International Order Flows:
Explaining Equity and Exchange Rate Returns

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Abstract

Macroeconomic models of equity and exchange rate returns perform poorly. The proportion of daily returns that these models explain is essentially zero. Instead of relying on macroeconomic determinants, we model equity price and exchange rate behavior based on a concept from microstructure - order flow. The international order flows are derived from belief changes of different investor groups in a two country setting. We obtain a structural relationship between equity returns, exchange rate returns and their relationship to home and foreign market order flow. To test the model we construct daily aggregate order flow data from all equity trades in the U.S. and France from 1999 to 2003. Almost 60 percent of the daily returns in the S&P100 index is explained jointly by exchange rate returns and aggregate order flows. The model implications are also validated for intraday returns.
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Macroeconomic models of equity and exchange rate returns perform poorly. The proportion of daily returns that these models explain is essentially zero. Instead of relying on macroeconomic determinants, we model equity price and exchange rate behavior based on a concept from microstructure - order flow. The international order flows are derived from belief changes of different investor groups in a two country setting. We obtain a structural relationship between equity returns, exchange rate returns and their relationship to home and foreign market order flow. To test the model we construct daily aggregate order flow data from all equity trades in the U.S. and France from 1999 to 2003. Almost 60 percent of the daily returns in the S&P100 index is explained jointly by exchange rate returns and aggregate order flows. The model implications are also validated for intraday returns.
1 Introduction

The aggregate stock market index and the exchange rate are known to have a very low correlation with any other measurable macroeconomic variable (Frankel and Rose (1995), Rogoff (2001)). Financial economists interpret this very lack of predictability as evidence for efficiency, whereby only unpredictable news should move prices. But even gathering proxy variables for news ex-post does not seem to substantially increase the explanatory power of asset pricing models (Roll (1988)). This motivates us to examine a new financial market variable called order flow in its relationship to stock and exchange rate returns. Order flow is the net of buy minus sell initiated orders. In the foreign exchange market, daily exchange rate returns and daily order flow show a remarkably high correlation (Evans and Lyons (2002a, 2002b, 2002c)) and even permanent changes in the exchange rate appear to be explained by order flow. Unfortunately, most of the microstructure literature features order flow as an exogenous variable in a single market setting. Its very origin remains unexplained and this lack of economic structure constrains the analysis. In particular, issues of market interdependence between different international stock markets are generally ignored.

This paper contributes to the existing literature in four dimensions. First, we provide a market model in which order flow is the result of belief changes of various investor groups. This allows for a structural interpretation of order flow regressions. Second, we model a two country multi-market setting in which we can explore the relationship between equity, exchange rate and bond markets. In particular, we obtain testable restrictions which link equity returns to the various order flows. Thirdly, we show that our empirical framework explains up to 60 percent of the daily return variations in the S&P 100 index. In accordance with the theory, both exchange rate returns and order flow into the overseas market have explanatory power for the domestic stock market returns. Fourth, our model can explain the asymmetry in the correlation structure of equity returns and exchange rates. U.S. equity market appreciations typically come with U.S. dollar appreciations, while European equity market returns correlate negatively with Euro appreciations.

The starting point of our analysis is a coherent interpretation of order flow itself. What motivates trades through market orders as opposed to limit orders? In most microstructure model of limit order markets those market participants with private asset valuations removed from the current midprice tend to pursue market order strategies. The intuition is straightforward. Execution uncertainty related to limit order submission is a multiplicative factor for the expected benefit of a trade. In the absence of risk aversion, the probability of non-execution reduces the expected trade benefit linearly as the difference between current midprice and the private value increases. The cost of market order submission by contrast is an additive cost related to

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1 Occasionally, order flow is also referred to as order imbalance.

the effective spread. It is unchanged by more extreme private asset valuations. A large change in the asset valuation by a segment of market participants will therefore tend to trigger predominantly market orders. This feature of modern limit order markets makes order flow a suitable proxy for (substantial) investor belief changes. Our simple market model captures this aspect, namely order flow is simply a linear function of belief changes. Hence, order flows can be used to identify heterogenous belief changes within a segmented investor population. We do not deny that other trade motivations like (urgent) hedging or liquidation needs might also come with a preference for market over limit order implementation of the transaction. These trades are outside the model framework and feature as noise in the empirical analysis. We also highlight that we are agnostic about the source of the belief changes. These could be based on private information or have a behavioral explanation.

Previous work on the relationship between asset returns and order flow has typically been focused on a single asset market. The focus of our paper is the market interaction between equity and exchange rate markets in a partially segmented international asset market. Recent empirical and theoretical work has emphasized the limited market integration of the global equity market (Karolyi and Stulz (2002), Hau and Rey (2003), Stulz (2005)). The microstructure approach used here can be useful in understanding international market interdependence. We show that domestic equity returns should not only be highly correlated with domestic order flow, but exchange rate returns and order flow into the overseas market should have additional explanatory power for domestic equity returns. The additional explanatory power of overseas order flow is a direct consequence of international equity market segmentation. Belief changes of local overseas investors are reflected in the order flow of the overseas market and revealed in its equity price. Via an asset substitution effect, such belief changes also influences the home equity returns, but this substitution effect is public information and therefore not reflected in home market order flow. Hence the additional explanatory power of the foreign market order flow above the home market order flow. We highlight that the overall explanatory power for daily index returns is astonishing. For the S&P100 we are able to explain around 60 percent of the daily return variation and for the CAC40 approximately 40 percent.

International portfolio managers often highlight the asymmetry in the correlation structure of equity and exchange rate returns. Table 1 documents the negative correlation of the U.S. dollar return with the U.S. equity market index and the even more negative correlation of all European equity markets with the same exchange rate return. A symmetric setting should imply opposite signs for the respective correlations, hence the notion of asymmetry in the correlation structure. Our model framework can account for this asymmetry. The exchange rate correlation can be negative for both home and foreign country even after controlling for equity order flows. In particular, differences in the elasticity of counterparty asset supply faced by the international investor in the home and foreign market should generally result in the observed correlation asymmetry. The model framework also allows us to identify the correlation structure of belief changes for local and international investors. We find that local equity investors in France and U.S. have
almost uncorrelated belief innovations in their equity valuations, while international investors have valuation changes which appear negatively correlated to those of local investors. The relative independence of belief changes by local investors can be viewed as a source of international diversification benefits.

Our paper also relates to a recent literature which focuses on the role of order flow in the U.S. equity market. Hasbrouck and Seppi (2001) show that commonality in the order flows of individual stocks explains roughly two-thirds of the commonality in returns. But this paper restricts itself to high frequency intervals. Chordia and Subrahmanyam (2004) study the relationship between order flow and daily returns of individual stocks. Pastor and Stambaugh (2003) find that market wide liquidity is a state variable important for asset pricing at the daily frequency. Chordia, Roll and Subrahmanyam (2002) document for the period 1988 to 1998 that aggregate order flow in the NYSE is correlated with contemporaneous daily S&P500 returns. But their regressions are not based on any structural model and show much lower explanatory power. To the best of our knowledge, no paper has tried to formally model aggregate equity returns in terms of aggregate equity order flow or related cross country differences in equity order flows to exchange rate movements.

The following section presents the model. Section 3 summarizes and explains the resulting equilibrium relationships. The data is explained in Section 4. Section 5.1 discusses the estimation results for aggregate equity daily returns and section 5.2 for intraday returns. Section 6 concludes.

2 The Model

The model sketched in this section serves several purposes. First, it is designed to represent a multi-market setting in which different types of fund managers experience exogenous belief changes about the fundamental (or terminal) value of home and foreign equity. Our multi-market setting is a stylized representation of a partially integrated international equity and bond market linked through a common foreign exchange market. Secondly, we wish to capture how heterogenous belief changes about equity values trigger order flow in each market and leads to consecutive price changes. The market structure follows the simultaneous-trade approach (Evans and Lyons (1997, 2002a)) in which market orders precede the full price adjustment process. This framework is relatively simple and can be extended to the multimarket environment. Most importantly, it allows for a clear model-based definition of order flow. Thirdly, we wish to identify a structural relationship between order flow and directly unobservable belief changes. We can then substitute belief changes with order flows and obtain an empirically testable framework relating order flows to market returns.

The multi-market setting features equity and bond markets both at home and abroad. We assume that

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3Survey data suggests that international differences in equity return beliefs are in fact important. Shiller et al. (1996) document large aggregate differences of opinion on the price expectations of the Nikkei and S&P100 index across Japanese and American fund managers.

4On the other hand, the model abstracts from bid-ask spreads and the possibility of multiple limit orders submission which both arise naturally is a setting of asymmetric information.
there are four different types or groups of (atomistic) agents called ‘funds’.5 These four fund groups are listed in Table 1: International equity funds, investing in home and foreign equity; international bond funds, investing in home and foreign bonds; home country funds, investing in home equity and a home bond and foreign funds, investing in foreign equity and a foreign bond.6 Our model therefore features a segmented market in which the exchange rate risk is not diversified. The bonds in both markets are assumed to be in completely price inelastic supply with a constant return in local currency assumed to be \( r \) for both home and foreign bonds.7 The bond prices are normalized to 1 without loss of generality.

The market structure is illustrated in Figure 1. In round 1 funds first quote prices. Evans and Lyons (1997, 2002a) assumes an initial quoted price which is valid for any size of the subsequent market order. We modify this assumption and assume that the initial liquidity supply is price elastic. Hence, the execution price increases in the size of the market order. The elasticity of the supply is chosen such that execution

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5To justify competitive price taking behavior, we assume that each investor group is composed of a continuum of atomistic agents. For the price equilibrium only their aggregate risk tolerance matters. Belief heterogeneity within each group represents a less interesting extension as such heterogeneity will “wash out” under aggregation. We therefore focus on heterogeneity across groups.

6We fix notation as follows: the four funds are indicated with the subscripts \( h, f, e \) and \( b \) for home, foreign, international equity and international bond respectively; the superscripts \( H \) and \( F \) refer to holdings of home and foreign equities respectively and the superscripts \( B^H \) and \( B^F \) refer to holdings of home and foreign bonds respectively.

7We do not specify what pins down the riskless rate of interest in the model. In addition, there is no distinction between real and nominal returns. The reader may like to think of the riskless rate as being determined by the rate of time preference or a steady state marginal efficiency of capital. Finally, the assumption that the riskless rate is the same in both countries has no bearing on the results.

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Figure 1: Market Structure
price of the market order equals its equilibrium price impact. Large market orders with a large equilibrium price impact therefore face a large execution price. In the second step of round 1, funds change their beliefs about the liquidation values of the assets. Then they place optimal market orders which reflect their new beliefs and the (size contingent) execution price of the market order. The market orders do not account for the belief changes of other funds which are private information in round 1. At the beginning of round 2 all aggregate order flows are observed. This allows for the full revelation of all underlying belief changes. Under full information, new equilibrium prices are quoted. Since the market orders in round 1 are based on incomplete information, portfolio imbalances still exist. A further round of trading in round 2 leads to the equilibrium allocation of assets before the payoffs are realized. It is important to note that rebalancing under full information in round 2 does not allow any inference about the trade direction. Sell and buy initiated orders are equally likely in all markets. Hence, we expect no systematic effect of the trading in round 2 on the order flow statistics.

Let \( x_e^H \) and \( x_e^F \) denote the home and foreign equity market investment of the international fund, respectively, and \( x_h^H \) and \( x_f^F \) the domestic investments of the home and foreign fund. For simplicity, we assume that there are zero net stocks outstanding. The market clearing condition for the equity markets then takes the form

\[
\begin{align*}
x_e^H + x_h^H &= 0 \\
x_e^F + x_f^F &= 0.
\end{align*}
\]

The foreign exchange market clears for a demand \( x_e^H \) for home currency on the part of the international equity market and a home currency demand \( x_b^H \) from the international bond fund. Under zero net balances, we have

\[
x_e^H + x_b^H = 0
\]

We assume that all four funds are fully leveraged and that their net asset position is zero. In combination with the zero net equity supply, this assumption implies that we can neglect risk premia in the analysis. The exchange rate, \( E \), is defined as the ratio of home (U.S.) to foreign currency (Euro), hence an increase in \( E \) corresponds to a dollar depreciation.

The investment behavior of the four funds is defined by the following assumption:

**Assumption 1: Fund Objectives**

The four groups of investment funds \( i = e,b,h,f \) competitively pursue investment objectives which maximize a CARA objective function given by

\[
U_i = E_i(\Delta \Pi_i|I) - \frac{1}{2} \mu_i \text{Var}(\Delta \Pi_i|I).
\]

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8 The expected trading profits of liquidity supply with respect to period 2 equilibrium prices are therefore zero.

9 We could have equivalently expressed the foreign exchange market clearing condition in terms of the demand for foreign currency.
where the expected net payoffs $\mathcal{E}_i(\Delta \Pi_i \mid I)$ and payoff variance $\text{Var}(\Delta \Pi_i \mid I)$ are conditional on equity prices and the exchange rate $(I = \{P^H, P^F, E\})$ and $1/\rho_i$ denotes the aggregate risk tolerance of each group.

1) International equity funds $(e)$ choose optimal home and foreign equity holdings $(x^H_e, x^F_e)$ subject to a budget constraint

$$0 = x^H_e P^H + x^F_e E P^F.$$ 

2) International bond funds $(b)$ choose optimal home and foreign bond holdings $(x^H_b, x^F_b)$ subject to a budget constraint

$$0 = x^H_b P^H + E x^F_b.$$ 

3) Home funds $(h)$ choose optimal home equity and home bond holdings $(x^H_h, x^F_h)$ subject to the budget constraint

$$0 = x^H_h P^H + x^F_h.$$ 

4) Foreign funds $(f)$ chooses optimal foreign equity and foreign bond holdings $(x^F_f, x^F_f)$ subject to the budget constraint

$$0 = x^F_f P^F + x^F_f.$$ 

The mean-variance framework allows for a particularly straightforward closed-form solution. Our main interest is not the steady state solution of the price system, but its reaction to perturbations. In particular, we are interested in the price and order flow effects if each fund manager changes his belief about the fundamental value of equity in a heterogenous manner. We assume a single stochastic belief change around the correct expected liquidation values of equity. Formally, we have:

**Assumption 2: Heterogenous Belief Changes**

Managers for the fund types $i = e, b, h, f$ undergo heterogenous (type specific) stochastic belief changes $\mu = (\mu^H_e, \mu^F_e, \mu^H_h, \mu^F_f)$ about the liquidation value of home and foreign equity\(^{10}\). Starting from initially correct beliefs about the steady state liquidation values $(\bar{V}^H, \bar{V}^F)$ of home and foreign equity, the new conditional beliefs $(I = \{P^H, P^F, E\})$ about the fundamental equity value can be expressed as

$$\mathcal{E}_e(V^H \mid I) = \bar{V}^H + \mu^H_e$$
$$\mathcal{E}_e(V^F \mid I) = \bar{V}^F + \mu^F_e$$
$$\mathcal{E}_h(V^H \mid I) = \bar{V}^H + \mu^H_h$$
$$\mathcal{E}_f(V^F \mid I) = \bar{V}^F + \mu^F_f.$$ 

\(^{10}\) Obviously, holders of the international bond fund do not suffer from misperceptions about equity prices because they never invest in that asset.
Heterogenous belief changes concern only equity valuation. Relative to bonds with predefined cash flows, equity is notoriously difficult to value and might therefore be more exposed to belief changes. These belief changes only concern the first moment. The funds hold identical and correct beliefs concerning the variances. The liquidation value of both the home and foreign equity has a variance $\sigma^2$ and the liquidation value of currency a variance $\sigma^2_E$.

3 Equilibrium Relationships

The objective function of each fund is defined in terms of mean and variance of the terminal payoff. This allows for simple linear asset demand functions for each fund. Combined with the market clearing condition for the two equity markets and the forex market, we therefore obtain a linear system of three equations which characterizes the equilibrium prices and returns as a function of the belief changes. The three markets in our model are interrelated in the sense that a belief shock in one market affects the equilibrium price in the other two. For example, a positive belief shock for the home fund manager will increase the price of domestic equity. Higher prices in domestic equity induce a substitution effect on the part of the international equity fund, which will increase its demand for foreign equity and reduce its home country equity holdings. This increases the foreign equity price and at the same time increases the demand for foreign exchange balances. The foreign currency will therefore appreciate.

The equilibrium return impact of general belief change on the part of all equity investors is summarized in Proposition 1:

Proposition 1: Returns and Heterogenous Beliefs

The equilibrium returns $\mathbf{R} = (R^H, R^F, R^E)$ for home equity, foreign equity, and the exchange rate, respectively are linearly related to belief changes $\mathbf{\mu} = (\mu^H_e, \mu^F_e, \mu^H_h, \mu^F_f)$ about the home and the foreign equity value according to

$$\mathbf{AR} = \mathbf{B}\mathbf{\mu}$$

for matrices $\mathbf{A}$ and $\mathbf{B}$ defined as

$$\mathbf{A} = \begin{bmatrix}
(1 + \lambda_h) & -1 & -1 \\
-1 & (1 + \lambda_f) & 1 \\
-1 & 1 & (1 + \lambda_b)
\end{bmatrix}, \quad \mathbf{B} = \frac{1}{1 + r} \begin{bmatrix}
1 & -1 & \lambda_h & 0 \\
-1 & 1 & 0 & \lambda_f \\
-1 & 1 & 0 & 0
\end{bmatrix}$$

a riskless rate $r$, parameters defined as

$$\lambda_h = \frac{\rho_e \left[2\sigma^2 + (1 + r)^2 \sigma^2_E\right]}{\rho_h \sigma^2}, \quad \lambda_f = \frac{\rho_e \left[2\sigma^2 + (1 + r)^2 \sigma^2_E\right]}{\rho_f \sigma^2}, \quad \lambda_b = \frac{\rho_e \left[2\sigma^2 + (1 + r)^2 \sigma^2_E\right]}{\rho_b (1 + r)^2 \sigma^2_E}.$$

Proof: See Appendix.
The return vector $\mathbf{R}$ is uniquely determined by the belief changes $\boldsymbol{\mu}$ as long as the matrix $\mathbf{A}$ is non-singular. The three equilibrium equations result directly from the market clearing condition in the home and foreign equity markets and in the foreign exchange market. The belief changes $\mu^H_e, \mu^F_e$ for the international equity fund always appear symmetrically in the term $\mathbf{B}_e \boldsymbol{\mu}$ but with opposite sign, hence only the relative belief of the international equity investor change $\mu^H_e - \mu^F_e$ matters for the price determination. The parameters $\lambda_h, \lambda_f$ and $\lambda_b$ denote ratios of asset supply elasticities. For example, $\lambda_h$ denotes the aggregate supply elasticity of the home equity investors (proportional to $1/\rho_h \sigma^2$) relative to the aggregate supply elasticity of the international equity investors. A lower risk aversion of the home investor or lower home price variance imply a more price inelastic home asset supply and therefore a large parameter $\lambda_b$. Belief shocks by the international equity fund then have a more modest home return effect. We also note that the belief changes of the home and foreign fund only enter the first and second equation, respectively, while belief change of the international equity fund affects all three market clearing conditions simultaneously.

Exogenous belief changes are the only source of price change in our model. But such belief changes are not directly observable. But in our stylized trading model, belief changes lead directly to market orders and are therefore revealed through order flow. Generally, belief changes create a motive for each fund to rebalance its portfolio. Theoretically, such rebalancing could occur through a passive limit order submission strategy only. The fund which desires to sell equity would try to maintain the most competitive ask price and reduce its position successively over time as this sell offer is repeatedly executed. In this case the belief shift would not translate into a corresponding (negative) order flow. In practice, however, fund managers typically pursue more active strategies by directly submitting market sell orders. Active order placement tends to accelerate the portfolio rebalancing and avoids front running by other investors. The belief change is then clearly associated with a corresponding (negative) order flow. Recent empirical work on order execution strategies indeed confirm that the likelihood of a market order increases with an investor’s valuation distance from the spread midpoint (Hollifield et al., 2004). This is captured in our model framework. In round 1 funds react to the belief changes with market orders which result in an aggregate order flow stated in the following proposition:

**Proposition 2: Equity Order Flows**

Belief changes $\boldsymbol{\mu} = (\mu^H_e, \mu^F_e, \mu^H_h, \mu^F_f)$ trigger market orders resulting in equity market order flow $(OF^H, OF^F)$ for the home and foreign, respectively, given by

$$OF^H = k [\mu^H_e + \mu^H_h - \mu^F_e]$$

$$OF^F = k [\mu^F_f + \mu^F_e - \mu^H_e]$$
where the parameters are defined as:

\[
k = \frac{1}{\Delta} \times \frac{\lambda_h \lambda_f \lambda_b}{\rho_e [2\sigma^2 + (1 + r)^2 \sigma_h^2]} > 0
\]

\[
\Delta = \lambda_h \lambda_f \lambda_b + \lambda_h \lambda_f + \lambda_b \lambda_f + \lambda_f \lambda_b
\]

Proof: See Appendix.

Order flow in the home and foreign equity market is proportional to the belief change \(\mu^H, \mu^F\) of the home and foreign fund, respectively. And in each case order flow depends linearly (with opposite signs) on the relative belief change, \(\mu^H - \mu^F\), of the international equity fund. Hence, as for returns, only the relative belief change is identified though the order flow. It may appear surprising that the belief changes of the international and domestic investor carry equal weights given by \(k\) in the measure of total order flow. Intuitively, a less risk averse investor group should submit a larger market order and therefore account for a larger share of the order flow. However, this argument does not account for different price elasticities in the counterparty’s asset supply curve. A less risk averse investor group faces a more price inelastic liquidity supply because of its larger equilibrium price impact. The steeper asset supply curve therefore discourages the more risk neutral investor group to submit larger market orders and implies equal weights for all belief changes. Differences in risk aversion across investor groups are therefore important for explaining return patterns, but not for the structure of order flows as long as the parameter \(k\) is constant.

The international equity fund is also assumed to practice active order placement in the foreign exchange market as a corollary to its rebalancing in the two equity markets. The rebalancing in the two equity markets depends on its own relative belief change \(\mu^H - \mu^F\) as well as belief revision of the local funds \((\mu^H, \mu^F)\) which translate into local equity price changes. The resulting foreign exchange order flow is given in proposition 3:

**Proposition 3: Foreign Exchange Order Flows**

The international equity fund initiates (in round 1) foreign exchange transactions in order to finance overseas equity investments. Their overseas investment is determined by their belief change relative to the local equity investors, therefore foreign exchange order flow \(OF^E\) follows as

\[
OF^E = k[\mu_e^F - \mu_e^H].
\]

Proof: See Appendix.

The theoretical linkage between order flow in round 1 and the belief changes allow us to restate the structural model in proposition 1 in terms of observable variables only. In particular, belief changes can be substituted by equity order flows and we obtained a reduced form structure summarized as follows:
Proposition 4: Reduced Form Structure

The home and foreign equity returns, $R^H$ and $R^F$, are linearly related to the exchange rate return $R^E$ and the home and foreign equity order flows, $OF^H$ and $OF^F$, according to:

$$
R^H = \frac{1}{3} \left[ (1 + \lambda_h) + \lambda_b \left( \frac{\lambda_h - 2\lambda_f}{\lambda_h \lambda_f} \right) \right] R^E + \frac{1}{3k(1+r)} (2OF^H + OF^F) \tag{3}
$$

$$
R^F = \frac{1}{3} \left[ - (1 + \lambda_h) + \lambda_b \left( \frac{2\lambda_h - \lambda_f}{\lambda_h \lambda_f} \right) \right] R^E + \frac{1}{3k(1+r)} (OF^H + 2OF^F) \tag{4}
$$

where $\lambda_h, \lambda_f, \lambda_b > 0$ and $k > 0$ are the previously defined parameters.

Proof: See Appendix.

The reduced form implies that both home and foreign equity returns can be represented as a linear combination of the exchange rate return and both home and foreign equity market order flows. Moreover, local equity returns are more sensitive to local order flow than the order flow in the overseas equity market. The local order flow has a coefficient exactly twice as large as the overseas order flow. An important advantage of the reduced form is that it can be directly estimated. But before we proceed to estimation, we provide an intuitive explanation for the reduced form structure.

Why does overseas order flow help explain the local market equity return? The foreign market order flow partly captures belief shifts of the foreign equity fund. These belief shifts affect the foreign equity price and via substitution effects of the international equity fund also positively influence the home equity price. On the other hand, this substitution effects occur only in round 2 after the initial price effect in the foreign market is revealed. The substitution effect therefore occurs under public information without any particular order flow implication in the home market. Only the foreign order flow captures the initial price impact of the belief innovations of the foreign equity fund and the consecutive substitution effect. Hence the additional explanatory power of the foreign order flow for the home return even after controlling for home market order flow. The same applies symmetrically for foreign equity return.

Next we explain why the exchange rate has further explanatory power for the equity returns. Consider first the possibility that risk aversion towards equities is the same for the home and foreign fund. This means that $\lambda_h = \lambda_f = \lambda$. In this case, the exchange rate coefficient in the home equation is $\frac{1}{3} \left[ 1 + \lambda_b (1 - 1/\lambda) \right]$ while that in the foreign equation is equal but opposite in sign. Deviations from this symmetry occur when equity supply elasticity (governed by the risk aversion) of the domestic equity fund differs across the two countries. Note that the sum of the two exchange rate coefficients is $\lambda_b [1/\lambda_f - 1/\lambda_h]$. If risk aversion for the home equity fund is lower than that of the foreign equity fund, this sum is positive and vice-versa. For $\lambda_h = \lambda_f = \lambda$, we can then express the sum of the two returns as a linear function of the sum of the two equity order flows only,

$$
R^H + R^F = \frac{1}{k(1+r)} [OF^H + OF^F].
$$
Belief changes by the international equity fund creates off-setting negative and positive order flow and return effects which do not affect the sum of order flow and return. However, this pre-supposes that both equity markets have the same asset supply elasticities. Generally, we have

\[ R^H + R^E = \lambda_b \left( \frac{1}{\lambda_f} - \frac{1}{\lambda_h} \right) R^E + \frac{1}{k(1+r)} \left[ OF^H + OF^E \right]. \]

The exchange rate return \( R^E \) now enters as an additional term to explain the sum of equity returns. To develop an intuition consider the case of higher risk aversion for the home than foreign equity fund (or alternatively higher price variance), so that \( \lambda_f > \lambda_h \). Belief changes by the international investor have now a larger price effect in the home than in the foreign equity market because of a relatively steeper home asset supply curve. But as pointed out in propositions 2 and 3, order flows reflect belief changes equally across investor groups of different risk aversion. For a constant parameter \( k \), relative changes in the risk aversion of the home and foreign fund do not alter the order flows. We therefore need the exchange rate return as an additional statistics to fully explain the sum of return changes. Relative optimism of the international investor about the home market, \( \mu^H_e - \mu^F_e > 0 \), implies a negative exchange rate return (dollar appreciation), \( R^E < 0 \). Multiplication with a negative coefficient \( \lambda_b [1/\lambda_f - 1/\lambda_h] \) implies a positive contribution in explaining the sum of the returns. This sum should be larger for \( \mu^H_e - \mu^F_e > 0 \) as the positive return effect of the home inflows is exceeding the negative return effect of the foreign outflows due to the relatively steeper asset supply in the home country.

Our model can therefore explain why the exchange rate should have explanatory power for equity index returns even after controlling for the order flows in both markets. Indeed both exchange rate coefficients in equations (3) and (4) can be negative if the asset supply elasticity of the home equity fund is relatively low. This corresponds to a situation in which home equity investors find it relatively unattractive to substitute between home equity and home bonds. The negative exchange rate coefficients correspond to a negative correlation between index returns and exchange rates conditional on order flows. Whenever the equity order flows are also uncorrelated with the exchange rate (as is actually the case), it follows that the unconditional correlations should also be negative. This corresponds to the evidence presented in Table 1.

Finally, it is interesting to characterize the structure of belief changes implied by the various order flows. As already highlighted, belief changes for the international investor are only identified as the relative belief change \( \mu^H_e - \mu^F_e \). We therefore define a vector of belief innovations as \( \nu = (\mu^H_e - \mu^F_e, \mu^H_h, \mu^F_f)^t \). Using the order flow relationships in propositions 2 and 3, we obtain for the covariance matrix \( \Sigma \) of the belief innovations

\[ \Sigma = \mathcal{E}(\nu \nu^t) = \frac{1}{k^2} \mathcal{C}^{-1} \mathcal{E}(OF^HOF^F) \mathcal{C}^{-1} \]

where \( OF = (OF^H, OF^F, OF^E)^t \) represents the order flow vector, \( k \) an unidentified scaling parameter and
\( C \) an involutory matrix given by

\[
C = \begin{bmatrix}
1 & 1 & 0 \\
-1 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}.
\]

Since, the belief innovations are presented here as a linear combination of the order flows, the time series properties of the order flows carry over to the belief changes. In particular, belief changes should have the same low degree of autocorrelation as order flow innovations. Section 5 discusses the implied structure of the belief innovations.

### 4 Data

An empirical test of the above model would ideally involve many country pairs with developed equity markets. While equity return data is available for almost all countries, the information needed to construct order flow data can only be obtained for a small number of countries. The United States and France are the two largest OECD countries for which individual transaction data on a large part of the domestic equity trading volume is publicly available. We therefore take the U.S. to be the home country and France to be the foreign country. The relevant exchange rate is then the Euro-Dollar rate and we assume that the French equity market is representative of the consolidated euro-zone equity market both in terms of returns and order flow characteristics. Our data spans the five year period from January 1999 to December 2003 and therefore start with the creation of the common European currency.

#### 4.1 U.S. Equity Data

The U.S. order flow data is constructed from the TAQ database with the help of Wharton Research Data Services. We restrict attention to the stocks in the Standard&Poors 100 index and accounted for all their trades on AMEX, NASDAQ and NYSE over the 5 year period, approximately 600 million trades in total. All trades are signed as buyer- or seller-initiated depending on whether the executed price was higher or lower than the midpoint between the ask and bid quote respectively. Trades executed at the mid-point are not signed. The value of all buy trades in all of the 100 stocks in each day are accumulated to create a single aggregate daily buyer-initiated equity trade series for the U.S. Corresponding series are constructed for the seller-initiated and unsigned trades. The raw aggregate home equity order flow series \( ROF_H \) is then derived as the buyer-initiated series minus the seller-initiated series. Trading volume \( VOL_H \) is derived as the sum of the buyer-initiated, seller-initiated and the unsigned trades series. We define the aggregate normalized order flow series as the ratio of order flow to volume \( OF_H \). The home equity returns series \( R_H \) is the first difference of the log of the New York closing value for the S&P100. It was obtained from Datastream.

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11The method used by WRDS restricts itself to quotes that have been in effect for at least five seconds when the trade occurs (see Lee and Ready, 1991).
We also examine the model implications for intraday returns. For this purpose we distinguish the $1\frac{1}{2}$ hour period of parallel equity trading (subscripted $p$) in the U.S. and France from the remaining $22\frac{1}{2}$ hours for which equity trading occurs sequentially. The intraday return and order flow corresponding to the parallel trading period are denoted by $R^H_p$ and $OF^H_p$ respectively.

### 4.2 French Equity Data

French order flow data is constructed based on transaction and quote data from Euronext (Données de Marché Historiques). The reference universe consists of all stocks in the French CAC40 index. Again, we use the Lee and Ready (1991) algorithm to sign trades. Analogously to the U.S. data, we obtain daily raw aggregate equity order flow series $(ROF^F)$, daily volume series $(VOL^F)$ and the daily normalized order flow $(OF^F)$. French aggregate daily equity returns $(R^F)$ were defined as difference in the log of the Paris closing price for the CAC40. The French data accounts for approximately 200 million transactions.

Naturally, the home order flow is denominated in U.S. Dollars and the foreign order flow in Euros. We note that the scale of the U.S. market exceeds that of France by almost an order of magnitude. The use of normalized order flow addresses both issues simultaneously as normalized order flow is strictly bounded between -1 and 1. Normalized order flow is also without currency denomination. Again, we divided the trading day into a period of parallel trading when the U.S. equity market is open and the remainder of the day. Hence, $OF^F_p$ refers to late afternoon order flow in the French market with a corresponding return of $R^F_p$.

### 4.3 Foreign Exchange Data

Daily foreign exchange order flow was obtained directly from Electronic Broking Services (EBS). There are three types of trades in the forex market: customer-dealer trades, direct inter-dealer trades, and brokered inter-dealer trades. Customers are non-financial firms and non-dealers in financial firms (e.g., corporate treasurers, hedge funds, mutual funds, pension funds, proprietary trading desks, etc.). Dealers are market-makers employed in banks worldwide, of which the largest 10 dealing banks account for more than half of the volume in major currencies.

Our data come from the third trade type: brokered inter-dealer trading. There are two main inter-dealer broking systems, EBS and Reuters Dealing 2000-2. Both offer competing central market places through electronic terminals. Estimates by the Bank of England (2001) suggest that electronic brokering was used for 66 percent of all transactions in 2001, up from 30 percent in 1998. Similarly, the Federal Reserve Bank of New York (2001) estimates the market share of electronic trading systems at 71 percent in 2001. Discussions with industry specialists indicate that EBS has a two-thirds market share in the brokered inter-dealer dollar-euro market. Our data set includes the daily value of purchases and sales in the dollar-euro market for first year of our sample, 1999. They are measured in millions of Euros. Unlike the equities data, no algorithm
was needed to sign trades (ex post) since this occurs electronically at the moment of execution. Each trading
day (weekday) covers the 24 hour period starting at 21.00 GMT. The daily raw foreign exchange order
flow series \( ROF^E \) is calculated as the value of buy trades minus the value of sell trades. The daily volume series
\( VOL^E \) represents the sum of the value of buy and sell trades and the daily normalized order flow \( OF^E \)
is again defined as the ratio of \( ROF^E \) to \( VOL^E \). The dollar-euro exchange rate at the New York close was
obtained from Datastream. It is defined as the dollar price of euro. The daily foreign exchange return \( R^E \)
follows as the difference in the log of the exchange rate level. The spot return during the intraday period
of parallel trading is denoted as \( R_{Ep} \).

Table 2 provides descriptive statistics for the variables used in the estimation. Panel A features the daily
variables \( ROF^E, VOL^E, OF^E, ROF^F, VOL^F, OF^F, ROF^H, VOL^H, OF^H \) and the three daily returns,
\( R^F, R^H, R^E \). Panel B details the following variables during the parallel trading period: \( R_{Ep}^F, R_{Ep}^H, R_{Ep}^E, OF_{Ep}^H \)
and \( OF_{Ep}^F \). For each variable, the table shows the mean, the standard deviation and the first order autocorrelation coefficient.

5 Estimation Results

5.1 The Reduced Form for Daily Returns

Unlike the structural form in Proposition 1, the reduced form in Proposition 4 can be directly estimated.
The unobservable belief changes \( \mu = (\mu^H_e, \mu^F_e, \mu^H_f, \mu^F_f) \) are substituted for order flow variables which proxy
for the belief changes. We note, however, that in the system of two equations, the three parameters \( \lambda \)
cannot be separately identified. Moreover, the identification rests on the assumption that the equity funds
undergoing belief changes implement their portfolio change through active order placement strategies. Since
we aggregate over a large number of daily transactions in many different stocks to obtain daily order flow, the
proxy character of order flow for belief changes should still be preserved if a certain proportion of portfolio
change is achieved through passive limit order submission.

Before we estimate the reduced form system, it is instructive to examine how equity returns are affected
by own-order flow. Figures 2 and 3 show scatter diagrams for both the U.S. and France. They show
a strong positive correlation. This means that equity index returns are strongly related to aggregate or
macroeconomic order flow. But our theory asserts much more. Non-local (or overseas) order flow and the
exchange rate return also influence the local equity return. In each equation, the equity return is affected by
both home and foreign market order flow. Moreover, local equity returns are more sensitive to local order
flow than the order flows into the overseas equity market.

Table 3 displays the estimation results. The upper and lower panels show the results for equations (3)
and (4). The first column reports results using Ordinary Least Squares (OLS). The own-order flow is highly
significant in both equations with t-statistics of 27.58 and 16.69 in the home and foreign return equation,
respectively. The overseas order flow is also highly significant in both equations with t-statistics of 9.12
and 8.39, respectively. All four order flow coefficient have the correct sign. As predicted by the theory, the magnitude of the coefficient for order flow into the overseas market is less than for own market order flow. The exchange rate return is also significant in both equations. Since both coefficients are negative, we conclude that $\lambda_f > \lambda_b$. Hence, the counterparty asset supply elasticity of domestic relative to international investors is larger for the French than for the U.S. market.\textsuperscript{12} We also note that the $R^2$ in both equations is very high. We succeed in explaining almost 60% of daily aggregate U.S. stock returns. Ljung-Box Q tests for both equations show that there is no evidence of autocorrelation up to 5th order.

The negative OLS coefficient for the exchange rate in equations (3) and (4) implies a negative correlation between the equity and exchange rate returns conditional on the order flow variables. However, we also find that daily order flows and daily exchange rate returns have a low, but negative correlation. These correlations are $-0.18$ and $-0.16$ for home and foreign market order flow, respectively. Omitting the order flow as a control variable creates a negative bias in the unconditional correlation relative to the conditional correlation. Therefore, negative conditional correlations also imply the negative unconditional correlations as reported in Table 1 for a wide cross section of OECD countries.

An obvious criticism against the OLS estimation concerns the endogeneity of exchange rate returns, which implies a simultaneity bias for the coefficients.\textsuperscript{13} Finding a suitable instrument for an asset price or its return is generally difficult. However, we know from Evans and Lyons (2002a) that foreign exchange order flow is highly correlated with the exchange rate return. This is confirmed for this particular data set in Hau, Killeen and Moore (2002). We can therefore use foreign exchange order flow as an instrument for the 12 months of 1999.\textsuperscript{14} Estimation proceeds by Two Stage Least Squares (2SLS). Columns 3 and 4 give both the OLS and the 2SLS estimates for the year 1999. Comparing the two sets of estimates for equation 1, it is hard to find any difference between the OLS and 2SLS cases for the first year of the sample. This is formally confirmed by the result of the Hausmann specification test in the upper panel. For the foreign returns equation, the only estimate that appears to change is the exchange rate returns coefficient. Nevertheless, it is still negative and significant and again the Hausmann specification test rejects the endogeneity of the exchange rate, at least for this instrument. We conclude that the OLS estimates for the full sample are confirmed by the instrumental variable procedure.\textsuperscript{15}

Next, we examine the intertemporal robustness of the return equations. The last line in each of the panels

\textsuperscript{12}A narrow interpretation within the two country framework would be that the French domestic investor is less risk averse than the U.S. domestic investor. More generally, differences in the short-run equity supply elasticities may also be influenced by microstructure effects. The centralized limit order book in the French market (Euronext) is reputed for its high degree of liquidity, which could explain a relatively lower price effect of international order flows.

\textsuperscript{13}Killeen, Lyons and Moore (forthcoming, 2005) show that order flow is weakly exogenous with respect to exchange rate returns. Secondly, they show that order flow is also strongly exogenous with respect to exchange rate returns. Finally, they show strict exogeneity (exchange rate and order flow innovations are orthogonal).

\textsuperscript{14}The data on foreign exchange order flow is available only for the first year of the sample.

\textsuperscript{15}We also estimated the two equations as a system using Seemingly Unrelated Regressions (SUR). It makes almost no difference to neither the estimated coefficients nor the standard errors.
of Table 3 reports the Chow tests on the parameters between 1999 and the rest of the sample. In the upper panel, the estimated coefficients on both the exchange return and foreign order flow appear to be stable. However, the price impact of home equity order flow is smaller for 1999 than for the whole sample. But it is still large, positive and significant. A formal Chow test confirms the absence of intertemporal instability for the U.S. returns equation. For the foreign returns equation, the exchange rate return coefficient is also stable, but both order flow coefficients are smaller in 1999. The upward trend for the order flow coefficients is confirmed by the rejection of the stability assumption underlying the Chow test. Despite this unfavorable statistical result, it is difficult to argue that the 1999 sub-period displays any economically significant difference relative to the whole sample period.

To evaluate the explanatory power of our parsimonious model specification, we confront the data with the exact parameter restrictions. The own order flow coefficient should be precisely twice the overseas order flow coefficient in both equations. Furthermore, the equivalent coefficients should be the same in both equations. Obviously, the point estimates do not observe these restrictions. A straightforward Wald Test yields a $\chi^2(3)$ test statistic of 397.66. This is a statistically very strong rejection of the parameter restrictions on order flow. But it is natural to ask if model estimation under the exact theoretical parameter restrictions preserves considerable explanatory power for equity returns. In column 2 of Table 3, we show the results of estimating both the home and foreign return equation jointly using Seemingly Unrelated Regressions (SUR). Both the within and cross equation restrictions are now imposed. The explanatory power in the French equation is barely affected by the restrictions, while the $R^2$ for U.S. return falls to just under 47 percent. The restricted order flow coefficients have the theoretically correct sign and remain statistically and economically significant. The exchange rate coefficients are also still negative and statistically significant. It is interesting to note that the sum of the exchange rate coefficients is essentially the same under the restricted and unrestricted model estimation. Hence both procedures produce the same implication for the relationship between risk aversions of domestic investors at home and in the foreign country.

Finally, we calculate the structure of belief innovations $\nu = (\mu^H_e - \mu^F_e, \mu^H_h, \mu^F_f)'$ implied by the order flows. We can estimate the covariance matrix $\Sigma = E(\nu\nu')$ directly from the order flow vector as described in section 3. But since the scaling parameter $k$ of the covariance matrix is not identified, we only report the correlation matrix given by

$$Corr(\nu, \nu') = \begin{bmatrix} 1.00 & -0.24 & 0.37 \\ -0.24 & 1.00 & 0.03 \\ 0.37 & 0.03 & 1.00 \end{bmatrix}.$$ 

The implied belief changes $\mu^H_h$ and $\mu^F_f$ of the representative home and foreign funds are almost uncorrelated with a correlation as low as 0.03. For the correlations with daily belief innovations of the international fund

$^{16}$The 1% critical value is 11.345
$^{17}$Note that these estimates are obtained only for the year 1999, the only year for which we have daily exchange rate order flow data.
with those of the local investors we find $\text{Corr}(\mu^H\mu^E, \mu^H\mu^E) = -0.24$ and $\text{Corr}(\mu^H\mu^E, \mu^F\mu^F) = -0.37$, respectively. Based on our order flow identification, international equity investors seem to be characterized by belief innovations which are contrarian to those of local equity investors. In particular, we have $\text{Corr}(\mu^H\mu^E, \mu^H\mu^E) = -0.44$. This suggests that the international investor has a stabilizing effect on equity prices. Moreover, the near independence of valuation changes by the local investors provides a valuable source of diversification benefits for the international equity funds.

5.2 Intraday Results

Our model is predicated on the idea that all markets are open simultaneously. Though the foreign exchange market is open continuously, the bulk of equity trades are executed during formal opening hours in both France and the U.S. Because of the six hour time difference, parallel equity trading occurs for only $1\frac{1}{2}$ hours of U.S. morning and French afternoon trading. We refer to this interval as parallel trading indexed by $p$.

Defining the intraday periods of parallel trading allows us to estimate equations (3) and (4) again:

$$R^H_p = \frac{1}{3} \left[ (1 + \lambda_b) + \lambda_b \frac{(2\lambda_h - \lambda_f)}{\lambda_h\lambda_f} \right] R^E_p + \frac{1}{3k(1+r)} (2OF^H_p + OF^F_p) $$

$$R^F_p = \frac{1}{3} \left[ -(1 + \lambda_b) + \lambda_b \frac{(2\lambda_h - \lambda_f)}{\lambda_h\lambda_f} \right] R^E_p + \frac{1}{3k(1+r)} (OF^H_p + 2OF^F_p) $$

Equations (5) and (6) explain intraday U.S. and French equity returns for the period in which both equity markets are open. To this intraday period the model applies most directly.

In Table 4, the estimation results for the equations (5) and (6) are reported. We have placed most credence on the unrestricted OLS estimates of Table 3, and repeat these results here for intraday returns. It is straightforward to summarize the overall result. The general picture conveyed by Table 3 remains unchanged. For both home and foreign equity returns, the exchange rate effect is still negative and even higher in absolute value for the French case. The coefficients are always significant. Own and cross order flows remain positive and highly significant in all cases though the coefficients are somewhat reduced in comparison to the daily equation. The only noticeable deviation from the results of Table 3 is that for French equities, the impact of own order flow is less than that of overseas order flow (see equation (6)). The regression $R^2$ are noticeably lower for the U.S. case but is still higher than for France where the explanatory power is essentially unchanged. Again there is no evidence of autocorrelation. Overall, the most striking feature is the robustness of the results.

6 Conclusion

We presented a model in which equity order flow is the expression of heterogeneous belief shifts by different investors. The multimarket setting provides not only for enough observable prices and order flows to identify heterogeneous belief shifts, but it also implies testable restrictions for international market interdependence. We derived a closed-form solution for equity returns in both equity markets, which relates equity returns
to the exchange rate and to order flows in both the local and the overseas market. The model can explain asymmetry across countries in the correlations between domestic equity returns and the exchange rate return conditional on order flows.

We confront the model with 5 years of daily U.S. and French equity data. The respective daily order flows for the S&P100 and the CAC40 index are constructed based on the aggregation of approximately 800 million individual equity transactions. We find that an extraordinarily high percentage of aggregate equity return variation is explained jointly by exchange rate returns and macroeconomic order flows. Our model can explain approximately 60 percent of the daily variation in the S&P100 return and 40 percent of the CAC40 return fluctuations. As predicted by theory, both returns are strongly and positively influenced not only by own market order flow, but also by the order flow in the overseas market. The results are essentially unchanged when estimation is limited to the intraday periods of parallel equity trading in France and the U.S. In summary, heterogeneous belief changes as identified by order flows provide a promising paradigm for future research on equity index movements, exchange rates and international financial market interdependence.
References


Appendix

Proposition 1: Returns and Heterogenous Beliefs

The four investment funds \( i = c, b, h, f \) pursue investment objectives which maximize a CARA objective function. For normally distributed payoff this simplifies to a utility function in conditional mean and conditional variance of the payoff. In general, for price changes \( \Delta P_1 \) and \( \Delta P_2 \) in two assets and corresponding asset holdings \( x_1 \) and \( x_2 \) the payoff is given by \( \Delta \Pi = x_1 \Delta P_1 + x_2 \Delta P_2 \). For a budget constraint \( x_1 P_1 + x_2 P_2 = 0 \), we obtain \( \Delta \Pi = x_1 \left[ \Delta P_1 - \frac{P_1}{P_2} \Delta P_2 \right] \) and the optimal asset demand can be stated as

\[
x_1 = \frac{\mathcal{E}(\Delta P_1 - \frac{P_1}{P_2} \Delta P_2 | I)}{\rho V ar \left( \Delta P_1 - \frac{P_1}{P_2} \Delta P_2 | I \right)}
\]

where \( \mathcal{E}(\cdot | I) \) denotes conditional expectation, \( V ar(\cdot | I) \) the conditional variance and \( \rho \) the coefficient of absolute risk aversion.

We solve the model by first considering the full information price equilibrium which arises in round 2. At this stage, the belief changes of all funds are public information and this allows for a straightforward calculation of the equilibrium price. Let \( V^H, V^F \) and \( V^E \) denote the liquidation value of the home equity, the foreign equity and the foreign currency respectively. The corresponding equilibrium prices are \( P^H, P^F \) and \( E \). For the home and foreign equity fund, the second asset is a bond with a unit price \( (P_2 = 1) \) and a return \( \Delta P_2 = r \). Hence, their optimal asset demands are given by

\[
x^H_b = \frac{\mathcal{E}_f \left[ V^H - P^H - P^H \rho^H | I \right]}{\rho_b V ar_b (V^H | I)} = \frac{\bar{V}^H + \rho^H - (1 + r^H)P^H}{\rho_b \sigma^2} \quad (A1)
\]

\[
x^F_f = \frac{\mathcal{E}_f \left[ V^F - P^F - P^F \rho^F | I \right]}{\rho_f V ar_f (V^F | I)} = \frac{\bar{V}^F + \rho^F - (1 + r^F)P^F}{\rho_f \sigma^2} \quad (A2)
\]

For the international equity fund the payoff is given by

\[
\Delta \Pi_c = x_1 \left[ \Delta P_1 - \frac{P_1}{P_2} \Delta P_2 \right] = x^H_c \left[ V^H - P^H - \frac{P^H}{E P^F} (V^E V^F - E P^F) \right].
\]

For steady state values \( \bar{V}^H, \bar{P}^H, \bar{V}^E, E, \bar{V}^F, \bar{P}^F \), we can linearize the excess return on home equity as

\[
V^H - P^H - \frac{P^H}{E P^F} (V^E V^F - E P^F) = \nu^H - P^H - \frac{P^H}{E P^F} (V^E V^F - E P^F) + \nu^H \frac{P^H}{E} \frac{V^F}{P^F} (V^E - E) + \nu^H \frac{V^F}{E} \frac{P^H}{P^F} (E - V^E) - \nu^H \frac{V^F}{E} \frac{P^H}{P^F} (V^F - V^E) =
\]

\[
(V^H - \bar{V}^H) - (P^H - \bar{P}^H) - \nu^H \frac{V^F}{P^F} (P^H - \bar{P}^H) + \nu^H \frac{V^F}{P^F} (E - \bar{E}) - \nu^H \frac{V^F}{P^F} (V^E - \bar{V}^E) - \nu^H \frac{V^F}{E} \frac{P^H}{P^F} (V^F - \bar{V}^F) =
\]

\[
(V^H - \bar{V}^H) - \nu^H (P^H - \bar{P}^H) + \nu^H (P^F - \bar{P}^F) + \nu^H (E - 1) - \nu^H (V^E - 1) - (V^F - \bar{V}^F),
\]
The final equality is obtained (for reasons of model symmetry\textsuperscript{18}) by using $V^H = V^F$, $\bar{V}^E = E = 1$, $P^H = P^F$.

For the international equity fund the optimal asset demand follows as

$$x^H_e = \frac{\xi_i [(V^H - \bar{V}^H) - (V^F - \bar{V}^F) - \frac{\sigma}{P^F}(P^H - \bar{P}^H) + \frac{\sigma}{P^F}(P^F - \bar{P}^F) + \bar{V}^F (E - 1) - \bar{V}^F (V^E - 1) \mid I]}{\rho \cdot \text{Var}_{\xi}((V^H - \bar{V}^H) - (V^F - \bar{V}^F) (V^E - 1) - (V^F - \bar{V}^F) \mid I)}$$

$$= \frac{\mu^H_e - \mu^F_e - \frac{\sigma}{P^F}(P^H - \bar{P}^H) + \frac{\sigma}{P^F}(P^F - \bar{P}^F) + \bar{V}^F (E - 1)}{\rho \cdot \text{Var}_{\xi}((V^H - \bar{V}^H) - (V^F - \bar{V}^F) \mid I)}$$

(A3)

For the international bond fund, the payoff is given by

$$\Delta \Pi_b = x_b^H \left[ r^H - \frac{1}{E}(V^E (1 + r^F) - E) \right] \approx x_b^H (1 + r)(E - V^E),$$

where bond prices are normalized to 1 and $r^H = r^F = r$. The optimal demand of the international bond investor follows as

$$x_b^{H*} = \frac{\xi_b [(1 + r)(E - V^E) \mid I]}{\rho_b \cdot \text{Var}_{\xi}((1 + r)(E - V^E) \mid I)} = \frac{E - 1}{\rho_b (1 + r) \bar{\sigma}^2}.$$  \hspace{1cm} (A4)

The steady state (marked by upper bars) follows for $\mu_e^H = \mu_e^F = \mu_b^H = \mu_b^F = 0$. Then market clearing implies

$$0 = \frac{-\bar{V}^F (E - 1)}{\rho \cdot \text{Var}_{\xi}((V^H - \bar{V}^H) - (V^F - \bar{V}^F) \mid I)} + \frac{\bar{V}^H - (1 + r) \bar{P}^H}{\rho \cdot \text{Var}_{\xi}((V^H - \bar{V}^H) - (V^F - \bar{V}^F) \mid I)}.$$  

Inspection of the above equation reveals that $\bar{V}^H = (1 + r) \bar{P}^H$, $\bar{V}^F = (1 + r) \bar{P}^F$ and $E = 1$ is a solution. This implies $\pi^H_e = \pi^F_e = 0$ and from the budget constraint of the international equity fund, $x_e^H \bar{P}^H + x_e^F \bar{E} \bar{P}^F = 0$, we get $\pi^H_e = \pi^F_e = 0$. Therefore $\bar{V}^F = (1 + r) \bar{P}^F$. We can further simplify by setting $\bar{P}^H = \bar{P}^F = 1$. Finally, we can also derive a simple expression for the international equity fund’s foreign equity demand, $x^F_e$. From its budget constraint, we get through linearization around the steady state holdings $(\pi^H_e = \pi^F_e = 0)$ directly

$$x_e^F = -x_e^H \frac{\bar{P}^H}{\bar{E} \bar{P}^F} = -x_e^H.$$

(A5)

Next we solve for the equilibrium prices $P^H$, $P^F$ and $E$ under general belief changes $\mu = (\mu^H_e, \mu^F_e, \mu^H_b, \mu^F_b)$. Market clearing in the two equity markets implies, (using 1, A1, A2, A3 and A5)

\textsuperscript{18}There is one asymmetry in the model. This is the choice of currency in which the international fund is denominated. However, this has a second order impact on the model. We abstract from this.
Market clearing in the currency market occurs between the two international funds and yields, (using 2, A3 and A5)

\[ 0 = x_e^H + x_b^B = \frac{\mu_e^H - \mu_e^F - (1 + r)(P^H - 1) + (1 + r)(P^F - 1) + (1 + r)(E - 1)}{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma_E^2 \right]} + V^H + \frac{\mu_h^H - (1 + r)P^H}{\rho_h \sigma^2} \]

\[ 0 = x_i^F + x_f^F = \frac{-\mu_e^H - \mu_e^F - (1 + r)(P^H - 1) + (1 + r)(P^F - 1) + (1 + r)(E - 1)}{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma_E^2 \right]} + V^F + \frac{\mu_f^F - (1 + r)P^F}{\rho_f \sigma^2} \]

It is straightforward to show that in the absence of any belief changes (\( \mu_e^H = \mu_e^F = \mu_h^H = \mu_f^F = 0 \)), the equilibrium price is given by the unit vector, \( P_0 = (P_0^H, P_0^F, E_0) = (1, 1, 1) \). Note that these represent the optimal price quotes at the beginning of round 1. In round 2, however, the equilibrium price depends on the realized belief changes.

We define strictly positive parameters

\[
\lambda_h = \frac{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma_E^2 \right]}{\rho_h \sigma^2}, \quad \lambda_f = \frac{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma_E^2 \right]}{\rho_f \sigma^2}, \quad \lambda_b = \frac{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma_E^2 \right]}{\rho_b (1 + r)^2 \sigma_E^2}.
\]

For asset returns \( R^H = P^H - 1, R^F = P^F - 1 \) and \( R^E = E - 1 \) between the initial state and the full information price equilibrium of round 2, we obtain a linear system of three equations given by

\[
\begin{pmatrix}
(1 + \lambda_h) & -1 & -1 \\
-1 & (1 + \lambda_f) & 1 \\
-1 & 1 & (1 + \lambda_b)
\end{pmatrix}
\begin{pmatrix}
R^H \\
R^F \\
R^E
\end{pmatrix} = \frac{1}{1 + r}
\begin{pmatrix}
\mu_e^H - \mu_e^F + \mu_h^H \lambda_h \\
\mu_f^F - \mu_e^H + \mu_f^F \lambda_f \\
\mu_e^H - \mu_e^F
\end{pmatrix}.
\]

The return vector \( \mathbf{R} = (R^H, R^F, R^E) \) can then be expressed linearly in terms of belief changes \( \bm{\mu} = (\mu_e^H, \mu_e^F, \mu_h^H, \mu_f^F) \) as \( \mathbf{R} = \mathbf{A}^{-1} \mathbf{B} \bm{\mu} \). This proves proposition 1.

**Proposition 2: Equity Order Flow**

At the end of round 1, funds place market orders to acquire positions in accordance with their belief changes. The liquidity supply of their counterparty is price elastic and takes into account the equilibrium price effect of these demand change. The residual liquidity supply elasticity in turn conditions the optimal size of the fund’s market order. We denote the execution price of these market orders in terms of return functions \( R^H(\cdot), R^F(\cdot), \) and \( R^E(\cdot) \). The execution price depends only on the size of a fund’s market. The combined order flow in the home equity market is given as the sum \( OF^H = OF_h^H + OF_e^H \) of the order flow
by the home and international fund. Similarly we have \( OF^F = OF^F_f + OF^F_e \). The respective order flows follow as

\[
OF^H_h = \frac{\mu^H_h - (1 + r)R^H \langle \mu^H_e \rangle}{\rho_e \sigma^2} = x^H, \text{ for } \mu^F = \mu^H_e = \mu^F_e = 0 \text{ (from A1)}
\]

\[
OF^F_f = \frac{\mu^F_h - (1 + r)R^F \langle \mu^F_e \rangle}{\rho_f \sigma^2} = x^F, \text{ for } \mu^H = \mu^H_e = \mu^F_e = 0 \text{ (from A2)}
\]

\[
OF^H_e = \frac{\mu^H_e - \mu^F_e - (1 + r) (R^H \langle \mu^H_e - \mu^F_e \rangle) + (1 + r) (R^F \langle \mu^H_e - \mu^F_e \rangle) + (1 + r)R^E \langle \mu^H_e - \mu^F_e \rangle}{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma^2_e \right]} = x^H, \text{ for } \mu^H = \mu^F_e = 0 \text{ (from A3)}
\]

\[
OF^F_e = \frac{\mu^H_e - \mu^F_e - (1 + r) (R^H \langle \mu^H_e - \mu^F_e \rangle) + (1 + r) (R^F \langle \mu^H_e - \mu^F_e \rangle) + (1 + r)R^E \langle \mu^H_e - \mu^F_e \rangle}{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma^2_e \right]} = x^F, \text{ for } \mu^H = \mu^F_e = 0 \text{ (from A5)}
\]

The equilibrium returns are given by

\[
R^H = \frac{\lambda_f \lambda_b}{(1 + r) \Delta} (\mu^H_e - \mu^F_e) + \frac{\lambda_h \{(1 + \lambda_f)(1 + \lambda_b) - 1\}}{(1 + \lambda_f) \Delta} \mu^H_e + \frac{\lambda_b \lambda_f}{(1 + \lambda_f) \Delta} \mu^F_e
\]

\[
R^F = \frac{-\lambda_b \lambda_b}{(1 + r) \Delta} (\mu^H_e - \mu^F_e) + \frac{\lambda_b \lambda_h}{(1 + r) \Delta} \mu^H_e + \frac{\lambda_f \{(1 + \lambda_b)(1 + \lambda_f) - 1\}}{(1 + \lambda_f) \Delta} \mu^F_e
\]

\[
R^E = \frac{-\lambda_f \lambda_b}{(1 + r) \Delta} (\mu^H_e - \mu^F_e) + \frac{\lambda_b \lambda_f}{(1 + r) \Delta} (\mu^H_e - \mu^F_e)
\]

and execution prices expressed as returns relative to the steady state are given by

\[
R^H \langle \mu^H_e \rangle = \frac{\lambda_h \{(1 + \lambda_f)(1 + \lambda_b) - 1\}}{(1 + r) \Delta} \mu^H_e = R^H, \text{ for } \mu^F = \mu^H_e = \mu^F_e = 0
\]

\[
R^F \langle \mu^F_e \rangle = \frac{\lambda_f \{(1 + \lambda_b)(1 + \lambda_f) - 1\}}{(1 + r) \Delta} \mu^F_e = R^F, \text{ for } \mu^H = \mu^F_e = \mu^H_e = 0
\]

\[
R^H \langle \mu^H_e - \mu^F_e \rangle = \frac{\lambda_f \lambda_b}{(1 + r) \Delta} (\mu^H_e - \mu^F_e) = R^H, \text{ for } \mu^H = \mu^F_e = 0
\]

\[
R^F \langle \mu^H_e - \mu^F_e \rangle = \frac{-\lambda_b \lambda_b}{(1 + r) \Delta} (\mu^H_e - \mu^F_e) = R^F, \text{ for } \mu^H = \mu^F_e = 0
\]

We obtain the equity order flows (demands at the end of round 1) as

\[
OF^H = k [\mu^H + \mu^H_e - \mu^F_e]
\]

\[
OF^F = k [\mu^H_f + \mu^F_e - \mu^H_e]
\]

where we defined

\[
k = \frac{1}{\Delta} \times \frac{\lambda_h \lambda_f \lambda_b}{\rho_e \left[ 2\sigma^2 + (1 + r)^2 \sigma^2_e \right]} > 0
\]

\[
\Delta = \lambda_h \lambda_f \lambda_b + \lambda_h \lambda_f + \lambda_h \lambda_b + \lambda_f \lambda_b.
\]
Proposition 3: FX Order Flow

The international bond fund is the liquidity provider in the FX market. It does not experience any belief changes. Order flow in the FX market, \( OF^E \), therefore arises only from the market orders of the international equity fund. Hence,

\[
OF^E = k[\mu_e^F - \mu_e^H].
\]

Proposition 4: Reduced Form Structure

The above system of equations (from proposition 1) can be rewritten as

\[
\begin{bmatrix}
(1 + \lambda_h)\lambda_f & -\lambda_f & -\lambda_f \\
-\lambda_h & (1 + \lambda_f)\lambda_h & \lambda_h \\
-1 & 1 & (1 + \lambda_b)
\end{bmatrix}
\begin{bmatrix}
R^H \\
R^F \\
R^E
\end{bmatrix}
= \frac{1}{1 + r}
\begin{bmatrix}
(\mu_e^H - \mu_e^F)\lambda_f + \mu_e^H\lambda_h\lambda_f \\
(\mu_e^F - \mu_e^H)\lambda_h + \mu_e^F\lambda_f\lambda_h \\
\mu_e^F - \mu_e^H
\end{bmatrix}.
\]

Adding the first two equations yields

\[
\begin{bmatrix}
(1 + \lambda_h)\lambda_f - \lambda_h & (1 + \lambda_f)\lambda_h - \lambda_f & \lambda_h - \lambda_f \\
-1 & 1 & (1 + \lambda_b)
\end{bmatrix}
\begin{bmatrix}
R^H \\
R^F \\
R^E
\end{bmatrix}
= \frac{1}{1 + r}
\begin{bmatrix}
(\mu_e^H - \mu_e^F)(\lambda_f - \lambda_h) + (\mu_e^H + \mu_e^F)\lambda_h\lambda_f \\
\mu_e^F - \mu_e^H
\end{bmatrix}.
\]

and adding \( (\lambda_f - \lambda_h) \) times the last equation gives

\[
\begin{bmatrix}
\lambda_h\lambda_f & \lambda_f\lambda_h & (\lambda_f - \lambda_h)\lambda_b \\
-1 & 1 & (1 + \lambda_b)
\end{bmatrix}
\begin{bmatrix}
R^H \\
R^F \\
R^E
\end{bmatrix}
= \frac{1}{1 + r}
\begin{bmatrix}
(\mu_e^H + \mu_e^F)\lambda_h\lambda_f \\
\mu_e^F - \mu_e^H
\end{bmatrix}.
\]

Finally, dividing the first equation by \( \lambda_h\lambda_f \), we obtain

\[
\begin{bmatrix}
1 & 1 & \frac{(\lambda_f - \lambda_h)\lambda_b}{\lambda_h\lambda_f} \\
1 & -1 & - (1 + \lambda_b)
\end{bmatrix}
\begin{bmatrix}
R^H \\
R^F \\
R^E
\end{bmatrix}
= \frac{1}{1 + r}
\begin{bmatrix}
\mu_h^H + \mu_f^F \\
\mu_e^H - \mu_e^F
\end{bmatrix}.
\]

The order flow definitions allow us to rewrite

\[
\begin{align*}
OF^H + OF^F &= k \left[ \mu_h^H + \mu_f^F \right] \\
OF^H - OF^F &= k \left[ (\mu_h^H - \mu_f^F) + 2(\mu_e^H - \mu_e^F) \right].
\end{align*}
\]

Note that the exchange rate return can be expressed as

\[
R^E = \frac{-\lambda_f\lambda_h}{(1 + r)} \left[ \mu_e^H - \mu_e^F \right] + \frac{\lambda_h\lambda_f}{(1 + r)} \left[ \mu_h^H - \mu_f^F \right].
\]

25
or

\[
\frac{1}{\rho_b(1 + r)\sigma_E^2} R^{E} = -k(\mu_e^H - \mu_e^F) + k(\mu_h^H - \mu_f^F) \\
= -3k(\mu_e^H - \mu_e^F) + (OF^H - OF^F) \\
= 3OF^E + (OF^H - OF^F)
\]

or

\[
\mu_e^H - \mu_e^F = \frac{1}{3k} (OF^H - OF^F) - \frac{1}{3k(1 + r)\rho_b\sigma_E^2} R^{E}.
\]

Substitution then implies

\[
\begin{bmatrix}
1 & 1 & \lambda_e (\lambda_f - \lambda_h) \\
1 & -1 & -1 (1 + \lambda_b)
\end{bmatrix}
\begin{bmatrix}
R^H \\
R^F \\
R^E
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{3k} (OF^H + OF^F) \\
\frac{1}{3k(1 + r)} (OF^H - OF^F) - \frac{1}{3k\rho_b(1 + r)\sigma_E^2} R^{E}
\end{bmatrix}
\]

Adding the two equations implies the following expression for home returns

\[
R^H = \frac{1}{3} \left[ (1 + \lambda_b) + \lambda_b \frac{(\lambda_h - 2\lambda_f)}{\lambda_h \lambda_f}\right] R^E + \frac{1}{3k(1 + r)} (2OF^H + OF^F)
\]

and subtracting gives

\[
R^F = \frac{1}{3} \left[ -(1 + \lambda_b) + \lambda_b \frac{(2\lambda_h - \lambda_f)}{\lambda_h \lambda_f}\right] R^E + \frac{1}{3k(1 + r)} (OF^H + 2OF^F)
\]

where we used

\[
(1 + \lambda_b) - \frac{1}{3k\rho_b(1 + r)^2 \sigma_E^2} = (1 + \lambda_b) - \frac{1}{3} \Delta \frac{(\lambda_h + \lambda_f)}{\lambda_h \lambda_f}
\]

\[
= (1 + \lambda_b) - \frac{1}{3} \left[ (1 + \lambda_e) + \lambda_e \frac{(\lambda_h + \lambda_f)}{\lambda_h \lambda_f}\right]
\]

\[
= \frac{2}{3} (1 + \lambda_b) - \frac{1}{3} \lambda_e \frac{(\lambda_h + \lambda_f)}{\lambda_h \lambda_f}.
\]
Table 1: Asymmetry of Exchange Rate Correlations with Stock Indices

Reported are correlations of daily bilateral dollar (log) exchange rate returns, $R^E$, with (1) the US stock index return, $R^H$, and (2) the foreign country stock index return, $R^F$ (in local currency), for 17 OECD countries for the pre-Euro period 01/01/1992 to 31/12/1998 (Panel A) and the post-Euro period 01/01/1999 to 31/06/2006 (Panel B). We report a non-parametric z-test based on Kendall rank correlations for the null hypothesis that the correlation is zero.

### Panel A: Pre-Euro Period 01/01/1992 to 31/12/1998

<table>
<thead>
<tr>
<th>Foreign Country</th>
<th>(1) US Stock Index</th>
<th>(2) Foreign Stock Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Corr } [R^H, R^E]$</td>
<td>$Z$-Test</td>
</tr>
<tr>
<td>Australia</td>
<td>0.014</td>
<td>-0.159</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.127</td>
<td>-6.148</td>
</tr>
<tr>
<td>Belgium-Lux</td>
<td>-0.152</td>
<td>-6.400</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.155</td>
<td>-6.922</td>
</tr>
<tr>
<td>Finland</td>
<td>-0.111</td>
<td>-5.329</td>
</tr>
<tr>
<td>France</td>
<td>-0.162</td>
<td>-6.898</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.165</td>
<td>-7.044</td>
</tr>
<tr>
<td>Ireland</td>
<td>-0.131</td>
<td>-5.855</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.129</td>
<td>-5.416</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.047</td>
<td>-2.878</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.161</td>
<td>-7.050</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.133</td>
<td>-5.650</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.122</td>
<td>-6.349</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.129</td>
<td>-5.624</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.077</td>
<td>-3.000</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.17</td>
<td>-7.254</td>
</tr>
<tr>
<td>UK</td>
<td>-0.11</td>
<td>-4.461</td>
</tr>
<tr>
<td>Average</td>
<td>-0.122</td>
<td>-0.121</td>
</tr>
</tbody>
</table>

### Panel B: Post-Euro Period 01/01/1999 to 31/06/2006

<table>
<thead>
<tr>
<th>Foreign Country</th>
<th>(1) US Stock Index</th>
<th>(1) Foreign Stock Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Corr } [R^H, R^E]$</td>
<td>$Z$-Test</td>
</tr>
<tr>
<td>Australia</td>
<td>0.067</td>
<td>1.753</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.190</td>
<td>-7.427</td>
</tr>
<tr>
<td>Belgium-Lux</td>
<td>-0.009</td>
<td>-7.372</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.183</td>
<td>-7.044</td>
</tr>
<tr>
<td>Finland</td>
<td>-0.192</td>
<td>-7.482</td>
</tr>
<tr>
<td>France</td>
<td>-0.192</td>
<td>-7.558</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.191</td>
<td>-7.457</td>
</tr>
<tr>
<td>Ireland</td>
<td>-0.192</td>
<td>-7.502</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.187</td>
<td>-7.458</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.065</td>
<td>-2.531</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.186</td>
<td>-7.340</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.129</td>
<td>-5.570</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.084</td>
<td>-7.421</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.190</td>
<td>-7.405</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.033</td>
<td>-1.583</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.241</td>
<td>-9.247</td>
</tr>
<tr>
<td>UK</td>
<td>-0.126</td>
<td>-5.836</td>
</tr>
<tr>
<td>Average</td>
<td>-0.139</td>
<td>-0.089</td>
</tr>
</tbody>
</table>
Table 2: Investment Opportunities

Represented are the investment opportunities for the four funds $i = e, b, h, f$ in the four markets (Yes/No) and the notation for their respective belief innovation in the equity market.

<table>
<thead>
<tr>
<th>Equity Markets</th>
<th>International Funds</th>
<th>Domestic Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Home country</strong></td>
<td>Yes</td>
<td>$\mu^H_e$</td>
</tr>
<tr>
<td><strong>Foreign country</strong></td>
<td>Yes</td>
<td>$\mu^F_e$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bond Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Foreign country</strong></td>
</tr>
<tr>
<td><strong>Home country</strong></td>
</tr>
</tbody>
</table>
Table 3: Summary Statistics

For the five year period 01/1999 to 12/2003 we report for the U.S. \((H)\) and French \((F)\) equity market, as well as dollar/euro foreign exchange market \((E)\) the mean, standard deviation \(\text{S.D.}\), and first-order auto-correlation \(\text{AR}(1)\) of the stock market returns, \(R^H\) (S&P100) and \(R^F\) (CAC40), the dollar-euro exchange rate return \((R^E)\), the raw daily order flows \((ROF)\), daily trade volume \((VOL)\), and the normalized order flow \((OF)\) defined as the ratio of raw order flow and volume. The daily exchange rate order flow is available only for 12 months from 01/1999 to 12/1999. Panel A reports these summary statistics for a daily sampling frequency and panel B reports the intraday statistics. We identify the \(1\frac{1}{2}\) hours of parallel trading \((p)\) when both the U.S. and French equity operate simultaneously.

### Panel A: Daily Data

<table>
<thead>
<tr>
<th>Daily Volume</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
<th>Daily Order Flow</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(VOL^H) $ millions</td>
<td>23206</td>
<td>7591</td>
<td>0.77</td>
<td>(ROF^H) $ millions</td>
<td>1412</td>
<td>1075</td>
<td>0.16</td>
</tr>
<tr>
<td>(VOL^F) € millions</td>
<td>3153</td>
<td>1101</td>
<td>0.51</td>
<td>(ROF^F) € millions</td>
<td>37</td>
<td>344</td>
<td>0.17</td>
</tr>
<tr>
<td>(VOL^E) € millions</td>
<td>37217</td>
<td>11527</td>
<td>0.45</td>
<td>(ROF^E) € millions</td>
<td>519</td>
<td>1103</td>
<td>0.26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Daily Returns</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
<th>Daily Order Flow</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^H) %</td>
<td>-0.007</td>
<td>1.40</td>
<td>-0.03</td>
<td>(OF^H) %</td>
<td>0.06</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>(R^F) %</td>
<td>-0.008</td>
<td>1.66</td>
<td>0.002</td>
<td>(OF^F) %</td>
<td>0.01</td>
<td>0.10</td>
<td>0.18</td>
</tr>
<tr>
<td>(R^E) %</td>
<td>0.005</td>
<td>0.67</td>
<td>-0.07</td>
<td>(OF^E) %</td>
<td>0.01</td>
<td>0.03</td>
<td>0.23</td>
</tr>
</tbody>
</table>

### Panel B: Intraday Data

<table>
<thead>
<tr>
<th>Intraday Returns</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
<th>Intraday Order Flows</th>
<th>Mean</th>
<th>S.D.</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^H_p) %</td>
<td>-0.007</td>
<td>0.90</td>
<td>0.009</td>
<td>(OF^H_p) %</td>
<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>(R^F_p) %</td>
<td>-0.033</td>
<td>0.72</td>
<td>-0.012</td>
<td>(OF^F_p) %</td>
<td>0.02</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>(R^E_p) %</td>
<td>0.009</td>
<td>0.27</td>
<td>-0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Reduced Form Estimates for Daily Data

U.S. and French stock returns, \( R^H \) (S&P100) and \( R^F \) (CAC40), respectively, are each regressed on the daily dollar-euro exchange rate return \( R^E \) as well as daily U.S. \( (OF^H) \) and French \( (OF^F) \) equity order flow. The equations are estimated using ordinary least squares for the whole sample (OLS, 01/1999-12/2003), ordinary least squares for 1999 only (OLS, 01/1999-12/1999), and two stage least squares for 1999 using foreign exchange order flow as an instrument for exchange rate returns (2SLS, 01/1999-12/1999). We also report a seemingly unrelated regressions (SUR (Restricted), 01/1999-12/2003) which imposes the exact parameter restrictions. T-tests in parentheses use Newey West robust standard errors.

Panel A: U.S. Equity Returns

\[
R^H = \frac{1}{3} \left[ (1 + \lambda_b) + \lambda_b \left( \frac{\lambda_h - 2\lambda_f}{\lambda_h\lambda_f} \right) \right] R^E + \frac{1}{3k(1+r)} (2OF^H + OF^F)
\]

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>SUR (Restricted)</th>
<th>OLS</th>
<th>2SLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^E )</td>
<td>-0.22 (4.61)</td>
<td>-0.27 (6.04)</td>
<td>-0.22 (2.71)</td>
<td>-0.23 (1.71)</td>
</tr>
<tr>
<td>( OF^H )</td>
<td>21.50 (27.58)</td>
<td>11.58 (40.63)</td>
<td>16.81 (17.75)</td>
<td>16.77 (15.95)</td>
</tr>
<tr>
<td>( OF^F )</td>
<td>2.49 (9.12)</td>
<td>5.79 (40.63)</td>
<td>2.47 (4.21)</td>
<td>2.45 (4.02)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>58.9%</td>
<td>46.5%</td>
<td>63.0%</td>
<td>62.8%</td>
</tr>
<tr>
<td>Q(5)</td>
<td>1.66</td>
<td>1.66</td>
<td>4.30</td>
<td>4.27</td>
</tr>
<tr>
<td>Hausm. Test</td>
<td>( \chi^2(4) = 0.05 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chow Test</td>
<td>( \chi^2(3) = 4.39 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: French Equity Returns

\[
R^F = \frac{1}{3} \left[ - (1 + \lambda_b) + \lambda_b \left( \frac{2\lambda_h - \lambda_f}{\lambda_h\lambda_f} \right) \right] R^E + \frac{1}{3k(1+r)} (OF^H + 2OF^F)
\]

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>SUR (Restricted)</th>
<th>OLS</th>
<th>2SLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^E )</td>
<td>-0.17 (2.89)</td>
<td>-0.13 (2.34)</td>
<td>-0.20 (2.12)</td>
<td>-0.38 (2.33)</td>
</tr>
<tr>
<td>( OF^H )</td>
<td>7.32 (8.39)</td>
<td>5.79 (40.63)</td>
<td>3.86 (3.34)</td>
<td>3.08 (2.48)</td>
</tr>
<tr>
<td>( OF^F )</td>
<td>9.19 (16.69)</td>
<td>11.58 (40.63)</td>
<td>6.86 (7.58)</td>
<td>6.61 (7.28)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>40.1%</td>
<td>38.2%</td>
<td>46.4%</td>
<td>44.5%</td>
</tr>
<tr>
<td>Q(5)</td>
<td>3.58</td>
<td>3.58</td>
<td>5.81</td>
<td>4.64</td>
</tr>
<tr>
<td>Hausm. Test</td>
<td>( \chi^2(2) = 3.79 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chow Test</td>
<td>( \chi^2(3) = 22.62 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

30
Equations (5) and (6) explain intraday U.S. and French equity returns for the $1\frac{1}{2}$ hour period in which both equity markets are open in parallel ($p$). The equations are estimated using ordinary least squares (OLS) for the whole sample. T-tests in parentheses use Newey West robust standard errors.

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Equation (5)</th>
<th>Equation (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>$R_p^H$</td>
<td>$R_p^F$</td>
</tr>
<tr>
<td>01/1999-12/2003</td>
<td>$-0.18$</td>
<td>$(2.06)$</td>
</tr>
<tr>
<td></td>
<td>$-0.73$</td>
<td>$(9.02)$</td>
</tr>
<tr>
<td>$OF_p^H$</td>
<td>8.23</td>
<td>$(13.84)$</td>
</tr>
<tr>
<td></td>
<td>3.38</td>
<td>$(9.33)$</td>
</tr>
<tr>
<td>$OF_p^F$</td>
<td>1.84</td>
<td>$(9.45)$</td>
</tr>
<tr>
<td></td>
<td>1.86</td>
<td>$(10.36)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>42.2%</td>
<td>38.0%</td>
</tr>
<tr>
<td>$Q(5)$</td>
<td>3.57</td>
<td>6.03</td>
</tr>
</tbody>
</table>
Figure 2: Plotted are daily returns in the S&P100 index against normalized daily order flow into the U.S. equity market for the five year period 1999 to 2003.
Figure 3: Plotted are daily returns in the CAC40 index against normalized daily order flow into the French equity market for the five year period 1999 to 2003.
Figure 4: Plotted are daily returns in the Dollar/Euro exchange rate against normalized daily order flow in the EBS trading system for the year 1999.