Incorporating taxation in the valuation of variable annuity contracts: the case of the guaranteed minimum accumulation benefit

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14^{th} International Longevity Risk and Capital Markets Solutions Conference (Amsterdam, The Netherlands)
Thursday September 20^{th} 2018

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Variable Annuities

Variable Annuities (VAs) were first introduced in the early 1950s and various 'GMxBs' have become available since:

- Guaranteed Minimum Death Benefit introduced in 1980s.
- Guaranteed Minimum Living Benefits introduced in late 1990s.
  - GMAB - Accumulation
  - GMIB - Income
  - GMWB - Withdrawal
  - (GLWB - Lifelong form of GMWB)
- VA industry is large: US$1.98 trillion in the U.S. as of 2015 (IRI 2015)

Research questions

- What is the impact of tax on surrender behavior? *Allowing for losses to offset gains is beneficial to policyholders and insurers*
- How does this impact pricing (from policyholder's and insurer's perspective)? *Policyholder is willing to pay less if losses cannot offset gains*
- And what if capital losses do offset gains? *policyholder is willing to pay more as losses are also beneficial them at the expense of the government*

Surrender behavior

- GMABs promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract
- Typically, the valuation frameworks study the effect of the underlying equity distribution (GBM, Levy, etc) on the fee
- Recently, the surrender behavior is studied more closely in the literature (e.g. Bernard et al. (2014); Kang and Ziveyi (2018)) as underpricing lapse risk has resulted in significant losses for insurers (Service 2017)
- **Here:** the contract can be surrendered at any time prior to maturity. and the payments are liable for **taxes** (policyholder perspective)

Importance of Incorporating Tax

- One of the main attractive features of VAs is their tax-advantaged investing (Milevsky and Panyagometh 2001; Brown and Poterba 2006)
- Incorporating taxation in riders such as GMWB reconciles empirically observed fees with the theory (Moenig and Bauer 2015)
- The financial planning literature has long looked at ways to provide rules to follow to maximise post-tax returns (Sumutka et al. 2012; Horan and Robinson 2008)
- **Here:** we examine the impact of tax on the optimal surrender boundaries for a GMAB and its impact in pricing

Two tax regimes

- In Moenig and Bauer (2015) the authors study the effect of tax on capital gains only [GMWB]
- They see that it reconciles the *theoretical* fees with those found in the US market.
- However, in some tax regimes capital losses can offset capital gains, lowering the total tax liability
- **Here:** we study the policyholder behavior without tax [classical academic assumption], with tax on capital gains only [recent development] and when capital losses can offset gains [*novelty*]

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GMAB product

- Policyholder invests an initial amount x_0 and at maturity receives the greater of the guarantee G and the fund value
- To finance the guarantee, the insurer charges a continuously compounded fee, q, as a percentage of the fund
- The income of the policyholder is taxable, however they are not taxed until early surrender or maturity
- The taxable income of the policyholder at maturity can be re-written as:

$$\overbrace{\max(G, x_T)}^{\text{guarantee}} - \underbrace{(x_0 + C_0)}_{\text{initial payment + upfront cost}} - \overbrace{y(T)}^{\text{total fees}} . \qquad (1)$$

- If tax only on capital gains: $\max[\text{Equation (1)}]_+$,
- If losses offset gains: Equation (1).

Surrender

- The GMAB contract permits the policyholder to surrender early
- Policyholders are not eligible for the guarantee if they surrender early (Kang and Ziveyi 2018).
- Upon surrender, the insurer will pay $\gamma_\nu x_\nu$, where $(1-\gamma_\nu)$ is the surrender penalty.
- In the event of early surrender at time ν, the taxable income will thus be

$$[\gamma_\nu x_\nu - x_0 - C_0 - y(\nu)]_+. \tag{2}$$

The governing PDE

- Value of the GMAB: $u(x, y, \nu)$ with x fund value, y total fees paid and ν time elapsed since purchase
- The fund evolves as a Geometric Brownian Motion
- If t represents the contract's time to maturity, u will satisfy the PDE:

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + x \cdot q \cdot u_y + (r - q) \cdot x \cdot u_x - r \cdot u - u_t = 0 \qquad (3)$$

- The boundary conditions capture the taxes paid upon surrender and maturity With tax-gains only With tax-losses offset
- Observe that for sufficiently large fees paid and no offset, the taxable amount is zero $\rightarrow$ no taxation case No tax
- The boundaries will change when looking at the insurer's perspective Insurer

Solving methodology

- We use the Method of Lines to solve Equation (3) (Meyer and Van der Hoek 1997)
- The PDE is discretised in t and y, while continuity is maintained in x
- The differentials u_t and u_y are re-expressed using a finite different approximation, e.g.

$$u_t = \begin{cases} \frac{u - u_{k,n-1}}{\Delta t} & \text{if } n = 1, 2 \\ \frac{3}{2}\frac{u - u_{k,n-1}}{\Delta t} - \frac{1}{2}\frac{u_{k,n-1} - u_{k,n-2}}{\Delta t} & \text{if } n \geq 3 \end{cases} \quad (4)$$

- This is a fast and accurate methodology (Meyer and Van der Hoek 1997; Chiarella et al. 2009) and has already proved useful in the VA space (Kang and Ziveyi 2018).

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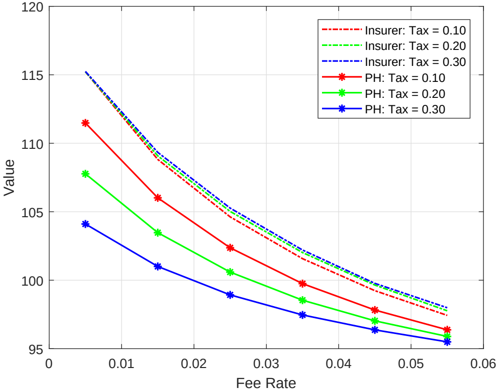
Financial base case parameters

Parameter	Value	
r	0.02	risk-free rate
σ	0.25	fund volatility
τ	0.10	tax rate
x_0	100	initial premium
G	100	guarantee
T	15	maturity
κ	0.005	penalty

- Weekly time discretisation
- Maximum possible fund value and total fees set to $4 \cdot G$

Capital gains - Value to PH and insurer

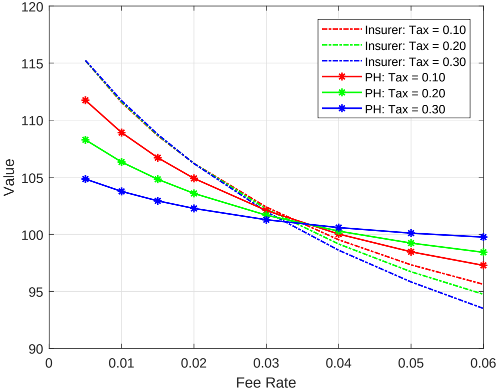


A market in this tax setting may not exist!

Surrender boundary

- Value to insurer's and policyholder's decrease with fees $\rightarrow$ higher fees will incentivize early surrender (loss-loss situation)
- Value to the policyholder decreases with tax: for a given fee, all gains are taxed and losses cannot offset them
- Value to the insurer increases (slightly) with tax: policyholder will behave as to maximize post-tax value. Higher tax will delay surrender and increase fees to the insurer.

Capital losses can offset gains - Value to PH and insurer



Higher (than fair) fees increase “losses” which benefit the policyholder and insurer at the expense of the government!

Surrender boundary

Insurer liability curve shifts down

- as tax increases → policyholders are more likely to delay surrender to obtain a certain post-tax value → higher fee income
- as fee increases → higher fee income

However, for the policyholder

- if fee $<$ fair fee → higher potential capital gains → value decreases with tax
- if fee $>$ fair fee → higher potential losses → value increases with tax (tax back)

The difference between the insurer and policyholder value is the value to the government. The point at which they meet is where the value to the government is zero.

Fair Fees under different tax regimes

Tax regime	q^*_{Ph} (% p.a.)	q^*_{Ins} (% p.a.)
No tax	3.91	3.91
$\tau = 0.30$, offset allowed	5.25	3.56
$\tau = 0.30$, no offsets	1.94	4.32

Notes: Surrender boundary no tax

- No tax vs offset: the policyholder is willing to pay more than in the no tax regime as any losses will benefit them. Similarly, the insurer is willing to enter the contract at a lower rate at the expense of the government.
- No tax vs no offset: the policyholder is willing to pay a much lower fee to have higher gains $\rightarrow$ higher post-tax value. Similarly, the insurer needs a higher fee to compensate the surrender behavior.

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Conclusion

- Tax is a key aspect of financial planning.
- We illustrate the impact of various tax systems, including the realistic case when losses can offset gains.
- The relationship between fees, behavior and contract value vary across systems: e.g., when losses offset gains then the contract is interesting for both parties at the expense of the government.
- The method of lines used enables us to efficiently determine optimal surrender boundaries, contract values and fair fees.
- *Next steps:* adding withdrawals or regular premiums?

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Thanks

Thank you for your attention

Questions?

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Boundary conditions with tax

In order to obtain the contract value from the policyholder perspective, we solve equation (3) subject to the following boundary conditions:

$$u(x,y,0) = \max(x,G) - \tau\big[\max(x,G) - y - x_0 - C_0\big]_+, \tag{5}$$

$$u(s(t,y),y,t) = s(t,y)\gamma_t - \tau\big[s(t,y)\gamma_t - y - x_0 - C_0]_+, \tag{6}$$

$$u(0,y,t) = (G - \tau[G - y - x_0 - C_0]_+)e^{-rt}, \tag{7}$$

$$u_x(s(t,y),y,t) = \gamma_t - \tau\gamma_t \mathbb{I}\big\{s(t,y)\gamma_t - y - x_0 - C_0 > 0\big\}, \tag{8}$$

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Boundary conditions without tax

Putting $u_y = 0$ into equation (3), we recover the following 2 dimensional PDE

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + (r-q)\cdot x u_x - ru - u_t = 0. \tag{9}$$

which must be solved subject to the following boundary conditions:

$$u(x, Y, 0) = \max(x, G), \tag{10}$$
$$u(s(t, Y), Y, t) = s(t, Y)\gamma_t, \tag{11}$$
$$u(0, Y, t) = Ge^{-rt}, \tag{12}$$
$$u_x(s(t, Y), Y, t) = \gamma_t. \tag{13}$$

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Boundary conditions for the insurer's liabilities

To obtain the value of the contract from the insurer's perspective, henceforth to be referred to as the insurer's liabilities, the partial differential equation (3) must be solved subject to boundary conditions which reflect the total before tax payments the insurer must make to the policyholder:

$$u_{Ins}(x, y, 0) = \max(x, G) \tag{14}$$

$$u_{Ins}(s(t, y), y, t) = s(t, y)\gamma_t \tag{15}$$

$$u_{Ins}(0, y, t) = Ge^{-rt} \tag{16}$$

$$\frac{\partial u_{Ins}(s(t, y), y, t)}{\partial x} = \gamma_t \tag{17}$$

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Boundary conditions when losses can offset capital gains

We explore the case in which the capital losses on the GMAB product can be used to offset other income sources, as is the case for nonqualified plans in the US. Mathematically, this entails the following replacement:

$$\tau(\gamma_t x - y - x_0)_+ \to \tau(\gamma_t x - y - x_0)$$

in equation the boundary conditions (5),(6), (7) and (8). Therefore, the new problem requires us to solve the PDE (3) subject to the boundary conditions:

$$u(x, y, 0) = \max(x, G) - \tau(\max(x, G) - y - x_0 - C_0), \tag{18}$$

$$u(s(t, y), y, t) = s(t, y)\gamma_t - \tau(s(t, y)\gamma_t - y - x_0 - C_0), \tag{19}$$

$$u(0, y, t) = [G - \tau(G - y - x_0 - C_0)]e^{-rt}, \tag{20}$$

$$u_x(s(t, y), y, t) = \gamma_t - \tau\gamma_t. \tag{21}$$

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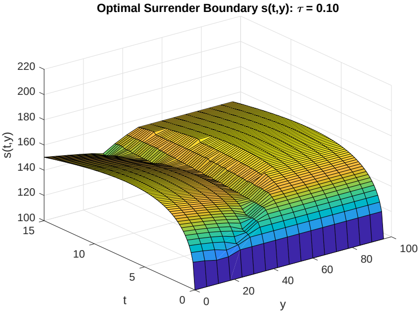
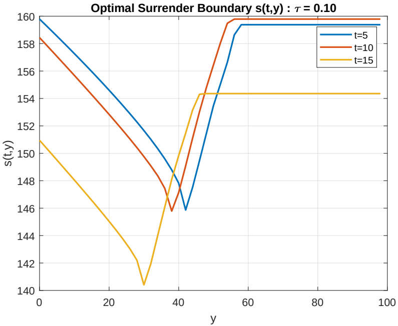
Surrender boundary **no** offset ($\tau = 0.10$)





The 'valley of surrender', a combination of values of cumulative fees paid y and time to maturity t is driven by the fact that capital losses cannot be claimed on the product.

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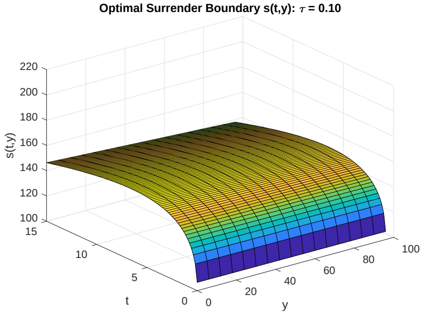
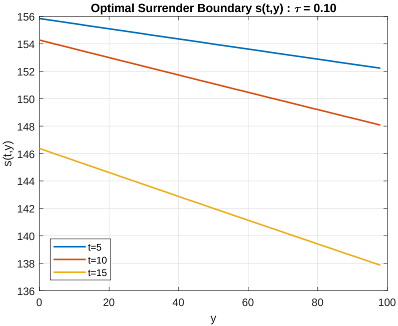
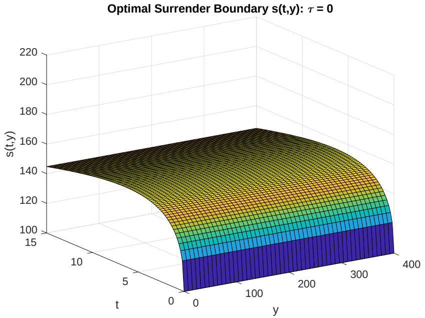
Surrender boundary **with** offset ($\tau = 0.10$)

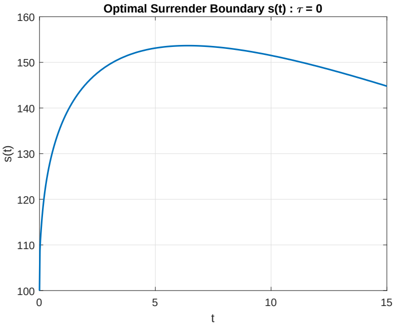




The surrender surface $s(t, y)$ is monotonically decreasing in y. This is because all else equal, having already paid a greater sum of fees will reduce taxable income. Back to Main story

Surrender surface when $\tau = 0$





The surrender boundary is independent of the cumulative fees paid y because there is no tax. The shape agrees with those presented by Bernard et al. (2014).

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