Longevity-Linked Annuities: Benefit Structures and Risk Sharing Profiles

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Introduction

Longevity guarantees

- Individuals are exposed to the risk of outliving their own resources
  - $\Rightarrow$ Individual longevity risk & financial risk
- A number of post-retirement income products/arrangements, with different types and levels of guarantees
- Critical issue: The cost of guarantees, impacted by
  - Reduction of interest rates & volatility of financial markets
  - Reduction of mortality rates & major unanticipated mortality improvements ($\Rightarrow$ "Aggregate" longevity risk, for the provider)

Post-retirement income products/arrangements – I

Traditional, immediate life annuities

- Longevity guarantee (lifelong payment) & financial guarantee (fixed or minimum annual amount)

Self-annuitization (Income drawdown)

- No guarantee

Variable annuities

- Several guarantees available, typically financial

Delayed and contingent life annuities (e.g., ALDA, RCLA)

- Longevity guarantee at older ages only, possibly contingent on adverse scenarios

Post-retirement income products/arrangements – II

Group Self-Annuitization (GSA), pooled annuities and tontine arrangements

- Longevity risk sharing within a pool, without guarantees
- Literature
  GSA: [Piggott et al., 2005], [Valdez et al., 2006], [Bravo et al., 2009], [Qiao and Sherris, 2012], [Boyle et al., 2015]
  Pooled annuities: [Stamos, 2008], [Donnelly et al., 2013], [Donnelly et al., 2014], [Donnelly, 2015]
  Tontine arrangements: [McKeever, 2009], [Baker and Peter Siegelman, 2010], [Sabin, 2010], [Milevsky, 2014], [Milevsky and Salisbury, 2015], [Milevsky and Salisbury, 2016], [Weinert and Gruendl, 2016], [Chen et al., 2018]

Mortality/longevity-linked life annuities

- Longevity risk sharing within an annuity, with partial guarantees
- Literature
  [Lüthy et al., 2001], [de Melo, 2008], [Denuit et al., 2011], [Richter and Weber, 2011], [Maurer et al., 2013], [Denuit et al., 2015], [Weale and van de Ven, 2016], [Bravo and de Freitas, 2018]

Longevity-linked benefits

Participating structure

- The benefit amount is allowed to fluctuate, depending on a given longevity experience
- Guarantees can be included (for example: a minimum benefit amount)

Benefit at time t

$$b_t = b_{t-1} \cdot \text{adj}_{(t-1,t)}$$

or

$$b_t = b_0 \cdot \text{adj}_{(0,t)}$$

or

$$b_t = b_{t-k} \cdot \text{adj}_{(t-k,t)} \qquad \text{every } k \text{ years}$$

- $\text{adj}_{(t-1,t)}$, $\text{adj}_{(0,t)}$, $\text{adj}_{(t-k,t)}$: Adjustment coefficients at time t, expressing a longevity experience, respectively in $(t-1,t)$, $(0,t)$ or $(t-k,t)$

Adjustment coefficient

Alternatives

	Portfolio/Indemnity-based	*Index-based*
Number of survivors or Survival rates (observed vs expected)	In an appropriate portfolio	In a reference population
Actuarial quantities	Required portfolio reserve vs Available assets	Actuarial value of the annuity with updated life tables

In the literature

- [Richter and Weber, 2011], [Maurer et al., 2013], [Lüthy et al., 2001]: Actuarial values (namely, comparison between the required and the available reserve)
- [de Melo, 2008], GSA: Assets vs reserve
- [Denuit et al., 2011], [Bravo and de Freitas, 2018]: Survival rates (index-based)
- [Denuit et al., 2015]: Expected lifetime (i.e., actuarial value 0% discount rate)

A general framework – I

Actuarial balance in year $(t-1, t)$ . . .

(One policy, in-force at time $t-1$)

. . . in terms of the conditions applied to the annuitant

$$\underbrace{\overbrace{b_{t-1} \cdot a_{x+t-1(\tau')}}^{\text{Reserve invested at time } t-1} \cdot (1+g_t)}_{\text{"Assets" at time } t} = \underbrace{b_t \cdot (1 + a_{x+t(\tau'')}) \cdot \tilde{p}_{x+t-1}}_{\text{Payment + Reserve at time } t\text{, if alive}}$$

- $a_{x+h(\tau)}$: Actuarial value at time h of a unitary annuity, based on the best-estimate assumptions at time τ, $0 \le \tau \le h$ (In particular: $0 \le \tau' \le t-1$, $0 \le \tau'' \le t$)
- $\tilde{p}_{x+t-1}$: Survival rate assigned to the annuitant for year $(t-1, t)$
- g_t: Financial return assigned to the annuitant for year $(t-1, t)$

A general framework – II

Benefit at time t

$$b_t = b_{t-1} \cdot \frac{\overbrace{a_{x+t-1(\tau')} \cdot (1+g_t)}^{\text{Available assets}}}{\underbrace{\left(1 + a_{x+t(\tau'')}\right) \cdot \tilde{p}_{x+t-1}}_{\text{(Payment +) Required reserve}}}$$

- Typical structure in self-insured arrangements or when no guarantee is provided
- In this case:

$\tau' = t-1$	Latest best-estimate
$\tau'' = t$	Current best-estimate
$\tilde{p}_{x+t-1} = \tilde{p}^{[\text{ptf}]}_{x+t-1}$	Observed in the pool $\Rightarrow$ Indemnity-based
$g_t = \tilde{\imath}_t$	Realized return

A general framework – III

Equivalently: Benefit at time t

$$b_t = b_{t-1} \cdot \underbrace{\frac{1+g_t}{1+i_{(\tau')}}}_{\substack{\text{Return} \\ \text{on investments}}} \cdot \underbrace{\frac{p_{x+t-1(\tau')}}{\tilde{p}_{x+t-1}}}_{\text{Survival rate}} \cdot \underbrace{\frac{1+a_{x+t(\tau')}}{1+a_{x+t(\tau'')}}}_{\substack{\text{Actuarial value} \\ \text{of the annuity}}}$$

- $p_{x+t-1(\tau')}$: Survival rate based on the best-estimate assumptions at time τ', $0 \leq \tau' \leq t-1$
- $i_{(\tau')}$: Interest rate based on best-estimate assumptions at time τ'

- Appropriate structure in insured arrangements
- In this case, it is also appropriate to link the adjustment only to the survival rate or only to the actuarial value of the annuity $\Rightarrow$ Some risk is retained by the provider

Example: Linking based on the survival rate

Particular choices

- $b_t = b_{t-1} \cdot \frac{p_{x+t-1(0)}}{\tilde{p}^{[\text{pop}]}_{x+t-1}} = \cdots = b_0 \cdot \frac{{}_t p_{x(0)}}{{}_t \tilde{p}^{[\text{pop}]}_x}$ Target: Best-estimate at time 0
- $b_t = b_{t-1} \cdot \frac{p_{x+t-1(t-1)}}{\tilde{p}^{[\text{pop}]}_{x+t-1}}$ Target: Latest best-estimate

where $\tilde{p}^{[\text{pop}]}$ is observed in a reference population $\Rightarrow$ Index-based

Guarantees

Can be introduced by setting minimum/maximum values for

- The ratio $\frac{p_{(\tau')}}{\tilde{p}}$
- The probabilities $\tilde{p}$
- The benefit amount b_t
- The age of adjustment
- ...

Such bounds can also serve to avoid the transfer of larger profits

What to assess

Targets of a longevity-linking arrangement

For the provider

- Default probability
- Business value
- Deviations in annual payouts and annual profits wrt a target
- Portfolio reserve vs available assets
- ...

For the individual

- Premium loading
- Longevity guarantee
  - Duration of the annuity
  - Stability of the path of the benefit amounts
- ...

Some results – I

Arrangements

- Fixed benefit
- GSA arrangement
- Linking based on the survival rates
  - Mortality experience measured in a reference population (index-based linking)
  - Target survival rate: either the best-estimate at time 0 or the latest best-estimate
  - Maximum age for benefit adjustment: $x_{\max} = 95$
  - Maximum variation of the benefit amount (in respect of the initial amount): $\pm 25\%$
- Linking based on the actuarial value of the annuity
  - Target actuarial value: either the best-estimate at time 0 or the latest best-estimate
  - Other conditions as above

Basic parameters

- One cohort
- Initial age: $x = 65$. Maximum attainable age: $\omega = 100$
- No financial return, no financial risk
- Annuity immediate

Some results – II

Mortality model

- A time-discrete model
- Mortality rate: $Q_{x,t} = q^*_{x,t} \cdot Z_{x,t}$, where
  - $q^*_{x,t}$: Best-estimate mortality rate (at issue)
  - $Z_{x,t}$: Random coefficient expressing unanticipated mortality improvements
- $Z_{x,t} \simeq \text{Gamma}(\alpha_{x,t}, \beta_{x,t})$
  - The parameters are updated in time, learning from the mortality experience, through an inferential procedure
  - The initial value of the parameters is set so to have a priori an expected lifetime in line with the current projected life tables
  - The severity of the longevity risk can be modelled through the initial dispersion of the Gamma distribution
- Details in: [Olivieri and Pitacco, 2009]
- Mortality rates are generated for a reference population and for a portfolio
  $\Rightarrow$ Basis risk (but the only difference is the size of the population)

Some results – III

Safety loading

- Pricing rule

$$b_0 = S \cdot \frac{1}{a_{x(0)} \cdot (1 + \pi)}$$

  - π: Premium loading
  - S: Initial capital

- π assessed such that the provider's probability of loss is 10%, excluding basis risk

	Benefit type	Moderate longevity risk	Major longevity risk
FB	Fixed benefit	1.760%	5.692%
L-SRt	Survival rate (Target: latest BE)	1.708%	5.528%
L-AV0	Actuarial value (Target: BE at time 0)	0.306%	0.962%
L-SR0	Survival rate (Target: BE at time 0)	0.076%	0.242%
L-AVt	Actuarial value (Target: latest BE)	0.055%	0.154%
GSA	Group Self-Annuitization	0.000%	0.000%

(% of the actuarial value of a unitary annuity, based on best-estimate assumption at time 0)

Some results – IV

Present Value of Future Benefits (PVFB) and Present Value of provider's Future Profits (PVFP)

Moderate longevity risk					
		PVFB		**PVFP**	
Benefit type		**Exp. value**	**99%-Conf. interval**	**Exp. value**	**99%-Conf. interval**
FB	Fixed benefit	98.301	(95.194,101.416)	1.699	(–1.416,4.806)
L-SRt	Survival rate (Target: latest BE)	98.350	(95.337,101.372)	1.650	(–1.372,4.663)
L-AV0	Actuarial value (Target: BE at time 0)	99.692	(99.140,100.264)	0.308	(–0.264,0.860)
L-SR0	Survival rate (Target: BE at time 0)	99.925	(99.786,100.062)	0.075	(–0.062,0.214)
L-AVt	Actuarial value (Target: latest BE)	99.945	(99.843,100.044)	0.055	(–0.044,0.157)
GSA	Group Self-Annuitization	100.000	(100.000,100.000)	0.000	(0.000,0.000)
Major longevity risk					
		PVFB		**PVFP**	
Benefit type		**Exp. value**	**99%-Conf. interval**	**Exp. value**	**99%-Conf. interval**
FB	Fixed benefit	94.791	(85.736,104.234)	5.209	(–4.234,14.264)
L-SRt	Survival rate (Target: latest BE)	94.935	(86.151,104.099)	5.065	(–4.099,13.849)
L-AV0	Actuarial value (Target: BE at time 0)	99.034	(97.384,100.800)	0.966	(–0.800,2.616)
L-SR0	Survival rate (Target: BE at time 0)	99.716	(97.871,100.724)	0.284	(–0.724,2.129)
L-AVt	Actuarial value (Target: latest BE)	99.845	(99.537,100.120)	0.155	(–0.120,0.463)
GSA	Group Self-Annuitization	100.000	(100.000,100.000)	0.000	(0.000,0.000)

(Values per policy issued and per 100 units of initial capital)

Discount rate: 0%; No basis risk

Summary

- We address annuity designs in which the benefit is updated to the mortality experience
- A general framework is described, providing several particular cases (just few are discussed here)
- Main issues:
  - Choice of the parameters ensuring a satisfactory risk/return trade-off, for the individual and the provider
  - Individual preferences about the benefit path
  - Cost of capital and value created for the provider
  - Annual results, in respect of both a target cash flow and a target profit
  - Smoothing of the benefit amounts
  - Interaction with other risks, financial risk in particular
  - Pricing, identifying the embedded options
  - Mortality model
  - Solidarity effects, in case of a heterogeneous population
  - . . .

Many thanks for your kind attention!

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