Reverse Mortgage Pricing and Risk Analysis Allowing for Idiosyncratic House Price Risk and Longevity Risk

Adam Wenqiang Shao, Katja Hanewald, Michael Sherris

ARC Centre of Excellence in Population Ageing Research (CEPAR) and School of Risk & Actuarial Studies, University of New South Wales, Sydney

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Introduction

Pricing Framework

Idiosyncratic House Price Risk

Termination Model

Pricing and Risk Analysis

Conclusions
Introduction

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Conclusions
Population ageing: how to finance health care in retirement?
- large component of household wealth in home equity
- role for reverse mortgages and other equity release products

Growing literature on reverse mortgage pricing and risk management
- impact of longevity risk (Wang et al., 2008; Li et al., 2010; Yang, 2011)
- new pricing framework (Alai et al., 2013; Cho et al., 2013)
- house price risk typically analysed using time series models based on one market-wide index (e.g., Chen et al., 2010; Yang, 2011; Lee et al., 2012)

This study: idiosyncratic house price risk and longevity risk
- portfolio of options (not: option on portfolio)
- idiosyncratic house price risk substantial (Hanewald and Sherris, 2013)
- disaggregated house price indices: more volatility and different trends (Shao et al., 2013)
Accumulated loan amount

Consider a reverse mortgage loan with:

- variable interest rate
- lump-sum payment
- single borrower

The accumulated loan amount at termination:

\[ L_{T_x} = L_0 \exp \left( \sum_{t=1}^{K_x+1} (y_t^1 + \varphi + \pi) \right) \]

- \( T_x \): random termination time for age \((x)\), \( K_x = \lfloor T_x \rfloor \)
- \( L_0 \): initial loan amount
- \( y_t^1 \): quarterly risk-free rate
- \( \varphi \): quarterly lending margin
- \( \pi \): mortgage insurance premium
No-Negative Equity Guarantee (NNEG)

At termination, the repayment amount is \( \min\{L_{T_x}, H_{T_x}\} \)

Borrower’s net equity is:

\[
\text{Net Equity}_{T_x} = H_{T_x} - \min\{L_{T_x}, H_{T_x}\} \\
= \max\{H_{T_x} - L_{T_x}, 0\}
\]

Present value of lender’s loss:

\[
\text{Loss}_{T_x} = \max\{L_{T_x} - (1 - c)H_{T_x}, 0\} \prod_{s=1}^{K_x+1} m_s
\]

- \( c \): transaction cost
- \( m_s \): risk-adjusted stochastic discount factor

The value of NNEG is:

\[
\text{NNEG} = \sum_{t=0}^{\omega-x-1} E\left[ t|q_x^c \text{ Loss}_{t+1} \right]
\]

- \( t|q_x^c \): probability of terminating between \( t \) and \( t + 1 \)
Mortgage insurance premium

Premium accumulation:

\[ dP_t = \pi L_t dt \]

EPV of charged premium is:

\[ \text{MIP} = \pi \sum_{t=0}^{w-x-1} E \left[ t p_x^c L_t \prod_{s=0}^{t} m_s \right] \quad (2) \]

- \( t p_x^c \): in-force probability

Solve for \( \pi \) by equating expectations of MIP and NNEG: Equations (1) and (2).

\[ \text{SF} = \sum_{t=0}^{w-x-1} \left\{ t q_x^c E [\text{Loss}_{t+1}] - t p_x^c \pi E \left[ L_t \prod_{s=0}^{t} m_s \right] \right\} \quad (3) \]
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Hybrid house price model (Shao et al., 2013)

\[ V_{it} = \alpha + T'\beta + X'\gamma + X'\Delta T' + \eta_i + \xi_{it} \] (4)

- \( \beta \): coefficients for time dummy variables \( T \)
- \( \gamma \): coefficients for house characteristic variables \( X \)
- \( \Delta \): coefficients for the interactions between time dummy and house characteristic variables
- \( \eta_i \): individual house specific error, uncorrelated with \( \xi \)

Differencing Equation (4):

\[ V_{jt} - V_{js} = D'\beta + X'\Delta D + \xi_{jt} - \xi_{js} \] (5)

- \( D \): differenced time dummy variables
- No data on changes of \( X \)
Economic scenario generation

- **Model**: VAR(2)

\[ Y_t = \kappa + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Sigma^{1/2} Z_t \]

- **State variables included in** \( Y_t \):
  1. aggregate house price index growth rates
  2. rental yields
  3. GDP growth rates
  4. 1-quarter zero-coupon bond yield rates
  5. spread of 5-year over 1-quarter bond yield rates

- **\( Z_t \)**: vector of independent standard normal variables

- Derive bond yield curve based on the development of the state variables under the VAR(2) model.

- Derive risk-adjusted stochastic discount factors \( (m_t) \) assuming no arbitrage (Cochrane and Piazzesi, 2002; Alai et al., 2013).
Projection of individual house prices

- Model: VARX(1,0)

\[ IHG_t = \tilde{\kappa} + \tilde{\Phi} IHG_{t-1} + \tilde{\beta} HG_t + \tilde{\Sigma}^{1/2} \tilde{Z}_t, \]

- \( IHG_t \): disaggregated house price growth rate
- \( HG_t \): aggregate house price growth rate

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### Aggregate

- Historical Index
- Forecasted Index

### Central Business District

- Historical Index
- Forecasted Index
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Termination triggers (Ji et al., 2012)

Four major triggers of repayment:

- $\lambda$: age-specific factor to scale down at-home mortality
- $\kappa$: age-specific factor that relates LTC incidence to mortality
- $q_{i}^{pre}$: duration-dependent annual prepayment probability
- $q_{i}^{ref}$: duration-dependent annual refinancing probability

In-force probability of the reverse mortgage contract is:

$$tp_x^c = \exp \left\{ \int_0^t (\lambda_{x+s} + \kappa_{x+s})\hat{\mu}_{x+s} ds \right\} \prod_{i=1}^{t} \left[ (1 - q_{i}^{pre})(1 - q_{i}^{ref}) \right]^{1/4} \tag{6}$$
Stochastic mortality: two factor CBD model

\[
\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})
\] (7)

- \( q(t, x) \): death probability
- \( \bar{x} \): average age

Males (left) and females (right), + (white) and - (black)
Stochastic mortality: Wills-Sherris model

\[ \Delta_c \ln(\mu(x, t)) = \ln(\mu(x, t)) - \ln(\mu(x - 1, t - 1)) = ax + b + \sigma \varepsilon(x, t) \]  

- \( \mu(x, t) \): force of mortality for a cohort aged \( x \) at time \( t \)
- \( a, b \) and \( \sigma \): parameters to be estimated
- \( \varepsilon(x, t) \): a standard normal variable

Age dependence:

\[ [\varepsilon(x_1, t), \varepsilon(x_2, t), \ldots, \varepsilon(x_N, t)]' = \Omega^{\frac{1}{2}} W_t \]
Stochastic mortality: Wills-Sherris model (cont’d)

Parameter estimation:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{a} (\times 10^{-4}) )</td>
<td>-2.36</td>
<td>4.02**</td>
</tr>
<tr>
<td>( \hat{b} (\times 10^{-2}) )</td>
<td>8.38***</td>
<td>4.59***</td>
</tr>
<tr>
<td>( \hat{\sigma} (\times 10^{-2}) )</td>
<td>5.76***</td>
<td>6.52***</td>
</tr>
</tbody>
</table>

Males (left) and females (right), + (white) and - (black)
Projected survival curve: three models

Figure: Simulated survival probabilities of 65-year-old females.
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NNEG and mortgage insurance premium (Age 65, LTV 0.4): house price risk
NNEG and mortgage insurance premium (Age 65, LTV 0.4): longevity risk
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Conclusions

- Pricing and risk analysis of reverse mortgages
  - idiosyncratic house price risk
  - longevity risk

- Considerable house price risk
  - using an aggregate index underestimates the risk
  - risk factors associated with house characteristics should be used

- Longevity risk
  - mortality improvement modelling has a large impact
  - effect smaller than that of house price risk

- Improved insights into product pricing and risk analysis
References


Appendix A: Australian market for reverse mortgages

Figure: Australian Market for Reverse Mortgages, data source: Deloitte and SEQUAL media release 2012
### Appendix B: NNEG and mortgage insurance premium

<table>
<thead>
<tr>
<th>Model</th>
<th>Deterministic</th>
<th>Wills-Sherris</th>
<th>Cairns-Blake-Dowd</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTV</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>( \pi ) (p.a.)</td>
<td>0.003%</td>
<td>0.230%</td>
<td>3.246%</td>
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<tr>
<td>NNEG</td>
<td>71</td>
<td>12,794</td>
<td>400,017</td>
</tr>
<tr>
<td>S.E.</td>
<td>17</td>
<td>498</td>
<td>2,131</td>
</tr>
<tr>
<td>TVaR</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A. Overall Sydney house price index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi ) (p.a.)</td>
<td>0.218%</td>
<td>0.720%</td>
<td>1.829%</td>
</tr>
<tr>
<td>NNEG</td>
<td>6,043</td>
<td>42,421</td>
<td>186,092</td>
</tr>
<tr>
<td>S.E.</td>
<td>470</td>
<td>1,673</td>
<td>4,092</td>
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<tr>
<td>TVaR</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>B. Houses near the central business district</td>
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<td></td>
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<tr>
<td>( \pi ) (p.a.)</td>
<td>0.076%</td>
<td>0.255%</td>
<td>1.184%</td>
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<tr>
<td>NNEG</td>
<td>2,062</td>
<td>14,238</td>
<td>110,932</td>
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<tr>
<td>S.E.</td>
<td>289</td>
<td>879</td>
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<td>TVaR</td>
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<td>0.000</td>
<td>0.000</td>
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<tr>
<td>C. Houses near to coastlines</td>
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<td></td>
</tr>
<tr>
<td>( \pi ) (p.a.)</td>
<td>0.010%</td>
<td>0.247%</td>
<td>3.078%</td>
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<tr>
<td>NNEG</td>
<td>269</td>
<td>13,752</td>
<td>370,431</td>
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<tr>
<td>S.E.</td>
<td>86</td>
<td>566</td>
<td>2,239</td>
</tr>
<tr>
<td>TVaR</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>D. Houses with less than or equal to two bathrooms</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \pi ) (p.a.)</td>
<td>0.058%</td>
<td>0.418%</td>
<td>2.868%</td>
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<tr>
<td>NNEG</td>
<td>1,577</td>
<td>23,759</td>
<td>335,272</td>
</tr>
<tr>
<td>S.E.</td>
<td>209</td>
<td>893</td>
<td>2,871</td>
</tr>
<tr>
<td>TVaR</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E. Houses with more than two bathrooms</td>
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<td></td>
</tr>
<tr>
<td>( \pi ) (p.a.)</td>
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