Value Hedging with an Uncertain Market Price of Longevity Risk

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Introduction

Mortality model

Insurer

Results

Conclusions
Motivation

Presentation: Focus on results; Maths in paper.
Large exposure to longevity risk (pension funds & insurers).
No hedging products available.
Cash flow matching:
2004: EIB/BNP longevity bond withdrawn prior issue:
- Duration: 25 years;
- High capital relative to the risk exposure;
- Parameter and model risk;
- Not flexible.

Capital markets:
Liquid market & flexible products; q-forwards building blocks:
- Maturity;
- Gender;
- Age group.
Value hedging

Q-forwards shown to be effective hedge.

Typical assumption: constant price longevity risk over time?

Stochastic volatility risk premium (Bollerslev, Gibson, and Zhou, 2012).
Risk premium known

Unhedged

Hedged

Profit/Loss

Probability
Risk premium 50% of the cases 0

- **Unhedged**
- **Hedged**
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GWIW & Bayesian

The parameters in the random walk with drift are estimates.

Limited data $\Rightarrow$ estimates are uncertain.

New information $\Rightarrow$ update estimates.

- Wishart distribution: multivariate $\chi^2$ distribution;
- Inverse Wishart distribution: ensures positive definite $\Sigma$;
- Gaussian Inverse Wishart distribution: uncertain mean & variance;
- Generalized Gaussian Inverse Wishart distribution: different number of observations.
Cairns Blake Dowd Model

CBD model:

\[ p(x, t) = 1 - q(x, t) \]
\[ q(x, t) = \frac{\exp(A_1(t) + x \cdot A_2(t))}{1 + \exp(A_1(t) + x \cdot A_2(t))}. \]

Two stochastic processes (random walk with drift):

\[ A(t) = [A_1(t) \ A_2(t)]^\top \]
\[ D(t) = A(t) - A(t - 1) \]
\[ = \mu + CZ(t). \]  

Longevity risk premium

Mortality dynamics (including parameter risk):

\[ V|D \sim \text{Inverse Wishart}_2(T, \hat{V}) \]  \hspace{1cm} (2)

\[ \mu|V, D \sim N_2(\hat{\mu}, T^{-1}V). \]  \hspace{1cm} (3)

Change of measure:

\[ A(t + 1) - A(t) = \mu + C(\tilde{Z}(t + 1) - \lambda) \]
\[ = \tilde{\mu} + C\tilde{Z}(t + 1), \]

where \( \tilde{\mu} = \mu - C\lambda. \)

Dynamics for \( \lambda \):

- Similar GIW as mortality dynamics;
- Allow for parameter risk in covariance.
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Issuing an annuity to a 65 year old male.

(Real) payment of annuity is 1 if insured is alive and 0 otherwise.

(Real) interest rate is set at 2%.

No idiosyncratic longevity risk.

No financial risk.
Insurer

Value hedging for 3 years.

Two q-forwards:
- 75 years, maturity 3 years;
- 85 years, maturity 3 years.

Insurer minimizes portfolio variance by optimally selecting # q-forwards.

Net asset value includes:
- risk premium (q-forward);
- payments (via assets);
- liability value at time 3 (surviving, mortality dynamic & risk premium).
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Scenarios

Risk adjusted process is unknown.

In base case we set:
\[ \hat{\lambda} = [0.1167, 0.1167] \Rightarrow \text{risk premium 5%}. \]
\[ \tau = 5 \text{ (not much information)} \]
\[ \hat{V} \text{ such that at } t = 3 \text{ the effect on the risk premium of an annuity:} \]
- 25% – 75%CI: -0.5% – 0.5%;
- 10% – 90%CI: -1.1% – 1.2%.
25% – 75%CI of \( \lambda \) at year 3: 0.102-0.131.

Robustness checks:
- Use \( \tau \) of 10 & 30;
- Increase standard deviations by 50%, decrease by 33%.
### Portfolios and risk reduction

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\beta_{75}$</th>
<th>$\beta_{85}$</th>
<th>std</th>
<th>std hedged</th>
<th>reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>No risk premium</td>
<td>101</td>
<td>30</td>
<td>0.228</td>
<td>0.010</td>
<td>95%</td>
</tr>
<tr>
<td>Known risk premium</td>
<td>97</td>
<td>36</td>
<td>0.249</td>
<td>0.017</td>
<td>93%</td>
</tr>
<tr>
<td>Risk premium 50%</td>
<td>93</td>
<td>35</td>
<td>0.375</td>
<td>0.290</td>
<td>23%</td>
</tr>
<tr>
<td>Base case</td>
<td>99</td>
<td>35</td>
<td>0.290</td>
<td>0.152</td>
<td>48%</td>
</tr>
<tr>
<td>More information $\tau = 10$</td>
<td>92</td>
<td>37</td>
<td>0.264</td>
<td>0.101</td>
<td>62%</td>
</tr>
<tr>
<td>Confident $\tau = 30$</td>
<td>99</td>
<td>35</td>
<td>0.259</td>
<td>0.090</td>
<td>65%</td>
</tr>
<tr>
<td>High variance</td>
<td>95</td>
<td>36</td>
<td>0.340</td>
<td>0.236</td>
<td>30%</td>
</tr>
<tr>
<td>Low variance</td>
<td>100</td>
<td>35</td>
<td>0.266</td>
<td>0.103</td>
<td>61%</td>
</tr>
</tbody>
</table>

Optimal hedging portfolio robust to longevity risk premium;

Knowing uncertainty in variance more important than level of the variance.
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Longevity risk can effectively be hedged using q-forwards if market price of longevity risk is known.

Optimal hedging portfolio robust to longevity risk premium.

Value hedging less effective is market price of longevity risk becomes uncertain in the future. Risk-Reward tradeoff?

Knowing uncertainty in variance more important than level of the variance.

Value hedging: Could have potential, but there are risks!

Academics need information on market price!