Common Periodic Cycles and Multicointegration in Daily Airport Transit Data.

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Introduction

⋆ For any agent maximizing profits or minimizing costs the efficient use of the capacity of the agents production facilities is of utmost importance.

⋆ In many cases utilization of capacity is varying over time. This may, for instance, be due to seasonal variations of the year, variation over weekdays, time of day variations etc.

⋆ Obviously, modelling the variation may help determine the optimal capacity.

⋆ In some cases the problem faced has two import aspects.

  ◇ The optimal size of the facilities handling the inflows to the system and the outflows from the system.

  ◇ The optimal size of the facilities handling the stock within the system.

⋆ An example of such a system is the tourist industry in, for instance, Mallorca.

  ◇ The inflow and outflow are mainly determined by the "airport capacity" at the island. That is airlines, aircraft support services, passenger services, security, baggage logistics etc.

  ◇ while the "hotel capacity", that is hotels, apartments and houses for short term rents, restaurants etc., is the main supply side determinant of the number of tourists in Mallorca.

The data describing the system is

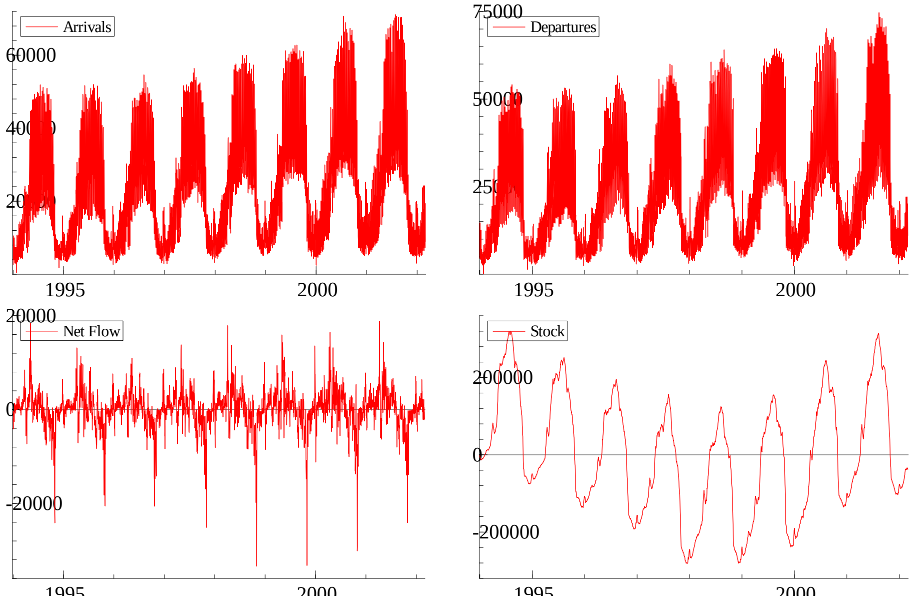
⋆ the number of passengers flying in to Mallorca each day, a_t

⋆ the number of passengers flying out each day,d_t

⋆ the net inflow of passengers, f_t

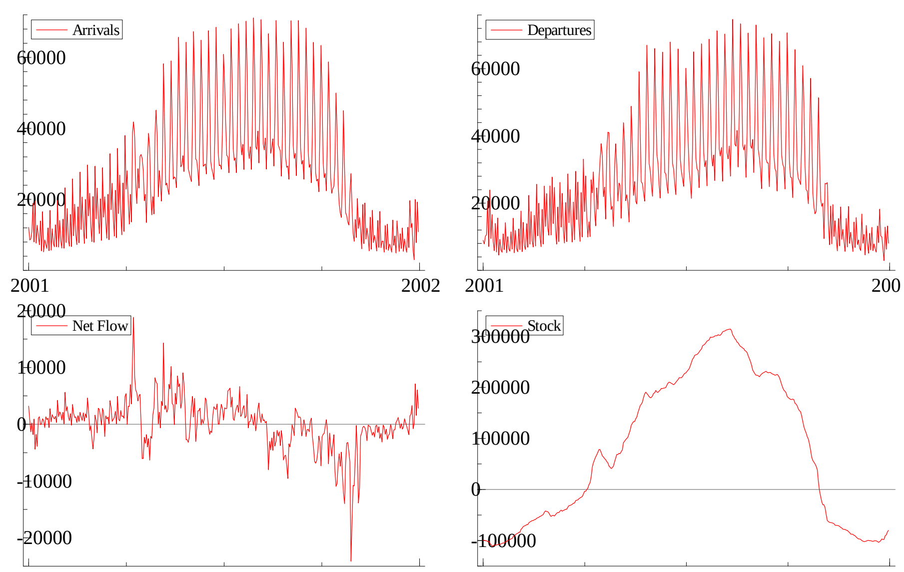
⋆ and the cumulation of the net inflow, that is the stock of passengers on the island of Mallorca, s_t.

The arrivals a, departures d, net inflow f, and stock s are strongly seasonal



a_t, d_t, f_t, and s_t (1994.01.01-2002.02.28)

and the arrivals a, and the departures d have a strong weekly pattern as well, while the stock s, has no weekly pattern



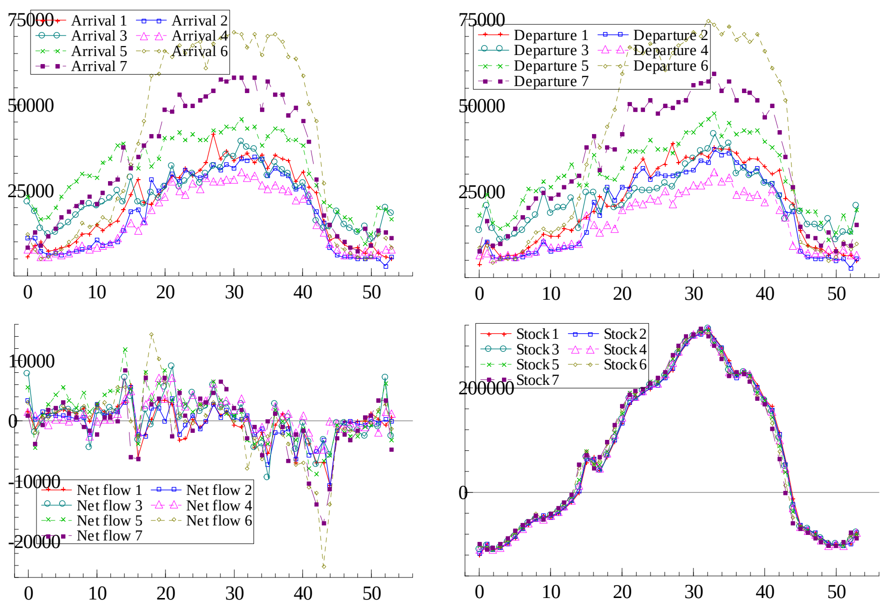
a_t, d_t, f_t, and s_t (2001.01.01-2001.12.31)

⋆ The figures above immediately tell us that

◇ the variation in the utilization of airport capacity varies more over the week than over the year, especially in the summer time,

◇ the variation in the utilization of the hotel capacity varies much more over the year than over the week.

From the following figure it is learned that Saturdays and Sundays are the big arrival and departure dates during the summer months.



Day-of-the-week a_t, d_t, f_t, and s_t (2001.01-2001.52)

The following features are also apparent from the figures above:

⋆ A very close co-movement of arrivals and departures suggesting a strong common seasonal pattern in the two series over the year. A feature that is expected in "charter tourism".

⋆ The arrivals and departures have strong day-of-week effects and this feature seems to change over the year. This might indicate that these series potentially can be modelled as changing seasonal or periodic seasonal processes.

⋆ Multicointegration in daily transit data.

- ◇ A further aspect of the present data set concerns the possibility of a multicointegration like feature amongst the series. If we assume that the arrivals and departures series are cointegrated in some sense, then it is of interest to look at the cumulated net flow series, i.e. the stock variable generated from arrivals and departures.
- ◇ It appears from figures 1 and 2 that although the stock series has much less weekly variation, the level around some trend co-varies with both the arrivals and departures series. This is an interesting phenomenon because it allows for the possibility of more than just one cointegrating relationship existing between just two series.
- ◇ The property is often being referred to as multicointegration.

Empirical Questions

1. Changing intra-week seasonality, and interaction between weekly seasonality, annual pattern and trend of flow variables: ARE THE ARRIVALS AND DEPARTURES PERIODICALLY INTEGRATED?

2. Net flows do not show the features of arrivals and departures: ARE THE ARRIVALS AND DEPARTURES COINTEGRATED? AND IS COINTEGRATION PERIODIC OR NONPERIODIC?

3. Stock of visitors seems to share the same annual pattern/trend of the flow variables: ARE THE ARRIVALS AND DEPARTURES ALSO PERIODICALLY MULTICOINTEGRATED?

4. Day-of-week series share annual seasonality and business cycle: HAS THE SHORT-RUN COMPONENT OF THE DAILY AIRPORT DATA COLINEAL PERIODIC AUTOCORRELATIONS?

Empirical Model: Weekly Representation

Consider the weekly representation of $\mathbf{Y}_n = (y_{1,n}, ..., y_{7,n})'$ to make inference on different properties of the daily process

$$\underbrace{\mathbf{\Gamma}(L^7)}_{\substack{\text{ANNUAL SEAS.}\\ \text{BUSINESS CYCLES}}} \Delta_7 \mathbf{Y}_n = \underbrace{\boldsymbol{\mu} + \mathbf{\Psi}\mathbf{d}_n}_{\text{DET. SEAS.}} + \underbrace{\mathbf{\Theta}\mathbf{cal}_n}_{\text{CAL.EFF.}} + \underbrace{\mathbf{\Pi}}_{\substack{\text{WEEKLY. SEAS}\\ \text{LONG-RUN}}} \mathbf{Y}_{n-1} + \mathbf{U}_n$$

1. d_t is periodically integrated <–> $Y_n = (d_{1,n}, ..., d_{7,n})'$ is a cointegrated system with 6 cointegrating relations.

2. $(a_t, d_t)'$ is periodically cointegrated <–> $Y_n = (a_{1,n}, ..., a_{7,n}, d_{1,n}, ..., d_{7,n})'$ is a cointegrated system with 13 cointegrating relations.

3. $(a_t, d_t)'$ is periodically multicointegrated <–> $Y_n = (d_{1,n}, ..., d_{7,n}, \frac{1}{7}\Sigma_{s=1}^{7} s_{s,n})'$ is a cointegrated system with 7 cointegrating relations.

4. The PeACF of $(1 - \varphi_s L)d_t$ is colineal <–> Y_n has (polynomial) weak form of Serial Common Correlation Features/Common Periodic Features.

Periodic Integration?

The intra-week[1] seasonality is represented by means of the relations among the day-of-the-week processes $Y_n \equiv (y_{1,n}, ..., y_{7,n})'$:

$$\Phi_0 Y_n = C + \Phi_1 Y_{n-1} + ... + \Phi_P Y_{n-P} + E_n$$

$C \equiv (c_1, ..., c_7)'$, $E_n \equiv (\varepsilon_{1n}, ..., \varepsilon_{7n})'$i.i.d$(0, \Sigma)$, $\Sigma = diag(\sigma_1^2, ..., \sigma_7^2)$, and

$$\Phi_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -\phi_{2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\phi_{7,6} & \cdots & -\phi_{7,1} & 1 \end{bmatrix}, \Phi_k = \begin{bmatrix} \phi_{1,7k} & \cdots & \cdots & \phi_{1,7k-6} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \phi_{7,7k+6} & \cdots & \cdots & \phi_{7,7k} \end{bmatrix}$$

Periodic Integration?

$$|\Phi(L)| Y_n = \Phi^*(1) C + \Phi^*(L) E_n$$

y_t is first-order unit root nonstationary when some roots of $|\Phi(z)| = 0$ lie on the unit circle while all the others lie outside.

Different types of first-order unit root nonstationary are associated with different cointegration[2] relations among $y_{s,n}$:

$$\Gamma(L^7)\Delta_7 Y_n = \widetilde{C} + \alpha\beta' Y_{n-1} + U_n$$

[1] see Gladyshev (1961)

[2] Osborn (1993, *JE*) and Franses (1994, *JE*)

Periodic Integration?

$y_t \sim I(1)$ (no periodic/seasonal integration) when $rank(\Pi) = 6$ and

$$\beta' = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\Delta y_t \tilde{} PI(0)$$

Periodic Integration?

$y_t \sim PI(1)$ (periodic integration, Osborn, 1988, *JAE*) when $rank(\Pi) = 6$ and

$$\begin{aligned} \beta' &= \begin{pmatrix} -\varphi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varphi_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varphi_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\varphi_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varphi_6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varphi_7 & 1 \end{pmatrix} \\ \varphi_1 &= (\varphi_2\varphi_3\varphi_4\varphi_5\varphi_6\varphi_7)^{-1} \\ \varphi_s &\neq 1 \text{ for some } s \end{aligned}$$

$$(1 - \varphi_s L) y_t \tilde{} PI(0) \text{ for all } s$$

Periodic Integration?

$y_t \sim SI(d_0, d_1, d_2, d_3)$ (seasonal integration at not all the frequencies, Hylleberg, Engle, Granger, and Yoo 1990, JE) when $0 < rank(\Pi) < 6$, and

$$(1-L)^{d_0}(1 - 2\cos\frac{2\pi}{7}L + L^2)^{d_1}(1 - 2\cos\frac{4\pi}{7}L + L^2)^{d_2} *$$
$$(1 - 2\cos\frac{6\pi}{7}L + L^2)^{d_3} y_t \tilde{} PI(0)$$

$y_t \sim SI(1,1,1,1)$ when Y_n is a noncointegrated I(1) process, and

$$\Delta_7 y_t \tilde{} PI(0)$$

Periodic Integration Analysis

	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
a_t	168.51^{***}	114.49^{***}	76.70^{***}	50.11^{***}	28.54^{***}	11.41^{**}	1.28
d_t	181.64^{***}	115.55^{***}	80.39^{***}	52.99^{***}	32.09^{***}	14.65^{***}	2.31
f_t	985.82^{***}	733.18^{***}	545.96^{***}	397.70^{***}	272.08^{***}	168.67^{***}	70.55^{***}
s_t	850.82^{***}	634.46^{***}	438.52^{***}	271.06^{***}	167.71^{***}	73.15^{***}	2.91
	$\widehat{\varphi}_1^i$	$\widehat{\varphi}_2^i$	$\widehat{\varphi}_3^i$	$\widehat{\varphi}_4^i$	$\widehat{\varphi}_5^i$	$\widehat{\varphi}_6^i$	$\widehat{\varphi}_7^i$
a_t	0.910	0.561	3.039	0.683	1.100	1.390	0.617
d_t	0.889	0.551	3.087	0.602	1.044	1.583	0.665
s_t	0.999	0.999	1.000	0.998	0.997	1.001	1.006

- ⋆ Arrivals and departures are PI(1) with very similar periodic integration coefficients. Therefore, the accumulation of shocks has changed the intra-week pattern of the flow variables (Franses, 1996).
- ⋆ Net Flows is PI(0), suggesting nonperiodic cointegration between arrivals and departures, and stock of visitors is I(1).

Periodic Cointegration?

a_t, d_t˜PI(1) and cointegrated ($a_t - k_s d_t \sim$PI(0)) (Franses, 1994, *JE*)
$\mathbf{Y}_n \equiv (A_n, D_n)'$: where $A_n = (a_{1,n}, ..., a_{7,n})'$ and $D_n = (d_{1,n}, ..., d_{7,n})'$:

$$\mathbf{\Gamma}(L^7)\Delta_7 \begin{bmatrix} A_n \\ D_n \end{bmatrix} = \mathbf{C} + \boldsymbol{\alpha} \begin{bmatrix} \mathbf{I}_7 & \mathbf{K}' \\ \mathbf{0} & \beta' \end{bmatrix} \begin{bmatrix} A_{n-1} \\ D_{n-1} \end{bmatrix} + \mathbf{U}_n$$

$\mathbf{I}_7$ is the 7-dimensional identity matrix, $\mathbf{0}$ is the 6×7-dimensional null matrix, $\mathbf{K} \equiv diag(-k_1, ..., -k_7)$ (s=1,...,7)) and $\beta' D_n$˜$I(0)$.

- ⋆ There is cointegration between all the pairs ($a_{s,n}$,$d_{s,n}$) (Osborn, 2002, WP).
- ⋆ The cointegrating vectors $k_s \neq k$ (at least for some s) may be different.
- ⋆ When $\varphi_s^1 = \varphi_s^2$ for all s, cointegration is not periodic ($k_s = k$ all s).

Periodic Cointegration Analysis (1)

Cointegration can be analyzed with a reduced system: $(A_n, \frac{1}{7}\Sigma_{s=1}^{7} d_{,n})'$:

LR_0	LR_1	LR_2	LR_3
330.33***	166.53***	113.51***	78.319
LR_4	LR_5	LR_6	LR_7
50.965***	27.616***	11.329***	1.207

⋆ We do not reject cointegration between the arrivals and departures.

Periodic Cointegration Analysis (2)

Nonperiodic versus Periodic Cointegration only can be analyzed with the whole system $(A_n, D_n)'$:

	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
	769.77***	594.74***	475.16***	376.21***	300.01***	232.61***	175.08***
	LR_7	LR_8	LR_9	LR_{10}	LR_{11}	LR_{12}	LR_{13}
	124.88***	90.72***	61.25***	40.81***	22.20***	9.40	1.35
	$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$
	1.003	1.043	1.013	1.105	1.237	1.030	0.977
	$\widehat{\varphi}_1^i$	$\widehat{\varphi}_2^i$	$\widehat{\varphi}_3^i$	$\widehat{\varphi}_4^i$	$\widehat{\varphi}_5^i$	$\widehat{\varphi}_6^i$	$\widehat{\varphi}_7^i$
a_t	0.895	0.592	3.125	0.646	1.156	1.331	0.609
d_t	0.872	0.569	3.219	0.592	1.032	1.598	0.642

⋆ The likelihood ratio test ($LR_{12} = 9.40$) is rather close to the 10% critical value 9.67.

⋆ Contrary to the univariate analysis, nonperiodic Cointegration is rejected at 1%.

Periodic Multicointegration?

Jones and Brelsford (1967, *Biometrika*):

$$y_t = c_s + \underbrace{\Sigma_{j=1}^{p} \lambda_{0,j} y_{t-j}}_{\text{NON PERIODIC COMP.}} + \underbrace{\Sigma_{k=1}^{3} \Sigma_{j=1}^{p} (\lambda_{k,j} y_{\alpha k,t-j} + \gamma_{k,j} y_{\beta k,t-j})}_{\text{PERIODIC COMP.}} + \varepsilon_t,$$

$y_{\alpha k,t} = \cos\left(\frac{2\pi k}{7} t\right) y_t$, $y_{\beta k,t} = \sin\left(\frac{2\pi k}{7} t\right) y_t$, $(k = 1, 2, 3)$.

The cyclical parameters $\boldsymbol{\psi}_j = (\lambda_{0,j}, \lambda_{1,j}, \gamma_{1,j}, \lambda_{2,j}, \gamma_{2,j}\lambda_{3,j}, \gamma_{3,j})'$ and the periodic parameters $\boldsymbol{\phi}_j = (\phi_{1,j}, \phi_{2,j}, ..., \phi_{7,j})'$ satisfy $\boldsymbol{\psi}_j = R\boldsymbol{\phi}_j$

$$R = \frac{1}{7} \left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \\ 2\cos(1\frac{2\pi}{7}) & 2\cos(2\frac{2\pi}{7}) & \cdots & 2\cos(7\frac{2\pi}{7}) \\ \vdots & \vdots & & \vdots \\ 2\sin(1\frac{2\pi}{7}3) & 2\sin(2\frac{2\pi}{7}3) & \cdots & 2\sin(7\frac{2\pi}{7}3) \end{array} \right]$$

Ex.: $\lambda_{0,j} = \frac{1}{7}\Sigma_{s=1}^{7}\phi_{s,j}$.

Periodic Multicointegration?

$d_t \sim PI(1)$ and $s_t \sim I(1)$ and Periodic $CI(1,1)$

- ⋆ $(D_n, S_n^C)'$: where $S_n^C = RS_n = (s_{\alpha 0,n}, ..., s_{\beta 3,n})'$ are the systematically sampled weekly version of the HEGY variables (Hylleberg et al., 1990, Osborn, 2002), with $s_{\alpha 0,n} = \frac{1}{7}\Sigma_{s=1}^{7} s_{s,n}$:

$$\boldsymbol{\Gamma}(L^7)\Delta_7 \left[\begin{array}{c} D_n \\ S_n^C \end{array} \right] = \mathbf{C}^* + \boldsymbol{\alpha}^* \left[\begin{array}{ccc} \mathbf{I}_7 & \mathbf{k} & \boldsymbol{\eta} \\ \mathbf{0} & \mathbf{0} & \theta_2' \end{array} \right] \left[\begin{array}{c} D_{n-1} \\ S_{n-1}^C \end{array} \right] + \mathbf{U}_n^*,$$

$\mathbf{k} = (-k_1, ..., -k_7)'$.

$$\boldsymbol{\Pi}(L^7)\Delta_7 \left[\begin{array}{c} D_n \\ s_{\alpha 0,n} \end{array} \right] = \mathbf{C}^{**} + \boldsymbol{\alpha}^{**} [I_7, \mathbf{k}] \left[\begin{array}{c} D_{n-1} \\ s_{\alpha 0,n-1} \end{array} \right] + \mathbf{U}_n^{**}.$$

Periodic Multicointegration Analysis

Multicointegration is analyzed with $(D_n, s_{\alpha 0,n})'$:

LR_0	LR_1	LR_2	LR_3
293.69***	211.14***	141.63***	84.741***
$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$
0.013	0.004	0.006	0.011

LR_4	LR_5	LR_6	LR_7
56.983^{***}	34.333^{***}	16.059^{***}	0.001
$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$	
0.006	0.007	0.022	

- ⋆ The cointegration rank is 7 and hence suggesting the stock to cointegrate with the departure series.
- ⋆ Arrivals and departures are periodically multicointegrated.

Common Periodic Features?

Serial Correlation Common Features (SCCF): Engle and Kozicki (1993, *JBES*), Vahid and Engle (1993, *JAE*)

Y_n has Weak Form of SCCF (Hecq, Palm, & Urbain, 2004, forth. *JE*) if there exists a $7 \times s_1$ $(0 < s_1 < 7)$ cofeature matrix $\alpha_{1\perp}$, $\alpha'_{1\perp}\Gamma_j = 0$ $(j = 1, ..., P-1)$.

Then $r_1 (\equiv 7 - s_1) \times 1$ vector of dynamic factors generate the short-run dynamics of all $y_{s,n}$ (and all $y_{\alpha k,n}, y_{\beta k,n}$)

$$\Delta_7 Y_n = \widetilde{C} + \alpha\beta' Y_{n-1} + \alpha_1 \underbrace{(\beta'_1 \Delta_7 Y_{n-1} + ... + \beta'_{P-1} \Delta_7 Y_{n-P-1})}_{\text{r}_1 \text{ COMMON FEATURES}} + U_n$$

$\alpha'_{1\perp}\alpha_1 = 0.$

$$\underbrace{0}_{\text{NO COMMON FEATURES}} < s_1 \leq \underbrace{6}_{\text{ONE DYNAMIC FACTOR}}$$

Common Periodic Features?

Codependent Cycles: Vahid and Engle (1997, *JE*), Polynomial SCCF: Cubadda and Hecq, 2001, *EL*)

Y_n has Weak Form of PSCCF (Hecq, Palm, & Urbain, 2004)

$$\Delta_7 Y_n = \widetilde{C} + \alpha\beta' Y_{n-1} + \Gamma_1 \Delta_7 Y_{n-1} + \alpha_2 \underbrace{(\beta'_2 \Delta_7 Y_{n-2} + ... + \beta'_{P-1} \Delta_7 Y_{n-P-1})}_{\text{r}_2 \text{ COMMON DYNAMIC FACTORS}} + U_n$$

Common Periodic Features Analysis: Arrivals

s	$\xi_{m=0}(s)$	$\xi_{m=1}(s)$	$\xi_{m=2}(s)$	$\xi_{m=3}(s)$	$\xi_{m=4}(s)$	$\xi_{m=5}(s)$	$\xi_{m=6}(s)$
1	50.39	33.38	28.34	15.34	10.71	4.37	0.02
2	107.22*	77.47	65.00	35.78	27.82	14.28	0.54
3	196.99***	139.77*	113.24*	62.61	47.59	26.05	1.82
4	306.10***	210.00***	167.96**	107.96	78.75	43.57	10.00
5	432.96***	303.89***	228.06***	154.76*	122.33**	64.56	23.61
6	581.74***	412.46***	307.06***	228.13***	185.20***	108.84**	44.82
7	862.96***	637.36***	503.77***	337.59***	277.53***	168.61***	75.92***

⋆ Day-of-week arrivals have 1 PSCCF(0), 2 PSCCF(1), 4 PSCCF(4), 5 PSCCF(5) and 6 PSCCF(6) features.

Common Periodic Features Analysis: Departures

s	$\xi_{m=0}(s)$	$\xi_{m=1}(s)$	$\xi_{m=2}(s)$	$\xi_{m=3}(s)$	$\xi_{m=4}(s)$	$\xi_{m=5}(s)$	$\xi_{m=6}(s)$
1	49.12	36.53	24.49	11.65	9.31	2.56	0.01
2	120.19**	81.42	58.56	40.49	30.80	13.84	1.14
3	211.65***	137.26*	107.46	81.31	58.17	31.55	4.67
4	307.36***	212.90***	173.78***	141.49***	102.82***	50.98	15.22
5	441.22***	306.19***	254.94***	209.20***	160.48***	88.58***	29.24
6	581.41***	422.86***	345.65***	292.86***	229.93***	138.92***	58.77***

⋆ Day-of-week departures have 1 PSCCF(0), 2 PSCCF(1), 3 PSCCF(2), 4 PSCCF(5), and 5 PSCCF(6).

Common Periodic Features Analysis: Stock of Visitors'

s	$\xi_{m=0}(s)$	$\xi_{m=1}(s)$	$\xi_{m=2}(s)$
1	4.90	2.10	0.04
2	17.61	8.54	0.49
3	37.09	18.73	1.54
4	59.75	32.11	3.48
5	92.30	48.72	11.32
6	135.96	72.61	31.02

⋆ Day-of-week Stock of visitors have 6 PSCCF(0).

Concluding Remarks

⋆ First evidence on daily PI(1)ness of economic data. (Daily Dutch tax revenues: Koopman and Ooms (2004, WP), Hourly Return volatility: PGARCH(1,1) Bollerslev and Ghysels (1996, *JBES*), and S&P500 Composite Index: PAR(p)+PGARCH(1,1): Franses and Paap (2004)).

⋆ Empirical Evidence on daily nonperiodic cointegration and periodic multi-cointegration.

⋆ Extension of common features to periodic models.

⋆ Daily model for an airport transit/tourism variable.