons can have important asset pricing effects for the simple reason that different horizons imply different trading frequencies. More specifically, liquidity plays a distinct role for investors with diverse horizons insofar as trading costs only matter when trading actually takes place. The investment horizon then becomes a key element in the asset pricing effects of liquidity.

We explicitly take this standpoint and derive a new liquidity-based asset pricing model featuring investors with heterogeneous investment horizons and stochastic transaction costs. Investors with longer investment horizons are clearly less concerned about trading costs, because they do not trade every period. A longer investment horizon then may allow to earn higher returns that can cover the higher trading costs of illiquid assets.

Previous theories of liquidity and asset prices have largely ignored heterogeneity in investor horizons, with the exception of the seminal paper of Amihud and Mendelson (1986), who study the existence of *clienteles* that have different liquidity preferences in a setting where transaction costs are constant. However, there is large empirical evidence that liquidity is time-varying. The most influential asset pricing model with liquidity risk, Acharya and Pedersen (2005), features stochastic transactions costs, but includes only one type of investors, who trade every period. Our paper effectively bridges these two seminal papers, because our model entails different clienteles, as in Amihud and Mendelson (1986), with stochastic illiquidity, as in Acharya and Pedersen (2005).

This theoretical setup delivers a number of novel and interesting predictions in two basic alternative equilibriums. If transaction costs are relatively small, we obtain an equilibrium where all investors trade all assets (integration). In this case, the importance of liquidity risk relative to standard CAPM market risk is lower than in a setting where all investors trade every period, because of the presence of investors with longer investment horizons. More specifically, the relative importance of the two risk premiums depends crucially on the risk aversion of long-term versus short-term investors. For example, if long-term investors are more risk-averse, liquidity risk becomes more important because short-term investors, who care more about liquidity, hold a relatively larger fraction of the asset supply in equilibrium.

Another important prediction of this setup is that, given that some investors do not trade every period, the effect of expected liquidity is smaller and varies in the cross-section of stocks according to the covariance between returns and illiquidity costs. Interestingly, we identify cases in the cross-section in which high liquidity risk may actually lead to a lower premium on expected liquidity due to the presence of long-term investors.

Alternatively, if transaction costs on some assets are sufficiently large, we obtain an equilibrium where short-term investors do not invest in some more illiquid assets (partial segmentation). In this case, our model shows that expected stock returns contain again a larger proportion of market risk premiums relative to liquidity risk premium, plus an additional segmentation premium reflecting the supply for the segmented asset. This segmentation premium is due to imperfect risk sharing: only long-term investors hold these illiquid assets. We also find that the effect of expected liquidity on returns is naturally larger for the assets that are held by all investors. In this setup, an increasing and concave relationship between expected returns and trading costs arises naturally, since excessive trading costs exclude the clientele that is more sensitive to liquidity costs.

These theoretical predictions are borne out in the empirical estimation. Specifically, we explore the cross-sectional predictions of the model using the cross-section of U.S. stocks over the period 1964 to 2009. We use the illiquidity measure of Amihud (2002) to proxy for liquidity costs. We find that our heterogeneous-horizon asset pricing model fares better than a standard or a liquidity-adjusted CAPM in terms of R-squared for the cross-section of expected returns.

We also find that segmentation is an important feature of our model with heterogenous horizons. Specifically, when some investors do not invest in the most illiquid assets, the model delivers a much more accurate cross-sectional fit. Interestingly, our model empirical estimates can also be used to make inferences about the implied number of investors in each horizon class. We also find that the estimated effect of expected liquidity on returns is much larger when we allow for segmentation.

Our paper contributes to the existing literature on liquidity and asset pricing along three general dimensions. First, our model is related to other theoretical papers on the effects of liquidity on portfolio choice and prices, besides Amihud and Mendelson (1986) and Acharya and Pedersen (2005). Starting with the work of Constantinides (1986), several researchers have examined multi-period portfolio choice in the presence of transaction costs. Our key distinguishing feature is the introduction of heterogenous investment horizons and liquidity risk, while we simplify the analysis in some other less crucial dimensions to obtain a tractable asset pricing model. In particular, we assume that long-term investors do not rebalance at intermediate dates and that transaction costs are i.i.d. over time.

Our empirical results contribute to a rich literature that has shown the asset pricing implications of liquidity and liquidity risk. Amihud (2002) finds that stock returns are increasing in the level of illiquidity both in the cross-section (consistent with Amihud and Mendelson, 1986) and in the time-series. Pástor and Stambaugh (2003) show that the sensitivity of stock returns to aggregate liquidity is priced. Acharya and Pedersen (2005) integrate these effects into a liquidity-adjusted CAPM that performs better empirically than the standard CAPM. The liquidity-adjusted CAPM is such that, in addition to the standard CAPM effects, the return on a security increases with the level of illiquidity and is influenced by three different liquidity risk covariances. Several articles build on these seminal papers and document the pricing of liquidity and liquidity risk in various asset classes.[1] However, none of these papers study the liquidity effects of heterogenous investment horizons.

Finally, our paper is also related to research showing the relations between liquidity and investors' holding periods. For example, Chalmers and Kadlec (1998) find evidence that it is not the spread, but the amortized spread that is more relevant as a measure of transaction costs, as it takes into account the length of investors' holding periods. Cremers and Pareek (2009) study how investment horizons of institutional investors affect market efficiency. Cella, Ellul, and Giannetti (2011) demonstrate that investors' short horizons amplify the effects of market-wide negative shocks. All these articles use turnover data for stocks and investors to capture investment horizons. In contrast, we estimate the degree of heterogeneity in investment horizons by fitting our asset pricing model to the cross-section of U.S. stock returns.

The remainder of the paper is organized as follows. Section 2 illustrates our multi-period liquidity asset pricing model in the most intuitive setting with two investment horizons (one-period and two-periods) and two assets. Section 3 generalizes the model to arbitrarily many investment horizons and assets. We describe our estimation methodology in Section 4. Section 5 illustrates the data and Section 6 presents our empirical findings. We conclude with a summary of our findings in Section 7.

[1]For example, Bekaert, Harvey, and Lundblad (2007) focus on emerging markets, Sadka (2010) studies hedge funds, Franzoni, Nowak, and Phalippou (2011) focus on private equity, Bao, Pan, and Wang (2011) study corporate bonds, and Bongaerts, De Jong, and Driessen (2011) focus on credit default swaps.

2 Simple Example: Two Assets and Two Horizons

In this section we present a simple version of our asset pricing model, with two investor types and two assets. We have overlapping generations, and assume that two one-period investors and one two-period investor enter the market in each period. Asset i pays a per-period dividend D_{it} and selling the asset costs C_{it}. The first investor type has a one-period horizon and mean-variance preferences with risk-aversion A_1. At time t, these one-period agents solve a maximization problem where they choose the quantity of stocks purchased y_{1t} (a vector with one element for each asset) to maximize utility

$$\begin{aligned} & \max_{y_{1t}} \ \mathbb{E}[W_{t+1}] - \frac{1}{2} A_1 \mathrm{Var}(W_{t+1}) \\ W_{t+1} &= (P_{t+1} + D_{t+1} - C_{t+1})' y_{1t} + R_f (e_1 - P_t' y_{1t}), \end{aligned} \tag{1}$$

where R_f is the gross risk-free rate, W_{t+1} is wealth at time $t+1$, P_{t+1} is the vector of prices, and e_1 is the endowment.

The two-period investors are also mean-variance optimizers, but care about their wealth two periods ahead. For simplicity, we do not allow these two-period agents to rebalance after one period. In essence, we assume that rebalancing trades of long-term investors are relatively small and can be ignored. The utility maximization is then given by[2]

$$\begin{aligned} & \max_{y_{2t}} \ \mathbb{E}[W_{t+2}] - \frac{1}{2} A_2 \mathrm{Var}(W_{t+2}) \\ W_{t+2} &= (P_{t+2} + R_f D_{t+1} + D_{t+2} - C_{t+2})' y_{2t} + R_f^2 (e_2 - P_t' y_{2t}). \end{aligned} \tag{2}$$

For simplicity, we assume that both dividends and costs are i.i.d. Then, given that optimal demand is independent of wealth, given a fixed asset supply, and with the same type of investors entering the market each period, we obtain a stationary equilibrium where the price of each asset P_{it} will be constant over time. At any point in time, the market clears with new investors buying the supply of stocks minus the amount held by the two-period investor that entered the market one period ago,

$$2y_{1t} + y_{2t} = S - y_{2,t-1}, \tag{3}$$

where S is vector with supply of assets (in amount of each of the assets). Given the i.i.d. setting, we have constant demand over time, $y_{1t} = y_{1,t-1}$ and $y_{2t} = y_{2,t-1}$.

[2] It is assumed that the proceeds of the dividend at $t+1$ are added to the risk-free deposit.

Below, we work out the equilibrium expected returns for various cases. To set the stage, we start with the case where all investors have the same horizon. Then we allow for horizon heterogeneity, and consider two potential equilibria. In the first case (*integration*), both investors have strictly positive holdings in both assets. In the second case (*partial segmentation*), the short-term investor only invests in the asset with low transaction costs (i.e., his optimal position in the high-cost asset is equal to zero, since the transaction costs prevent this investor from buying or selling the asset).

2.1 Case 0, *homogeneity*: both investors have the same horizon

If all investors have the same one-period horizon, we obtain an i.i.d. version of Acharya and Pedersen's liquidity CAPM. This can be seen as follows. The optimal demand of the investor is

$$y_1 = \frac{1}{A_1}\text{diag}(P_t)^{-1}\text{Var}(R_{t+1} - c_{t+1})^{-1}(\mathbb{E}[R_{t+1} - c_{t+1}] - R_f) \tag{4}$$

where R_{t+1} denotes the vector of gross asset returns, $R_{i,t+1} = (D_{i,t+1} + P_{i,t+1})/P_{it}$, and c the percentage costs, $c_{it} = C_{it}/P_{it}$. Solving the equilibrium condition $2y_{1t} = S$, with two investors entering the market each period, gives

$$\mathbb{E}[R_{t+1}] - R_f = \mathbb{E}[c_{t+1}] + \frac{A_1}{2}\widetilde{S}'\iota\text{Cov}(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}) \tag{5}$$

where $\widetilde{S} = \text{diag}(P_t)S$ is dollar supply (which is constant over time given that prices are constant over time), and where $R^m = \widetilde{S}'R/\widetilde{S}'\iota$.

Alternatively, if all investors are two-period investors (with a new two-period investor entering the market each period), the Appendix shows that the optimal demand is

$$y_2 = \frac{1}{A_2}\text{diag}(P_t)^{-1}\text{Var}_t(R_f R_{t+1} + R_{t+2} - c_{t+2})^{-1}(\mathbb{E}[R_f R_{t+1} + R_{t+2} - R_f - c_{t+2}] - R_f^2) \tag{6}$$

Using the equality $\text{Var}(R_f R_{t+1} + R_{t+2} - c_{t+2}) = \text{Var}(R_f R_{t+1}) + \text{Var}(R_{t+2} - c_{t+2})$, valid in our i.i.d. setting, equilibrium expected returns,

$$\begin{aligned}
\mathbb{E}[R_{t+1}] - R_f \quad &= \quad \frac{1}{1+R_f}\mathbb{E}[c_{t+1}] + \qquad\qquad (7)\\
&\quad \frac{A_2}{2(1+R_f)}\widetilde{S}'\iota(R_f^2\text{Cov}(R_{t+1}, R^m_{t+1}) + \text{Cov}(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1})).
\end{aligned}$$

Comparing equilibrium expected returns in equation (7) to the one-period case in equation (5), we observe that, due to the longer horizon, the coefficient on expected liquidity decreases from 1 to $1/(1+R_f)$ (and we have $1/(1+R_f) \leq 0.5$ if $R_f \geq 1$). In addition, the role of liquidity risk is smaller, given that the first-period return is not affected by liquidity costs.

2.2 Case 1, *integration*: both investors invest in both assets

We now turn to the case with heterogeneous horizons. We first consider the case where the optimal demands y_1 and y_2 are strictly positive, so that both investor types have positive holdings of both assets. This corresponds to a situation where the liquidity costs are sufficiently small. Using the market clearing condition (3) and optimal demands in (4) and (6), the Appendix derives the equilibrium expected returns

$$\mathbb{E}[R] - R_f = \Phi\mathbb{E}[c] + (\lambda_1 + \lambda_2)\mathrm{Cov}(R - c, R^m - c^m) + \lambda_2\mathrm{Cov}(R, R^m), \tag{8}$$

where

$$\Phi = \gamma_1 I - \gamma_2(r_f^2 \mathrm{Var}(R-c)^{-1}\mathrm{Var}(R) + I)^{-1} \tag{9}$$

and where we suppress all time subscripts given the i.i.d. nature of the equilibrium. The Appendix shows that λ_1, λ_2, γ_1, and γ_2 are scalars that are functions of the risk aversion levels and covariance matrices of returns and costs and that $\lambda_1 > 0$, $\lambda_2 > 0$, $\gamma_1 > 0$ and $\gamma_2 > 0$.

Equation (8) shows that the risk premium term is a mixture of the net-of-cost covariance and the regular CAPM covariance. This is the expected outcome of the presence of long-term investors, who care more about regular market risk and relatively less about liquidity risk. The weights on these two covariances depend, amongst others, on the risk aversion of the one-period and two-period investors. For example, in the Appendix we show that as the long-term investors become relatively more risk averse (or short-term agents become less risk averse), the liquidity risk covariance becomes more important relative to the market covariance (formally $\frac{\lambda_1+\lambda_2}{(\lambda_1+\lambda_2)+\lambda_2} \uparrow$ as $A_2/A_1 \uparrow$). This makes intuitive sense. When long-term investors are more risk averse (or short-term investors less risk averse), the long-term investors hold a relatively smaller fraction of the supply in equilibrium, and hence the demand of the short-term investors is the predominant factor determining expected returns. Since short-term investors care more about liquidity risk, the liquidity risk premium becomes relatively more important in equilibrium.

Next, we turn to the loading on expected liquidity, as defined by the matrix Φ in equation (9). This term provides two important insights. First, if there is no liquidity risk, i.e. $\mathrm{Var}(c) = 0$, we obtain $\mathrm{Var}(R-c)^{-1}\mathrm{Var}(R) = I$, and the effect of expected liquidity is the same for both assets and equal to $\gamma_1 - \frac{1}{1+R_f^2}\gamma_2$. The Appendix shows that

$\gamma_1 - \frac{1}{1+R_f^2}\gamma_2 < 1$, so that the coefficient on expected liquidity is smaller than 1 (which is the coefficient in the baseline one-period model in Section 2.1), due to the presence of two-period investors who care less about expected liquidity.

If $\mathrm{Var}(c) > 0$, the coefficients on expected liquidity may vary across the two assets due to covariance between costs and returns. This reflects the fact that short-term investors care more about liquidity risk covariances than long-term investors. For example, suppose the second asset has no liquidity risk ($\mathrm{Var}(c_2) = 0$), while the first asset has liquidity risk ($\mathrm{Var}(c_1) > 0$). In addition, suppose that for asset 1, $\mathrm{Cov}(R_1, c_1) < 0$, so that $\mathrm{Var}(R_1 - c_1) > \mathrm{Var}(R_1)$. The first asset is then less attractive for short-term investors since high costs coincide with low returns, while this liquidity risk is less important for long-term investors. Consider for simplicity the case that both $\mathrm{Var}(R)$ and $\mathrm{Var}(R - c)$ are diagonal. It then follows directly that the expected liquidity matrix Φ is diagonal, with the coefficient on expected liquidity for asset 2 equal to $\Phi_{2,2} = \gamma_1 - \frac{1}{1+R_f^2}\gamma_2$, while for asset 1 we obtain

$$\Phi_{1,1} = \gamma_1 - \frac{1}{1 + R_f^2 \mathrm{Var}(R_1)/\mathrm{Var}(R_1 - c_1)}\gamma_2 < \Phi_{2,2}. \tag{10}$$

It thus follows that for asset 1 the coefficient on expected liquidity is smaller than for asset 2: since the first asset is relatively less attractive for short-term investors, it will be held in equilibrium mostly by long-term investors that care less about liquidity, leading to a smaller coefficient for the expected liquidity effect. Hence, we see that in the cross-section higher liquidity risk may actually lead to a smaller expected liquidity premium.

2.3 Case 2, *partial segmentation*: only the long-term investor invests in both assets

We now turn to the case where the costs on one asset are so high that, in equilibrium, the one-period investors optimally invest only in the low-cost asset and have a zero position in the high-cost asset. Suppose asset 1 has higher costs than asset 2. In fact, costs are so high that, in equilibrium, $y_1(1) < 0$ (and $y_1(2) > 0$). This means that short-term investors do not want to buy asset 1. Of course, it is still possible that the investor wants to short asset 1, but this is unlikely given the high transaction costs. To see this formally, if the optimal position in asset 1 were negative (and positive for asset 2), the optimal portfolio would be

$$z_1 = \frac{1}{A_1}\mathrm{diag}(P_t)^{-1}\mathrm{Var}(R_{t+1} - \delta_1 c_{t+1})^{-1}(\mathbb{E}[R_{t+1} - \delta_1 c_{t+1}] - R_f) \tag{11}$$

where $\delta_1 = \text{diag}(-1, 1)$, hence δ_1 is a diagonal matrix with elements equal to 1 if the investor is long in the respective asset, and -1 if he is short (see Bongaerts, De Jong, and Driessen, 2011). If $z_1(1) < 0$, this is indeed the solution to the optimal portfolio rule, but this is unlikely if costs are high for this asset. In turn, if $z_1(1) > 0$ and $y_1(1) < 0$, it is optimal for the investor to have a zero position in asset 1. We thus focus here on the case in which costs are high enough so that the investor optimally has a zero position in asset 1.

This simplifies the optimal allocation of agent 1,

$$\widetilde{y}_1 = \begin{pmatrix} 0 \\ \frac{1}{A_1}\text{Var}(R_2 - c_2)^{-1}(\mathbb{E}[R_2 - c_2] - R_f) \end{pmatrix} \tag{12}$$

The demand of agent 2 is unchanged from above. The Appendix derives the equilibrium expected returns,

$$\begin{aligned} \mathbb{E}[R] - R_f &= \Lambda_1^{-1}\Lambda_2\mathbb{E}[c] + \\ &\quad \phi_1(R_f^2\text{Cov}(R, R^m) + \text{Cov}(R - c, R^m - c^m)) + \begin{pmatrix} \phi_2\widetilde{S}_1 \\ 0 \end{pmatrix}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \Lambda_1 &= A_1^{-1}\begin{pmatrix} 0 & 0 \\ 0 & \text{Var}(R_2 - c_2)^{-1} \end{pmatrix} + (1 + R_f)A_2^{-1}(R_f^2\text{Var}(R) + \text{Var}(R - c))^{-1}) \\ \Lambda_2 &= A_1^{-1}\begin{pmatrix} 0 & 0 \\ 0 & \text{Var}(R_2 - c_2)^{-1} \end{pmatrix} + A_2^{-1}(R_f^2\text{Var}(R) + \text{Var}(R - c))^{-1}) \end{aligned}$$

and where the parameters ϕ_i are scalars, and the Appendix shows that $\phi_1 > 0$ and $\phi_2 > 0$.

This shows that we get two deviations from the case of homogeneity of investors. First, the effect of the liquidity risk covariances is smaller (relative to the market covariance). Second, we get a segmentation result. The expected return on the first asset is higher by an extra term that reflects the fact that only a subset of the investors holds this asset. This is in the spirit of the international asset pricing literature (e.g. de Jong and de Roon, 2005), where segmentation also leads to additional effects on expected returns, that depend on the size of the supply of the segmented asset ($\widetilde{S}_1$). The Appendix shows that the coefficient on the segmented supply, ϕ_2, increases with the risk-aversion of long-term investors A_2, since these are the investors that have to hold this asset in equilibrium.

In Figure 1 we illustrate a numerical example about the segmentation effect ϕ_2. using variation in the risk aversion of the long-term investor (A_2) and in the degree of idiosyncratic risk of the illiquid asset. More specifically, we assume $R_1 = R_2 + \varepsilon$, where ε captures the degree of idiosyncratic risk (relative to the liquid asset). Figure 1 shows that higher variance of ε implies a higher segmentation coefficient ϕ_2. This effect makes intuitive sense. As the illiquid asset is only held by the long-term investor, its idiosyncratic risk is priced due to imperfect risk sharing. Furthermore, Figure 1 also confirms our analytical result that ϕ_2 depends positively on the risk aversion of the long-term investor.

Then we turn to the expected liquidity coefficients, $\Lambda_1^{-1}\Lambda_2$. To obtain some intuition, consider the case where there is zero covariance between returns on the two assets (both before and after costs). In this case, in the Appendix we show that

$$\Lambda_1^{-1}\Lambda_2 = \begin{pmatrix} \frac{1}{1+R_f} & 0 \\ 0 & \frac{1+\eta}{1+R_f+\eta} \end{pmatrix} \tag{14}$$

with $\eta = \frac{A_2}{A_1}\frac{\mathrm{Var}(R_2-c_2)+R_f^2\mathrm{Var}(R_2)}{\mathrm{Var}(R_2-c_2)} > 0$. This setup reveals several interesting effects. First, we see that the coefficient on expected liquidity is larger for the low-cost asset 2. Thus if we graph the relation between expected returns and expected costs, we get a piecewise linear and concave relation, like in Amihud and Mendelson (1986). Intuitively, when costs on an asset are too high, short-term investors drop out and only long-term investors invest in the asset. Given that long-term investors care less about liquidity, the effect of liquidity on expected returns is smaller. In this two-period, two-asset example, the coefficient is equal to $1/(1+R_f) \leq 1/2$ for asset 1, because the holding period of all investors holding this asset is two periods. Hence the expected liquidity coefficient for asset 1 is the same as in the homogenous two-period model (equation (7)).

Finally, note that Amihud and Mendelson (1986) find that long-term investors only invest in high-cost assets, and not in the low-cost assets. This is because they assume risk-neutrality. In our model with risk averse agents, long-term investors will diversify and invest in low-cost assets as well.

3 Full Model with Multiple Assets and Horizons

The baseline model with two assets and two investors can be generalized to a setting with many assets and many investors with heterogenous investment horizons. This general framework is more realistic and, most importantly, is suitable for empirical estimation. We now list the assumptions in our general setting:

- There are K assets, with asset i paying each period a dividend D_{it}. Selling the asset costs C_{it}. Transaction costs and dividends are i.i.d. in order to obtain a stationary equilibrium.
- We model N classes of investors, indexed by $j = 1, 2, \ldots, N$, with distinct investment horizons $h_1, h_2, \ldots, h_N$.
- Investors have mean-variance utility over terminal wealth with risk aversion A_j for type j.
- We have an overlapping generations (OLG) setup. Each period, a fixed quantity $Q_j > 0$ of type j investors enters the market and invests in some or all of the K assets.
- Investors with horizon h_j only trade when they enter the market and at their terminal date, hence they do not rebalance their portfolio at intermediate dates.

As before, we let R_{t+1} denote the $K \times 1$ vector of gross asset returns, and c_{t+1} the $K \times 1$ vector of percentage costs. The Appendix shows that under these assumptions we obtain a stationary equilibrium with the following equilibrium expected returns

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right] - R_f = &\left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j \mathbb{E}\left[c_{t+1}\right] \\ &+ \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j\right)^{-1} \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right),\end{aligned} \tag{15}$$

where $\gamma_j = Q_j/(A_j \widetilde{S}'\iota)$ (where supply $\widetilde{S}_t$ is constant over time), $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j - k}$ and

$$V_j = h_j \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j - k} R_{t+k} - c_{t+h_j}\right)^{-1}.$$

We compute the long-term covariance matrices using the i.i.d. assumption. The details are given in the Appendix. Regarding the interpretation of V_j, we consider the case of diagonal covariance matrices. We may then write the ℓ-th diagonal element of V_j, that corresponds to asset ℓ, as

$$V_{\ell,j} = \frac{\operatorname{Var}\left(R_{t+1} - c_{t+1}\right)/1}{\operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j - k} R_{t+k} - c_{t+h_j}\right)/h_j}.$$

That is, we have a variance ratio between the return variances corresponding to horizons of 1 month and h_j months.

Equation (15) corresponds to the case of integration described in the basic version of the model in Section 2.2. We now consider the alternative case of segmentation, i.e. the possibility that some classes of investors do not hold some assets because the associated trading costs are too high relative to the expected return over the investment horizon. To this end, we introduce sets D_j $(j = 1, \ldots, N)$ that are subsets of $1, \ldots, K$, where K is the number of tradable assets. The set D_j represents the set of assets that investors j will invest in in equilibrium. As discussed in Section 2, a short-horizon investor will endogenously avoid investing in assets for which the associated transaction costs are too large. The sets D_j thus depend on the level of transaction costs of the assets. Note that without loss of generality we may assume that for some j it holds that $D_j = \{1, \ldots, K\}$. That is, there are some investors who may invest in all assets.

We denote by A_{D_j} the $|D_j| \times |D_j|$ matrix containing only the rows and columns of A that are in D_j. In addition, we write $A^{-1}_{D_j,p}$ for the inverse of A_{D_j} with zeros inserted at the locations where rows and columns of A were removed. The Appendix shows that, in this setting, equilibrium excess returns are defined by the following equation:

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right] - R_f = & \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+h_j}\right] \\ & + \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right),\end{aligned} \tag{16}$$

where $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$, $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j - k}$, and

$$V_j^{D_j} = h_j \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)^{-1}_{D_j,p}.$$

Regarding the interpretation, the dependence on the parameters remains qualitatively the same. The only difference lies in the sensitivity of different classes of investors to the variances and covariances of returns and costs. To see this in more detail, consider the case of diagonal covariance matrices. In that case we obtain for asset ℓ that

$$\begin{aligned}\mathbb{E}\left[R_{\ell,t+1}\right]-R_f = {} & \frac{\sum_{j=1}^{N}\gamma_j V_{\ell,j}^{D_j}}{\sum_{j=1}^{N}\gamma_j\rho_j V_{\ell,j}^{D_j}}\mathbb{E}\left[c_{\ell,t+1}\right] \\ & +\frac{1}{\sum_{j=1}^{N}\gamma_j\rho_j V_{\ell,j}^{D_j}}\operatorname{Cov}\left(R_{\ell,t+1}-c_{\ell,t+1},R_{t+1}^{m}-c_{t+1}^{m}\right),\end{aligned} \tag{17}$$

where

$$V_{\ell,j}^{D_j}=\mathbf{1}_{\{\ell\in D_j\}}\frac{h_j\operatorname{Var}\left(R_{\ell,t+1}-c_{\ell,t+1}\right)}{\operatorname{Var}\left(\sum_{k=1}^{h_j}R_f^{h_j-k}R_{\ell,t+k}-c_{\ell,t+h_j}\right)}.$$

This shows the effect of segmentation in a simple way. The coefficient on the covariance term $(1/\sum_{j=1}^{N}\gamma_j\rho_j V_{\ell,j}^{D_j})$ is larger for assets that are not held by all investors as $V_{\ell,j}^{D_j}$ will be zero for investors j that do not hold this asset. This additional effect generates the segmentation premium. One can also derive easily that the coefficient on expected liquidity is smaller for the segmented assets (if the variances of the different assets are of similar size). Both results are in line with the simple example in Section 2.

The full model derived in this section can be directly related to the liquidity CAPM of Acharya and Pedersen (2005). It is instructive to consider the case where $N=1$ and $h_1=1$, so that there is just one class consisting of one-month investors and no segmentation. For ease of comparison, we write it in beta form. This gives

$$\mathbb{E}\left[R_{t+1}\right]-R_f=\mathbb{E}\left[c_{t+1}\right]+\frac{\operatorname{Var}\left(R_{t+1}^{m}-c_{t+1}^{m}\right)}{\gamma_1}\frac{\operatorname{Cov}\left(R_{t+1}-c_{t+1},R_{t+1}^{m}-c_{t+1}^{m}\right)}{\operatorname{Var}\left(R_{t+1}^{m}-c_{t+1}^{m}\right)}, \tag{18}$$

which is an i.i.d. version of the equilibrium relation found in Acharya and Pedersen (2005). Empirically, we follow AP and allow for a slope coefficient κ on $\mathbb{E}\left[c_{t+1}\right]$, although formally the AP model implies a coefficient on expected liquidity equal to one.

4 Empirical Methodology

The theoretical setup developed in the previous section generates two liquidity-based asset pricing models, with either integration or segmentation. In this section, we explain how these models can be estimated. We also explore the economic mechanism that allows the identification of the parameters. We then discuss alternative approaches for a robust computation of standard errors.

4.1 GMM Estimation

We use a Generalized Method of Moments (GMM) methodology to estimate the equilibrium condition in the general case, as defined by equation (15). The key estimated parameters are the risk aversion coefficients of the different classes of investors or, more generally, risk aversion divided by the number of agents per holding period. More specifically, we estimate $\gamma_j = Q_j/(A_j \widetilde{S}'\iota)$. Based on the asset pricing model in case of segmentation (which has the integration model as special case), we can define the vector of pricing errors of all assets $g(\psi, \gamma)$ as

$$g(\psi,\gamma) = \mathbb{E}\left[R_{t+1}\right] - R_f - \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+1}\right] \\ - \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right), \tag{19}$$

where γ is the vector of parameters, and ψ is a vector containing the underlying expectations and covariances that enter the pricing errors. Specifically, ψ contains all expected returns, expected costs, covariances entering the $V_j^{D_j}$ matrices, and the covariances with the market return. In a first step, we estimate all elements of ψ by their sample moments. In a second step, we then perform a first-step GMM estimation of γ, using an identity weighting matrix across all assets. We thus minimize the sum of squared pricing errors over γ,

$$\min_{\gamma} \; g(\widehat{\psi},\gamma)' g(\widehat{\psi},\gamma). \tag{20}$$

In the Appendix we derive the asymptotic covariance matrix of this GMM estimator, which incorporates the estimation error in the underlying sample moments. The resulting standard errors take into account the estimation error in all sample moments in ψ, in line with the approach of Shanken (1992).

4.2 Identification

To gain insight into the economic mechanism that allow the identification of the parameters, it is useful to illustrate some comparative statics results. Specifically, a change in γ_i means that the horizon h_i investors become either more numerous, or less risk averse, or both. In the segmented model (which has the integrated model as special case), the effect of such a change on expected returns is given by

$$\frac{\partial\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)}{\partial\gamma_i}=\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j^{D_j}\right)^{-1}V_i^{D_i}\left(\mathbb{E}\left[c_{t+1}\right]-\rho_i\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)\right). \tag{21}$$

We see that there are two opposing effects. The first effect is an increase in the risk premium due to the impact of liquidity. The second effect is the increased risk sharing, which leads to a decrease in the risk premium proportional to the original risk premium. As $\rho_i > 1$ for all i (assuming that $R_f > 1$), ρ_i increases with h_i, and the level of liquidity is comparable in magnitude to the risk premium (see Table 1), we see that for the longer horizon investors, the risk sharing effect tends to dominate the liquidity effect. This is what we would expect: they incur the liquidity cost less frequently than the short term investors and hence care less about liquidity. As the matrix premultiplying the difference between the liquidity cost and the scaled risk premium can reverse the sign, we see that hedging considerations could also play a different role for short-term versus long-term investors.

To obtain more insight, we consider the case of diagonal covariance matrices, in which case we obtain

$$\frac{\partial\left(\mathbb{E}\left[R_{\ell,t+1}\right]-R_f\right)}{\partial\gamma_i}=\frac{V_{\ell,i}^{D_i}}{\sum_{j=1}^{N}\gamma_j\rho_j V_{\ell,j}^{D_j}}\left(\mathbb{E}\left[c_{\ell,t+1}\right]-\rho_i\left(\mathbb{E}\left[R_{\ell,t+1}\right]-R_f\right)\right). \tag{22}$$

Since the factor multiplying the cost and excess return terms is positive, we find that the sign of the comparative statics for γ_i is completely determined by the size of the scaled risk premium compared to the transaction cost. An additional effect is that in the case of segmentation, a change in risk aversion of investor class i has a zero effect on the expected return of asset ℓ if investors i do not invest in that asset. With non-diagonal covariance matrixes, this is not necessarily the case. The cross-asset hedging effect can have an indirect impact on the assets in which investors i do not invest through the allocations of other investors who do invest in those assets.

4.3 Bootstrap Standard Errors

To check the robustness of our results, we employ two kinds of bootstrap tests. For each test we generate bootstrap samples by resampling the data and then carrying out the first step of the estimation procedure so that we have estimates for the various moments that arise in the vector of pricing errors. The first test is a bootstrap t-test based on the bootstrap estimate of the standard error. The second test is based on the quantiles of the distribution of the bootstrap estimates. The latter approach is known as the percentile method. Both of these tests do not provide asymptotic refinement, but they have the advantage that they do not require direct computation of asymptotically consistent standard errors.

Test based on bootstrap standard error

Let B be the number of bootstrap replications, yielding estimates $\widehat{\gamma}_1^*, \ldots, \widehat{\gamma}_B^*$. For testing $\gamma_i = \gamma_{i,0}$ we compute

$$t_i = \frac{\widehat{\gamma}_i - \gamma_{i,0}}{s_{\widehat{\gamma}_i,\text{boot}}}, \tag{23}$$

where

$$s^2_{\widehat{\gamma}_i,\text{boot}} = \frac{1}{B-1}\sum_{b=1}^{B}(\widehat{\gamma}^*_{i,b} - \bar{\widehat{\gamma}}^*_i)^2. \tag{24}$$

The t-statistic is compared to critical values from the standard normal distribution.

Percentile method test

For this test we find the 2.5% and 97.5% quantiles of the bootstrap estimates $\widehat{\gamma}_1^*, \ldots, \widehat{\gamma}_B^*$. If $\gamma_{i,0}$ falls outside these quantiles, the null hypothesis that $\gamma_i = \gamma_{i,0}$ is rejected.

5 Data

We largely follow Acharya and Pedersen (2005) in our data selection and construction. We use daily stock return and volume data from CRSP from 1964 until 2009 for all common shares listed on NYSE and AMEX. As our empirical measures of liquidity rely on volume, we do not include Nasdaq since the volume data includes interdealer trades (and only starts in 1982). Overall, we consider a number of stocks ranging from 1056 to 3358, depending on the month. To correct for survivorship bias, we adjust the returns for stock delisting (see Shumway, 1997; Acharya and Pedersen, 2005). Some descriptive statistics are given in Table 1.

The relative illiquidity cost is computed as in Acharya and Pedersen (2005). The starting point is the Amihud (2002) illiquidity measure, which is defined as

$$ILLIQ_t^j = \frac{1}{D_t^j}\sum_{d=1}^{D_t^j}\frac{|R_{td}^j|}{V_{td}^j} \tag{25}$$

for stock j in month t, where D_t^j denotes the number of observations available in month t, R_{td}^j and V_{td}^j denote the volume in millions of dollars on day d in month t, respectively.

We follow Acharya and Pedersen (2005) and define a normalized measure of illiquidity that deals with non-stationarity and is a direct measure of trading costs, consistent with the model specification. The normalized illiquidity measure can be interpreted as the dollar cost per dollar invested and is defined by

$$c_t^j = \min\left\{0.25 + 0.30 ILLIQ_t^j P_{t-1}^m, 30.00\right\}, \tag{26}$$

where P_{t-1}^m is equal to the market capitalization of the market portfolio at the end of month $t-1$ divided by the value at the end of July 1962. The product with P_{t-1}^m makes the cost series c_t^j relatively stationary and the coefficients 0.30 and 0.25 are chosen as in Acharya and Pedersen (2005) to match approximately the level and variance of c_t^j for the size portfolios to those of the effective half spread reported by Chalmers and Kadlec (1998). The value of normalized liquidity c_t^j is capped at 30% to make sure the empirical results are not driven by outliers.

Turnover is computed as dollar volume divided by market capitalization. As the monthly turnover series contains some outliers (e.g. exchange traded funds with relatively low market capitalization), we censor the turnover series at 500%. This affects 1023 data points.

We obtain the book-to-market ratio using fiscal year-end balance sheet data from COMPUSTAT in the same manner as Ang and Chen (2002). They follow Fama and French (1993) in defining the book value of a firm as the sum of common stockholders' equity, deferred taxes, and investment credit minus the book value of preferred stocks. The ratio is obtained by dividing the book value by the fiscal year-end market value.

We construct the market portfolio on a monthly basis and only use stocks that have a price on the first trading day of the corresponding month between \$5 and \$1000. We include only stocks that have at least 15 observations of return and volume during the month.

We construct 25 illiquidity portfolios, 25 illiquidity variation portfolios, and 25 book-to-market and size portfolios, similarly to Acharya and Pedersen (2005). The portfolios are formed on an annual basis. For these portfolios, we require again for the stock price on the first trading day of the corresponding month to be between \$5 and \$1000. For the illiquidity and illiquidity variation portfolios, we require to have at least 100 observations of the illiquidity measure in the previous year.

Table 1 shows the estimated average costs and average returns across the 25 illiquidity portfolios. The values correspond closely to those found in Table 1 of Acharya and Pedersen (2005). Most importantly, we see that average returns tend to be higher for illiquid assets. Also, the table shows that returns on more illiquid portfolios are more volatile. This holds for returns net of costs as well. The returns (net of costs) on more illiquid portfolios tend to co-move more strongly with market returns (also net of costs).

6 Empirical Results

In this section, we take the model to the data. First, we estimate the parameters of the models for both the integrated and the segmented case. We also explore the implications of the estimates for the importance of the different components of expected returns. We then study the robustness of our results to the choice of the investor horizon, to the extent of segmentation, and to pricing different set of portfolios.

6.1 Estimation of the Integrated and Segmented Models

We estimate the parameters of the equilibrium relations corresponding to the integrated case of equation (15) and the segmented case of equation (16) for the sample period 1964–2009 using the GMM methodology outlined in Section 4.1. The sample consists of 25 portfolios of stocks listed on NYSE and AMEX, sorted on illiquidity. In the next subsection, we also estimate the model for 25 illiquidity-variation portfolios and 25 Book/Market-by-Size portfolios.

Our benchmark estimation hinges on two classes of investors.[3] The first class (short horizon) has an investment horizon h_1 of one month, the second class (long horizon) has an investment horizon h_2 of 240 months (20 years). We estimate the parameters $\gamma_j = Q_j/(A_j\widetilde{S}'\iota)$, and the constant term α. We denote the model without the constant term as specification 1 (INT) and the model including the constant term as specification 2 (INT). The interpretation of the estimated parameters can offer interesting insights. In fact, the parameters γ_j are equal to the ratio of the number Q_j of investors with horizon h_j entering the market each period to their risk aversion A_j, scaled by the inverse of the total market capitalization $\widetilde{S}'\iota$.

We also estimate a segmented version of the model, where the one-month investors invest only in the 19 most liquid portfolios. We choose this threshold by maximizing the cross-sectional R^2 across all possible thresholds. A segmentation threshold equal to 19 is in line with Table 1. For the six least liquid portfolios, the expected costs are roughly 2 to 9 times higher than the expected monthly excess return. As the one-month investors

[3]Adding a third class of investors does not yield substantial empirical improvement. In the case of integration, the coefficient of the additional investment horizon class goes to zero, effectively removing the impact of the new class of investors. In the case of segmentation, the coefficient does not necessarily go to zero, but the R^2 remains essentially unchanged, with little gain in terms of explanatory power.

incur the costs each period, these assets can be seen as prohibitively costly. We denote the segmented model specifications as 3 (SEG) (without the constant term) and 4 (SEG) (including the constant term).

We compare our model with a baseline Acharya and Pedersen (2005) version (18), where $N = 1$ and $h_1 = 1$. In this case, we allow for a coefficient κ on the cost term. This coefficient is necessary in Acharya and Pedersen (2005) to obtain sensible estimates and to provide a fair comparison to the heterogeneous horizon specification. We denote these specifications 5 (AP) (without the constant term) and 6 (AP) (with the constant term included). We also include the conditional version of the Acharya and Pedersen (2005) model, using AR(2) residuals to compute the covariances. We denote by 7 (APc) and 8 (APc) these specifications without and with a constant term, respectively.

Table 2 shows the results for the illiquidity portfolios. We find that the first specification of the segmented model 3 (SEG), without a constant term, improves the R^2 obtained by Acharya and Pedersen (2005) by about 20%. Importantly, this improvement is achieved retaining the parsimony of the original model – both models depend on two parameters. The fit is graphically displayed in Figure 2. The graphs indicate that accounting for segmentation leads to smaller pricing errors in the case of the more illiquid portfolios. The descriptives in Table 1 give an additional indication why this is so. The risk premium levels off after portfolio 19, but the cost term keeps rising. The AP model implies a linear relation between costs and expected returns, and thus has difficulty fitting the cross-section of liquid versus illiquid portfolios. Our model with segmentation reduces the impact of the expected liquidity term on the least liquid portfolios, thus improving the fit in those cases (see also Figure 2).

The estimates in Table 2 also yield some interesting economic implications. For example, if we assume for simplicity that risk aversion is constant across investor classes (i.e., $A_1 = A_2$), we can make inferences about the number of investors in each class. More specifically, we examine the ratio $(h_1\gamma_1)/(h_2\gamma_2) = (h_1Q_1)/(h_2Q_2)$.[4] The results for the first and second specification (non-segmented model without and with constant term) both indicate that there are about 20 times as many long horizon investors as there are short horizon investors. If we use the segmented version, we see that the difference is smaller. For specifications three and four (segmented without and with constant term), we see that the estimates imply that there are respectively 2.8 and 3.5 times as many long horizon investors as there are short horizon investors. What we do see is that the direction of the difference is consistent across all specifications: the number of long term

[4]As Q_j investors with horizon h_j enter each period, at each point in time the total number of type-j investors equals h_jQ_j. Also note that including $\widetilde{S}'\iota$ in the γ_j does not influence our comparison.

investors is larger than the number of short term investors. Of course, an alternative interpretation would be that the long-term investors are less risk averse than short-term investors (if one assumes that there are as many long-term investors as short-term investors ($h_1 Q_1 = h_2 Q_2$)).

We use the estimated model to decompose expected returns in the two fundamental terms of the equilibrium equation. We depict these decomposition for different portfolios and different model specifications in Figure 3 and Figure 4. We notice that the covariance term provides the largest contribution to the expected excess returns, in most cases by far. The impact of the cost term increases with the level of illiquidity, as expected. In the segmented case, we observe a drop in the impact of the cost term after portfolio 20, which shows that the level of illiquidity is indeed of lesser importance to long horizon investors. Moreover, the total model-implied expected return drops when we move from portfolio 19 to 20. This drop indicates that the decrease for the lower importance of illiquidity more than offsets the increase in the risk premium due to lower risk sharing (the segmentation premium).

The impact of segmentation on risk premia is shown in Figure 5. In the figure we see risk premia for (i) the observed returns; (ii) the integrated model; (iii) the segmented model with the coefficients from the integrated version; and (iv) the segmented model. As the total effect is composed of an liquidity level effect and a covariance effect, we show these in Figure 6. The decomposition indicates clearly the source of the changes. The impact of the liquidity level is increased, while the impact of the covariance is decreased. It is noteworthy that, compared to the integrated case, the liquidity level impact is much less pronounced for the most illiquid portfolios due to the segmentation effect. The graph shows that the increase in importance of the cost term is caused by the segmented estimates, which imply a much greater number of short-term investors (or much more risk averse long-term investors). This makes sense, as the short-term investors are the ones who care about liquidity the most.

The comparative statics for each model parameter (see (21) for the analytical expression) are shown in Figure 7. The graphs show the impact on the risk premium of an increase in the quantity of a certain class of investors, a decrease in their risk aversion, or both. For long-term investors, the effect of an increase in γ_2 is always negative. This is consistent with the theoretical result that the risk sharing effect dominates the liquidity effect for long term investors (absent hedging considerations). In other words, the finding confirms empirically that long term investors are less concerned about liquidity. For the short term investors, we even see that the effect is positive for the most illiquid portfolios. Here we have another indication that they are very concerned about liquidity to the extent that the compensation that they require crowds out the risk sharing benefit. This also further motivates our choice of the segmentation structure, where the short term investors do not invest in the six most illiquid portfolios. In sum, these comparative statics show that γ_1 and γ_2 have quite different effects on expected returns, which shows that these parameters are well identified empirically.

6.2 Robustness across horizons, segmentation, and portfolios

To test the sensitivity of model performance to the choice of the long term investor horizon, we compute the R^2 for $h_2 = 30, 60, 120, 240, 480$. The results are given in Table 3, Panel A, and show that the explanatory power of the model is relatively insensitive to the choice of horizon. In addition, the coefficients do not vary much across the different choices. The performance is also robust to varying h_1, the short term investor horizon, as long as it does not grow too large. More specifically, $h_1 = 6$ yields results similar to the case where $h_1 = 1$, but for $h_1 = 30$ the performance deteriorates. This shows that including a short horizon class is material to the explanatory power of the model. On the other hand, performance is not harmed if the short horizon is increased to the point where deviations from the i.i.d. assumption should not be too large. In unreported results, we find that the time-series persistence in liquidity decreases with horizon: there is much less persistence in liquidity at a semi-annual frequency than at a monthly frequency.

Table 3, Panel B, shows the sensitivity of the R^2 to the choice of the segmentation threshold. The results show that the R^2 is not too sensitive to the choice, but has a clear maximum at 19 for both of the segmented specifications.

The bootstrap t-values, which are given in Table 4, Panel A, indicate that for the non-segmented model the t-values are in the same range as those for the Acharya and Pedersen (2005) specification. In the case of specification 4 (SEG), we obtain lower t-values. The bootstrap 95% confidence intervals are given in Table 4, Panel B.

Table 5, Panel A, shows similar improvements in the cross-sectional fit of our integrated and segmented model for the σ(illiquidity) portfolios. For the B/M-by-size portfolios, the improvement is slightly less pronounced (see Table 5, Panel B).

7 Conclusions

Heterogeneous investment horizons can have important asset pricing effects through the distinct role of liquidity, simply because trading costs only matter when trading actually takes place.

We model investors with heterogeneous investment horizons and stochastic illiquidity costs. Investors with longer investment horizons are less concerned about trading costs because they do not trade every period and earn larger risk premia that can potentially cover the higher trading costs of the more illiquid assets.

We find that in an equilibrium where all investors trade all assets (integration), the existence of investors with heterogeneous horizons, as opposed to homogeneity of horizons, reduces the importance of liquidity risk relative to the standard CAPM market risk. More specifically, the relative importance of the two risk premiums depends crucially on the risk aversion of long-term versus short-term investors. For example, if long-term investors are more risk-averse, liquidity risk becomes more important because short-term investors, who care more about liquidity, hold a relatively larger fraction of the asset supply in equilibrium.

In an equilibrium where short-term investors do not invest in some more illiquid assets (partial segmentation), our model shows that expected stock returns contain an additional segmentation premium reflecting the extent of supply for the segmented asset. The effect of expected liquidity on returns is naturally larger for the assets that are traded by all investors. In this setup, an increasing and concave relationship between expected returns and trading costs arises naturally, since excessive trading costs exclude the clientele that is more sensitive to liquidity costs.

Empirically, our heterogeneous-horizon asset pricing model fares better than a standard or a liquidity-adjusted CAPM, with a covariance liquidity risk term providing the largest contribution to expected returns. The segmented model delivers a more accurate cross-sectional fit and improved parameter significance.

Segmentation is an important feature of our model with heterogenous horizons. Specifically, when some investors do not invest in the most illiquid assets, the model delivers a much more accurate cross-sectional fit and improved parameter significance. Interestingly, our model empirical estimates can also be used to make inferences about the implied number of investors in each horizon class.

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8 Appendix

8.1 Two-period two-asset model

In this Appendix we provide derivations for several equations in Section 2.

Optimal demand two-period investors

Two-period agents solve

$$\begin{aligned} & \max_{y_2} \ \mathbb{E}[W_{t+2}] - \frac{1}{2}A_2 \mathrm{Var}(W_{t+2}) \qquad (27) \\ W_{t+2} &= (P_{t+2} + R_f D_{t+1} + D_{t+2} - C_{t+2})' y_2 + R_f^2 (e_2 - P_t' y_2) \end{aligned}$$

The solution to this problem is

$$\begin{aligned} y_2 = {} & \frac{1}{A_2} \mathrm{Var}(P_{t+2} + R_f D_{t+1} + D_{t+2} - C_{t+2})^{-1} \qquad (28) \\ & \times (\mathbb{E}[P_{t+2} + R_f D_{t+1} + D_{t+2} - C_{t+2}] - R_f^2 P_t) \end{aligned}$$

or

$$\begin{aligned} y_2 = {} & \frac{1}{A_2} \mathrm{diag}(P_t)^{-1} \mathrm{Var}(R_f R_{t+1} + R_{t+2} \frac{P_{t+1}}{P_t} - R_f - \frac{P_{t+2}}{P_t} c_{t+2})^{-1} \qquad (29) \\ & \times (\mathbb{E}[R_f R_{t+1} + R_{t+2} \frac{P_{t+1}}{P_t} - R_f - \frac{P_{t+2}}{P_t} c_{t+2}] - R_f^2) \end{aligned}$$

In equilibrium, prices P_t are constant over time, and we obtain equation (6).

Equilibrium in case of integration

Filling in the optimal demands into the equilibrium condition $2y_1 + y_2 = S - y_2$ and defining $\widetilde{S} = \mathrm{diag}(P_t)S$ gives

$$\begin{aligned} \widetilde{S} = {} & \frac{2}{A_1} \mathrm{Var}(R_{t+1} - c_{t+1})^{-1} (\mathbb{E}[R_{t+1} - c_{t+1}] - R_f) \qquad (30) \\ & + \frac{2}{A_2} \mathrm{Var}(R_f R_{t+1} + R_{t+2} - c_{t+2})^{-1} (\mathbb{E}[R_f (R_{t+1} - R_f) + R_{t+2} - c_{t+2}] - R_f) \end{aligned}$$

Dropping time subscripts, this equilibrium condition can be rewritten as

$$\begin{aligned} \mathbb{E}[R] - R_f = {} & \left(A_1^{-1} \mathrm{Var}(R-c)^{-1} + \frac{1+R_f}{A_2} (R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R-c))^{-1} \right)^{-1} \frac{\widetilde{S}}{2} \qquad (31) \\ & + \left(A_1^{-1} \mathrm{Var}(R-c)^{-1} + (1+R_f) A_2^{-1} (R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R-c))^{-1} \right)^{-1} \\ & \times \left(A_1^{-1} \mathrm{Var}(R-c)^{-1} + A_2^{-1} (R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R-c))^{-1} \right) \mathbb{E}[c] \end{aligned}$$

In the two-asset case, the term

$$\left(A_1^{-1}\mathrm{Var}(R-c)^{-1}+(1+R_f)A_2^{-1}(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c))^{-1})\right)^{-1}\widetilde{S}/2$$

can be written as

$$\frac{1}{d_0}(\frac{1}{d_1}+\frac{1}{d_2})\frac{\widetilde{S}'\iota}{2}\mathrm{Cov}(R-c,R^m-c^m)+\frac{1}{d_0d_2}\frac{\widetilde{S}'\iota}{2}\mathrm{Cov}(R,R^m)$$

using

$$\mathrm{Var}(R_{t+1}-c_{t+1})\widetilde{S}=\widetilde{S}'\iota\mathrm{Cov}(R_{t+1}-c_{t+1},R_{t+1}^m-c_{t+1}^m) \tag{32}$$

with $R^m=\widetilde{S}'R/\widetilde{S}'\iota$, and with

$$\begin{aligned} d_0 &= \det(A_1^{-1}\mathrm{Var}(R-c)^{-1}+(1+R_f)A_2^{-1}(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c))^{-1}) \qquad (33)\\ d_1 &= A_1\det(\mathrm{Var}(R-c))\\ d_2 &= \frac{1}{1+R_f}A_2\det(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c)) \end{aligned}$$

The equilibrium terms for $\mathrm{Cov}(R,R^m)$ and $\mathrm{Cov}(R-c,R^m-c^m)$ in equation (8) then follow directly with $\lambda_1=\frac{1}{d_0d_1}\frac{\widetilde{S}'\iota}{2}$ and $\lambda_2=\frac{1}{d_0d_2}\frac{\widetilde{S}'\iota}{2}$. Both λ_1 and λ_2 are positive because the determinants of covariance matrices are positive. It is easy to see that the liquidity premium, relative to the total risk premium, $\frac{\lambda_1+\lambda_2}{(\lambda_1+\lambda_2)+\lambda_2}$, can be written as $1-\frac{1}{d_2/d_1+2}$ which is increasing in A_2/A_1.

Next we turn to the expected liquidity effect

$$\begin{aligned}&\left(A_1^{-1}\mathrm{Var}(R-c)^{-1}+(1+R_f)A_2^{-1}(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c))^{-1})\right)^{-1}\\ &\quad\times\left(A_1^{-1}\mathrm{Var}(R-c)^{-1}+A_2^{-1}(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c))^{-1})\right)\mathbb{E}[c]\end{aligned}$$

which in the two-asset case can be rewritten as

$$\begin{aligned}I\times\mathbb{E}[c]-&\left(\frac{1}{d_0}(\frac{1}{d_1}+\frac{1}{d_2})\mathrm{Var}(R-c)+\frac{1}{d_0}\frac{1}{d_2}\mathrm{Var}(R)\right)\\ &\times\left(A_2^{-1}(R_f^2\mathrm{Var}(R)+\mathrm{Var}(R-c))^{-1})\right)\mathbb{E}[c]\end{aligned}$$

which can be simplified into

$$(1-\frac{1}{d_0d_2A_2})I\times\mathbb{E}[c]-(\frac{1}{d_0d_1A_2}-\frac{R_f^2-1}{d_0d_2A_2})(R_f^2\mathrm{Var}(R-c)^{-1}\mathrm{Var}(R)+I)^{-1}\mathbb{E}[c]$$

With $\gamma_1=1-\frac{1}{d_0d_2A_2}$ and $\gamma_2=(\frac{1}{d_0d_1A_2}-\frac{R_f^2-1}{d_0d_2A_2})$ we then obtain the expression for Φ in equation (8). Finally, we show that $\gamma_1 - \frac{1}{2}\gamma_2 < 1$. This inequality follows directly as all determinants d_i are positive.

Equilibrium in case of segmentation

In this case the equilibrium condition is

$$\Lambda_1 \left(\mathbb{E}[R] - R_f\right) = \widetilde{S}/2 + \Lambda_2 \mathbb{E}[c], \tag{34}$$

where

$$\Lambda_1 = \left(A_1^{-1} \begin{pmatrix} 0 & 0 \\ 0 & \mathrm{Var}(R_2 - c_2)^{-1} \end{pmatrix} + (1+R_f) A_2^{-1} (R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R-c))^{-1}) \right)$$

$$\Lambda_2 = \left(A_1^{-1} \begin{pmatrix} 0 & 0 \\ 0 & \mathrm{Var}(R_2 - c_2)^{-1} \end{pmatrix} + A_2^{-1} (R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R-c))^{-1}) \right)$$

First, the liquidity risk implications follow from working out the terms in the matrix Λ_1,

$$\Lambda_1^{-1} \widetilde{S}/2 = \phi_1 (R_f^2 \mathrm{Cov}(R, R^m) + \mathrm{Cov}(R - c, R^m - c^m)) + \begin{pmatrix} \phi_2 \widetilde{S}_1 \\ 0 \end{pmatrix} \tag{35}$$

where ϕ_1 and ϕ_2 are scalars, with

$$\phi_1 = \frac{2}{(1+R_f) A_2 d_3 d_4} \widetilde{S}' \iota > 0 \tag{36}$$

$$\phi_2 = \frac{1}{2 A_1 d_4 \mathrm{Var}(R_2 - c_2)} > 0 \tag{37}$$

$$d_3 = \det(R_f^2 \mathrm{Var}(R) + \mathrm{Var}(R - c)) \tag{38}$$

$$d_4 = \det(\Lambda_1) \tag{39}$$

From the definition of d_4 it directly follows that ϕ_2 is increasing in A_2. Then we turn to the expected liquidity coefficients, $\Lambda_1^{-1}\Lambda_2$. If the covariances are zero, we obtain(after some algebra)

$$\Lambda_1^{-1} \Lambda_2 = \begin{pmatrix} \frac{1}{1+R_f} & 0 \\ 0 & \frac{1+\eta}{1+R_f+\eta} \end{pmatrix} \tag{40}$$

with $\eta = \frac{A_2}{A_1} \frac{\mathrm{Var}(R_2 - c_2) + R_f^2 \mathrm{Var}(R_2)}{\mathrm{Var}(R_2 - c_2)} > 0$.

8.2 Multi-period multi-asset model

Equilibrium in case of integration

The equilibrium for equation (15) follows as a special case of the equilibrium in case of segmentation, which we discuss below. It corresponds to the case where $D_j = \{1, \ldots, K\}$ for $j = 1, \ldots, N$.

Equilibrium in case of segmentation

We start by introducing sets D_j ($j = 1, \ldots, N$) that represent the assets that investor j considers for his or her portfolio. We let the D_j be subsets of $1, \ldots, K$, where K is the number of assets. To motivate this setup, think of a short horizon investor, who is able to rule out certain assets a priori on the basis of their high transaction cost. Without loss of generality we assume that for some j it holds that $D_j = \{1, \ldots, K\}$.

To derive the results, we first need some notation. For a $K \times K$ matrix A, we denote by A_{D_j} the $|D_j| \times |D_j|$ (with $|\cdot|$ the cardinality of a set) matrix with the rows and columns that are elements of $\{1, \ldots, K\} \setminus D_j$ removed. As it will be used frequently, we also introduce the notation $A^{-1}_{D_j,p}$ for the inverse of A_{D_j} with zeros inserted at the locations where rows and columns of A were removed, so that $A^{-1}_{D_j,p}$ is a $K \times K$ matrix. Note that formally, for $A^{-1}_{D_j,p}$ to be well-defined, it is not necessary that A be invertible. It is only required that A_{D_j} be invertible.

For example, let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

and let $D_j = \{1, 3\}$. Then

$$A_{D_j} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

and $\det A_{D_j} = 1$, which implies $A^{-1}_{D_j} = \operatorname{adj} A_{D_j}$, so that

$$A^{-1}_{D_j} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}.$$

It follows that

$$A^{-1}_{D_j,p} = \begin{bmatrix} 7 & 0 & -2 \\ 0 & 0 & 0 \\ -3 & 0 & 1 \end{bmatrix}.$$

To derive the equilibrium, we first consider each investor's optimization problem. For the investors with horizon h_j it is given by

$$\max_{y_{j,t}} \ \mathbb{E}\left[W_{j,t+h_j}\right] - \frac{1}{2}A_j \mathrm{Var}\left(W_{j,t+h_j}\right) \tag{41}$$

$$W_{j,t+h_j} = \left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)' y_{j,t} + R_f^{h_j}\left(\iota - P_t' y_{j,t}\right).$$

Taking into account the restriction that the investor only invests in assets that are elements of D_j, the solution is

$$y_{j,t} = \frac{1}{A_j}\mathrm{Var}\left(P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right)^{-1}_{D_j,p} \tag{42}$$

$$\times\left(\mathbb{E}\left[P_{t+h_j} + \sum_{k=1}^{h_j} R_f^{h_j-k} D_{t+k} - C_{t+h_j}\right] - R_f^{h_j} P_t\right).$$

Using the i.i.d. assumption, we obtain a stationary equilibrium with constant prices and i.i.d. returns. It is then straightforward to derive that $y_{j,t}$ can be written as (derivation available on request)

$$y_{j,t} = \frac{1}{A_j}\,\mathrm{diag}\left(P_t\right)^{-1}\mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)^{-1}_{D_j,p} \tag{43}$$

$$\times\left(\mathbb{E}\left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k}\right).$$

Similarly, it is also straightforward to show that

$$\mathbb{E}\left[\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k}\right] - \sum_{k=0}^{h_j-1} R_f^{h_j-k} = \rho_j\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right), \tag{44}$$

where $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j-k}$. Making further use of the i.i.d. assumption by which $\mathbb{E}(c_{t+h_j}) =$

$\mathbb{E}(c_{t+k})$ for all j and k, the allocations can thus be written as

$$y_{j,t} = \frac{1}{A_j} \operatorname{diag}(P_t)^{-1} \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j} \right)^{-1}_{D_j,p} \times \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right). \tag{45}$$

Each period a fixed quantity $Q_j > 0$ of type j investors enters the market. The equilibrium condition at time t is

$$\sum_{j=1}^{N} Q_j y_{j,t} = S_t - \sum_{j=1}^{N} \sum_{k=1}^{h_j-1} Q_j y_{j,t-k}, \tag{46}$$

which is equivalent to

$$\sum_{j=1}^{N} \sum_{k=0}^{h_j-1} Q_j y_{j,t-k} = S_t. \tag{47}$$

Under the iid assumption we have $y_{j,t-k} = y_{j,t}$ for all k, so that

$$\sum_{j=1}^{N} h_j Q_j y_{j,t} = S_t. \tag{48}$$

Scaling by price we obtain

$$\sum_{j=1}^{N} h_j Q_j \operatorname{diag}(P_t)\, y_{j,t} = \widetilde{S}_t. \tag{49}$$

Multiplying both sides by $(1/\widetilde{S}_t'\iota)\mathrm{Var}\left(R_{t+1} - c_{t+1}\right)$, using the expression for the allocations and noting that, as we assume that $\widetilde{S}_t$ is constant over time,

$$\begin{aligned} \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \widetilde{S}_t &= \operatorname{Cov}\left(R_{t+1} - c_{t+1}, \widetilde{S}_t' R_{t+1} - \widetilde{S}_t' c_{t+1}\right) \\ &= \widetilde{S}_t'\iota \operatorname{Cov}\left(R_{t+1} - c_{t+1}, \frac{\widetilde{S}_t' R_{t+1}}{\widetilde{S}_t'\iota} - \frac{\widetilde{S}_t' c_{t+1}}{\widetilde{S}_t'\iota}\right) \\ &= \widetilde{S}_t'\iota \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right), \end{aligned} \tag{50}$$

where $R_{t+1}^m = \widetilde{S}_t' R_{t+1}/\widetilde{S}_t'\iota$, and $c_{t+1}^m = \widetilde{S}_t' c_{t+1}/\widetilde{S}_t'\iota$, it now follows that

$$\sum_{j=1}^{N} h_j \frac{Q_j}{A_j \widetilde{S}_t'\iota} \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+1} \right)^{-1}_{D_j,p} \times \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right). \tag{51}$$

We define $\gamma_j = Q_j/(A_j \widetilde{S}'\iota)$ (given that $\widetilde{S}_t$ is constant over time) and

$$V_j^{D_j} = h_j \mathrm{Var}\left(R_{t+1} - c_{t+1}\right) \mathrm{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j-k} R_{t+k} - c_{t+h_j}\right)_{D_j,p}^{-1}.$$

This allows us to write

$$\sum_{j=1}^{N} \gamma_j V_j^{D_j} \left(\rho_j \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) - \mathbb{E}\left[c_{t+1}\right]\right) = \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right). \tag{52}$$

From this we find that

$$\begin{aligned}\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j} \left(\mathbb{E}\left[R_{t+1}\right] - R_f\right) = \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+1}\right] \\ + \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).\end{aligned} \tag{53}$$

We can now write the equilibrium condition as

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right] - R_f = \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+1}\right] \\ + \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).\end{aligned} \tag{54}$$

Comparative statics

We consider an increase in γ_i, so that the horizon h_i investors become either more numerous, or less risk averse, or both. We find

$$\begin{aligned}\frac{\partial\left(\mathbb{E}\left[R_{t+1}\right] - R_f\right)}{\partial \gamma_i} = -\left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \left(\rho_i V_i^{D_i}\right) \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \\ \times \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+1}\right] \\ + \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} V_i^{D_i} \mathbb{E}\left[c_{t+1}\right] \\ - \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \left(\rho_i V_i^{D_i}\right) \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j}\right)^{-1} \\ \times \mathrm{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right).\end{aligned} \tag{55}$$

Rearranging gives

$$\frac{\partial\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)}{\partial\gamma_i}=\left(\sum_{j=1}^{N}\gamma_j\rho_j V_j^{D_j}\right)^{-1}V_i^{D_i}\left(\mathbb{E}\left[c_{t+1}\right]-\rho_i\left(\mathbb{E}\left[R_{t+1}\right]-R_f\right)\right). \tag{56}$$

Computing the long term covariance matrix

We use the i.i.d. assumption to rewrite part of the moment conditions as follows

$$\begin{aligned}
&\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\operatorname{Var}\left(\sum_{k=1}^{h_j}R_f^{h_j-k}R_{t+k}-c_{t+h_j}\right)^{-1} \qquad (57)\\
&=\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\operatorname{Var}\left(\sum_{k=1}^{h_j-1}R_f^{h_j-k}R_{t+k}+R_{t+h_j}-c_{t+h_j}\right)^{-1}\\
&=\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\left(\operatorname{Var}\left(\sum_{k=1}^{h_j-1}R_f^{h_j-k}R_{t+k}\right)+\operatorname{Var}\left(R_{t+h_j}-c_{t+h_j}\right)\right)^{-1}\\
&=\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\left(\sum_{k=1}^{h_j-1}R_f^{2(h_j-k)}\operatorname{Var}\left(R_{t+k}\right)+\operatorname{Var}\left(R_{t+h_j}-c_{t+h_j}\right)\right)^{-1}\\
&=\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\left(\left(\sum_{k=1}^{h_j-1}R_f^{2(h_j-k)}\right)\operatorname{Var}\left(R_{t+1}\right)+\operatorname{Var}\left(R_{t+1}-c_{t+1}\right)\right)^{-1}.
\end{aligned}$$

This allows us to compute the covariance terms using only one-period covariances, which leads to greater numerical stability.

8.3 Estimation Methodology: Obtaining Standard Errors

We denote the required moments that enter the asset pricing model by the vector ψ. This vector contains expected returns, expected costs, and all required covariances of returns and costs. It is straightforward to derive the asymptotic covariance matrix of the sample estimator of these moments (since covariances can be written as second moments plus products of first moments),

$$\sqrt{T}\left(\widehat{\psi}-\psi\right)\xrightarrow{d}\mathcal{N}\left(0,S_{\psi}\right). \tag{58}$$

We can now use the delta method to find the standard errors for $\widehat{\gamma}$, as $\widehat{\gamma}$ is implicitly given as the solution of the elementary zero function in the second stage or, equivalently, as the solution of the GMM minimization problem.

We start with the GMM minimization problem

$$\min_{\gamma}\ g(\widehat{\psi},\gamma)'g(\widehat{\psi},\gamma), \tag{59}$$

which has solution

$$2G(\widehat{\psi},\gamma)'g(\widehat{\psi},\gamma)=0, \tag{60}$$

where

$$G_{\gamma}(\psi,\gamma)=\frac{\partial g(\psi,\gamma)}{\partial\gamma}. \tag{61}$$

Dividing both sides of (60) by 2 and evaluating at $\widehat{\gamma}$, we may write

$$G_{\gamma}(\widehat{\psi},\widehat{\gamma})'g(\widehat{\psi},\gamma_0)+G_{\gamma}(\widehat{\psi},\widehat{\gamma})'\left(g(\widehat{\psi},\widehat{\gamma})-g(\widehat{\psi},\gamma_0)\right)=0. \tag{62}$$

Next, we expand $g(\widehat{\psi},\widehat{\gamma})$ around γ_0:

$$g(\widehat{\psi},\widehat{\gamma})-g(\widehat{\psi},\gamma_0)\approx G_{\gamma}(\widehat{\psi},\widehat{\gamma})\left(\widehat{\gamma}-\gamma_0\right). \tag{63}$$

It follows that

$$G_{\gamma}(\widehat{\psi},\widehat{\gamma})'g(\widehat{\psi},\gamma_0)+G_{\gamma}(\widehat{\psi},\widehat{\gamma})'G_{\gamma}(\widehat{\psi},\widehat{\gamma})\left(\widehat{\gamma}-\gamma_0\right)=0. \tag{64}$$

We now expand $g(\widehat{\psi},\gamma_0)$ around ψ_0 and use the fact that $g(\psi_0,\gamma_0)=0$:

$$g(\widehat{\psi},\gamma_0)\approx G_{\psi}(\widehat{\psi},\widehat{\gamma})\left(\widehat{\psi}-\psi_0\right), \tag{65}$$

where

$$G_{\psi}(\psi,\gamma)=\frac{\partial g(\psi,\gamma)}{\partial\psi}. \tag{66}$$

Hence

$$G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\gamma(\widehat{\psi},\widehat{\gamma})\,(\widehat{\gamma}-\gamma_0) = -G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\psi(\widehat{\psi},\widehat{\gamma})\left(\widehat{\psi}-\psi_0\right). \tag{67}$$

Using this result we obtain

$$\sqrt{T}\,(\widehat{\gamma}-\gamma_0) \approx -\left(G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\gamma(\widehat{\psi},\widehat{\gamma})\right)^{-1} G_\gamma(\widehat{\psi},\widehat{\gamma})'G_\psi(\widehat{\psi},\widehat{\gamma})\sqrt{T}\left(\widehat{\psi}-\psi_0\right). \tag{68}$$

It follows that

$$\sqrt{T}\,(\widehat{\gamma}-\gamma_0) \xrightarrow{d} \mathcal{N}\left(0, \left(G_\gamma' G_\gamma\right)^{-1} G_\gamma' G_\psi S_\psi G_\psi' G_\gamma \left(G_\gamma' G_\gamma\right)^{-1}\right). \tag{69}$$

This result allows us to compute standard errors for the γ estimates taking into account the pre-estimation of the various moments ψ. For the final estimation procedure, we restrict the γ_j pertaining to the horizons h_j to be positive by estimating the logs. We use the usual, additional, delta method correction for the computation of the standard errors.

Table 1: Descriptive statistics.

This table shows some descriptive statistics pertaining to the data that are used to estimate the model. The data used are monthly data corresponding to 25 value-weighted portfolios sorted on illiquidity with sample period 1964–2009. The average excess return $\mathbb{E}\left[R_{t+1}\right] - R_f$, standard deviation of returns $\sigma\left(R_{t+1}\right)$, standard deviation of returns net of costs $\sigma\left(R_{t+1} - c_{t+1}\right)$, and covariance between portfolio and market level returns net of costs $\mathrm{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right)$ are computed from the time-series observations using the corresponding method of moments estimators.

Portfolio	$\mathbb{E}\left[R_{t+1}\right] - R_f$ (%)	$\mathbb{E}\left[c_{t+1}\right]$ (%)	$\sigma\left(R_{t+1}\right)$ (%)	$\sigma\left(R_{t+1} - c_{t+1}\right)$ (%)	$\mathrm{Cov}\left(R_{t+1} - c_{t+1}, R^m_{t+1} - c^m_{t+1}\right)$ ($\cdot 100$)
1	0.3282	0.2519	4.2063	4.2065	0.1772
2	0.3748	0.2568	4.6173	4.6182	0.2138
3	0.4253	0.2611	4.6465	4.6479	0.2213
4	0.5580	0.2664	4.7630	4.7651	0.2287
5	0.5892	0.2732	5.0549	5.0585	0.2447
6	0.5365	0.2828	4.8567	4.8613	0.2374
7	0.5531	0.2928	4.8044	4.8092	0.2380
8	0.5405	0.3084	4.9590	4.9679	0.2444
9	0.6029	0.3252	4.9624	4.9733	0.2476
10	0.6232	0.3449	4.8947	4.9079	0.2444
11	0.6646	0.3727	5.1488	5.1660	0.2576
12	0.5538	0.4039	4.8977	4.9184	0.2447
13	0.6466	0.4379	4.8907	4.9177	0.2464
14	0.7310	0.4762	5.0650	5.0967	0.2541
15	0.6452	0.5455	5.0824	5.1289	0.2562
16	0.6244	0.6216	5.0356	5.0894	0.2528
17	0.7943	0.7243	5.0766	5.1531	0.2532
18	0.6752	0.8290	5.0326	5.1159	0.2507
19	0.8586	0.9753	5.1913	5.2908	0.2564
20	0.6753	1.2792	5.3195	5.4833	0.2596
21	0.8088	1.5535	5.3987	5.5626	0.2596
22	0.8322	1.9896	5.3905	5.5833	0.2556
23	0.8147	2.8093	5.4869	5.9004	0.2684
24	0.7453	4.4678	5.5583	6.3610	0.2733
25	0.8754	8.0711	6.0520	7.8366	0.2786

Table 2: Illiquidity portfolio regressions.

This table shows the results from estimation of the various specifications of the model. The estimates are based on monthly data corresponding to 25 value-weighted portfolios sorted on illiquidity with sample period 1964–2009. An equal-weighted market portfolio is used. The specifications are special cases of the relation

$$\begin{aligned}\mathbb{E}\left[R_{t+1}\right] - R_f = \alpha + \kappa \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j} \right)^{-1} \sum_{j=1}^{N} \gamma_j V_j^{D_j} \mathbb{E}\left[c_{t+h_j}\right] \\ + \left(\sum_{j=1}^{N} \gamma_j \rho_j V_j^{D_j} \right)^{-1} \operatorname{Cov}\left(R_{t+1} - c_{t+1}, R_{t+1}^m - c_{t+1}^m\right),\end{aligned}$$

where $\gamma_j = Q_j/(A_j \widetilde{S}' \iota)$, $\rho_j = \sum_{k=1}^{h_j} R_f^{h_j - k}$, and

$$V_j^{D_j} = h_j \operatorname{Var}\left(R_{t+1} - c_{t+1}\right) \operatorname{Var}\left(\sum_{k=1}^{h_j} R_f^{h_j - k} R_{t+k} - c_{t+h_j} \right)_{D_j, p}^{-1}.$$

We set $N = 2$, $h_1 = 1$, and $h_2 = 240$. The parameters are estimated using GMM. For each coefficient the t-value is given in parentheses. The pseudo-R^2 is reported in the rightmost column. The label INT denotes the integrated model. The segmented version, where short term investors invest only in the 19 most liquid portfolios, is denoted by SEG. AP indicates that the specification corresponds to a variant of the Acharya and Pedersen (2005) specification (18). APc is used to denote the original Acharya and Pedersen (2005) model, where AR(2) residuals of individual and market level returns and costs are used to compute the covariance. Where the value of κ is unreported, it is set to 1.

	γ_1	γ_2	α	κ	R^2
1 (INT)	0.0349	0.0028			0.6504
	(0.6264)	(2.5507)			
2 (INT)	0.0149	0.0013	-0.0075		0.7758
	(0.2589)	(1.3054)	(-0.7742)		
3 (SEG)	0.2261	0.0026			0.8247
	(0.6057)	(1.8843)			
4 (SEG)	0.0949	0.0014	-0.0046		0.8669
	(0.2922)	(1.1111)	(-0.5081)		
5 (AP)	0.4024			0.0299	0.6281
	(2.2212)			(0.1761)	
6 (AP)	0.1773		-0.0076	0.0104	0.7657
	(0.8494)		(-0.5435)	(0.0257)	
7 (APc)	0.3884			0.0377	0.6105
	(2.4514)			(1.4990)	
8 (APc)	0.1597		-0.0086	0.0267	0.7609
	(2.7018)		(-1.8791)	(1.1411)	

Table 3: Illiquidity portfolio regressions – sensitivity of R^2.

This table shows the sensitivity of the R^2 to varying the horizons and to varying the segmentation threshold. The data and the specifications are the same as in Table 2. Setting $h_1 = 1$, we let $h_2 = 30, 60, 120, 240, 480$. Alternatively, we fix $h_2 = 240$ and let $h_1 = 1, 3, 6, 12, 36$. For the segmentation level we take $h_1 = 1$, $h_2 = 240$ and let the short-term investors invest in the $16, \ldots, 25$ most liquid portfolios. The case of 25 corresponds to integration.

Panel A: Sensitivity of R^2 to choice of horizon					
$h_2 = 240$	$h_1 = 1$	$h_1 = 3$	$h_1 = 6$	$h_1 = 12$	$h_1 = 36$
1 (INT)	0.6504	0.6310	0.6237	0.6195	0.5707
2 (INT)	0.7758	0.7655	0.7612	0.7587	0.7214
3 (SEG)	0.8247	0.8140	0.7485	0.6885	0.6376
4 (SEG)	0.8669	0.8620	0.8206	0.7793	0.7430
$h_1 = 1$	$h_2 = 30$	$h_2 = 60$	$h_2 = 120$	$h_2 = 240$	$h_2 = 480$
1 (INT)	0.6294	0.6422	0.6478	0.6504	0.6514
2 (INT)	0.7635	0.7713	0.7745	0.7758	0.7763
3 (SEG)	0.7816	0.8220	0.8263	0.8247	0.8232
4 (SEG)	0.8475	0.8734	0.8714	0.8669	0.8642
Panel B: Sensitivity of R^2 to choice of segmentation threshold					
	16	17	18	19	20
3 (SEG)	0.7334	0.7770	0.7545	0.8247	0.7360
4 (SEG)	0.7918	0.8243	0.8076	0.8669	0.7922
	21	22	23	24	25
3 (SEG)	0.7528	0.7623	0.7109	0.6225	0.6504
4 (SEG)	0.8076	0.8305	0.7886	0.7269	0.7758

Table 4: Illiquidity portfolio regressions (bootstrap results).

This table shows the results from the bootstrap procedure. The data and the specifications are the same as in Table 2. The bootstrap procedure is based on 120 replications. In Panel A we report the bootstrap t-value for each coefficient in parentheses. In Panel B we give the bootstrap confidence intervals for each parameter.

Panel A: Bootstrap t-values					
	γ_1	γ_2	α	κ	R^2
1 (INT)	0.0349	0.0028			0.6504
	(1.4649)	(1.1855)			
2 (INT)	0.0149	0.0013	-0.0075		0.7758
	(1.1545)	(0.8687)	(-2.7977)		
3 (SEG)	0.2261	0.0026			0.8247
	(0.3881)	(0.4608)			
4 (SEG)	0.0949	0.0014	-0.0046		0.8669
	(0.4670)	(0.3667)	(-2.2218)		
5 (AP)	0.4024			0.0299	0.6281
	(1.1301)			(1.4297)	
6 (AP)	0.1773		-0.0076	0.0104	0.7657
	(0.8673)		(-3.0349)	(0.4702)	
7 (APc)	0.3884			0.0377	0.6105
	(1.1374)			(1.8214)	
8 (APc)	0.1597		-0.0086	0.0267	0.7609
	(0.9688)		(-3.1879)	(1.2253)	
Panel B: Bootstrap confidence intervals					
	γ_1	γ_2	α	κ	R^2
1 (INT)	[0.0125, 0.1070]	[0.0016, 0.0106]			0.6504
2 (INT)	[0.0054, 0.0564]	[0.0009, 0.0067]	[-0.0093, -0.0012]		0.7758
3 (SEG)	[0.0616, 2.3869]	[0.0013, 0.0208]			0.8247
4 (SEG)	[0.0353, 0.8159]	[0.0008, 0.0144]	[-0.0065, 0.0000]		
5 (AP)	[0.2507, 1.3973]			[0.0016, 0.0787]	
6 (AP)	[0.1362, 0.9252]		[-0.0088, -0.0010]	[-0.0167, 0.0697]	
7 (APc)	[0.2403, 1.3447]			[0.0073, 0.0807]	
8 (APc)	[0.1182, 0.7595]		[-0.0096, -0.0015]	[-0.0048, 0.0746]	

Table 5: σ(Illiquidity) and B/M-by-size portfolio regressions.

This table shows the results from estimation of the various specifications of the model for different portfolio types. The setup is the same as in Table 2. Panel A shows the results for 25 portfolios sorted on illiquidity variation. For Panel B 25 value-weighted portfolios sorted on book-to-market value and size are used.

	Panel A: σ(Illiquidity) portfolios				
	γ_1	γ_2	α	κ	R^2
1 (INT)	0.0290	0.0029			0.6469
	(0.4961)	(2.5975)			
2 (INT)	0.0107	0.0013	-0.0075		0.7773
	(0.1819)	(1.4548)	(-0.8410)		
3 (SEG)	0.2186	0.0027			0.8681
	(0.5680)	(1.9343)			
4 (SEG)	0.0890	0.0014	-0.0044		0.9064
	(0.3029)	(1.3999)	(-0.7564)		
5 (AP)	0.4043			0.0286	0.6425
	(2.2224)			(0.1792)	
6 (AP)	0.1787		-0.0075	0.0028	0.7824
	(0.9476)		(-0.6091)	(0.0073)	
7 (APc)	0.3901			0.0362	0.6203
	(2.4381)			(1.2600)	
8 (APc)	0.1638		-0.0082	0.0181	0.7659
	(2.7319)		(-1.8711)	(0.6898)	
	Panel B: B/M-by-size portfolios				
	γ_1	γ_2	α	κ	R^2
1 (INT)	0.0379	0.0032			0.2110
	(0.5607)	(2.2392)			
2 (INT)	0.0843	0.0057	0.0028		0.2671
	(0.7500)	(0.8208)	(0.9630)		
3 (SEG)	0.0578	0.0032			0.3075
	(0.7452)	(2.1224)			
4 (SEG)	0.1783	0.0080	0.0035		0.3923
	(0.5609)	(0.5692)	(1.1984)		
5 (AP)	0.4632			0.0351	0.2789
	(2.0754)			(0.2065)	
6 (AP)	1.0142		0.0031	0.0498	0.3491
	(0.6737)		(1.0357)	(0.6460)	
7 (APc)	0.4473			0.0449	0.2787
	(2.1795)			(1.5019)	
8 (APc)	0.9903		0.0031	0.0545	0.3444
	(0.6239)		(0.9761)	(2.0009)	

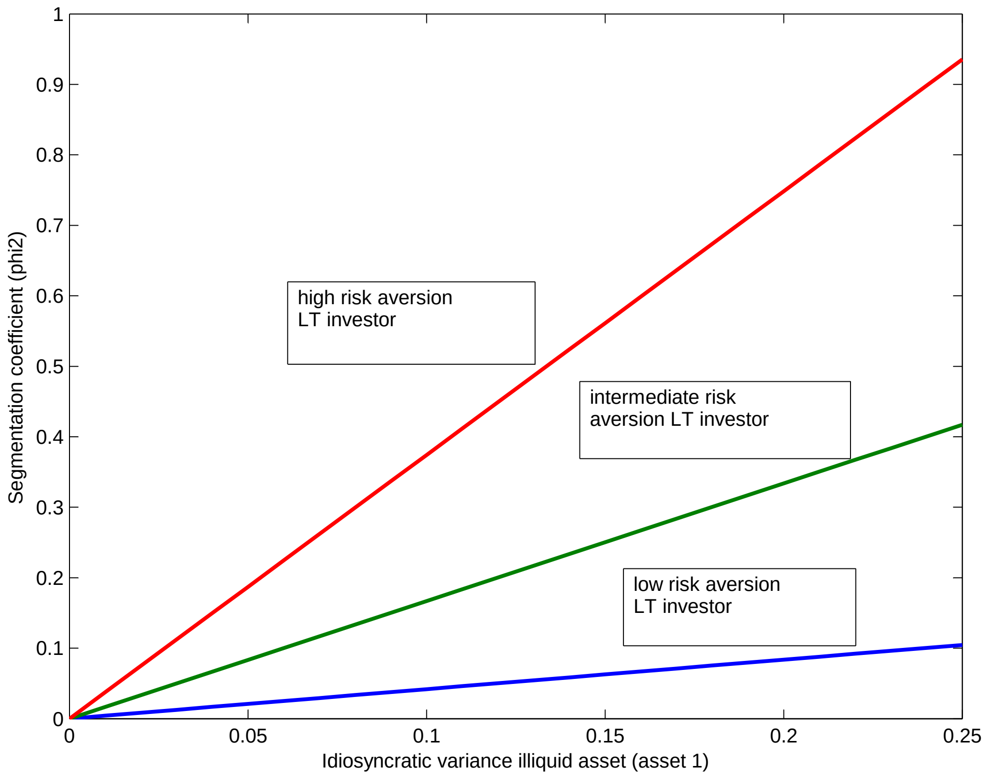

Figure 1: Segmentation effect in two-asset, two-investor model. This figure plots the segmentation coefficient ϕ_2 in equation (13) as a function of the risk aversion of the long-term investor (A_2) and the idiosyncratic variance of the illiquid asset (asset 1). The idiosyncratic variance is defined as $\mathrm{Var}(\varepsilon)$, with $R_1 = R_2 + \varepsilon$. This variance $\mathrm{Var}(\varepsilon)$ varies between 0 and $(0.5)^2$. The variance of R_2 is fixed at $(0.2)^2$, the transaction costs are assumed to be the same for both assets and equal to a constant minus $0.25R_2$. Also, we set $A_1 = 1$ and $R_f = 1.03$, while A_2 varies from 1 to 3.

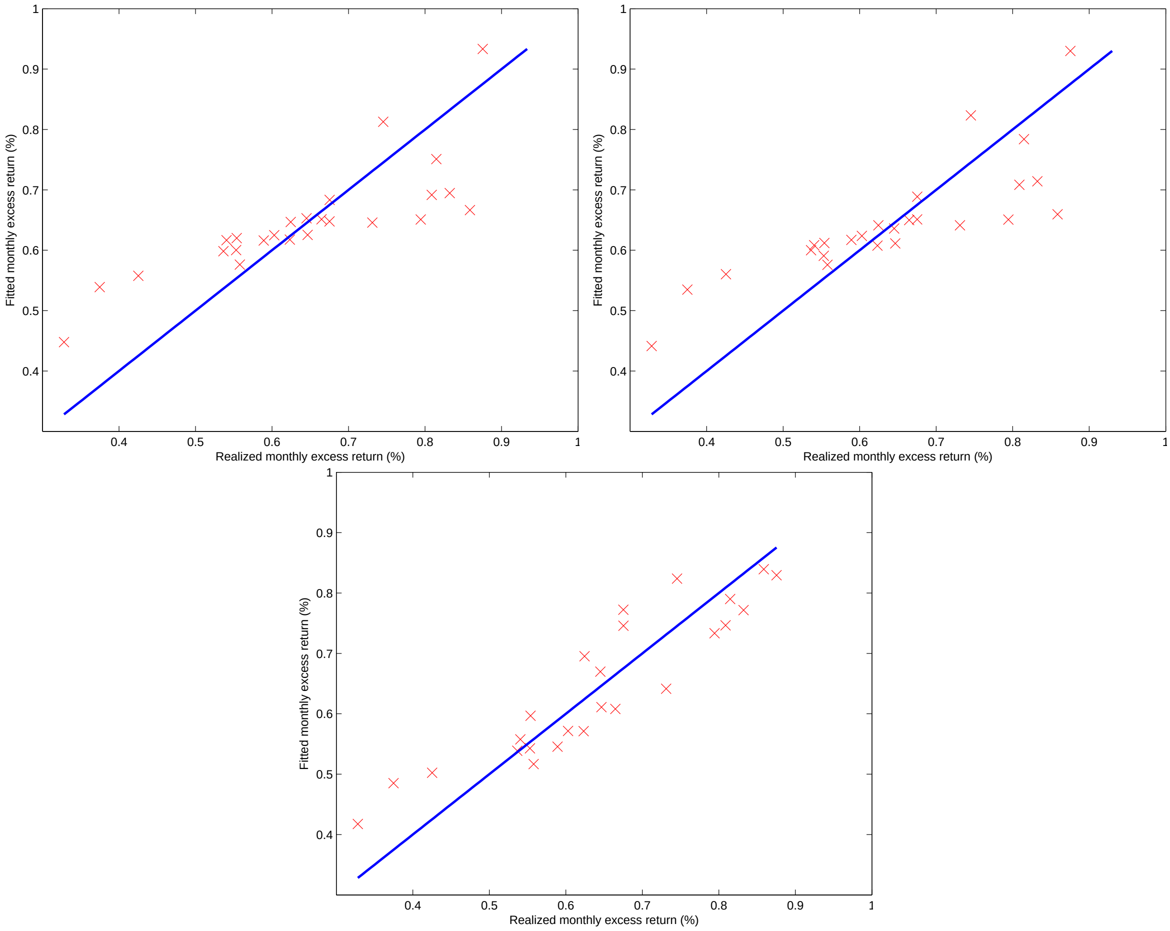

Figure 2: Fitted excess returns vs. realized excess returns. The top left panel shows the goodness of fit for the Acharya and Pedersen (2005) specification 5 (AP). The top-right panel shows the fit for the non-segmented specification 1 (INT). The bottom panel shows the fit for the segmented specification 3 (SEG). The graphs correspond to the estimation results as given in Table 2.

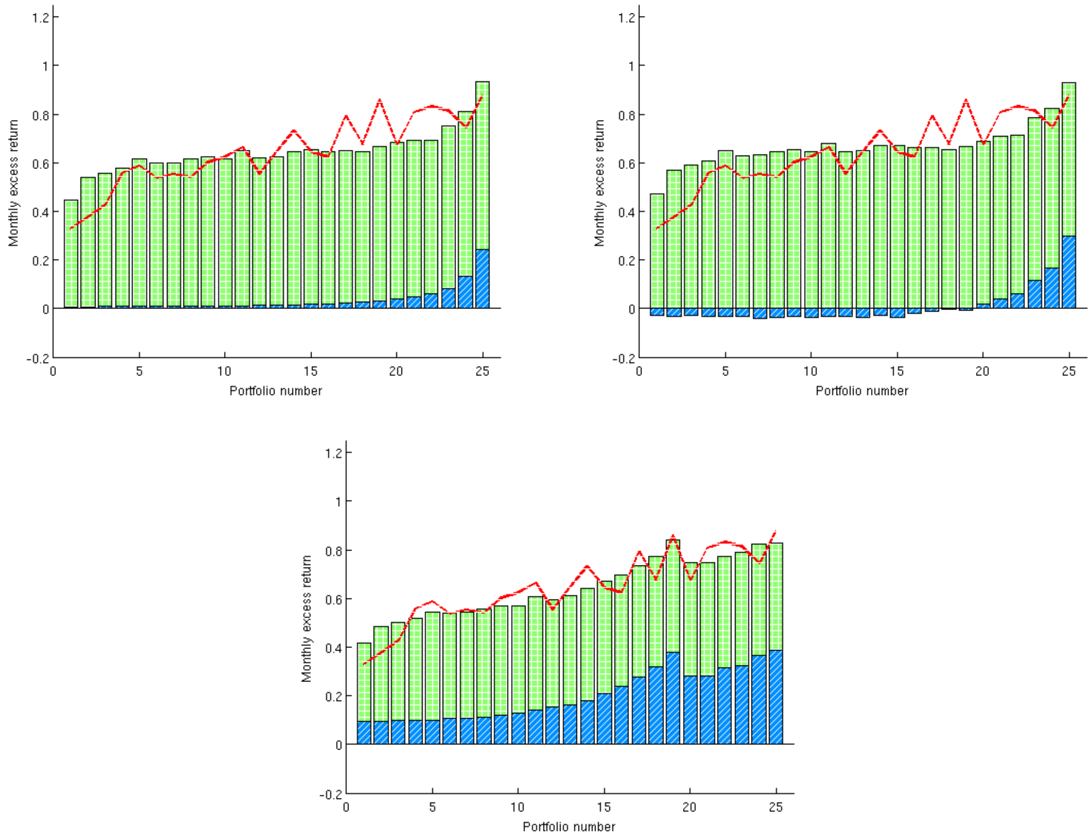

Figure 3: Decomposition of predicted excess returns in the cost term and the covariance term. In each panel the lower part shows the cost term and the upper part the covariance term. The line indicates the actual excess return. The top left panel shows the decomposition for the Acharya and Pedersen (2005) specification 5 (AP). The top-right panel shows the decomposition for the non-segmented specification 1 (INT). The bottom panel shows the decomposition for the segmented specification 3 (SEG). The graphs correspond to the estimation results as given in Table 2.

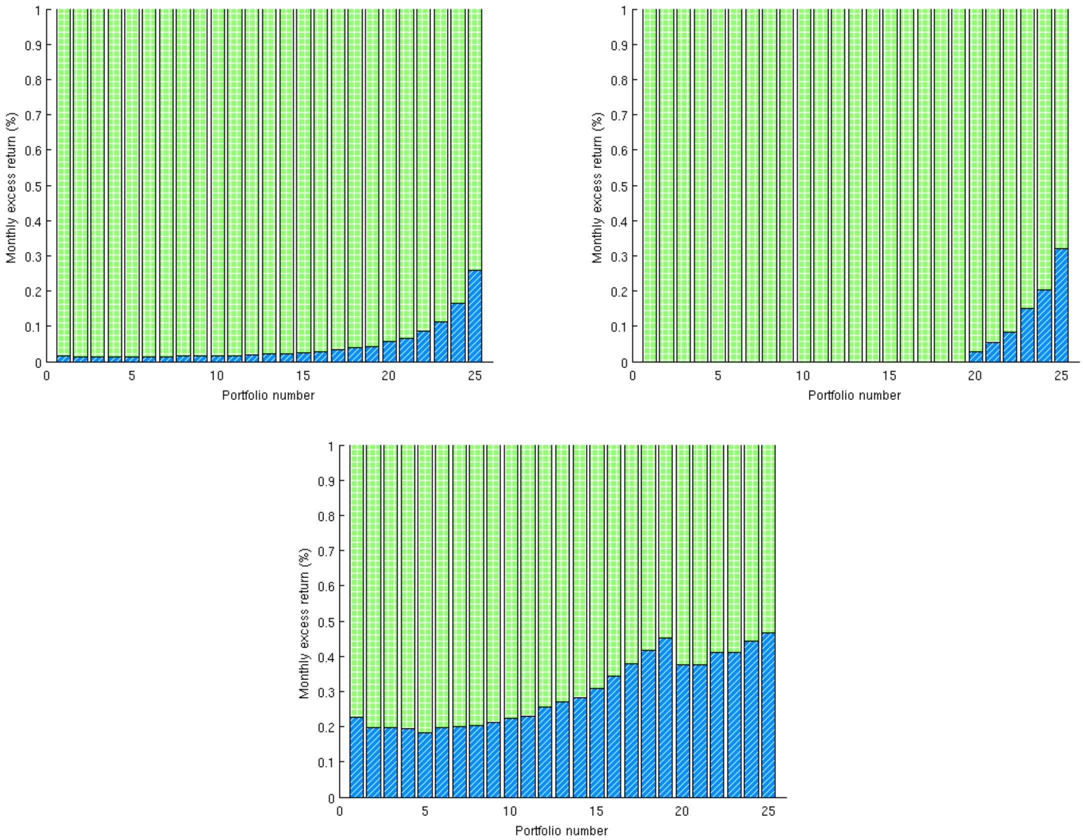

Figure 4: Decomposition of predicted excess returns in the cost term and the covariance term as a percentage of the total predicted excess return. In each panel the lower part shows the percentage of the risk premium generated by the cost term and the upper part the percentage generated by the covariance term. The top left panel shows the decomposition for the Acharya and Pedersen (2005) specification 5 (AP). The top-right panel shows the decomposition for the non-segmented specification 1 (INT). The bottom panel shows the decomposition for the segmented specification 3 (SEG). The graphs correspond to the estimation results as given in Table 2.

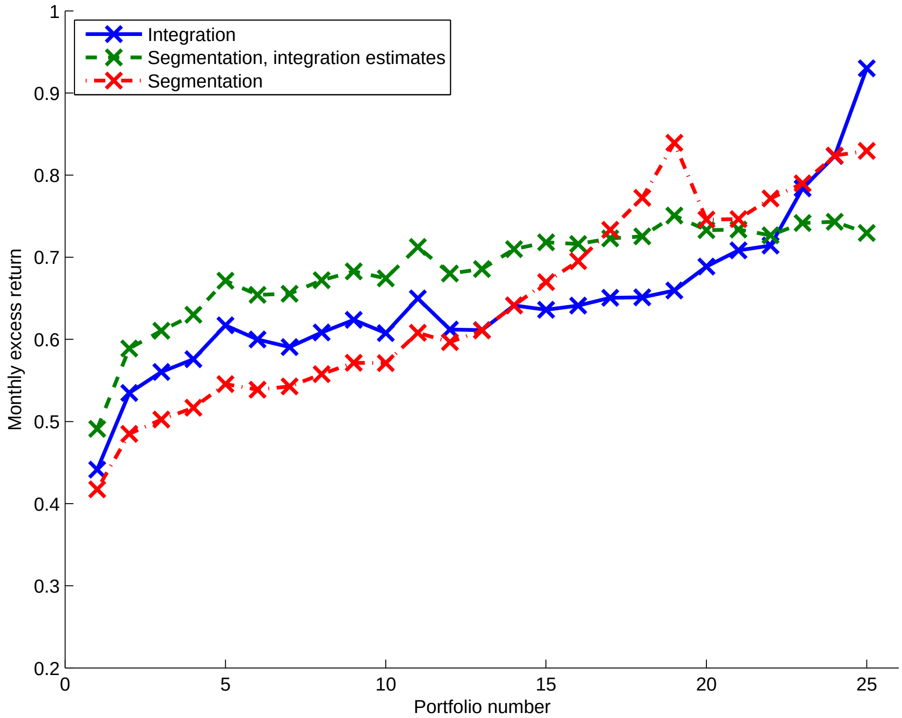

Figure 5: Impact of segmentation. The lines correspond to the risk premia generated by the integrated model 1 (INT), the segmented model with the coefficient values obtained from estimation of the integrated case, and the fully segmented model 3 (SEG). In all cases the specification without a constant term is used. The coefficient values correspond to the estimation results as given in Table 2.

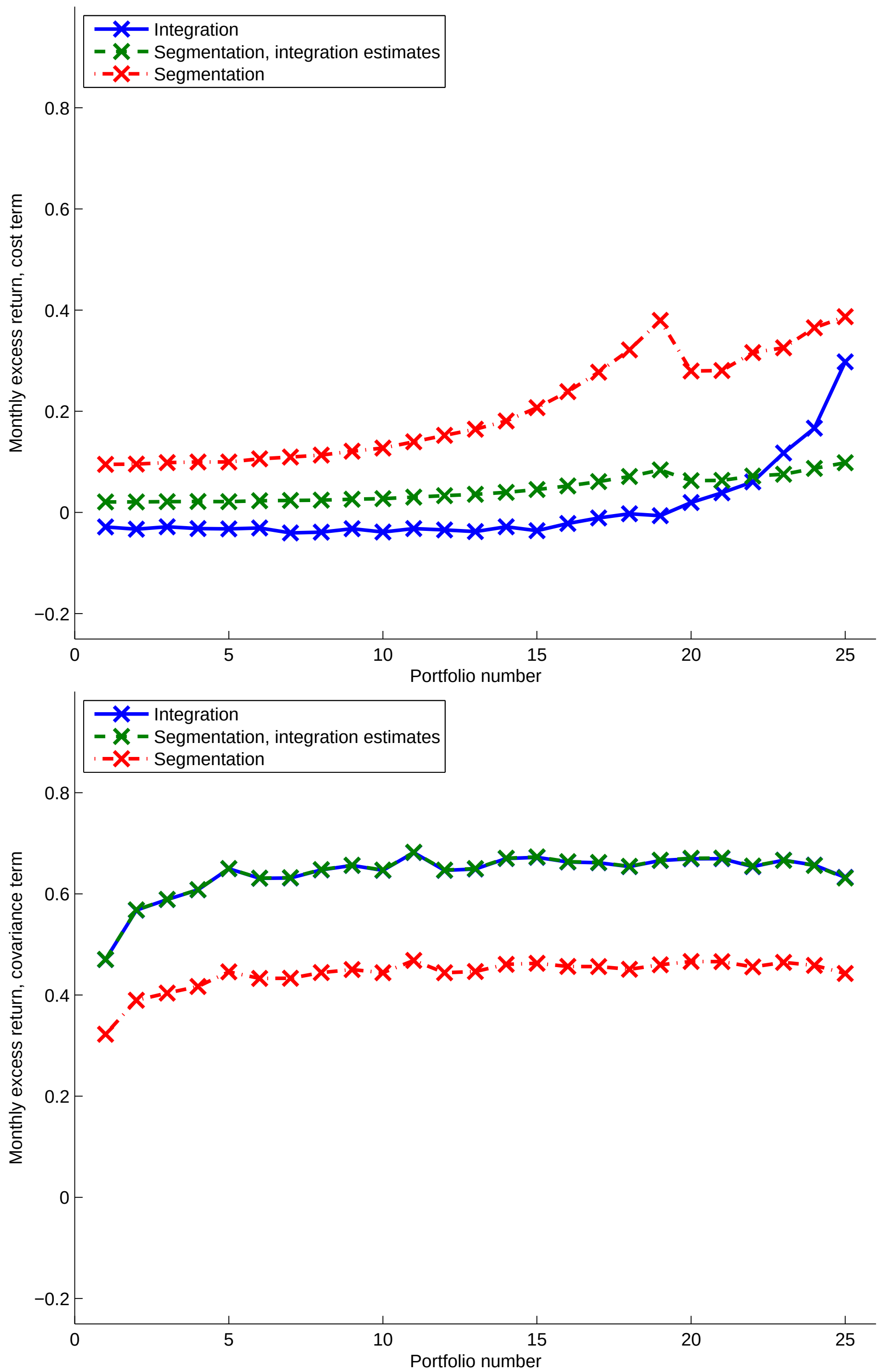

Figure 6: Impact of segmentation decomposed. The lines correspond to the risk premia generated by the integrated model 1 (INT), the segmented model with the coefficient values obtained from estimation of the integrated case, and the fully segmented model 3 (SEG). In all cases the specification without a constant term is used. The coefficient values correspond to the estimation results as given in Table 2. The top panel shows the impact of the cost term. The bottom panel shows the impact of the covariance term.

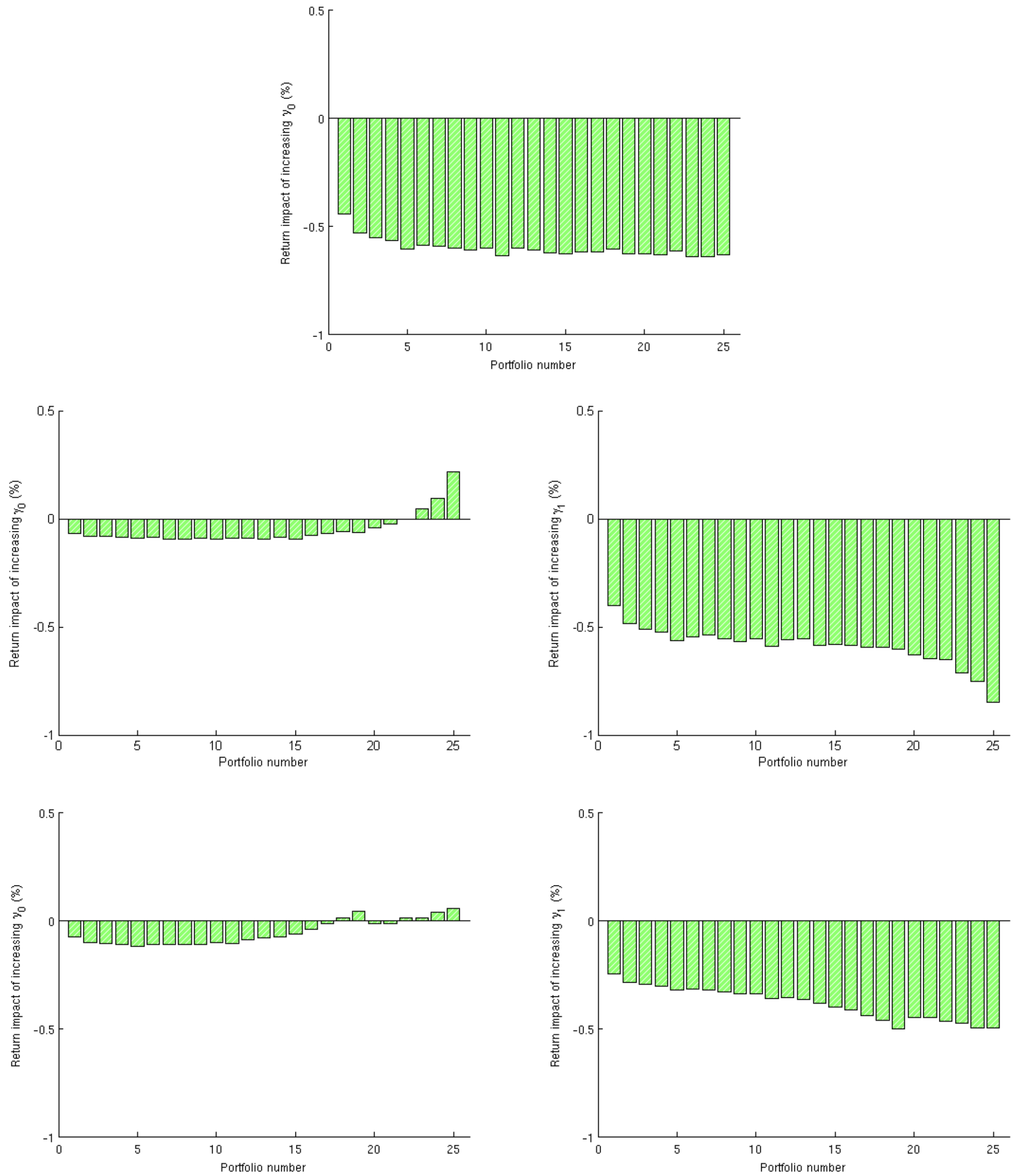

Figure 7: Comparative statics for both parameters. The comparative statics are computed according to (21). The top panel shows the comparative statics for the Acharya and Pedersen (2005) specification 5 (AP). The middle row shows the comparative statics for the non-segmented specification 1 (INT). The bottom row shows the comparative statics for the segmented specification 3 (SEG). The graphs correspond to the estimation results as given in Table 2.