(G)Rate Expectations: Risk Premia and Bond Excess Returns

Riccardo Rebonato – Spring 2015
Plan of the talk

Excess returns and risk premia

- The old received wisdom
- The new findings
- Analysis of the new evidence
- Extensions to real rates
- Robust, stable and interpretable return-predicting factors

Models to predict excess returns and risk premia

- Kim Wright
- New Fed Model
- PCA Affine (PIMCO) model
- Stochastic MPR model
- Common problems
- *Do we really need models?*
Excess Returns: Nominal Rates

The excess-return strategy:
1. Invest in the $n$-year maturity bond.
2. Fund the long position by borrowing for 1 year.
3. Sell the long bond after 1 year (when its residual maturity is $n-1$ years).
4. Repay the loan.
5. Record the profit or loss.
6. Rinse and repeat.
Observations

• If “forwards come true”, no money will be made. This is the (local) expectation hypothesis.
• An interpretation of roll-down and carry.
• The investor who engages in the buy-long-fund-short strategy will make or lose money for a variety of reasons, of which the term premium is only one.
• If the Fed unexpectedly decide to lower (hike) rates massively, the long position will make (lose) money on top of the risk premium, whose contribution will be totally swamped.
• The task at hand is to detect the effect of a tiny breeze of different intensity in the middle of a raging, never-stopping storm.
The Yield History
The Excess Returns (yearly, monthly overlapping)
One important observation

The forward rate today, \( f(t,T) \) is given by

\[
f(t,T) = \text{Spot}(t) + \text{RiskPrm}(t) + E_t[\Delta(\text{Spot})]
\]

If forwards come true, ie, if \( \text{Spot}(T) = f(t,T) \),
the buy-long/fund-short strategy makes no money.

I can make/lose money

1. because there is a term premium
2. because the expectation was wrong.

The second contributor to profitability is large and common.
Sharpe Ratios: 1955-2014

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<th>5 year ¹</th>
<th>10 year ¹ Sharpe Ratio</th>
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*From work by Vasant Naik and Mukundan Devarajan – PIMCO*
What We Now Know – 1

• Expected future interest rates are quite different from forward rates and therefore ‘[r]isk premia are large. [...] A pure forecasting approach leaves them as simple undigested residuals.’

• If the risk premium is assumed to be a constant, there is no correlation between excess returns and volatility, or any of its proxies. At first blush, this is puzzling: risk premia should be a compensation for uncertainty. If they are not linked to volatility, does our the simple picture break down?

• If we look the forecasting `errors' (residuals) obtained assuming a constant market price of risk we find that they are strongly negatively correlated with the slope of the yield curve.
What We Now Know – 2

• Some researchers claim that average expected excess returns to Treasury bonds are on average small (Cochrane and Piazzesi 2005.) By ‘on average' they mean that at some point in time excess returns may be very large, and at other times very negative, but that these high and low returns average out to a much smaller (positive) return.

• It should be stressed that the claims about the smallness of the excess returns have been made on the basis of studies that look at maturities out to five years. Excess returns are not so small when the maturities extend out to 10 years, and the attractive excess returns for long maturities do not come associated with unacceptably large variances of returns: the Sharpe ratios are relatively constant across maturities -- and, if anything, they slightly decline for the longest horizons.

• However, a more qualified statement along the following lines certainly remains valid: the average expected excess returns to Treasury bonds from the simple strategy always-buy-long-fund short are considerably lower than the excess returns from a smarter strategy when the magnitude and sign of the strategy is made to depend on some return-predicting factor.
What We Now Know – 3

• These unexciting unconditional excess returns can be reconciled with the statement that 'risk premia are large' if the market price of risk depends on a quantity of time-varying sign, giving at times a (possibly large) positive contribution and at time a (possibly large) negative contribution.

• **Explanation of the volatility conundrum.**

• The correlation of the residuals with the slope of the yield curve suggested to earlier researchers that this latter quantity could be the sign-changing quantity that can account for the large risk premia -- or, at least, that it might contain part of the explanation. **Carry and roll-down.**

• **Compensation for shocks other than level shocks are essentially zero**

• The finding that ‘the compensation for shocks other than level is essentially zero' means that what the investors are compensated for, however, is not ‘slope risk', but ‘level risk': the slope tells us the compensation per-unit-risk-of-something; the `something' is then the level of the yield curve.
What We Now Know – 3

• In general, the degree of compensation for bearing level risk is proportional to a "return-forecasting factor".
• Earlier studies indicated that this factor should be the slope of the yield curve.
• More recent work by Cochrane and Piazzesi (2005, 2008), Hellerstein (2011) and others suggests that this factor may be described by a special (tent-shaped) function, $f(\cdot)$, of five forward rates (and hence of five principal components):

$$\lambda(t) = f(x_1, x_2, \ldots, x_5).$$

• This more recent (‘tent’) return-forecasting factor is imperfectly spanned (captured) by level, slope and curvature.
What We Now Know – 4

• Dai, Singleton and Yang (2004) raise some doubts about the robustness of these results, and show that almost arbitrary choices about the interpolation method used to obtain the forward can alter -- significantly -- the loadings on the regressors.

• The tent disappears, and the pattern they (and we) find resembles more the wings of a gliding animal, that we choose to call a bat.

• They conclude that ‘in projections of excess returns onto forward rates, the data based on the relatively choppy forward curves will give rise to projection coefficients that are "biased" in the direction of having a tent-like shape'.

Understanding the empirical evidence

• Slope, Tents, bats, and other exotica – are they really different?

• To first order
  – Bats equivalent to tents
  – Slope (in yields) equivalent to tent (in forwards)

• Is the “second order” informative or noise?

• Can we identify a robust predictor?
The Basics: What Drives the XRs?

![Average Excess Returns vs Slope Graph](image-url)
What Drives the XRs

- **XR vs Slope (full)**: $y = 0.0505x + 0.0289$, $R^2 = 0.173$
- **XR vs Slope (1st half)**: $y = 0.0598x + 0.03$, $R^2 = 0.2652$
- **XR vs level (full sample)**: $y = 0.0645x + 0.2151$, $R^2 = 0.0029$
- **XR vs slope (2nd half)**: $y = 0.0708x + 0.0274$, $R^2 = 0.1076$
What Does The Return-Predicting Factor Look Like?
The Bat Shape

Regression coefficients (2- to 5-year strategies)
Forcing the recovery of the tent

Clearly this shape is at first blush very different from the tent pattern. In order to see how significant this difference is, we impose that the return-predicting factor should have the forward rates arranged in a tent-like shape. We do so by requiring that the return-predicting factor should be given by

$$RPF_{tent} = \alpha_0 + \sum_{k=1}^{5} \alpha_k f_k$$

with

- $\alpha_1 = -(k_1)^2$
- $\alpha_2 = (k_2)^2$
- $\alpha_4 = (k_4)^2$
- $\alpha_5 = -(k_5)^2$
- $\alpha_3 = \max\{\alpha_4, \alpha_2\} + (k_3)^2$
Results: The constrained loadings on the forward rates
How Similar? Predictions: Bat vs CP Coefficients

Excess returns: realized, tent- and bat-predicted
Predictions: Bat vs CP Coefficients

Differences between tent and bat predictions
From Slope in Yields to Bat in Forwards

The slope return-predicting factor for the $n$-maturity excess return for the investment periods starting at time $i$, $rpf_i^n$, can be written as

$$rpf_i^n = \text{Dur}^n \text{slope}_i$$

where, based on our regressions, we have assumed that the return-predicting factor can be expressed as a common slope factor times the duration, $\text{Dur}^n$, of the underlying, $n$-maturity, bond. \footnote{footnote}

The common slope factor is given (by definition of principal components) by

$$\text{slope}_i = \sum_{k=1}^{5} \gamma_k y_k^i$$

with the coefficients $\{\gamma\}$ shown in Fig \footnote{ref: gamma}. Given our assumption of the importance of the slope as a return-predicting factors we write for the excess return, $r_{x_{i,i+1}}^n$,

$$r_{x_{i,i+1}}^n = a_0 + a \times rpf_i^n = \sum_{k=1}^{5} \gamma_k y_k^i$$

$$= a_0 + a \times \text{Dur}^n \sum_{k=1}^{5} \gamma_k y_k^i$$
Finally, substituting from Equation (ref: yield) we have

\[
r_{x_{t+1}}^n = a_0 + a \times Dur^n \sum_{k=1}^{5} \gamma_k y_k^i =
\]

\[
= a_0 + a \times Dur^n \sum_{j=1}^{5} \sum_{k=j}^{5} \frac{\gamma_k}{k}
\]

Collecting terms gives

\[
r_{x_{t+1}}^n = a_0 +
\]

\[
f_1[a \times Dur^n \left( \gamma_1 + \frac{\gamma_2}{2} + \frac{\gamma_3}{3} + \frac{\gamma_4}{4} + \frac{\gamma_5}{5} \right)] +
\]

\[
f_2[a \times Dur^n \left( \frac{\gamma_2}{2} + \frac{\gamma_3}{3} + \frac{\gamma_4}{4} + \frac{\gamma_5}{5} \right)] +
\]

\[
f_3[a \times Dur^n \left( \frac{\gamma_3}{3} + \frac{\gamma_4}{4} + \frac{\gamma_5}{5} \right)] +
\]

\[
f_4[a \times Dur^n \left( \frac{\gamma_4}{4} + \frac{\gamma_5}{5} \right)] +
\]

\[
f_5[a \times Dur^n \left( \frac{\gamma_5}{5} \right)]
\]
From Slope in Yields to Bat in Forwards

Alternatively, we can regress directly the excess return on the forward rates, obtaining

\[ r_{t,t+1}^\gamma = b_0 + \sum_{k=1}^5 b_k f_k^t \]

which implies

\[ b_0 = a_0 \]

and

\[ b_k = a \times Dur^\gamma \sum_{i=1}^k \frac{\gamma_i}{S_i} \]

We can finally answer the following question: what does these relationships imply for the regression coefficients \( b_i \) when the \( \gamma_s \) are slope-like loadings in yield space as shown in stylized form in Fig gamma? So, we impose that the regression coefficients when yields are used as regressors should display a slope behaviour, as in Fig (ref: gamma), and we use Equations (ref: gamma2b) to determine what the regression coefficients would look like when the forward rates are used as regressors.
The input slope loadings
Translating a slope in yields to forward rates
Explaining slopes/bats (+/- signs)
Robustness

• Cocharne & Piazzesi’s tent may well explain more in-sample. But is it robust? Can we trust it for out-of-sample prediction? Or should we use the slope?

• To answer the question
  – we calculate the regression coefficients for slope using the first half of the sample;
  – we use them to predict excess returns in the second half;
  – we do the same with the other half;
  – we compare the out-of-sample predictions with the predictions obtained using the full sample;
  – we calculate the out-of-sample $R^2$ and compare.
Robustness: Tent

In- and out-of-sample regressions: slope

- Exret
- FullSI
- MixedSI
Robustness: Tent

In- and out-of-sample regressions: C & P
### Comparing $R^2$

<table>
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<tr>
<th>$R^2$</th>
<th>Slope (Full)</th>
<th>Slope (Mixed)</th>
<th>C&amp;P (full)</th>
<th>C&amp;P (mixed)</th>
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<td>0.014</td>
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<td>First Half</td>
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<td></td>
<td>0.3830</td>
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<td>Second Half</td>
<td>0.0772</td>
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<td>0.1440</td>
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Yes, the $R^2$ can become negative when it is out of sample!

Even if the slope is more robust, doesn’t it perform poorly out of sample? Not really, because in reality we would not use coefficients determined during 1960-1987 to make predictions in 2014.
Part II – The Models

Advantages of using models:
1. Regularizing noisy estimates
2. Ensuring via no-arbitrage consistency in risk premia across different maturities
3. Making use of cross-sectional information
4. Helping our “understanding”

Disadvantage of using models:
1. The model can only return an answer of the form it ‘knows about’ (eg, Vasicek)
2. It can only regurgitate the empirical information fed into it.
The New (ACM) Fed Model

• Traditional modelling has almost disappeared: any linear combination of yields (perhaps principal components) that can be estimated will do.

• Impose that the state variables should be affine

• Add the no-arbitrage conditions

• Where does the link with risk premia come from? From analysis of excess returns.
10-year term premium vs level
10-year term premium vs slope
Correlation average term premium vs slope/level

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<th>Corr(Term premium, lev)</th>
<th>Corr(Term premium, slope)</th>
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<tr>
<td>Level</td>
<td>66.1%</td>
<td>36.9%</td>
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<tr>
<td>Differences</td>
<td>30.0%</td>
<td>72.1%</td>
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These wrong correlations matter, because they give wrong predictions of when the risk premium is going to be high!
Deterministic link mpr/state variables

Model term premium: actual vs predicted

AvgTP
Pred
Expectation and Risk Premia

• Focus on the Bernanke tapering tantrums
• After the 22\textsuperscript{nd} May testimony, risk premium may have increased (with uncertainty), but we certainly expect rates expectations to have gone up.
• According to the model(s), did they?
Decomposition 1y + slope
Decomposition 3y + slope
Decomposition 5y + slope
Decomposition 10y + slope

10-year yield

- ACMY10
- ACMTP10
- ACMRNY10
- Slope

Dates:
- 18-Oct-12
- 26-Jan-13
- 6-May-13
- 14-Aug-13
- 22-Nov-13
- 2-Mar-14
- 10-Jun-14
- 18-Sep-14
- 27-Dec-14
The Kim Wright Model

- Latent-variable approach
- Good fit to nominal and (in augmented version) to real yields with constant parameters
- Liquidity factor has the answer largely ‘backed in’.
- Parameters unfortunately fitted to 1990-2007 period – does the behaviour of the model change because the world is different but the same parameters still apply, or because there has been a regime switch, (or both)?
- Dubious decomposition of yields into expectations and term premia
Risk Premia
KW: the ‘regime change’
KW: the ‘regime change’
KW: the ‘regime change’

expectation driven

risk-premium driven
KW: 10 year
Taper tantrum: KW 1 y
Taper tantrum: KW 3y

The change is all term premium

The term premium is (mainly) slope
Taper tantrum: KW 5y

If anything, after the Bernanke testimony the model expectations go *down* a bit!

The change is all term premium; the term premium is all slope
Taper tantrum: KW 10y

Slope, slope, slope...
Our PCA Affine Model

The Dynamics of the Problem

We impose that the principal components, $\mathbf{x}_t$, should follow an affine diffusion of the form:

$$d\mathbf{x}_t = \mathbf{K}(\mathbf{\bar{\theta}} - \mathbf{x}_t)dt + \mathbf{S}dz$$

and

$$E\left[ dz^T dz \right] = Idt$$

We refer to $\mathbf{K}$ as the reversion-speed matrix, to $\mathbf{S}$ as the diffusion matrix, and to $\mathbf{\bar{\theta}}$ as the reversion-level vector. For reasons that will become apparent in the following, we require the matrix $\mathbf{K}$ to be invertible and full rank. Since we want to interpret the factors, $\mathbf{x}_t$, as principal components, we require the matrix $\mathbf{S}$ to be diagonal:

$$\mathbf{S} = \text{diag}[s_1, s_2, \ldots, s_N]$$

and we impose

$$s_i = \sqrt{\lambda_i}$$

where $\{\lambda_i\}$ are the eigenvalues of the market covariance matrix (ref: mkt).
The model in a nutshell

• The model uses PCs (about which we know a lot) as state variables.

• It has essentially three degrees of freedom, that dictate the precise form of the reversion speed matrix if we want the state variables to be principal components.

• Yields and covariance matrix are always exactly recovered ‘on the knots’, but can behave in a complex way away from the reference maturities.
Different fits to the 1-to-5-y yields
Fit to the covariance matrix
For those who like numbers...

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Fit to yield volatilities (convexity)
How to determine the reversion levels
Evolution of expectation of yields
Convexity
10-year risk premia and expectations

Risk premium unconditionally correct, but locally far too high.
Predictions (PCA Affine Model)

Again, nominal yield all driven by risk premium, all driven by slope.
Predictions (PCA Affine) – 2

10-year yields (P and Q), risk premium (PCAffine) and slope
Models Compared – 10y risk premia and slope

Different models predict almost identical changes in risk premium. They differ in the intercept, which is the one more difficult to estimate.
The need for a different model

• Breaking the determinist link between slope and market price of risk
A Stochastic MPR Model

Market Price of risk is now stochastic – and mean-reverting

\[ dr_t^O = \kappa_r^P [\theta_t - r_t] dt + \sigma_r dz_t^r \]

\[ d\theta_t^O = \kappa_\theta^P \left( \hat{\theta}_t^P - \theta_t \right) dt + \lambda_\theta^0 \sigma_\theta dt + \sigma_\theta dz_t^r \]

\[ d\lambda_t^\theta = \kappa_\lambda \left( \hat{\lambda}_t - \lambda_t \right) dt + \sigma_\lambda dz_t^r \]

Investors are only compensated for long-term level risk
Evolution of the short rate, the reversion level and the mpr.
Market price of risk – Evolution
Slope and MPR

Correlated, but not joined at the hips.
Fit to Covariance Matrix

The market US$ covariance matrices for yields form 1 to 10
The model yield covariance matrices for yields from 1 to 20 years.
Fit to the yield curve – 20 and 30 years

Same as Fig Fit10, but with maturities extended out to 20 years.
Term Premia

Much more reasonable term premia: 30 bp at 10y, 55 bp at 20 years
Fig TPwhole
Fig TP10y

Risk Premium 10y

- Y-axis: 0.000% to 2.500%
- X-axis: Dates from 7-Sep-10 to 7-Jul-14
Fig TPFull
Expectations for the 2-, 3-, 5- and 10-y yields
Risk premia the the 2-, 3-, 5- and 10-y yields

Fig TapTanRP
Fig RPSlope

5- and 10y risk premia and yield curve slope

- 5y
- 10y
- Slope
Real Rates

• We want to extend the analysis to real yields.
• In particular, we want to extract the real-rate term premium: what does it depend on?
How the Inflation and Real Yields Are Calculated

• First one fits a discount (NSS) curve to nominal bonds.
• One observes the price of a linker, and one determines the inflation that, added to the contractual real coupon, equates the discounted (along the nominal curve) model price to the market price.
• The resulting inflation is a risk-adjusted (Q-measure) inflation. It contains a risk component.
• By subtracting the inflation thus obtained from the nominal yield we obtain the real yield.
• This real yield is also risk-adjusted (Q-measure). It contains its own risk-premium component.
• **It is this risk premium that we set out to discover.**
Qualitative Behaviour – 1
Qualitative Behaviour – 2

Time Series of BEI (2004-2014)
Qualitative Behaviour – 3

10y nominal yield, 10y BEI and VIX
PCA: nominal and real

The PCs have the usual shape, and lend themselves to the usual level, slope and curvature interpretation. However, the first eigenvalue of the real-rates covariance matrix explains even more of the total variance than the first eigenvalues from nominal rates (95.3% vs 92.0%).
What Drives Nominal Yields and BEI (10y)?

Explaining Nominals

- $R^2$ Nominal vs BEI = 8.3%
- $R^2$ Nominal vs REAL = 87.5%
- $R^2$ Nominal vs VIX = 1.3%

BEI dependence on VIX does not filter through to nominals: therefore it is due to TIPS-specific liquidity, not inflation risk premium.

Explaining BEI

- $R^2$ BEI vs Nominal = 8.3%
- $R^2$ BEI vs VIX = 56.5%
- $R^2$ BEI vs VIX+Nominal = 60.7%
- $R^2$ BEI vs VIX (02-Jun-04/30-Dec-07) = 0.00%
- $R^2$ BEI vs VIX (02-Jan-10/24-Jun-2014) = 18.9%
Results (NB: funding with 3-year real bond)
Actual and predicted excess returns – 10 y
Average excess returns and slope

![Average excess returns and slope regressor graph](image)
Results

• For all investment horizons, the most significant explanatory factor of real excess returns is the slope (2\textsuperscript{nd} PC) of the real curve.
• This lends itself again to a ‘carry-and-roll-down’ interpretation.
• The first and third principal components have no significant exploratory power.
• The explanatory power of the slope of the real curve is high 41\%.
• Parsimony, and the risk of over-fitting the signal given the short length of the time series and the overlapping nature of the procedure suggest that we should just use the real slope (2\textsuperscript{nd} PC) as the excess-return predicting factor.
Results - Sharpe Ratio
Results - Returns

Excess returns

Investment maturity at start (years)
Observations

• The regression coefficients are
  – very significant for the slope
  – marginally significant for the level
  – not at all significant for the curvature

• Keeping in mind the reservations due to the overlapping nature of the data, the regression coefficients are always strongly significant, both for the single and for the double regression.

• Excess returns have been very negative during the rate repression period. During this period the slope of the real curve approached zero.

• The $R^2$ decreases significantly as the maturity of the investment bond increases.
Nominal and real predicted excess returns
Predicted inflation (and liquidity) risk premium
Disentangling the liquidity contribution

• The break-even inflation is made up of
  – inflation expectation
  – inflation risk premium
  – liquidity
  – (liquidity risk premium)

• Can we disentangle the liquidity from the risk premia?

• Use VIX as a proxy for liquidity, and use as regressor.
Decomposing the liquidity component
Some conclusions

• After correcting for liquidity, in the 2004-2014 period most of the risk premium can be ascribed to compensation for bearing *real-rate risk*, not inflation risk. This makes sense in a period of well-controlled inflation. (The story may have been very different in the 1970s). It suggests that the market does not ascribe inflation risk to QE.
• The excess return associated with real risk is explained by the real slope.
• As the nominal excess return is mainly associated with real-rate risk, this is where the slope dependence for nominal is inherited.
The tapering period revisited

Breakeven Inflation =
Max[Expected inflation,0] +
Inflation Risk Premium +
Liquidity Adjustment

TIPS are floored – therefore the minimum price increment they can deliver is 0.

\[ Y(R) = Y(N) - (\text{max}[EI,0] + Pr(\text{Infl}) + Liq(\text{TIPS})) \]

Large \( Y(R) \) means:
1. Low inflation expectation
2. Low inflation risk premium
3. Low liquidity premium for TIPS
The taper period: nominal yields, real yields and BEI

After the 22\textsuperscript{nd} May, the real rate continues the trend it was displaying before the talk.

After the 22\textsuperscript{nd} May, BEI stops the downward trend it had displayed before the talk.

Before 22\textsuperscript{nd} May the rise in nominal comes from a tug of war between BEI and real rates: BEI falls, but real rises more quickly (different scales).

After 22\textsuperscript{nd} May all the “action” comes from the real rate.

What changes in the trend of the BEI components after 22\textsuperscript{nd} May?

- Higher inflation expectations?
- Higher inflation risk premium?
- Higher liquidity premium for TIPS?

One interpretation of what the May 2103 talk did, is that it stopped the decline in BEI – but no change to the upward grind of the real rates.