Case Study – Modeling Longevity Risk for Solvency II

Presented by
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Principal & Consulting Actuary

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Background

- **SOLVENCY II** – New Minimum Capital Requirements
  - Standard Formula - Stipulated Methodology
  - Internal Model … *may produce smaller amount*

- Internal Models Current Focus on Market Risks
  - Capture Asset Volatility including Asset-Related Liability Risks

- Fifth Quantitative Impact Study (QIS5)
  - Issued by EIOPA *(European Insurance and Occupational Pensions Authority)*
  - Analysis in advance of implementation of Solvency II in 2011
  - Identified Most Material Risk Modules for Life Undertakings:
    - After **Market Risk**, the next most material risk is:
      **Life Underwriting Risk** *(Lapse and Longevity)*
Solvency II Capital

- **Two Approaches:**
  - Standard Formula - Stipulated Methodology
  - Internal Model – *must satisfy certain standards*:
    - Widely used and plays important role in decision-making
    - Sufficiently sophisticated to support standards of statistical quality
    - Calibrated to external and internal trends and volatility
    - Back-tested to demonstrate sources of profit and loss
    - Validated regularly against results
    - Sufficient documentation including limits and deficiencies
Goals of this Presentation

- Demonstrate Advantages of Internal Model Using Volatile Mortality
  - Potential reduction in Required Capital
  - Better Understanding of Capital Requirements

This will be accomplished by a relatively simple case study…

The case study can be found at
Case Study

- Calculate Required Capital for Liabilities
- Ignore Market Risk
- Model Sample Portfolio
- Calculate Minimum Required Assets Under Solvency II as Sum of:
  1. Best Estimate Liability
  2. Solvency Risk Requirement (SCR) $\rightarrow$ 1-in-200-year event (99.5th Percentile)
  3. Risk Margin

- Used proprietary modeling software, Milliman REVEAL...
REVEAL stands for:

Risk and Economic Volatility Evaluation of Annuitant Longevity

REVEAL is a system developed to analyze longevity risk.

Stochastic model for pension & annuity liabilities with volatility:

• baseline mortality,
• mortality improvement,
• extreme mortality and longevity events, and
• plan participant behavior (e.g., retirement dates & benefit elections)

For more information about REVEAL, please visit:

Historical and Projected
General Population Annual Mortality Rate
(Male 70 years old, 25 scenarios)
Modeling Mortality Rate Volatility

1. Mortality improvement may be modeled as a combination of:
   – Long-term waves with lingering effects over multiple years, and
   – Random annual fluctuations.

2. May group ages to minimize offsetting fluctuations across model population

3. May designate random probability of the contingency of a significant long-term shift by specific cause of death related deaths (e.g., infection or cure)

4. May assign random probability of short-term mortality spike (e.g., epidemic or terrorism)
Modeling Mortality Rate Volatility

Long-Term Waves and Short-Term Fluctuations

1. Long-Term: Develop random average annualized mortality improvement factor for each T-year period.  
   *(e.g., T is 10 years in case study.)*

2. Short-Term: Develop random annual mortality improvement factor for each year that fluctuate around the random annualized long-term improvement factor for each T-year period.
Modeling Mortality Rate Volatility

Sample Projection of Long-Term Trend and Short-Term Volatility

Projected General Population Mortality Rates Based on Historical Annual and Long Term Mortality Improvements
(Male 70 years old, 3 Scenarios)
Calibrate the model, in this case, using moment fitting approach:

The stochastic mortality model is calibrated such as:

1. The expected value of the projected improvements matches with the average historical improvements (or current expectation);
2. The volatility of the long term improvement component as well as the short term improvement component match with the historical volatility (or current expectation);
3. The long term and short term correlation match with the historical correlation (or current expectation).
Modeling Mortality Rate Volatility
Historical Long-Term Mortality Improvement Correlation, UK population
Modeling Mortality Rate Volatility
Projecting Improvement by Specified Cause of Death

Historical and Projected General Population Annual Mortality Rate (75 year old UK Male, 2 scenarios with and without a 10% cancer improvement)
The Hypothetical Portfolio

“The Hypothetical Portfolio was designed to:

…be consistent with a typical block of in payment annuities held by an insurer.

…be sufficiently large to minimize random ‘basis’ volatility,

…highlight the effect of other volatility factors.”
Assumptions

- Valuation Date 12/31/2010
- Hypothetical Portfolio of 50,000 in-payment annuities
- Husband Age = Wife Age + 3
- 50% Benefit to Surviving Spouse
- Annualized Benefits Increase 5% Each Year
- Quarterly Payments
- Discount Interest at the 12/31/2009 Spot Rates
Assumptions

- **Best Estimate Mortality (before improvement):**
  - 90% of the PCMA00 and PCFA00 tables in the CMI Library are described: "Life Office Pensioners, Combined, amounts – ultimate"

- **Best Estimate Mortality Improvement:**
  - **Male:** CMI 2010 projection model, with a long term rate of 1.2% p.a., applied from 2000 onwards.
  - **Female:** CMI 2010 projection model, with a long term rate of 0.9% p.a., applied from 2000 onwards.
### Distribution of Hypothetical Portfolio 1
*by Annualized Benefit Amount*

<table>
<thead>
<tr>
<th>Measuring Life</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Annuitant</td>
<td>55%</td>
</tr>
<tr>
<td>Spouse (Widow/Widower)</td>
<td>45%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gender of Measuring Life</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>55%</td>
</tr>
<tr>
<td>Female</td>
<td>45%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Annualized Benefit Amount</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1,000</td>
<td>13%</td>
</tr>
<tr>
<td>1,000 – 4,999</td>
<td>22%</td>
</tr>
<tr>
<td>5,000 – 9,999</td>
<td>21%</td>
</tr>
<tr>
<td>10,000 – 19,999</td>
<td>7%</td>
</tr>
<tr>
<td>20,000 – 29,999</td>
<td>1%</td>
</tr>
<tr>
<td>30,000 +</td>
<td>2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Annual Benefits</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Indexed to CPI</td>
<td>84%</td>
</tr>
<tr>
<td>Fixed</td>
<td>16%</td>
</tr>
</tbody>
</table>
## Distribution of Hypothetical Portfolio 2

### by Annualized Benefit Amount

<table>
<thead>
<tr>
<th>Measuring Life Age Group</th>
<th>Age of Measuring Life</th>
<th>Benefits Currently Paid to Joint-Life or Surviving Spouse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages 60-64</td>
<td>13%</td>
<td>Ages 60-64</td>
</tr>
<tr>
<td>Ages 65-69</td>
<td>22%</td>
<td>Ages 65-69</td>
</tr>
<tr>
<td>Ages 70-74</td>
<td>21%</td>
<td>Ages 70-74</td>
</tr>
<tr>
<td>Ages 75-79</td>
<td>19%</td>
<td>Ages 75-79</td>
</tr>
<tr>
<td>Ages 80-84</td>
<td>14%</td>
<td>Ages 80-84</td>
</tr>
<tr>
<td>Ages 85-89</td>
<td>7%</td>
<td>Ages 85-89</td>
</tr>
<tr>
<td>Ages 90-94</td>
<td>2%</td>
<td>Ages 90-94</td>
</tr>
<tr>
<td>Ages 95-99</td>
<td>1%</td>
<td>Ages 95-99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ages 60-64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ages 65-69</td>
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<tr>
<td></td>
<td></td>
<td>Ages 70-74</td>
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<td>Ages 75-79</td>
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<td>Ages 80-84</td>
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<td>Ages 85-89</td>
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<tr>
<td></td>
<td></td>
<td>Ages 90-94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ages 95-99</td>
</tr>
</tbody>
</table>
Plan of Action – Values to Calculate

Best Estimate Liability

Standard Formula Liability

Standard Formula Solvency Capital Requirement

Standard Formula Risk Margin

Internal Model Excess over Best Estimate Liability

Best Estimate Liability
Best Estimate Liability

• Present Value of Expected Annuity Cash Flows,
  \( \text{(Estimated using average of stochastic projections on expected mortality)} \)

• Discounted using Risk-Free Spot Rates with 100% allowance for illiquidity premium.
**Best Estimate Liability**

\[ ia_t = \text{Annual Spot Rate from Risk-Free Curve with 100\% allowance for illiquidity premium.} \]

\[ BECF_t = \text{Average annual annuity payments projected to be paid in year (e.g., best estimate cash flow)} \]

\[ BE\ell_0 = \text{Best Estimate Liability at time zero} \]

\[
\begin{align*}
BE\ell_0 &= \sum_{t=0,1,2,...} \frac{BECF_t}{(1 + ia_t)^t} \\
&= 1,725.5 \text{ million}
\end{align*}
\]
Use Immediate & permanent 20% improvement in mortality rates. (i.e., Best Estimate Mortality multiplied by 80% in all years)

\[ \text{SCR} = \text{excess of} \]

(a) Standard Formula Liability
(Present value of cash flows reflecting the 20% margin),

over

(b) the Best Estimate Liability,
(discounted to Valuation Date using the spot curves with 100% allowance for the illiquidity premium.)

\[ \text{BEL}_0 = 1,725.5 \text{ million} \]
$\text{Standard Formula:} \quad \text{Solvency Capital Requirement}$

$SFCF_t$ = Avg annual annuity payments projected (Using the Standard Formula Mortality Assumption) to be paid in year.

$BEL_0 = 1,725.5$ million
$BEL_0 = 1,725.5 \text{ million}$

**Standard Formula:**

$SCR_0^{StdForm} = \text{Solvency Capital Requirement}$

$= \text{Standard Formula Liability less } BEL_0$

$= 1,884.1 \text{ million} - 1,725.5 \text{ million}$

$= 158.6 \text{ million}$
Standard Formula:
Range of Formulations under QIS5, including:

Amortize SCR proportional to Best Estimate Liability annuity cash flows:

\[
SCR_t^{StdForm} = \text{Amortized Solvency Capital Requirement}
\]

\[
= SCR_{t-1}^{StdForm} \times \frac{BECF_t}{BECF_{t-1}}
\]

\[
i_z t = \text{Annual Spot Rate from Risk-Free Curve with 0% allowance for illiquidity premium}
\]

\[
BEL_0 = 1,725.5 \text{ million}
\]

\[
SCR_0^{StdForm} = 158.6 \text{ million}
\]
\[
\begin{align*}
BEL_0 &= 1,725.5 \text{ million} \\
SCR_0^{\text{StdForm}} &= 158.6 \text{ million}
\end{align*}
\]

**Standard Formula:**

Risk Margin

\[
Risk Margin_0^{\text{StdForm}} = 6\% \text{ (i.e., proxy for cost of capital) of PV of future amortized } SCR_t \text{ rates, discounted at risk-free rates}
\]

\[
= 6\% \sum_{t=0,1,2,\ldots} \frac{SCR_t^{\text{StdForm}}}{(1 + i_z_t)^t}
\]

\[
= 181.0 \text{ million}
\]
\[
\begin{align*}
BEL_0 & = 1,725.5 \text{ million} \\
SCR_0^{\text{StdForm}} & = 158.6 \text{ million} \\
Risk\ Margin_0^{\text{StdForm}} & = 181.0 \text{ million}
\end{align*}
\]

**Standard Formula:** Excess Over Best Estimate

\[
ExBEL_0^{\text{StdForm}} = \text{Standard Formula SCR} + \text{Standard Formula Risk Margin}
\]

\[
ExBEL_0^{\text{StdForm}} = 339.6 \text{ million}
\]
Standard Formula:  

\[
\begin{align*}
SCR_0^{\text{StdForm}} &= 158.6 \text{ million} \\
\text{Risk Margin}_0^{\text{StdForm}} &= 181.0 \text{ million} \\
ExBEL_0^{\text{StdForm}} &= 339.6 \text{ million} \\
BEL_0 &= 1,725.5 \text{ million}
\end{align*}
\]
Modeling Stochastic Mortality

The stochastic projections reflect three sources of volatility:

1. Randomized Dates of Death
   *Monte Carlo Simulation applied to Scenario-Specific Mortality*

2. Future Mortality Improvement Trend Volatility
   Analysis of historical population mortality (e.g., UK 1979-2009) to create stochastic mortality improvement scenarios reproducing historic mean, standard deviation, and correlation over annual and adjacent longer-term (e.g., 10-year) periods

3. Potential Extreme Longevity Occurrences
   Risk of immediate long-term change for specific cause of death (e.g., a new highly effective treatment for cancer)
The Economic Capital Approach was performed two ways:

- **Volatility Assumptions A**
  
  volatility in the mortality curve based solely on historical mortality improvement trends, and

- **Volatility Assumptions B**
  
  volatility in the mortality curve based on both historical mortality improvement trends and the possibility of a significant reduction in cancer related deaths.
Principle-Based Economic Calculation

10,000 Scenarios for 50,000-Life Portfolio

The 99.5\textsuperscript{th} Percentile of PV Future Annuity Cash Flows:

\[
\begin{align*}
\text{ExBEL}_0 & = 339.6 \text{ million} \\
\text{Econ Model Vol A} & = 183.5 \text{ million} \\
\text{Econ Model Vol B} & = 190.8 \text{ million}
\end{align*}
\]
$BEL_0 = 1,725.5 \text{ million}$

**Internal Model Approach**

*For Each Scenario:*

1. Generate stochastic mortality improvement for first duration, and
2. Set the mortality improvement in years 2+ to:
   - reflecting simulated mortality improvement experience over the first year,
   - given a credibility factor of 10%.
Internal Model Approach

\[ \Delta q_{x,t} = \quad \text{Expected annual rate of mortality improvement at attained age, duration } t \]

\[ Q_x^{\text{scale}}(0) = \quad \text{Stochastic adjustment to mortality improvement at attained age } x, \text{ duration } 0 \]

\[ \Delta q_{x,t}^{\text{new}} = \] \[1 \quad \text{Expected annual rate of mortality improvement at attained age } x, \text{ duration } t, \]
\[\text{reflecting duration } 0 \text{ stochastic improvement} \]

\[ = \quad 1 \quad \left(1 - \Delta q_{x,t}\right) * Q_x^{\text{scale}}(0) \]
**Internal Model Approach**

\[
BEL_0 = 1,725.5 \text{ million}
\]

\[
c = \text{Credibility assigned to stochastic mortality improvement simulated in the first duration}
\]

\[
\Delta q_{x,t}^{Sol^2} = \text{Assumed annual rate of mort improvement at attained age } x, \text{ duration } t (t = 1, 2, \ldots) \text{ adjusted to reflect credibility of duration 0 stochastic improvement}
\]

\[
= (1 - c) \times \Delta q_{x,t} + c \times \Delta q_{x,t}^{new}
\]

**Note:** If credibility \( c \) equals zero, then the Assumed Improvement equals Best Estimate Expected Improvement. (The Case Study assumed \( c=10\% \))
$BEL_0 = 1,725.5$ million

**Internal Model Approach**

Our understanding of regulator-approved *internal model* under Solvency II:

Economic capital reflects once-in-200-years event

\[
\text{Internal Model SCR} = \text{VaR}_{99.5\%} \left( \frac{BEL_1}{1 + i(1)} \right) + \sum_{0 < t \leq 1} \frac{CF_t}{(1 + i(t))^t} - BEL_0, \text{ where}
\]

\[
\text{VaR}_{99.5\%}(x) = 99.5\% \text{ percentile of a random variable } x
\]

\[
BEL_1 = \text{Best Estimate Liability at Time } t=1
\]

*Using altered mortality expectation reflecting credibility of “simulated” experience from } t=0 \text{ to } t=1
\[ \text{BEL}_0 = 1,725.5 \text{ million} \]

### Internal Model Approach

<table>
<thead>
<tr>
<th></th>
<th>Volatility Assumptions A</th>
<th>Volatility Assumptions B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{SCR}^\text{IntModel}_0 )</td>
<td>140.5 million</td>
<td>140.9 million</td>
</tr>
<tr>
<td>( \text{Risk Margin}^\text{IntModel}_0 )</td>
<td>160.3 million</td>
<td>160.8 million</td>
</tr>
<tr>
<td>( \text{ExBEL}^\text{IntModel}_0 )</td>
<td>300.8 million</td>
<td>301.8 million</td>
</tr>
</tbody>
</table>
## Comparison:

<table>
<thead>
<tr>
<th></th>
<th>Standard Formula</th>
<th>Internal Model Volatility A</th>
<th>Internal Model Volatility B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SCR_0$</td>
<td>158.6 million</td>
<td>140.5 million</td>
<td>140.9 million</td>
</tr>
<tr>
<td>Risk Margin$_0$</td>
<td>181.0 million</td>
<td>160.3 million</td>
<td>160.8 million</td>
</tr>
<tr>
<td>$ExBEL_0$</td>
<td>339.6 million</td>
<td>300.8 million</td>
<td>301.8 million</td>
</tr>
<tr>
<td>$BEL_0$</td>
<td>1,725.5 million</td>
<td>1,725.5 million</td>
<td>1,725.5 million</td>
</tr>
</tbody>
</table>
## Solvency II Capital: Internal Model Savings

<table>
<thead>
<tr>
<th></th>
<th>Standard Formula</th>
<th>Reduction Under Internal Model Volatility A</th>
<th>Reduction Under Internal Model Volatility B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SCR_0$</td>
<td>158.6 million</td>
<td>18.1 million</td>
<td>17.7 million</td>
</tr>
<tr>
<td>$Risk Margin_0$</td>
<td>181.0 million</td>
<td>20.7 million</td>
<td>20.2 million</td>
</tr>
<tr>
<td>$ExBEL$</td>
<td>339.6 million</td>
<td>38.8 million</td>
<td>37.0 million</td>
</tr>
<tr>
<td>$BEL$</td>
<td>1,725.5 million</td>
<td>0 million</td>
<td>0 million</td>
</tr>
</tbody>
</table>
Components of Total Assets Requirement (TAR)

<table>
<thead>
<tr>
<th>Component</th>
<th>TAR (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Estimate Liability</td>
<td>1,725.5</td>
</tr>
<tr>
<td>Economic Capital Approach Under Volatility Assumption A</td>
<td>1,909.8</td>
</tr>
<tr>
<td>Economic Capital Approach Under Volatility Assumption B</td>
<td>1,921.3</td>
</tr>
<tr>
<td>Internal Model Approach Under Volatility Assumption A</td>
<td>2,026.3</td>
</tr>
<tr>
<td>Internal Model Approach Under Volatility Assumption B</td>
<td>2,027.3</td>
</tr>
<tr>
<td>Standard Formula</td>
<td>2,065.1</td>
</tr>
</tbody>
</table>
Summary – Excess Over Best Estimate Liability

- ExBEL - Standard Formula: 339.6
- ExBEL - Economic Capital Approach (A): 183.5
- ExBEL - Economic Capital Approach (B): 190.8
- ExBEL - Internal Model Approach (A): 300.8
- ExBEL - Internal Model Approach (B): 301.8

[Bar chart showing the values above]
Take-Aways

1. Internal Model may produce capital savings for Longevity Risk (relative to Standard Formula)

2. Internal Model may still produce higher capital costs than a principle-based economic calculation

3. Possible advantage of financial transactions to move longevity risk to more favorable regulatory environment.
Contact Information

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