Opening the Black Box: Structural Factor Models with Large Cross-Sections

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Abstract

This paper argues that large-dimensional dynamic factor models are suitable for structural analysis. We establish sufficient conditions for identification of the structural shocks and the associated impulse-response functions. In particular, we argue that, if the data follow an approximate factor structure, the “problem of fundamentalness”, which is intractable in structural VARs, can be solved provided that the impulse responses are sufficiently heterogeneous. Finally, we propose a consistent method (and $n, T$ rates of convergence) to estimate the impulse-response functions, as well as a bootstrapping procedure for statistical inference.

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1 Introduction

Recent literature has shown that large-dimensional approximate (or generalized) dynamic factor models can be used successfully to forecast macroeconomic variables (Forni, Hallin, Lippi and Reichlin, 2005, Stock and Watson, 2002a, 2002b, Boivin and Ng, 2003, Giannone, Reichlin and Sala, 2005). These models assume that each time series in the dataset can be expressed as the sum of two orthogonal components: the “common component”, capturing that part of the series which comove with the rest of the economy and the “idiosyncratic component” which is the residual. The vector of the common components is highly singular, i.e. is driven by a very small number (as compared to the number of variables) of shocks (the ”common shocks” or ”common factors”) which generate comovements between macro series. Indeed, evidence based on different datasets points to the robust finding that few shocks explain the bulk of dynamics of macro data (see Sargent and Sims, 1977 and Giannone, Reichlin and Sala, 2002 and 2005). If the common component of the variable to be predicted is large, a forecasting method based on a projection on linear combinations of these shocks performs well because, while being parsimonious, it captures the relevant comovements in the economy.

The present paper argues that the scope of dynamic factor models goes beyond forecasting. Our aim is to open the black box of these models and show how statistical constructs such as factors can be related to macroeconomic shocks and their propagation mechanisms.

We define macroeconomic shocks those structural sources of variation that are cross-sectionally pervasive, i.e. that significantly affect most of the variables of the economy, while we call idiosyncratic the shocks that are specific to a single variable or a small group of variables, hence capturing either sectoral-local dynamics (let us say ”micro” dynamics) or measurement error. This has a natural formalization within large-dimensional approximate factor models. More precisely, we assume that a \( q \)-dimensional vector of macroeconomic shocks drives the common components of a macroeconomic panel \( \mathbf{x}_t \) of size \( n \), with \( n \) very large with respect to \( q \). Our aim is the identification of the macroeconomic shocks and of the impulse response function of the common components of the \( x \)’s to \( \mathbf{u}_t \), whereas the idiosyncratic components are disregarded.

Firstly, we claim that ideas and methods of structural VAR analysis can be fruitfully imported in dynamic factor models. We start with the estimate of an autoregression of the common-components vector. Thus an autoregression of dimension \( n \), the size of the panel, with a residual vector of dimension \( q \), the number of factors. Calling \( \mathbf{v}_t \) the estimated residual vector, the vector of structural shocks, call it \( \mathbf{u}_t \), is then obtained as in structural VAR analysis (SVAR) by linearly transforming \( \mathbf{v}_t \) in order to fulfill restrictions that derive from economic theory. All the identification schemes proposed in the SVAR literature, such as
long-run or impact effects can be imposed. The key difference is that the number of shocks is smaller than the number of variables.

Secondly, we show that the fundamentalness problem, a weakness of VAR analysis, finds a satisfactory solution within our approach. Let us recall that in SVAR analysis, even when economic theory is sufficient to determine just one linear transformation of the estimated residuals, still identification is achieved by arbitrarily assuming that the structural shocks are fundamental with respect to the variables included in the model, i.e. that they can be obtained as linear combinations of present and past values of such variables. This assumption cannot hold true if economic agents have larger information (on the fundamentalness issue see Hansen and Sargent, 1991, Lippi and Reichlin, 1993 and 1994 and, more recently, Chari, Kehoe and Mccrattan, 2005, Fernandez-Villaverde, Rubio-Ramirez and Sargent, 2005, Giannone and Reichlin, 2006).

The fundamentalness problem depends on a somewhat artificial feature of the SVAR approach, namely that the number of variables used to estimate the structural vector \(\mathbf{u}_t\) must be equal to the dimension of \(\mathbf{u}_t\), so that the space spanned by present and past values of \(\mathbf{x}_t\) can be “too small” to recover \(\mathbf{u}_t\). This equal-dimension constraint is relaxed in the structural dynamic factor model proposed in this paper. We will argue that when the number of variables is large compared to the number of structural shocks, non fundamentalness of the structural shocks is unlikely, since it would require economically meaningless homogeneity restrictions on the impulse-response functions. The economic intuition of this claim is that in the factor model present and past information used to recover \(\mathbf{u}_t\) is not confined to \(q\) variables, as in VAR models, but ranges over the set of all available macroeconomic series, so that the ”superior information” argument no longer holds (on the importance of this feature for monetary models, see Bernanke and Boivin, 2003 and Giannone, Reichlin and Sala, 2002 and 2005).

Our work is closely related to the recently introduced FAVAR model (Bernanke, Boivin and Eliaasz, 2005). The FAVAR approach consists in augmenting the VAR by common factors precisely as a device to condition on a larger information set. We go one step further and give the factors themselves a structural interpretation.

The factor model employed here should be distinguished from what studied in the traditional factor literature (see Sargent and Sims, 1977, Geweke, 1977, Geweke and Singleton, 1981, Altug, 1989, Sargent, 1989, Giannone, Reichlin and Sala, 2003). Since our model is approximate and feasible for large panels we need less stringent assumptions to identify the common from the idiosyncratic component (we do not need to impose cross-sectional orthogonality of the idiosyncratic residuals).

The paper is organized as follows. In Section 2, we define the model and discuss the conditions needed to recover the common components from the panel. Section 3 develops the structural analysis by showing conditions needed for recovering fundamental shocks and identify them uniquely. Section 4 studies consis-
tency and rates of convergence for the estimation of the shocks and the impulse response functions. Section 5 analyses an empirical example on US macroeconomic data which revisits the results of King et al. (1991) in light of our discussion on fundamentalness.

2 The Model

The dynamic factor model used in this paper is a special case of the generalized dynamic factor model of Forni, Hallin, Lippi and Reichlin (2000) and Forni and Lippi (2001). Such model, and the one used here, differs from the traditional dynamic factor model of Sargent and Sims (1977) and Geweke (1977), in that the number of cross-sectional variables is infinite and the idiosyncratic components are allowed to be mutually correlated to some extent, along the lines of Chamberlain (1983), Chamberlain and Rothschild (1983) and Connor and Korajczyk (1988). Closely related models have been recently studied by Stock and Watson (2002a, 2002b), Bai and Ng (2002) and Bai (2003).

Denote by \( x^T \) an \( n \times T \) rectangular array of observations.

We make two preliminary assumptions:

PA1. \( x^T \) is a finite realization of a real-valued stochastic process

\[
X = \{ x_{it}, i \in \mathbb{N}, t \in \mathbb{Z}, x_{it} \in L_2(\Omega, \mathcal{F}, P) \}
\]

indexed by \( \mathbb{N} \times \mathbb{Z} \), where the \( n \)-dimensional vector processes

\[
\{ x_{nt} = (x_{1t} \ldots x_{nt})', t \in \mathbb{Z}, n \in \mathbb{N} \}
\]

are stationary, with zero mean and finite second-order moments \( \Gamma_{nk} = \mathbb{E}[x_{nt} x_{n,t-k}] \), \( k \in \mathbb{N} \).

PA2. For all \( n \in \mathbb{N} \), the process \( \{ x_{nt}, t \in \mathbb{Z} \} \) admits a Wold representation \( x_{nt} = \sum_{k=0}^{\infty} C^n_k w_{n,t-k} \), where the full-rank innovations \( w_{nt} \) have finite moments of order four, and the matrices \( C^n_k = (C^n_{ij,k}) \) satisfy \( \sum_{k=0}^{\infty} |C^n_{ij,k}| < \infty \) for all \( n, i, j \in \mathbb{N} \). We assume that each variable \( x_{it} \) is the sum of two unobservable components, the common component \( \chi_{it} \) and the idiosyncratic component \( \xi_{it} \). The common component is driven by \( q \) common shocks \( u_t = (u_{1t} u_{2t} \ldots u_{qt})' \). Note that \( q \) is independent of \( n \) (and small as compared to \( n \) in empirical applications). More precisely:

FM0. (Dynamic-factor structure of the model) Defining \( \chi_{nt} = (\chi_{1t} \ldots \chi_{nt})' \) and \( \xi_{nt} = (\xi_{1t} \ldots \xi_{nt})' \), we suppose that

\[
x_{nt} = \chi_{nt} + \xi_{nt} = B_n(L)u_t + \xi_{nt},
\]

(2.1)
where $\mathbf{u}_t$ is a $q$-dimensional orthonormal white noise vector.

Moreover, we assume that

$$B_n(L) = A_n N(L), \quad (2.2)$$

where (i) $N(L)$ is an $r \times q$ absolutely summable matrix function of $L$, (ii) $A_n$ is an $n \times r$ matrix, nested in $A_m$ for $m > n$. Defining the $r \times 1$ vector $\mathbf{f}_t$ as

$$\mathbf{f}_t = N(L) \mathbf{u}_t, \quad (2.3)$$

(2.1) can be rewritten in the static form

$$\mathbf{x}_{nt} = A_n \mathbf{f}_t + \xi_{nt} \quad (2.4)$$

In the sequel, we shall use the term static factors to denote the $r$ entries of $\mathbf{f}_t$, whereas the common shocks $\mathbf{u}_t$ will be also referred to as dynamic factors.

Note that under (2.2) all the variables $\chi_{it}, i = 1, \ldots, \infty$, belong to the finite dimensional vector space spanned by $\mathbf{f}_t$.

The common shocks $\mathbf{u}_t$ are assumed to be structural sources of variation. Therefore the model (2.1), (2.3), (2.4) is a structural factor model. We will establish conditions under which $\mathbf{u}_t$ can be identified and estimated by means of the observable variables $x_{it}$. We start in this section by recalling the assumptions necessary for identification and estimation of the common components $\chi_{it}$.

FM1. (Orthogonality of common and idiosyncratic components) $\mathbf{u}_t$ is orthogonal to $\xi_{it}, \ i \in \mathbb{N}, \ t \in \mathbb{Z}, \ \tau \in \mathbb{Z}$.

Indicate by $\Gamma_{nk}^{\chi}$ and $\Gamma_{nk}^{\xi}$ the $k$-lag covariance matrix of $\chi_{nt}$ and $\xi_{nt}$ respectively. Denote by $\mu_{nj}^{\chi}$ and $\mu_{nj}^{\xi}$ the $j$-th eigenvalue, in decreasing order, of $\Gamma_{n0}^{\chi}$ and $\Gamma_{n0}^{\xi}$ respectively.

FM2. (Pervasiveness of common dynamic and static factors)

(a) The matrix $N(e^{-i\theta})$ has (maximum) rank $q$ for $\theta$ almost everywhere in $[-\pi, \pi]$.

(b) There exists constants $\underline{\zeta}_r, \overline{\zeta}_1, \ldots, \underline{\xi}_r, \overline{\xi}_r$ such that

$$0 < \underline{\zeta}_r \leq \liminf_{n \to \infty} n^{-1} \mu_{nr}^{\chi} \leq \overline{\zeta}_r < \ldots < \underline{\zeta}_1 \leq \liminf_{n \to \infty} n^{-1} \mu_{n1}^{\chi} \leq \overline{\zeta}_1 < \infty$$

FM3. (Non-pervasiveness of the idiosyncratic components) There exists a real $\Lambda$ such that $\mu_{n1}^{\xi} \leq \Lambda$ for any $n \in \mathbb{N}$. 

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FM3 limits the cross-correlation generated by the idiosyncratic shock. It includes the case in which the idiosyncratic components are mutually orthogonal with an upper bound for the variances. Mutual orthogonality is a standard, though highly unrealistic assumption in factor models. Condition FM3 relaxes such assumption by allowing for a limited amount of cross-correlation among the idiosyncratic components.

Assumption FM2 implies that each common shock $u_{it}$ is pervasive in the sense that it affects all items of the cross-section as $n$ increases. Precisely, denoting by $\lambda_n^k(\theta)$, $k = 1, 2, \ldots, n$, the eigenvalues of the spectral density matrix $\Sigma_n^\chi(\theta)$, in decreasing order at each frequency, Assumption FM2 implies that $\lambda_n^k(\theta) \to \infty$ as $n \to \infty$, for $\theta$ a.e. in $[-\pi \pi]$. This implies that (I) the common components $\chi_{it}$ are identified (see Chamberlain and Rothschild, 1983), (II) the number $q$ is unique, i.e. a representation (2.1)-(2.4) with a different number of dynamic factors is not possible (see Forni and Lippi, 2001).

Note also that FM2(b) entails that, for $n$ sufficiently large, $A_n' A_n/n$ has full rank $r$. This, jointly with identification of the common components $\chi_{it}$, implies that the space spanned by the $r$ static factors $f_t$ is identified, or, equivalently, that the $r$ static factors $f_t$ are identified up to a linear contemporaneous transformation.

In conclusion, given a model of the form (2.1)-(2.4), then under FM0-FM3, the integers $q$ and $r$, the components $\chi_{it}$ and $\xi_{it}$, and the space spanned by the static factors $f_t$ are identified.

The following rational specification of model (2.1)-(2.4) provides a dynamic representation which is parsimonious and fairly general. Assume that the entries of $B_n(L)$ are rational functions and let $\phi_jn(L), j = 1, \ldots, q$, be the least common multiple of the denominators of the entries on the $j$-th column of $B_n(L)$. Elementary polynomial and matrix algebra shows that

$$B_n(L) = C_n(L)\Psi_n(L),$$

where $C_n(L)$ is a finite moving average $n \times q$ matrix and $\Psi_n(L)$ is the $q \times q$ diagonal matrix having

$$\begin{pmatrix}
\phi_1n(L)^{-1} & \phi_2n(L)^{-1} & \cdots & \phi_qn(L)^{-1}
\end{pmatrix}$$

on the main diagonal. Further assumptions are needed to ensure that all the variables $\chi_{it}$ belong to a finite dimensional vector space. These are:

(a) $C_n(L) = C_0^n + C_1^n L + \cdots + C_s^n L^s$, i.e. there exists a maximum for the length of the moving averages,

(b) $\Psi_n(L)$ is independent of $n$ and can therefore be denoted by $\Psi(L)$, with $\phi_j(L)^{-1}$ denoting its $(j, j)$ entry.

The rational specification of our model can then be written as
\[ \mathbf{x}_{nt} = C_n(L)\Psi(L)\mathbf{u}_t + \xi_{nt}. \]  

Model (2.5) can be tentatively put in the form (2.3)-(2.4) by setting \( r = q(s + 1) \), \( A_n = (C^0 C^1 \cdots C^n) \), \( \mathbf{f}_t = (\mathbf{u}'_{t-1} \cdots \mathbf{u}'_{t-s})' \) and

\[ N(L) = (\Psi(L)' \Psi(L) L \cdots \Psi(L)' L^s)'. \]

FM2(a) is trivially fulfilled. However, FM2(b) requires that the first \( q(s + 1) \) eigenvalues \( \mu_{t j} \) diverge as \( n \to \infty \). If no restrictions hold for the entries of the matrices \( C^n \) (assume for instance that they are independently drawn from the same distribution), then FM2(b) is fulfilled, otherwise \( r \) is smaller than \( q(s + 1) \) and the model for the static factors is less obvious. The following elementary specification of (2.5), will help to understand the interplay between assumption FM2(b) and the parameters \( q \) and \( r \).

**Example. Part A** Suppose that \( s = 1 \), \( q = 1 \) and \( \Psi = 1 \), so that the common components in (2.5) can be written as:

\[ \chi_{it} = a_i (1 - c_i L) u_t \]

The number of static factors \( r \) depends on the heterogeneity in the panel:

(i) Assume that the restriction \( c_i = c \) holds. In this case FM2(b) is fulfilled by the first eigenvalue provided that

\[ 0 < \underline{a} < \overline{a} \leq \frac{1}{n} \sum_{i=1}^n a_i^2 \leq \overline{a} < \infty \]

as \( n \to \infty \), but not by the second. As a consequence \( r = 1 \), \( f_t = (1 - cL)u_t \) and

\[ A_n = (a_1 a_2 \cdots a_n)' \]

(ii) If no restriction holds, then also the second eigenvalue fulfills FM2(b) provided that \( c_i \neq c_j \) for infinitely many couples \( (i, j) \). Thus \( r = 2 \), \( \mathbf{f}_t = (u_t, u_{t-1})' \) and

\[ A_n = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ a_1 c_1 & a_2 c_2 & \cdots & a_n c_n \end{pmatrix}' \]

Note that in case (i), with \( r = q = 1 \), though the static factor \( f_t = (1 - cL)u_t \) is identified, identification of \( u_t \) would require an assumption on \( c \). In Section

\footnote{We might assume that \( \Psi(L) = \Phi(L)^{-1} \), where \( \Phi(L) \) is any (not necessarily diagonal) invertible \( q \times q \) finite order matrix polynomial. However, as \( C_n(L)\Phi(L)^{-1} = [C_n(L)\Phi_{ad}(L)] [L, \det \Phi(L)^{-1}] \), which is (2.5) after simplifying some of the roots of \( \det \Phi(L) \), no gain in generality would be achieved.}
we will see that this difference between cases (i) and (ii) is crucial for the identification of the structural shocks.

Our short analysis of both model (2.5) and the example suggest that the more heterogeneous the dynamic responses of the $\chi$’s to $u_t$, the bigger is $r$ with respect to $q$, i.e. the bigger is the number of static factors which is necessary to transform representation (2.1) into (2.4).

To conclude this section, it only remains to observe that representation (2.3)-(2.4) is not unique under FM0-FM3. Identification of the structural shocks $u_t$ and the coefficients of the filter $B_n(L)$ calls for further informational and economic assumptions and will be thoroughly discussed in the next section.

3 Identification of the structural shocks

3.1 Response heterogeneity, $n$ large and fundamentalness

3.3.1 Let us begin by briefly recalling some basic notions on fundamental representations of stationary stochastic vectors. Assume that the $n$ stochastic vector $\mu_t$ admits a moving average representation, i.e. that there exist a $q$-dimensional white noise $v_t$ and an $n \times q$, one-sided, square-summable filter $K(L)$, such that

\[ \mu_t = K(L)v_t. \]  \hspace{1cm} (3.6)

If $v_t$ belongs to the space spanned by present and past values of $\mu_t$ we say that representation (3.6) is fundamental and that $v_t$ is fundamental for $\mu_t$ (the condition defining fundamentalness is also referred to as the miniphase assumption; see e.g. Hannan and Deistler, 1988, p. 25). With no substantial loss of generality we can suppose that $q \leq n$ and that $v_t$ is full rank. Moreover, for our purpose, we can suppose that the entries of $K(L)$ are rational functions of $L$ and that the rank of $K(z)$ is maximal, i.e. $q$, except for a finite number of complex numbers. Then:

(F) Representation (3.6) is fundamental if and only if the rank of $K(z)$ is $q$ for all $z$ such that $|z| < 1$ (see Rozanov, 1967, Ch. 1, Section 10, and Ch. 2, p. 76).

Assuming that (3.6) is fundamental, all fundamental white-noise vectors $z_t$ are linear transformations of $v_t$, i.e. $z_t = C v_t$ (see Proposition 2 below). Non fundamental white-noise vectors result from $v_t$ by means of linear filters that involve the so-called Blaschke matrices (see e.g. Lippi and Reichlin, 1994).

A fundamental white noise naturally arises with linear prediction. Precisely, the prediction error

\[ w_t = \mu_t - \text{Proj}(\mu_t|\mu_{t-1}, \mu_{t-2}, \ldots) \]
is white noise and fundamental for $\mu_t$. As a consequence, when estimating an ARMA with forecasting purposes, the MA matrix polynomial is always chosen to be invertible, which implies fundamentalness.

Fundamentalness plays also an important role for the identification of structural shocks in SVAR analysis. SVAR analysis starts with the projection of a full rank $n$-dimensional vector $\mu_t$ on its past, thus producing an $n$-dimensional full rank fundamental white noise $w_t$. The structural shocks are then obtained as a linear transformation $Aw_t$, the matrix $A$ resulting from economic theory statements, which is tantamount to assuming that the structural shocks are fundamental. Fundamentalness has here the effect that the identification problem is enormously simplified. However, as pointed out in the literature mentioned in the Introduction, economic theory, in general, does not provide support for fundamentalness, so that all representations that fulfill the same economic statements but are non fundamental are ruled out with no justification.

Our main point is that the situation changes dramatically if structural analysis is conducted assuming that $n > q$. Precisely, as we shall see below, non fundamentalness is a generic property for $n = q$, while it is non generic for $n > q$. Thus the question “why assuming fundamentalness?”, which is legitimately asked when $n = q$, is replaced by “why should we care about non fundamentalness?” when $n > q$.

An easy and effective illustration can be obtained assuming that $q = 1$, that the entries of $K(L) = (K_1(L) \ K_2(L) \cdots \ K_n(L))'$ are polynomials whose degree does not exceed $s$, so that $K(L)$ is parameterized in $\mathbb{R}^{n(s+1)}$. In this case, if $n = q = 1$, non fundamentalness translates into the condition that no root of $K_1(z)$ has modulus smaller than unity. Continuity of the roots of $K_1(z)$ implies that non fundamentalness is generic, i.e. that if it holds for a point $\kappa$ in the parameter space it holds also within a neighborhood of $\kappa$.

On the other hand, if $n > q$, by (F), non fundamentalness implies that the polynomials $K_j(z)$ have a common root. As a consequence, their coefficients must fulfill $n - 1$ equality constraints (see e.g. van der Waerden, 1953, p. 83). Non fundamentalness is therefore non generic.

This analytic argument has a forceful economic counterpart. Suppose for example that our variables are driven by two macroeconomic shocks, a monetary and a technology shock, so that the structural white noise $v_t$ is 2-dimensional. Let the first two variables in $\mu_t$ be the common components of aggregate output and consumption. We do not know in general if $v_t$ is fundamental for the 2-dimensional output-consumption vector, this is the fundamentalness issue. However, if $\mu_t$ contains other variables, say, the common components of investment, employment, industrial production, etc., then non fundamentalness of $v_t$, with respect to $\mu_t$, is possible only if the responses of all such variables to $v_t$ are forced to follow very special patterns. Thus in a framework in which the number of variables is larger than the number of shocks, a reasonable heterogeneity in the
way different variables respond to the shocks provides a sound motivation for
the fundamentalness assumption and for its consequences on identification (see
Section 3.2 for further details on this example).

3.1.2 The general discussion above will now be adapted to our specification of the
dynamic factor model. We have seen in Section 2 that under FM0 heterogeneity
of the dynamic responses implies that \( r \) is big as compared to \( q \). Further analysis
of heterogeneity in the example of Section 2 and the rational model (2.5) will
provide support to the assumption that \( N(L) \) is left invertible, i.e. there exists a
one-sided square-summable \( q \times r \) filter \( G(L) \) such that \( G(L)N(L) = I_q \).

Example. Part B Still assuming
\[
\chi_{it} = a_i(1 - c_i L)u_t,
\]
heterogeneity of the dynamic responses (no restrictions) implies \( r = 2 \). In this
case \( f_t = N(L)u_t \) takes the form
\[
\begin{pmatrix}
  u_t \\
  u_{t-1}
\end{pmatrix} = \begin{pmatrix} 1 \\ L \end{pmatrix} u_t.
\]
Obviously \( N(L) \) has the left inverse \( (1 \ 0) \), so that \( u_t \) is fundamental for \( f_t \).
Moreover, since \( r = 2 \), FM2 implies that for \( n \) large enough there must be a
couple \((i, j)\) such that \( a_i \neq 0, a_j \neq 0 \) and \( c_i \neq c_j \). Then
\[
\begin{align*}
  u_t &= a_j c_j \chi_{it} - a_i c_i \chi_{jt} \\
  &= \frac{a_j c_j \chi_{it}}{a_i a_j (c_j - c_i)},
\end{align*}
\]
so that \( u_t \) is fundamental for the whole set of the \( \chi \)'s (actually for the two-
dimensional vector \((\chi_{it} \ \chi_{jt})\)). Note that this result holds independently of the
values taken by the coefficients \( c_i \). It holds in particular even when \( c_i > 1 \) for all
\( i \), so that \( u_t \) is not fundamental for any of the \( \chi \)'s.

Conversely, the restriction \( c_i = c \), i.e. homogeneity, implies \( r = q = 1 \) and
\( f_t = N(L)u_t \) takes the form
\[
f_t = (1 - cL)u_t.
\]
Here we are precisely in the VAR situation. The system is square. Either some
extra information is available to motivate the assumption that \(|c| < 1\), or the
assumption that \( N(L) \) is invertible is ad hoc.

It is easily seen that the results obtained for the example, left invertibility
of \( N(L) \) in particular, generalize to model (2.5) in the case when no restrictions
hold. In that case the dynamic responses are most heterogeneous and therefore
\( r = q(s+1) \). As already seen in Section 2, \( N(L) = (\Psi(L)')^t \Psi(L)'L^s \cdots \Psi(L)'^t \).

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Setting \( G(L) = (\Psi(L)^{-1} \ 0_q \cdots 0_q) \), where \( 0_q \) is a \( q \times q \) matrix of zeros, we see that \( G(L)N(L) = I_q \). If restrictions hold among the entries of \( B_n(L), \ C_n(L) \) in the rational case, obtaining \( N(L) \) is less obvious. We do not need a detailed treatment of the problem. An example is the case \( c_i = c \) above.

The above discussion motivates Assumption FM4 as a most likely consequence of the heterogeneity of the dynamic responses to \( u_t \). Proposition 1 shows that FM4, jointly with FM2, imply fundamentalness.

(\text{FM4}) \ (\text{Fundamentalness}) \ There \ exists \ a \ \( q \times r \) one-sided filter \( G(L) \) such that \( G(L)N(L) = I_q \).

**Proposition 1** If FM0-FM4 are satisfied, \( u_t \) is fundamental for \( \chi_{nt} \) for \( n \) sufficiently large and therefore fundamental for \( \chi_{it}, \ i = 1, \ldots, \infty \). Moreover, \( u_t \) belongs to the space spanned by present and past values of \( x_{it}, \ i = 1, \ldots, \infty \), i.e. the shocks \( u_{ht} \) can be recovered as limits of linear combinations of the variables \( x_{it} \).

**Proof.** As already observed, FM2 implies that \( A_n' A_n \) is full rank for \( n \) sufficiently large. Setting, \( S_n(L) = G(L) (A_n' A_n)^{-1} A_n' \), where \( G(L) \) satisfies FM4, we have

\[
S_n(L)\chi_{nt} = G(L) (A_n' A_n)^{-1} A_n' \chi_{it} = G(L)f_t = G(L)N(L)u_t = u_t.
\]

Therefore \( u_t \) lies in the space spanned by present and past values of \( \chi_{nt} \). Moreover, \( S_n(L)\xi_{nt} = G(L) (A_n' A_n)^{-1} A_n' \xi_t \) converges to zero in mean square by assumptions FM2 and FM3.

Consider now the orthogonal projection of \( f_t \) on the space spanned by its past values:

\[
f_t = \text{Proj}(f_t \mid f_{t-1}, f_{t-2}, \ldots) + w_t,
\]

where \( w_t \) is the \( r \)-dimensional vector of the residuals. Under our assumptions, \( w_t \) has rank \( q \). Moreover, by the same argument used to prove Proposition 2 (see the next subsection), \( w_t = Ru_t \), where \( R \) is a maximum-rank \( r \times q \) matrix. Quite interestingly:

(a) For model (2.5), with \( \Psi(L) = I_q \) and no restrictions, the projection above requires only one lag. The intuition is that when \( r > q \) and the panel dynamics are very heterogenous, information contained in lagged values of \( f_{ht} \) can be substituted by cross-sectional information (just the same reason motivating fundamentality).

(b) Relaxing the assumption \( \Psi(L) = I_q \), as the reader can easily check the orthogonal projection requires only a finite number of lags, one lag being sufficient if the order of the polynomials appearing in the denominators of \( \Psi(L) \) is not greater than \( s + 1 \).
As a consequence, a specification of FM4 as
\[
f_t = F_t f_{t-1} + \cdots + F_m f_{t-m} + R u_t
\]
does not seem to cause a dramatic loss of generality, even when \( m = 1 \). In the sequel we will adopt the VAR(1) specification:

\begin{equation}
(FM4)' \text{ (Fundamentalness: VAR(1) specification)} \quad \text{The } r\text{-dimensional static factors } f_t \text{ admit a VAR(1) representation}
\end{equation}

\[
f_t = F f_{t-1} + R u_t
\]

where \( F \) is \( r \times r \) and \( R \) is a maximum-rank matrix of dimension \( r \times q \).

Summing up, a large \( n \) and heterogeneity of the dynamic responses of the \( \chi' \)'s to \( u_t \) makes fundamentalness of \( u_t \) with respect to the \( \chi' \)'s most plausible. In our model dynamic heterogeneity implies that \( r > q \) and that, most likely, \( N(L) \) is invertible, which implies fundamentalness. Lastly, with no significant loss of generality, the model for \( f_t \) can be written as a VAR(1).

### 3.2 Economic conditions for shocks identification

Proposition 1 ensures that under Assumptions FM0-FM4 \( u_t \) is fundamental for the common components \( \chi_{it} \) and can be recovered by using past and present values of the observable variables \( x_{it} \). Our next result shows that under the same assumptions \( u_t \) is identified up to a static rotation.

**Proposition 2** Consider the common components of model (2.1):

\[
\chi_{nt} = B_n(L)u_t.
\]

If

\[
\chi_{nt} = C_n(L)v_t
\]

for any \( n \in \mathbb{N} \), where \( v_t \) is a \( q \)-dimensional fundamental orthonormal white noise vector, then representation (3.9) is related to representation (3.8) by

\[
C_n(L) = B_n(L)H
\]

\[
v_t = H' u_t,
\]

where \( H \) is a \( q \times q \) unitary matrix, i.e. \( HH' = I_q \).

**Proof.** Projecting \( v_t \) entry by entry on the linear space \( U_t \) spanned by the present and the past of \( u_{ht} \), \( h = 1, \ldots, q \) we get

\[
v_t = \sum_{k=0}^{\infty} H_k u_{t-k} + r_t,
\]
where \( r_t \) is orthogonal to \( u_{t-k} \), \( k \geq 0 \). Now consider that \( U_t \) and the space spanned by present and past of the \( \chi_{it} \)'s, call it \( X_t \), are identical, because the entries of \( \chi_{t-k} \), \( k \leq 0 \), belong to \( U_t \) by equation (3.8), while the entries of \( u_{t-k} \), \( k \leq 0 \), belong to \( X_t \) by condition FM4. The same is true for \( X_t \) and the space spanned by present and past of the \( v_{ht} \)'s, call it \( V_t \), so that \( U_t = V_t \). Hence \( r_t = 0 \).

Moreover, serial non-correlation of the \( u_{ht} \)'s imply that \( \sum_{k=1}^{\infty} H_k u_{t-k} \) must be the projection of \( v_t \) on \( U_{t-1} \), which is zero because \( U_{t-1} = V_{t-1} \). It follows that \( v_t = H_0 u_t \). Orthonormality of \( v_t \) implies that \( H_0 \) is unitary \( H_0 H_0' = I \). QED

Since fundamentalness of the structural shocks can be assumed in the dynamic factor model framework, identification is reduced to the choice of a matrix \( H \) such that economically motivated restrictions on the matrix \( B_n(L) \) are fulfilled. For instance, identification can be achieved by maximizing or minimizing an objective function involving \( B_n(L)H \) (see, for example, Giannone, Reichlin and Sala, 2005). An alternative is to impose zero restrictions either on the impact effects \( B_n(0)H \) or the long-run effects \( B_n(1)H_0 \) or both. In this case we have to impose \( q(q-1)/2 \) restrictions (since orthonormality entails \( q(q+1)/2 \) restrictions). Notice that, once the conditions FM0-FM4 are satisfied, the number of economic identification restrictions we need to identify the shocks depend on \( q \) and not on \( n \). This is an advantage for structural analysis, since, provided \( q \) is small, we need few restrictions for identification while we are not limited on the informational assumptions (size of the panel).

A comparison with identification in SVAR analysis is in order here. To simplify the presentation, suppose, like in the example at the end of Section 3.3.1, that \( q = 2 \), that we are interested in the impulse-response functions of the first two common components to the structural shocks \( u_1 \) and \( u_2 \), and that our economic restrictions are sufficient to identify the matrix \( H \). We have \( \chi_{nt} = B_n(L)u_t \), with

\[
\begin{pmatrix}
\chi_{1t} \\
\chi_{2t}
\end{pmatrix} = B_2(L) 
\begin{pmatrix}
u_{1t} \\
\nu_{2t}
\end{pmatrix}
\] (3.12)

being the subsystem of interest. Now, \( (u_{1t} u_{2t})' \) is fundamental with respect to \( \chi_{nt} \), but, as already noted in Section 3.1, is not necessarily fundamental with respect to \( (\chi_{1t} \chi_{2t})' \), i.e. representation (3.12) is not necessarily fundamental. By contrast, if a VAR were estimated for the vector \( (\chi_{1t} \chi_{2t})' \),

\[
A(L) \begin{pmatrix}
\chi_{1t} \\
\chi_{2t}
\end{pmatrix} = \begin{pmatrix}
u_{1t} \\
\nu_{2t}
\end{pmatrix},
\]

the resulting MA representation,

\[
\begin{pmatrix}
\chi_{1t} \\
\chi_{2t}
\end{pmatrix} = A(L)^{-1} \begin{pmatrix}
u_{1t} \\
\nu_{2t}
\end{pmatrix},
\]

would be fundamental by definition. As a consequence, if \( B_2(L) \) were not fundamental, applying the same economic restrictions to rotate \( (\nu_{1t} \nu_{2t})' \) would never
allow recovering the structural shocks \((u_{1t}, u_{2t})'\). This point is further illustrated in Section 5, where an important empirical example of non-fundamentalness of the subsystem of interest is presented.

4 Estimation

Going back to equation (2.4) it is easily seen that the static factors \(f_t\) are identified only up to pre-multiplication by a non-singular \(r \times r\) matrix. Hence we cannot estimate \(f_t\). However, we can estimate the common-factor space, i.e. we can estimate an \(r\)-dimensional vector whose entries span the same linear space as the entries of \(f_t\). Such vector can be written as \(g_t = Gf_t\), were \(G\) is a non-singular matrix.

The static factor space can be consistently estimated by the first \(r\) principal components of the panel \(x_{nt}\) as in Stock and Watson, 2002a and 2002b\(^2\).

Precisely, the estimated static factors will be

\[
\hat{g}_t = \frac{1}{\sqrt{n}} W_n^T x_{nt}, \tag{4.13}
\]

where \(W_n^T\) is the \(n \times r\) matrix having on the columns the eigenvectors corresponding to the first \(r\) largest eigenvalues of the sample variance-covariance matrix of \(x_{nt}\), say \(\Gamma_{n0}^{xT}\). We do not normalize the factors to have unit variance. The estimated variance-covariance matrix of \(\hat{g}_t\) is the diagonal matrix having on the diagonal the normalized eigenvalues of \(\Gamma_{n0}^{xT}\) in descending order, \(\frac{1}{n} \Lambda_n^T = \frac{1}{n} W_n^T \Gamma_{n0} x_n W_n^T\).

The corresponding estimate of the common components is obtained by regressing \(x_{nt}\) on the estimated factors to get

\[
x_{nt}^T = W_n^T W_n^T x_{nt}. \tag{4.14}
\]

Having an estimate of \(g_t\), we have still to unveil the leading-lagging relations between its entries, in order to find out the underlying dynamic factors (or, better, a unitary transformation of such factors \(v_t = Hu_t\), with \(HH' = I_q\)). This can be done in our dynamic factor model by projecting \(g_t\) on its first lag. This approach is also followed in Giannone, Reichlin and Sala (2002, 2005).

4.1 Population formulas

By equation (3.7), any non-singular transformation of the common factors \(g_t = Gf_t\) has the VAR(1) representation

\[
g_t = GFG^{-1}g_{t-1} + \epsilon_t = Dg_{t-1} + \epsilon_t. \tag{4.15}
\]

\(^2\)Alternative \((n, T)\) consistent estimators proposed in the literature are Forni and Reichlin (1998), Boivin and Ng (2003) and Forni, Hallin, Lippi and Reichlin (2005).
Note that
\[ D = \Gamma^g_1 (\Gamma^g_0)^{-1}, \]  
(4.16)
where \( \Gamma^g_h = E(g_t g'_{t-h}) \), and
\[ \text{var}(\epsilon_t) = \Gamma^g_0 - D\Gamma^g_0 D'. \]  
(4.17)

By (3.7), the residual \( \epsilon_t \) can be written as
\[ \epsilon_t = GRu_t = (G RH') H u_t = K MH u_t, \]  
(4.18)
where
(i) \( M \) is the diagonal matrix having on the diagonal the square roots of the first \( q \) largest eigenvalues of the variance-covariance matrix of \( \epsilon_t \), i.e. the matrix \( GRG' = \Gamma^g_0 - D\Gamma^g_0 D' \), in descending order.
(ii) \( K \) is the \( r \times q \) matrix whose columns are the eigenvectors corresponding to such eigenvalues.
(iii) \( H \) is a \( q \times q \) unitary matrix;

By inverting the VAR we get
\[ g_t = (I - DL)^{-1} K MH u_t. \]

On the other hand, by equations (2.1) and (2.4)
\[ \chi_{nt} = B_n(L) u_t = A_n f_t = A_n G^{-1} g_t = Q_n g_t, \]  
(4.19)
where
\[ Q_n = E(\chi_{nt} g'_t) = E(\chi_{nt} g'_t). \]  
(4.20)
Hence, we have
\[ \chi_{nt} = B_n(L) u_t \]
\[ = Q_n(I - DL)^{-1} K MH u_t \]
\[ = Q_n(I + DL + D^2 L^2 + \cdots) K MH u_t. \]  
(4.21)

### 4.2 Estimators

By substituting \( \hat{g}_t = \frac{1}{\sqrt{n}} W^T_n x_{nt} \) for \( g_t \), it is quite natural to estimate \( Q_n \) by \( \frac{1}{\sqrt{n}} \Gamma^g_0 W^T_n \) (see equation (4.20)). Moreover, \( \Gamma^g_0 \), the variance-covariance matrix of \( g_t \), can be estimated by \( \frac{1}{n} W^T_n \Gamma^x_n W^T_n = \frac{1}{n} \Lambda^T_n \), and \( \Gamma^g_1 \) by \( \frac{1}{n} W^T_n \Gamma^x_n W^T_n \), so that, basing on equation (4.16), we estimate \( D_n \) by \( D^T_n = W^T_n \Gamma^x_n W^T_n (\Lambda^T_n)^{-1} \). Finally, to estimate the eigenvectors and eigenvalues in \( K_n \) and \( M_n \) we estimate
the variance-covariance matrix of $\epsilon_t$ by $\Sigma_n^T = \frac{1}{n}(\Lambda_n^T - D_n^T \Lambda_n D_n^T)$ (see equation (4.17)).

Summing up, in analogy with (4.21) we propose to estimate the impulse-response functions by

$$B_n^T(L) = Q_n^T \left( I + D_n^T L + (D_n^T)^2 L^2 + \cdots \right) K_n^T M_n^T H,$$

(4.22)

where

(i) $Q_n^T = \frac{1}{\sqrt{n}} \Gamma_{x0}^T W_n^T$, where $\Gamma_{x0}^T$ is the sample variance-covariance matrix of $x_{nt}$ and $W_n^T$ the $n \times r$ matrix having on the columns the eigenvectors corresponding to the first $r$ largest eigenvalues of $\Gamma_{x0}^T$;

(ii) $D_n^T = W_n^T \Gamma_{n1}^T \Gamma_{x0}^T W_n^T (\Lambda_n^T)^{-1}$, where $\Gamma_{n1}^T$ is the sample covariance matrix of $x_{nt}$ and $x_{nt-1}$;

(iii) $M_n^T$ is the diagonal matrix having on the diagonal the square roots of the first $q$ largest eigenvalues of the matrix $\frac{1}{n}(\Lambda_n^T - D_n^T \Lambda_n D_n^T)^T$, in descending order;

(iv) $K_n^T$ is the $r \times q$ matrix whose columns are the eigenvectors corresponding to such eigenvalues.

(v) $H$ is a unitary matrix to be fixed by the identifying restrictions.

In order to render operative the above procedure we need to set values for $r$ and $q$. Unfortunately, there are no criteria in the literature to fix jointly $q$ and $r$. Bai and Ng (2002) propose some consistent criteria to determine $r$. As regards the number of dynamic factors, we can follow a decision rule like that proposed in Forni, Hallin, Lippi and Reichlin (2000) i.e., we go on to add factors until the additional variance explained by the last dynamic principal component is less than a pre-specified fraction, say 5% or 10%, of total variance.

4.3 Consistency

Consistency of (4.22) as estimator of the impulse-response functions for large cross-sections and large sample size ($n, T \to \infty$) is shown in Proposition 3 below.

**Proposition 3** Under assumptions PA1-2, FM1-3, we have, as $\min(n, T) \to \infty$:

$$\sqrt{\delta_{nt}} |b_{ni}^T(L) - b_i(L)| = O_p(1), i = 1, \ldots, n.$$
Proof. See Appendix 1.

Proposition 3 shows that consistency is achieved along any path for \((n, T)\) with \(T\) and \(n\) both tending to infinity. The consistency rate is given by \(\min(\sqrt{T}, \sqrt{n})\).

This implies that if the cross-section dimension \(n\) is large relative to the sample size \(T\) \((T/n \to 0)\) the rate of consistency is \(\sqrt{T}\), the same we would obtain if the common components were observed, i.e. if the variables were not contaminated by idiosyncratic component. On the other hand, if \(n/T \to 0\), then the consistency rate is \(\sqrt{n}\) reflecting the fact that the common components are not observed but have to be estimated\(^3\).

4.4 Standard errors and confidence bands

To obtain confidence bands and standard errors we propose the following bootstrap procedure.

Firstly, compute \(\chi_{nt}^T\) and \(B_n^T(L)\) according to (4.14) and (4.22), and \(\xi_{nt}^T = x_{nt} - \chi_{nt}^T\).

Secondly, for each one of the estimated idiosyncratic components, estimate the univariate autoregressive model

\[
a_j(L)\xi_{jt}^T = \sigma_j\omega_{jt}, \quad j = 1, \ldots, n,
\]

whose order can be fixed by the Schwarz criterion, and take the estimated coefficients \(a_j^T(L)\) and \(\sigma_j^T\) and the unit variance residuals \(\omega_{jt}^T\).

Thirdly, generate new simulated series for the shocks, say \(u_{jt}^*\) and \(\omega_{jt}^*\), \(j = 1, \ldots, n\), by drawing from the standard normal. Use these new series to construct \(\chi_{nt}^* = B_n^T(L)u_{jt}^*, \xi_{jt}^* = a_j^T(L)^{-1}\sigma_j^T\omega_{jt}^*,\ j = 1, \ldots, n,\) and \(x_{nt}^* = \chi_{nt}^* + \xi_{nt}^*\).

Finally, compute new estimates of the impulse-response functions \(B_n^*(L)\) starting from \(x_{nt}^*\).

By repeating the two last steps \(N\) times we get a distribution of estimated values which can be used to obtain standard errors and confidence bands. Note that the estimates will in general be biased, since the estimation procedure involves implicitly the estimation of a VAR. An estimate of such bias is provided by the difference between the point estimate \(B_n^T(L)\) and the average of the \(N\) estimates \(B_n^*(L)\).

5 Empirical application

We illustrate our proposed structural factor model by revisiting a seminal work in the structural VAR literature, i.e. King et al., 1991 (KPSW from now on). To this

\(^3\)It should be pointed out that, under the model assumptions of Stock and Watson (2002a and 2002b) or Bai and Ng (2002), an alternative proof of consistency has been proposed by Giannone, Reichlin and Sala(2002).
end, we constructed a panel of macroeconomic series including the series used by KPSW, with the same sampling period. Just like KPSW, we identify a long-run shock by imposing long-run neutrality of all other shocks on per-capita output. The data are well described by three common shocks, so that the comparison with the three-variable exercise of KPSW is particularly appropriate. Having the same data, the same identification scheme and the same number of shocks, different results can only be due to the additional information coming from the other series in the panel.

5.1 The data

The data set was constructed by downloading mainly from the FRED II database of the Federal Reserve Bank of St. Louis and Datastream. The original data of KPSW have been downloaded from Mark Watson’s home page. We collected 89 series, including data from NIPA tables, price indexes, productivity, industrial production indexes, interest rates, money, financial data, employment, labor costs, shipments, and survey data. A larger $n$ would be desirable, but we were constrained by both the scarcity of series starting from 1949 (like in KPSW) and the need of balancing data of different groups. In order to use Datastream series we were forced to start from 1950:1 instead of 1949:1, so that the sampling period is 1950:1 - 1988:4. Monthly data are taken in quarterly averages. All data have been transformed to reach stationarity according to the ADF(4) test at the 5% level. Finally, the data were taken in deviation from the mean as required by our formulas, and divided by the standard deviation to render results independent of the units of measurement. A complete description of each series and the related transformations is reported in Appendix 2.

5.2 The choice of $r$ and the number of common shocks

As a first step we have to set $r$ and $q$. Let us begin with $r$. We computed the six consistent criteria suggested by Bai and Ng (2002) with $r = 1, \ldots, 30$. The criteria $IC_{p1}$ and $IC_{p3}$ do not work, since they do not reach a minimum for $r < 30$; $IC_{p2}$ has a minimum for $r = 12$. To compute $PC_{p1}$, $PC_{p2}$ and $PC_{p3}$ we estimated $\hat{\sigma}^2$ with $r = 15$ since with $r = 30$ none of the criteria reaches a minimum for $r < 30$. $PC_{p1}$ gives $r = 15$, $PC_{p2}$ gives $r = 14$ and $PC_{p3}$ gives $r = 20$. Below we report results for $r = 12$, $r = 15$ and $r = 18$, with more detailed statistics for $r = 15$. With $r = 15$, the common factors explain on average 79.7% of total variance. With reference to the variables of interest in KPSW, the common factors explain 85.6% of total variance for output, 84.4% for investment and 89.4% for consumption.

Regarding the choice of $q$, for comparison with the three variable VAR of KPSW we set $q = 3$. This choice is consistent with the decision rule proposed
in Forni, Hallin, Lippi and Reichlin (2000), since, with Bartlett lag window size 18, the overall variance explained by the third dynamic principal component is larger than 10% (10.2%), whereas the variance explained by the fourth one is less than 10% (6.8%). Given the illustrative purpose of this application, we do not use the more formal criteria for the choice of $q$ proposed in recent literature (Bai and Ng, 2005, Hallin and Liska, 2006 or Stock and Watson, 2005).

5.3 Fundamentalness

Now let us focus on the $3 \times 3$ impulse-response function system for the three variables of KPSW, i.e. per capita consumption, per capita income and per capita investment. As observed at the end of Section 3, we can compute the roots of the determinant of this system to check whether it is invertible or not. Figure 1 plots the moduli of the two smallest roots of the above determinant as a function of $r$, for $r$ varying over the range 3-30. Note that for $r = 3$ all roots must be larger than one in modulus, since they stem from a three-variate VAR. This is in fact the case for $r = 3$ and $r = 4$, but for $r \geq 5$ the smallest root is declining and lies always within the unit circle. For $r \geq 22$ the second smallest root becomes smaller than one in modulus.

Figure 1: The moduli of the first and the second smallest roots as functions of $r$

![Figure 1: The moduli of the first and the second smallest roots as functions of $r$](image)

Figure 2 reports the distribution of the modulus of the smallest root for $r = 15$ across 1000 bootstrapping replications. The mean value is 0.71, indicating a non-negligible upward bias, since our point estimate for $r = 15$ is 0.54. We shall come back to the estimation bias below. Here we limit ourselves to observe that if the smallest root is overestimated on average, the true value could be even smaller.

\[\text{Note that these roots (and therefore fundamentalness) are independent of the identification rule adopted and the rotation matrix } H.\]
than 0.54. Without any bias correction, the probability of an estimated value larger than one in modulus is less than 22%.

Figure 2: **Frequency distribution of the modulus of the smallest root**

![Graph](image)

We conclude that the true, structural impulse-response function system for the common components associated with these three variables is probably non-fundamental. As a consequence, such impulse response functions, as well as the associated structural shocks, cannot be recovered by estimating a three-dimensional VAR.

5.4 Impulse-response functions and variance decomposition

Coming to the impulse-response functions, as anticipated above we impose long-run neutrality of two shocks on per-capita output, like in KPSW. This is sufficient to reach a partial identification, i.e. to identify the long-run shock and its response functions on the three variables.

Figure 3 shows the response functions of per capita output for \( r = 12, 15, 18 \). The general shape does not change that much with \( r \). The productivity shock has positive effects declining with time on the output level. The response function reach its maximum value after 6-8 quarters with only negligible effects after two years. It should be observed that this simple distributed-lag shape is different from the one in KPSW, where there is a sharp decline during the second and the third year, which drives the overall effect back to the impact value.

In Figure 4 we concentrate on the case \( r = 15 \). We report the response functions with 90% confidence bands for output, consumption and investment respectively. Confidence bands are obtained with the procedure explained above (with 1000 replications). The shapes are similar for the three variables, with a positive impact effect followed by important, though declining, positive lagged effects.

Note that confidence bands are not centered around the point estimate, especially for consumption, suggesting the existence of a non-negligible bias. This
is not surprising, since formula (4.22) implicitly involves estimation of a VAR, where in addition the variable involved (the static factors) contain errors (a residual idiosyncratic term). Figure 5 shows the point estimate along with the mean of the bootstrap distribution for the output. Such a large bias is probably due to the small cross-sectional dimension. We have evidence of a much smaller bias for the larger data set of Giamone, Reichlin and Sala (2002). We do not make any attempt here to correct for the bias, but a procedure like the one suggested in Kilian (1998) could be appropriate.

Table 1 reports the fraction of the forecast-error variance attributed to the permanent shock for output, consumption and investment at different horizons. For ease of comparison we report the corresponding numbers obtained with the (restricted) VAR model and reported in Table 4 of KPSW.

At horizon 1, our estimates are smaller. The difference is important for consumption: only 0.30 according to the factor model as against 0.88 according to the KPSW model. But at horizons larger than or equal to 8 quarters our estimates are greater and the difference is very large for investment. At horizon 20 (5 years) the permanent shock explains 46% of investment variance according to KPSW as against 86% with the factor model. This result is interesting in that it solves a typical puzzle of the VAR literature: the finding that technological and other supply shocks explain a small fraction of investment variations even in the medium-long run.

6 Conclusions

In this paper we have argued that dynamic factor models are suitable for structural macroeconomic modeling and an interesting alternative to structural VARs.
Figure 4: The impulse response function of the long-run shock on output, consumption and investment for $r = 15$
We have shown that large information and a small number of shocks generating the comovement of many variables, allow the econometrician to recover the structural shocks driving the economy under the mild assumption that the structure of leads and lags is rich enough so that the cross-section can convey information on dynamic relations. Thus the fundamentalness problem, which has no solution in the VAR framework, where $n$ shocks must me recovered using present and past values of $n$ variables, becomes easily tractable when the number of variables exceeds the number of shocks.

Having established sufficient conditions for identification, we have proposed a procedure to estimate the impulse response functions. Moreover, we have shown consistency of such a procedure and have suggested a bootstrapping method for the construction of confidence bands and inference purposes.

In the empirical application, we have revisited the seminal paper by King et al. (1991, KPSW). We have designed a large data set including output, consumption and investment (the data analysed by KPSW) on the same sample period. We have estimated a large factor model with a three-shock specification and, after having identified the shocks as in KPSW, we have analysed impulse response functions on the three variables of interest: output, consumption and investment. We find that the smallest root of the determinant of the impulse-response functions formed by the three variables sub-system is non-fundamental and therefore could have not been obtained by estimating a VAR on these three variables alone. These impulse response functions imply a larger effect of the permanent shock on output and investment than those found by KPSW.
Table 1: Fraction of the forecast-error variance due to the long-run shock

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<th>Dynamic factor model</th>
<th>KPSW vector ECM</th>
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<td>Cons.</td>
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<tr>
<td>1</td>
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<td>0.30</td>
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<tr>
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<td>(0.12)</td>
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</table>

Appendix 1: Proof of Proposition 3

Let $A$ and $E$ be two $n \times n$ symmetric matrices and denote by $\sigma_j(\cdot), j = 1, \ldots, n$ the eigenvalues in decreasing order of magnitude. Throughout this section we will use the following inequalities due to Weyl (cfr. Stewart and Sun, 1990):

$$|\sigma_j(A + E) - \sigma_j(A)| \leq \sqrt{\sigma_1(E^2)} \leq \sqrt{\text{trace}(E^2)}$$

Denote by $\Lambda_n$ and $\Lambda_n^T$, the $r \times r$ diagonal matrices having on the diagonal elements the first $r$ largest eigenvalues of $\Gamma^\chi_{n0}$ and $\Gamma^\xi_{n0}$, respectively. Writing $W_n$ and $W_n^T$ for the $n \times r$ matrices having on the columns the corresponding eigenvectors, we have, by definition:

$$\Gamma^\chi_{n0}W_n = W_n\Lambda_n$$

$$\Gamma^{\xi^T}W_n^T = W_n^T\Lambda_n^T$$

Let us recall here our notation for the eigenvalues of the relevant matrices:

$$\mu^{\chi}_{nj} := \sigma_j(\Gamma^\chi_{n0}), \quad \mu^{\xi}_{nj} := \sigma_j(\Gamma^\xi_{n0}), \quad \mu^{\chi}_{nj} := \sigma_j(\Gamma^\chi_{n0}), \quad \mu^{\xi}_{nj} := \sigma_j(\Gamma^\xi_{n0}), \quad j = 1, \ldots, n$$
we have $\Lambda_n = \text{diag}(\mu_{n1}, \ldots, \mu_{nr})$ and $\Lambda_n^T = \text{diag}(\mu_{n1}^T, \ldots, \mu_{nr}^T)$

Using the following non-singular transformation of the common factors, $g_t = G_n f_t$ where $G_n = \frac{1}{\sqrt{n}} W_n' A_n$, we have (cfr. Section 4.1):

\[
Q_n = \frac{1}{\sqrt{n}} \Gamma_{n0} W_n, \quad D_n = W_n' \Gamma_{n1} W_n \Lambda_n^{-1} \quad \text{and} \quad \Sigma_n = \frac{1}{n} \Lambda_n - \frac{1}{n} D_n \Lambda_n D_n' \]

**Lemma 1** Under assumptions PA1-2, FM1-3, as $n, T \to \infty$, we have:

(i) $\text{trace} \left[ (\Gamma_{0n}^T - \Gamma_{0n}^x)^2 \right] = O_p \left( \frac{n^2}{T} \right)$, $k = 0, 1$

(ii) $\frac{1}{n} \mu_{nj}^x = \frac{1}{n} \mu_{nj}^x + O \left( \frac{1}{n} \right) + O_p \left( \frac{1}{\sqrt{T}} \right)$ for $k = 1, \ldots, n$

*Proof.* By assumption PA2, there exists a positive constant $K \leq \infty$, such that for all $T \in \mathbb{N}$ and $i, j \in \mathbb{N}$

\[
TE[(\hat{\gamma}_{0ij}^T - \gamma_{0ij}^x)^2] < K
\]
as $T \to \infty$, where $\gamma_{0ij}^T$ and $\gamma_{0ij}^x$ denote the $i, j$th entries of $\Gamma_{0n}^T$ and $\Gamma_{0n}^x$ respectively.

We have:

\[
\text{trace} \left[ (\Gamma_{0n}^x - \Gamma_{0n}^x)^2 \right] = \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x)^2
\]

Taking expectations, we obtain:

\[
E \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x)^2 \right] = \sum_{i=1}^{n} \sum_{j=1}^{n} E \left[ (\gamma_{0ij}^x - \gamma_{0ij}^x)^2 \right] = O_p \left( \frac{n^2}{T} \right)
\]

Result (i), for $k = 0$, follows from the Markov inequality. The result for $k = 1$ can be easily proved using the same arguments.

Turning to (ii), from the Weyl inequality, we have:

\[
\left( \mu_{nj}^x - \mu_{nj}^x \right)^2 \leq \text{trace} \left[ (\Gamma_{0n}^x - \Gamma_{0n}^x)^2 \right]
\]

moreover, from assumption FM0-3:

\[
\frac{1}{n} \mu_{nj}^x \leq \frac{1}{n} \mu_{nj}^x + \frac{1}{n} \mu_{nj}^x = \frac{1}{n} \mu_{nj}^x + O \left( \frac{1}{n} \right)
\]
The desired result follows. *Q.E.D.*
Corollary 1 Under assumptions PA1-2, FM1-3, as \( n, T \to \infty \), we have:

(i) \( \frac{1}{n} \Lambda_n^T = \frac{1}{n} \Lambda_n + O_p \left( \frac{1}{\sqrt{n}} \right) + O_p \left( \frac{1}{n} \right) \)

(ii) \( W_n^T W_n^T = I_r + O_p \left( \frac{1}{n} \right) + O_p \left( \frac{1}{\sqrt{T}} \right) \)

Proof. Result (i) trivially follows from Lemma 1. Turning to (ii), we have the following decomposition:

\[
\frac{1}{n} \Lambda_n^T = \frac{1}{n} W_n^T \Gamma_{n0}^T n_n W_n^T = \frac{1}{n} W_n^T W_n \Lambda_n W_n' W_n^T + \frac{1}{n} W_n^T \Gamma_{n0}^T n_n W_n^T + \frac{1}{n} W_n^T \left( \Gamma_{n0}^T - \Gamma_{n0}^x \right) W_n^T
\]

From results Lemma 1 (i) we get:

\[
\frac{1}{n} W_n^T \left( \Gamma_{n0}^T - \Gamma_{n0}^x \right) W_n^T \leq \frac{1}{n} \sqrt{\text{trace} \left[ (\Gamma_{n0}^T - \Gamma_{n0}^x)^2 \right]} = O \left( \frac{1}{\sqrt{T}} \right)
\]

Moreover, \( W_n^T \Gamma_{n0}^T W_n^T \leq \mu_{n1}^x = O_p(1) \) by assumption FM3. The desired result follows. Q.E.D..

Lemma 2 Under assumption PA1-2, FM1-FM3, as \( n, T \to \infty \), we have:

(i) \( Q_{ni}^T - Q_{ni} = O_p \left( \frac{1}{\sqrt{n}} \right) + O_p \left( \frac{1}{\sqrt{T}} \right) \)

(ii) \( D_n^T - D_n = O_p \left( \frac{1}{\sqrt{n}} \right) + O_p \left( \frac{1}{\sqrt{T}} \right) \)

(iii) \( \Sigma_n^T - \Sigma_n = O_p \left( \frac{1}{\sqrt{n}} \right) + O_p \left( \frac{1}{\sqrt{T}} \right) \)

where \( Q_{ni}^T \) and \( Q_{ni} \) denote the \( i \)th row of \( Q_n^T \) and \( Q_n \), respectively.

Proof. Let us start from result (i). We have the following decomposition

\[
Q_n^T = \frac{1}{\sqrt{n}} \Gamma_{n0}^T n_n W_n^T = \frac{1}{\sqrt{n}} \Gamma_{n0}^T n_n W_n^T + \frac{1}{\sqrt{n}} \Gamma_{n0}^x W_n^T + \frac{1}{\sqrt{n}} \left( \Gamma_{n0}^T - \Gamma_{n0}^x \right) W_n^T
\]

Write \( 1_{ni} \) for the \( n \) dimensional vector with entries equal to zero at the \( i \)th position and zero for the rest. Consequently:

\[
Q_{ni}^T = 1_{ni}' Q_n^T = \frac{1}{\sqrt{n}} 1_{ni}' \Gamma_{n0}^T n_n W_n^T = \frac{1}{\sqrt{n}} 1_{ni}' \Gamma_{n0}^T n_n W_n^T + \frac{1}{\sqrt{n}} 1_{ni}' \Gamma_{n0}^x W_n^T + \frac{1}{\sqrt{n}} 1_{ni}' \left( \Gamma_{n0}^T - \Gamma_{n0}^x \right) W_n^T
\]
Let us study separately each term of the right hand side. For the first term, Corollary 1 (ii), imply:

\[
\frac{1}{\sqrt{n}} 1_n' \Gamma_{n0}^x W_n^T = \frac{1}{\sqrt{n}} 1_n' \Gamma_{n0}^x W_n' W_n^T = Q_n W_n' W_n^T = Q_n + O_p \left( \frac{1}{\sqrt{n}} \right) + O_p \left( \frac{1}{\sqrt{T}} \right)
\]

since \( W_n W_n' A_n = A_n \) by Assumption FM0.

For the second term, we have:

\[
\frac{1}{\sqrt{n}} 1_n' \Gamma_{n0}^x W_n^T \leq \frac{1}{\sqrt{n}} \sqrt{1_n' \Gamma_{n0}^x 1_n} \sqrt{W_n^T \Gamma_{n0}^x W_n} \leq \frac{1}{\sqrt{n}} \mu_{n1} = O_p \left( \frac{1}{\sqrt{n}} \right)
\]

from assumption FM3.

Writing \( w_{jh}^T \) for the entry of \( W_n^T \) in the \( j \)th row and the \( h \)th columns, the third term can be written as:

\[
\frac{1}{\sqrt{n}} \left| 1_n' \left( \Gamma_{n0}^x - \Gamma_{n0}^x \right) W_n^T \right| \leq \frac{1}{\sqrt{n}} \left| \sum_{h=1}^{r} \left( \sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x) w_{jh}^T \right) \right|
\]

\[
\leq \frac{1}{\sqrt{n}} \sum_{h=1}^{r} \sqrt{\sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x)^2} \sqrt{\sum_{j=1}^{n} (w_{jh}^T)^2} = \frac{1}{\sqrt{n}} \sum_{h=1}^{r} \sqrt{\sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x)^2}
\]

since \( W_n^T \) is orthonormal. Because \( E \left[ \sum_{j=1}^{n} (\gamma_{0ij}^x - \gamma_{0ij}^x)^2 \right] = O_p \left( \frac{1}{T} \right) \), from the Markov’s inequality, we get

\[
\frac{1}{\sqrt{n}} 1_n' \left( \Gamma_{n0}^x - \Gamma_{n0}^x \right) W_n^T = O_p \left( \frac{1}{\sqrt{\mu}} \right)
\]

This proves result (i).

Turning to (ii), we have:

\[
\frac{1}{n} D_n^T A_n = \frac{1}{n} W_n^T \Gamma_{n1}^x W_n^T = \frac{1}{n} W_n^T \Gamma_{n1}^x W_n^T + \frac{1}{n} W_n^T \Gamma_{n1}^x W_n^T + \frac{1}{n} W_n^T (\Gamma_{n1}^x - \Gamma_{n1}^x) W_n
\]

From result (ii) of Corollary 1, we have:

\[
\frac{1}{n} W_n^T \Gamma_{n1}^x W_n^T = \frac{1}{n} (W_n^T W_n) W_n^T \Gamma_{n1}^x W_n W_n^T = \frac{1}{n} D_n A_n + O_p \left( \frac{1}{n} \right) + O_p \left( \frac{1}{\sqrt{T}} \right)
\]

since \( W_n W_n' A_n = A_n \) by Assumption FM0.

By assumptions PA1-2 and FM3, \( W_n^T \Gamma_{n1}^x W_n \) = \( O_p(1) \). Moreover, Lemma 1 (i) implies that: \( \frac{1}{n} W_n^T (\Gamma_{n1}^x - \Gamma_{n1}^x) W_n = O_p \left( \frac{1}{\sqrt{T}} \right) \). Result (ii), hence, follows from Corollary 1 (i) and Assumption FM2.
Finally, result (iii) is an immediate consequence of Lemma 1 (i) and result (ii) above.
Q.E.D.

Proof of Proposition 3
Note that the matrix $\Sigma_n$ is of fixed dimension $r$. Because of continuity of the eigenvalues and eigenvectors with respect to the matrix entries, by Lemma 2 (iii) and the continuous mapping theorem we have

$$M_n^T = M_n + O_p\left(\frac{1}{\sqrt{n}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) \quad \text{as } n, T \to \infty$$

and

$$K_n^T = K_n + O_p\left(\frac{1}{\sqrt{n}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) \quad \text{as } n, T \to \infty$$

Continuity of the matrix product (notice that $D_n$ has fixed dimension $r$), implies:

$$\left(D_n^T\right)^h = (D_n)^h + O_p\left(\frac{1}{\sqrt{n}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right) \quad \text{as } n, T \to \infty$$

Result (i) is hence an immediate consequence of Lemma 2 (i) and (ii).
Q.E.D.
## Appendix 2: Data description and data treatment

<table>
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<th>Database Source</th>
<th>ID Code in the Database Units</th>
<th>Orig. Seas. Freq. Adj. Treatment</th>
</tr>
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<td>Per Capita Real Consumption Expenditure</td>
<td>DLOG</td>
</tr>
<tr>
<td>2 MW Citibase</td>
<td>Per Capita Gross Private Domestic Fixed Investment</td>
<td>DLOG</td>
</tr>
<tr>
<td>3 MW Citibase</td>
<td>Per Capita Private Gross National Product</td>
<td>DLOG</td>
</tr>
<tr>
<td>4 MW Citibase</td>
<td>Per Capita Real M2 (M2 divided by P)</td>
<td>DLOG</td>
</tr>
<tr>
<td>5 MW Citibase</td>
<td>3-Month Treasury Bill Rate</td>
<td>D</td>
</tr>
<tr>
<td>6 MW Citibase</td>
<td>Implicit Price Deflator for Private GNP</td>
<td>DDLOG</td>
</tr>
<tr>
<td>7 Fred II BEA</td>
<td>Real Gross Domestic Product, 1 Decimal</td>
<td>DLOG</td>
</tr>
<tr>
<td>8 Fred II BEA</td>
<td>Real Final Sales of Domestic Product, 1 Decimal</td>
<td>DLOG</td>
</tr>
<tr>
<td>9 Fred II BEA</td>
<td>Real Gross Private Domestic Investment, 1 Decimal</td>
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<tr>
<td>10 Fred II BEA</td>
<td>Real State &amp; Local Cons. &amp; Expend. &amp; Gross Inv., 1 Dec.</td>
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<tr>
<td>11 Fred II BEA</td>
<td>Real Private Residential Fixed Investment, 1 Dec.</td>
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<td>12 Fred II BEA</td>
<td>Real Private Nonresidential Fixed Investment, 1 Dec.</td>
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<tr>
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<td>Real Nonresidential Inv. &amp; Equipment And Software, 1 Dec.</td>
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<td>Real Imports of Goods &amp; Services, 1 Dec.</td>
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<td>Real Federal Govt. Exp. &amp; Gross Inv., 1 Dec.</td>
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<td>Real Private Fixed Domestic Investment, 1 Decimal</td>
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<td>17 Fred II BEA</td>
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<td>Real Change in Private Inventories, 1 Dec.</td>
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<td>Real Personal Cons. Expenditures: Nondurable Goods</td>
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<td>Real Personal Cons. Expenditures: Durable Goods</td>
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<td>Personal Cons. Expenditures: Chain-type Price Index</td>
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<td>Nonfarm Business Sector: Real Compensation Per Hour</td>
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<td>31 Fred II BEA</td>
<td>Nonfarm Business Sector: Output Per Hour Of All Persons</td>
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<td>Nonfarm Business Sector: Compensation Per Hour</td>
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<td>Manufacturing Sector: Output Per Hour Of All Persons</td>
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<td>St. Louis St. Louis Adjusted Reserve</td>
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<td>37 Fred II BEA</td>
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<td>Moody's Moody's Seasoned Baa Corporate Bond Yield</td>
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<td>Currency Component of M1</td>
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<td>Real State &amp; Local Cons. &amp; Expend. &amp; Gross Inv., 1 Dec.</td>
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<td>Chicago Purchasing Manager Business Barometer</td>
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<td>Standard &amp; Poor’s 500 (monthly average)</td>
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<td>89 Datastream FT</td>
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Abbreviations:
MW: Mark Watson’s home page (http://www.wws.princeton.edu/~mwatson/publi.html)
Fred II: Fred II database of the Federal Reserve Bank of St. Louis
BEA: Bureau of Economic Analysis
BLS: Bureau of Labor Statistics
Fed: Federal Reserve Board
St Louis: Federal Reserve Bank of St. Louis
ISM: Institute for Supply Management
BC: Bureau of Census
S&P: Standard & Poor’s
FT: Financial Times
Q: Quarterly
M: Monthly (we take quarterly averages)
References


