GLS-based unit root tests with multiple structural breaks both under the null and the alternative hypotheses

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Introduction

- Non-stationarity in variance analysis that accounts for the presence of structural breaks has devoted a great interest in time series analysis
  1. Misspecification of deterministic function of the auxiliary regression that is used for either testing the null hypothesis of unit root or variance stationarity can lead to conclude in favour of variance non-stationarity
  2. This implied the design of test statistics that can accommodate the presence of structural breaks
  3. Earlier proposals were designed to account for one structural break, which could affect either the level and/or the slope of the deterministic time trend

Introduction

Allowance of one structural break would not be enough in all situations, and there are extensions in the literature to consider more than one break:

- Two structural breaks
    1. Garcia and Perron (1996) propose a pseudo F statistic that accounts for two structural breaks
    2. Lee (1996), and Lumsdaine and Papell (1997) – trending variables – and Carrion-i-Silvestre et al. (2004) – non-trending variables – generalize the approach in Zivot and Andrews (1992) to consider two structural breaks
    3. Clemente et al. (1998) extend the test in Perron and Vogelsang (1992): – non-trending variables allowing for two structural breaks
    4. Lee and Strazicich (2003) extend the statistic in Schmidt and Phillips (1992) to allow for two structural breaks both under the null and the alternative hypotheses, which can affect only the level, or both the level and the slope

Introduction

- Multiple structural breaks

  1. Ohara (1999) and Kapetanios (2005) generalize the approach in Zivot and Andrews (1992) through the consideration of multiple structural breaks using the DF statistic
  2. Gadea et al. (2004) design a pseudo F statistic to account for multiple level shifts for non-trending variables
  3. Bai and Carrion-i-Silvestre (2004) consider the square of the MSB statistic with multiple breaks affecting either the level and the slope of the time series

Introduction

- None of these papers use GLS detrending procedures to estimate the parameters of the model
  1. However, it has been shown that this estimation technique leads to test statistics with better properties – see Elliott et al. (1996) and Ng and Perron (2001)
  2. In this paper we extend the approach in Perron and Rodríguez (2003) for the case of multiple structural breaks that affect the slope of the time trend

The model

Let y_t be the stochastic process generated according to

$$y_t = d_t + u_t \quad (1)$$

$$u_t = \alpha u_{t-1} + v_t, \ \ t = 0, \ldots, T, \quad (2)$$

where $\{u_t\}$ is an unobserved stationary mean-zero process, $u_0 = 0$. We consider three models:

1. Model 0 (“level shift” or “crash”)
2. Model I (“slope change” or “changing growth”)
3. Model II (“mixed change”)

The model

The deterministic component in (1) $d_t = \psi' z_t(\lambda^0)$ is given by

$$d_t = z_t'(T_0^0)\psi_0 + z_t'(T_1^0)\psi_1 + \cdots + z_t'(T_m^0)\psi_m = z_t'(\lambda^0)\psi \qquad (3)$$

where

$$z_t(\lambda^0) = [z_t'(T_0^0), \ldots, z_t'(T_m^0)]' \text{ and } \psi = (\psi_0', \ldots, \psi_m')'$$

$$z_t(T_0^0) = (1, t),$$

with $\psi_0 = (\mu_0, \beta_0)'$ and, for $1 \le j \le m$,

$$z_t(T_j^0) = \begin{cases} DU_t(T_j^0), & \text{in Model 0} \\ DT_t^*(T_j^0), & \text{in Model I} \\ (DU_t(T_j^0), DT_t^*(T_j^0))', & \text{in Model II} \end{cases}$$

with $\psi_j = \mu_j$ in Model 0, $\psi_j = \beta_j$ in Model I, and $\psi_j = (\mu_j, \beta_j)'$ in Model II.

The model

We also consider the case where the magnitude of level shifts get large as the sample size grow

$$(\mu_1, \ldots, \mu_m) = T^{1/2+\eta}(\kappa_1, \ldots, \kappa_m); \quad \eta > 0$$

Call the models with this additional assumption as Models 0b and IIb

The model

Some remarks:

- In Models 0 and II, the level shifts belong to the class of "slowly evolving trend" defined by Elliott et al. (1996)
  1. Hence, ignoring these deterministic components in the unit root procedure has no effect on the asymptotic size and power of the tests...
  2. However this will clearly worsen the finite sample properties of the associated tests, especially when the magnitude of the shifts are non-negligible
  3. This typically implies that the derived asymptotic distribution is a bad approximation to the finite sample distribution
- In Models 0b and IIb, the level shifts do not belong to the class of "slowly evolving trend" and should not be ignored

The model

GLS detrended unit root test statistics are based on the use of the transformed data $y_t^{\bar{\alpha}}$ and $z_t^{\bar{\alpha}}(\lambda^0)$, where

$$\begin{aligned} y_t^{\bar{\alpha}} &= (y_1, (1-\bar{\alpha}L)\, y_t) \\ z_t^{\bar{\alpha}}(\lambda^0) &= \left(z_1(\lambda^0), (1-\bar{\alpha}L)\, z_t(\lambda^0)\right), \; t = 1, \ldots, T, \end{aligned}$$

with

$$\bar{\alpha} = 1 + \bar{c}/T$$

and $\bar{c}$ the non-centrality parameter to be defined below.

The model

The deterministic parameters ψ can be estimated through the minimization of the following objective function

$$S^* \left(\psi, \bar{\alpha}, \lambda^0\right) = \sum_{t=1}^{T} \left(y_t^{\bar{\alpha}} - \psi' z_t^{\bar{\alpha}}(\lambda^0)\right)^2 . \qquad (4)$$

The minimum of this function is denoted as $S\left(\bar{\alpha}, \lambda^0\right)$.

Feasible point optimal test with multiple structural breaks

The definition of the non-centrality parameter $\bar{c}$ is based on the point optimal statistic used in Elliott et al. (1996)

$$\begin{cases} H_0: & \alpha = 1 \\ H_1: & \alpha = \bar{\alpha} \end{cases} .$$

The feasible point optimal statistic is given by

$$P_T^{GLS}\left(c, \bar{c}, \lambda^0\right) = \left\{S\left(\bar{\alpha}, \lambda^0\right) - \bar{\alpha} S\left(1, \lambda^0\right)\right\} / s^2(\lambda^0), \tag{5}$$

where $S\left(\bar{\alpha}, \lambda^0\right)$ and $S\left(1, \lambda^0\right)$ are the sum of squared residuals (SSR) from a GLS regression with $\alpha = \bar{\alpha}$ and $\alpha = 1$, respectively, and $s^2(\lambda^0)$ is the autoregressive estimate of the spectral density at frequency zero of v_t.

Feasible point optimal test with multiple structural breaks

Theorem

Let y_t be the stochastic process generated according to (1) and (2) with $\alpha = 1 + c/T$. Let $P_T^{GLS}\left(c, \bar{c}, \lambda^0\right)$ be the statistic defined by (5) with the data obtained from local GLS detrending $(\tilde{y}_t)$ at $\bar{\alpha} = 1 + \bar{c}/T$. Also, let $s^2(\lambda^0)$ be a consistent estimate of σ^2. Then,
(i) Models 0 and 0b is given by

$$P_T^{GLS}\left(c, \bar{c}, \lambda^0\right) \Rightarrow \bar{c}^2 \int_0^1 V_{c,\bar{c}}^2(r)dr + (1-\bar{c})V_{c,\bar{c}}^2(1)$$

(ii) Models I, II, and IIb is given by

$$\begin{aligned} P_T^{GLS}\left(c, \bar{c}, \lambda^0\right) \quad &\Rightarrow \quad M\left(c, 0, \lambda^0\right) - M\left(c, \bar{c}, \lambda^0\right) - 2\bar{c}\int_0^1 W_c\left(r\right) dW\left(r\right) \\ &+ \left(\bar{c}^2 - 2\bar{c}c\right)\int_0^1 W_c\left(r\right)^2 dr - \bar{c} \end{aligned}$$

Feasible point optimal test with multiple structural breaks

Remarks:

- The limiting distribution in *(i)* is the same as that of the linear time trend model with no break, which can be found in Elliott et al. (1996)
  1. Because the timing of breaks are known in the current cases, the test statistic, $P_T^{GLS}\left(c, \bar{c}, \lambda^0\right)$ is exactly invariant to the break parameters
- There is no distinction between Models 0 and 0b, and between Models II and IIb
- The limiting distribution of the test statistic for Models I, II, and IIb depends both on the number of structural breaks and on the break fraction vector

Feasible point optimal test with multiple structural breaks

- Power envelope
  1. From this limiting distribution we can obtain the Gaussian power envelope for different values of $\bar{c}$, and select the $\bar{c}$ parameter so that the asymptotic local power of the test is tangent to the power envelope at 50% power
  2. For Models 0 and 0b the $\bar{c}$ parameter can be found in Elliott et al. (1996)
  3. For Models I, II, and IIb the $\bar{c}$ parameter varies both with the number of structural breaks and with their position

Feasible point optimal test with multiple structural breaks

We have approximated the asymptotic $\bar{c}$ parameter for up to $m = 5$ structural break points for all possible combinations of break fraction vectors $\lambda^0 = \left(\lambda_1^0, \cdots, \lambda_m^0\right)'$, $\lambda_i^0 = \{0.1, 0.2, \cdots, 0.9\}$

The information is summarized through the estimation of one response surface:

$$\bar{c}\left(\lambda_k^0\right) = \beta_{0,0} + \sum_{l=1}^{4}\sum_{i=1}^{m}\beta_{i,l}(\lambda_{i,k}^0)^l + \sum_{l=1}^{4}\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\gamma_{i,j,l}\left|\lambda_{i,k}^0 - \lambda_{j,k}^0\right|^l + \varepsilon_k, \tag{6}$$

Feasible point optimal test with multiple structural breaks

Using the definition of the $\bar{c}$ parameter we can compute the M-class tests in Ng and Perron (2001) allowing for multiple structural breaks:

$$MZ_{\alpha}^{GLS}\left(\lambda^{0}\right) = \left(T^{-1}\tilde{y}_{T}^{2} - s\left(\lambda^{0}\right)^{2}\right)\left(2T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^{2}\right)^{-1} \quad (7)$$

$$MSB^{GLS}\left(\lambda^{0}\right) = \left(s\left(\lambda^{0}\right)^{-2}T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^{2}\right)^{1/2} \quad (8)$$

$$MZ_{t}^{GLS}\left(\lambda^{0}\right) = \left(T^{-1}\tilde{y}_{T}^{2} - s\left(\lambda^{0}\right)^{2}\right)\left(4s\left(\lambda^{0}\right)^{2}T^{-2}\sum_{t=1}^{T}\tilde{y}_{t-1}^{2}\right)^{-1/2} \quad (9)$$

Feasible point optimal test with multiple structural breaks

Theorem

Let y_t be the stochastic process generated according to (1) and (2) with $\alpha = 1 + c/T$. Let $MZ_\alpha^{GLS}(\lambda^0)$, $MSB^{GLS}(\lambda^0)$ and $MZ_t^{GLS}(\lambda^0)$ be the statistics defined by (7)-(9) with the data obtained from local GLS detrending $(\tilde{y}_t)$ at $\bar{\alpha} = 1 + \bar{c}/T$. Also, let $s^2(\lambda^0)$ be a consistent estimate of σ^2. Then,
(i) Models 0 and 0b

$$MZ_\alpha^{GLS}(\lambda^0) \Rightarrow 0.5\left(V_{c,\bar{c}}(1)^2 - 1\right)\left(\int_0^1 V_{c,\bar{c}}(r)^2 dr\right)^{-1}$$

$$MSB^{GLS}(\lambda^0) \Rightarrow \left(\int_0^1 V_{c,\bar{c}}(r)^2 dr\right)^{1/2}$$

Feasible point optimal test with multiple structural breaks

Theorem

(continues)
(ii) Models I, II and IIb

$$MZ_{\alpha}^{GLS}(\lambda^0) \Rightarrow 0.5\left(V_{c,\bar{c}}(1,\lambda^0)^2 - 1\right)\left(\int_0^1 V_{c,\bar{c}}(r,\lambda^0)^2 dr\right)^{-1}$$

$$MSB^{GLS}(\lambda^0) \Rightarrow \left(\int_0^1 V_{c,\bar{c}}(r,\lambda^0)^2 dr\right)^{1/2}$$

(iii) The limiting distribution of $MZ_t^{GLS}\left(\lambda^0\right)$ in all models can be obtained in view of the fact that $MZ_t^{GLS}(\lambda^0) = MZ_{\alpha}^{GLS}(\lambda^0) \cdot MSB^{GLS}(\lambda^0)$, which is the same limiting distribution as that for the $ADF^{GLS}\left(\lambda^0\right)$ test.

Feasible point optimal test with multiple structural breaks

Remarks:

- Again, the limiting distribution in *(i)* is the same as that of the linear time trend model with no break given in Ng and Perron (2001)
    1. Note, thus, that the invariance to the break parameters holds for all test statistics for Models 0 and 0b
- This is not the case for Models I, II, and IIb, where their limiting distribution depends on the number and location of the break points

Feasible point optimal test with multiple structural breaks

- As above, we have summarized the 1, 2.5, 5 and 10% percentiles of the previous statistics using response surfaces:

$$
\begin{aligned}
cv\left(\lambda_k^0\right) &= \beta_{0,0} + \sum_{l=1}^{2}\sum_{i=1}^{m}\beta_{l,i}(\lambda_{i,k}^0)^l \\
&+ \sum_{l=1}^{2}\left(\gamma_{l,0} + \sum_{i=1}^{m}\gamma_{l,i}\lambda_{i,k}^0\right)\bar{c}\left(\lambda_k^0\right)^l \\
&+ \sum_{l=1}^{4}\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\delta_{i,j,l}\left|\lambda_{i,k}^0 - \lambda_{j,k}^0\right|^l \bar{c}\left(\lambda_k^0\right) + \varepsilon_k,
\end{aligned}
$$

- The estimates of the coefficients of the response surfaces are reported in the paper for up to $m = 5$ structural breaks

Unknown break points

For a given number of structural breaks $m > 0$, we propose to estimate the position of the break points through global minimization of the *SSR* of the GLS-detrended model:

$$S(\bar{\alpha}, \hat{\lambda}) = \min{}_{\lambda \in \Lambda(\varepsilon)} S\left(\bar{\alpha}, \lambda\right), \tag{10}$$

where the infimum is taken on all possible break fraction vectors defined on the set $\Lambda(\varepsilon)$, with ε being the amount of trimming

Therefore, the estimated vector of break fraction parameters are obtained as

$$\hat{\lambda} = \arg\min{}_{\lambda \in \Lambda(\varepsilon)} S\left(\bar{\alpha}, \lambda\right).$$

Unknown break points

Proposition: Let $\{y_t\}_{t=1}^{T}$ be the stochastic process generated according to (1) and (2) with $\alpha = 1$. Let us assume that $m > 0$ and ψ_j, $j = 1, \ldots, m$, so that there are structural breaks affecting y_t under the null hypothesis. Then, as $T \to \infty$:

(i) in Models I and II,

$$\left\| \hat{\lambda} - \lambda^0 \right\| = O_p\left(T^{-1}\right),$$

(ii) in Models 0b and IIb,

$$\left\| \hat{\lambda} - \lambda^0 \right\| = o_p\left(T^{-1}\right)$$

where $\hat{\lambda} = \arg\min_{\lambda \in \Lambda(\varepsilon)} S\left(\bar{\alpha}, \lambda\right)$.

Unknown break points

Proposition: Let $\{y_t\}_{t=1}^{T}$ be the stochastic process generated according to (1) and (2) with $\alpha = 1$. Let us assume that $m > 0$ and ψ_j, $j = 1, \cdots, m$. Then, provided that $s(\hat{\lambda})$ is a consistent estimate for σ,

(i) Models 0b and IIb, $P_T^{GLS}(c, \bar{c}, \hat{\lambda})$ has the same limiting distribution as $P_T^{GLS}(c, \bar{c}, \lambda^0)$.

(ii) Models 0b, I, II, and IIb: each of $MZ_{\alpha}^{GLS}(\hat{\lambda})$, $MSB^{GLS}(\hat{\lambda})$ and $MZ_t^{GLS}(\hat{\lambda})$ has the same limiting distribution as $MZ_{\alpha}^{GLS}(\lambda^0)$, $MSB^{GLS}(\lambda^0)$ and $MZ_t^{GLS}(\lambda^0)$, respectively.

Estimation of the break points: Dynamic algorithm

- Estimation of the multiple structural breaks is quite demanding especially when $m > 2$
- We have proposed a dynamic algorithm that minimizes the global Restricted SSR based on the approach in Bai and Perron (1998) and Perron and Qu (2005)

Estimation of the break points: Dynamic algorithm

- Details of the algorithm:

1. Compute initial estimated break dates, $\hat{\lambda} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_m)$ and the associated coefficients, $\hat{\psi} = (\hat{\psi}_0', \hat{\psi}_1', \ldots, \hat{\psi}_m')'$ by OLS using the dynamic algorithm in Bai and Perron (1998, 2003) applied to (1)

2. From the given break dates, get an initial value for $\bar{c}(\hat{\lambda})$ using (6)

3. Let $T^*(\psi, r, n) = (T_1^*(\psi, r, n), \ldots, T_r^*(\psi, r, n))$ be the vector of the optimal r break dates in the first n observations for a given vector of coefficients, ψ and $RSSR(T^*(\psi, r, n))$ be the associated restricted sum of squared residuals. Then, compute the restricted sum of squared residuals $RSSR(T^*(\psi, 1, n))$ for $2h \leq n \leq T - (m-1)h$. Then store the estimated break dates and update $\bar{c}(\hat{\lambda})$ accordingly.

4. Repeat steps 2 and 3 until convergence.

Finite sample performance: one structural break

Features of the Monte Carlo experiment:

- The analysis is conducted with and without pre-testing as proposed in Perron and Yabu (2005)
    1. Allows us to test whether structural breaks are present regardless of whether the time series is I(0) or I(1)

Finite sample performance: one structural break

- Empirical size $(\alpha = 1)$ and power $(\alpha = \bar{\alpha})$ is investigated using the following DGP:

$$y_t = d_t + u_t \quad (11)$$
$$d_t = \mu_b DU_t(T_1^0) + \beta_b DT_t^*(T_1^0) \quad (12)$$
$$u_t = \alpha u_{t-1} + v_t, \quad (13)$$

1. The magnitude of the level shift $\mu_b = \{0, 0.5, 1, 5\}$
2. β_b ranging from -4 to 4 in increments of 0.2
3. Three different values of the fraction $\lambda^0 = \{0.3, 0.5, 0.7\}$
4. The sample size is set at $T = \{100, 200, 300\}$
5. $v_t \sim iid\ N(0, 1)$, $u_0 = 0$.

Graphs for the empirical size of the statistics with $\lambda^0 = 0.5$ (no pre-testing)

Figure 8: Empirical size for the P_T test, $\lambda = 0.5$

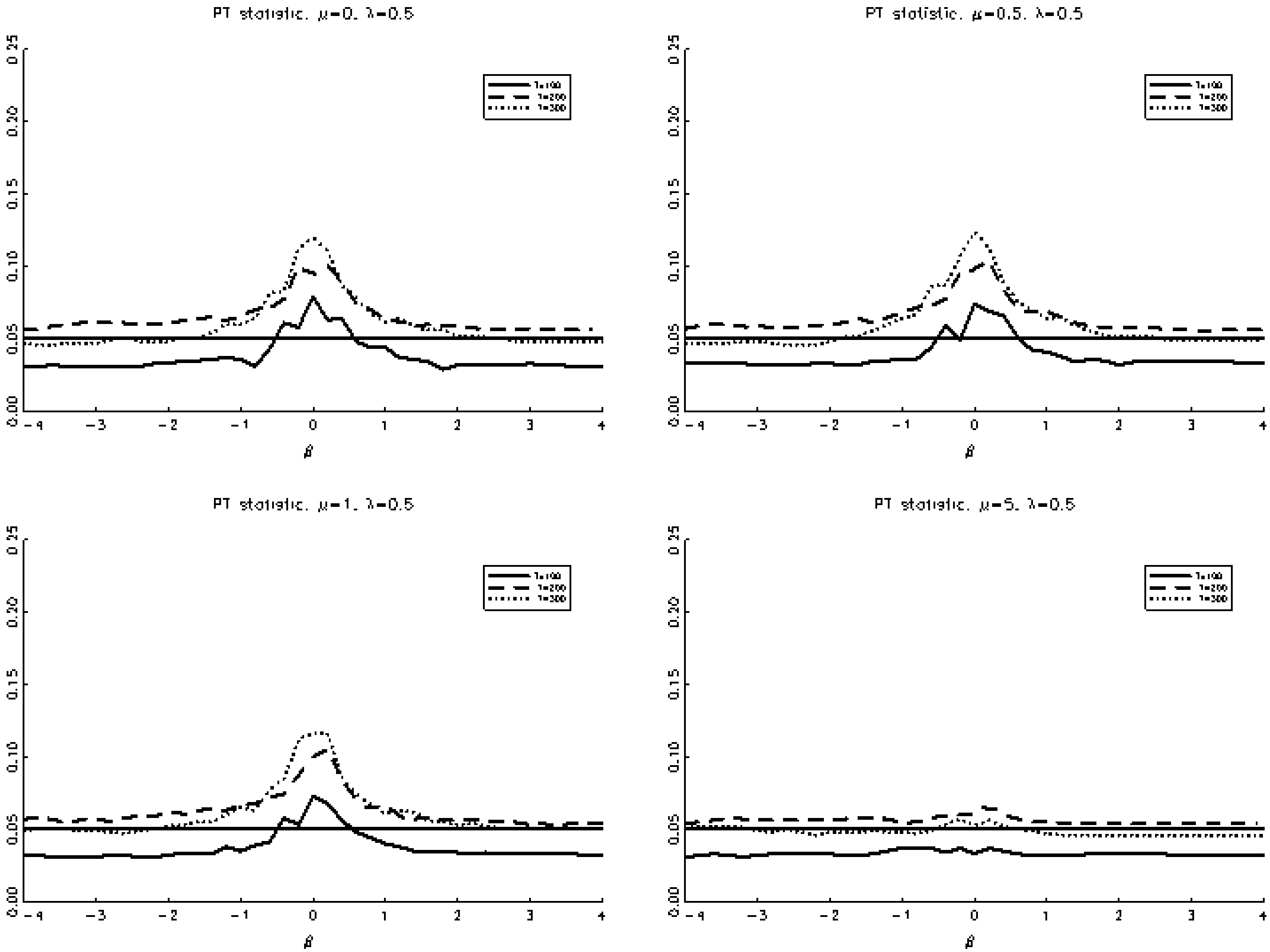


Figure 9: Empirical size for the MP_T test, $\lambda = 0.5$

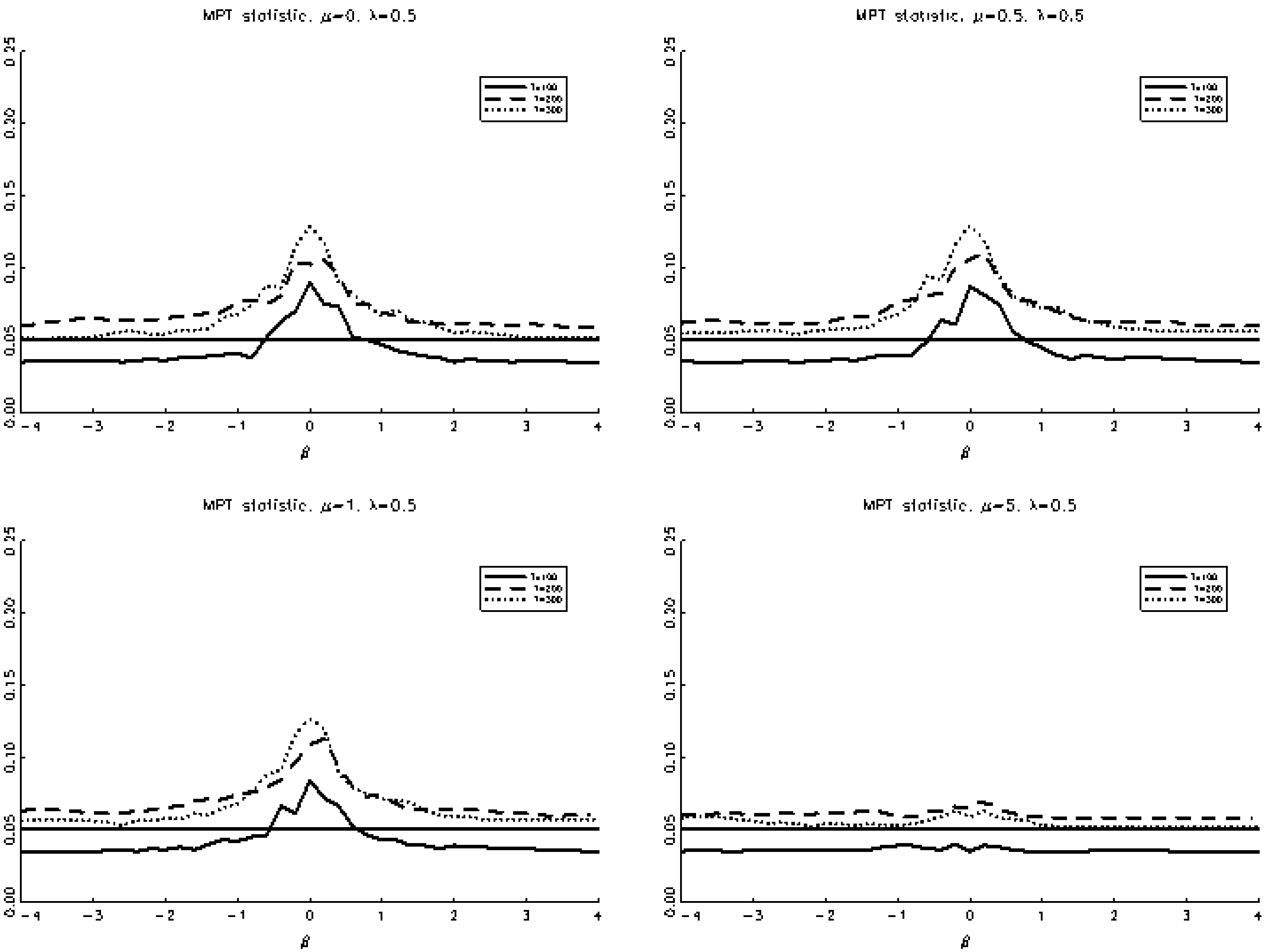


Figure 10: Empirical size for the ADF test, $\lambda = 0.5$

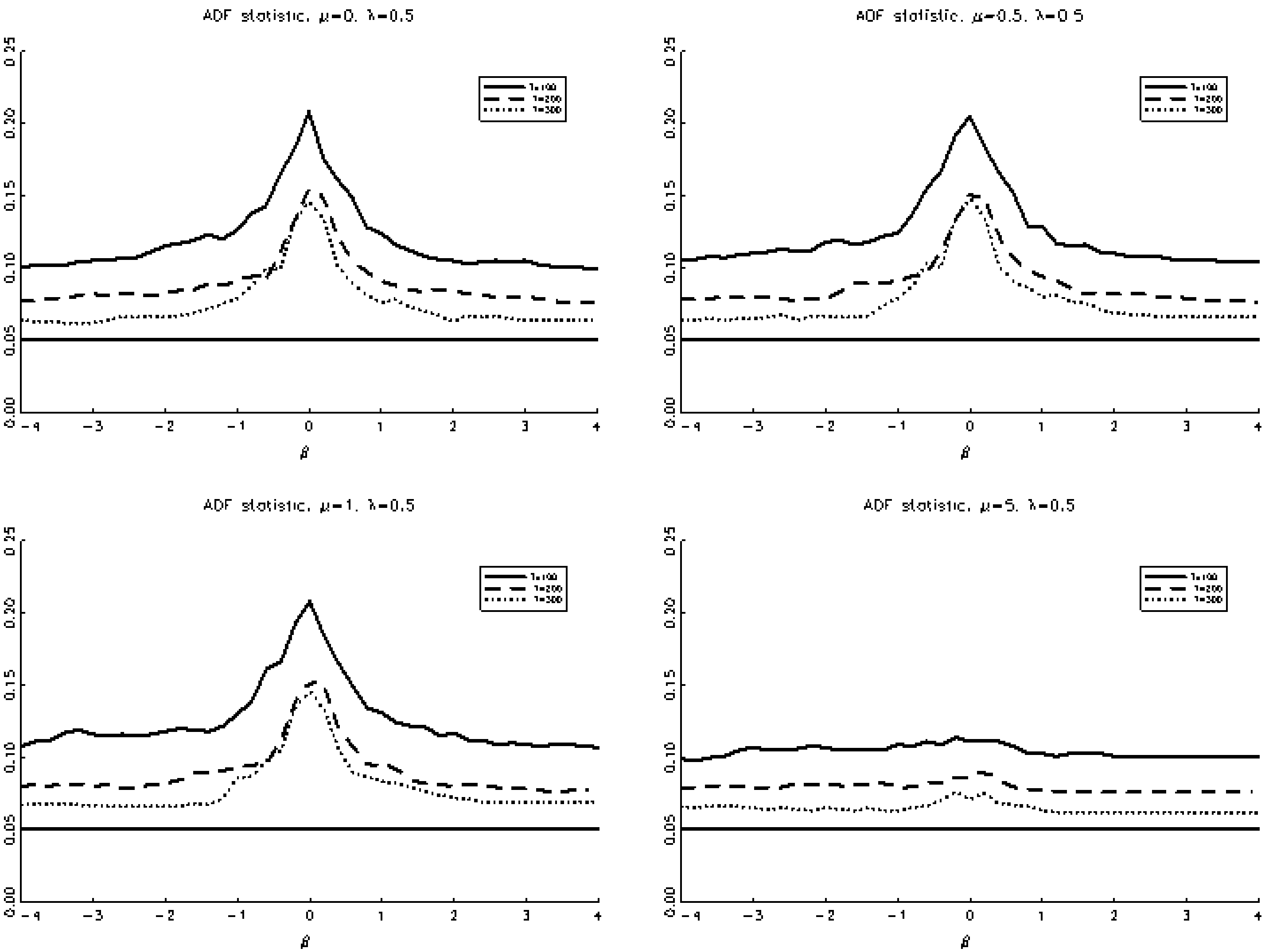


Figure 11: Empirical size for the ZA test, $\lambda = 0.5$

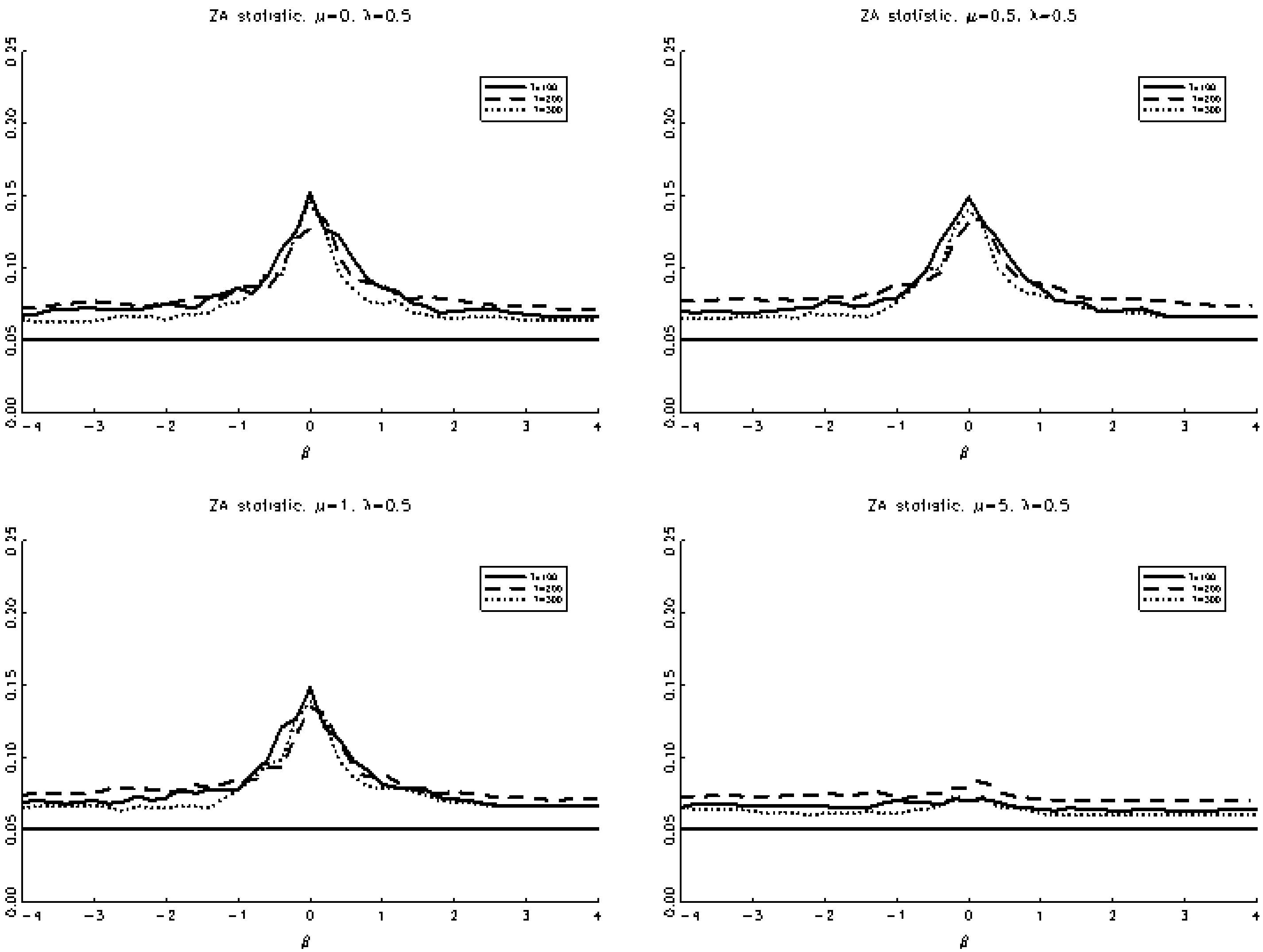


Figure 12: Empirical size for the MZA test, $\lambda = 0.5$

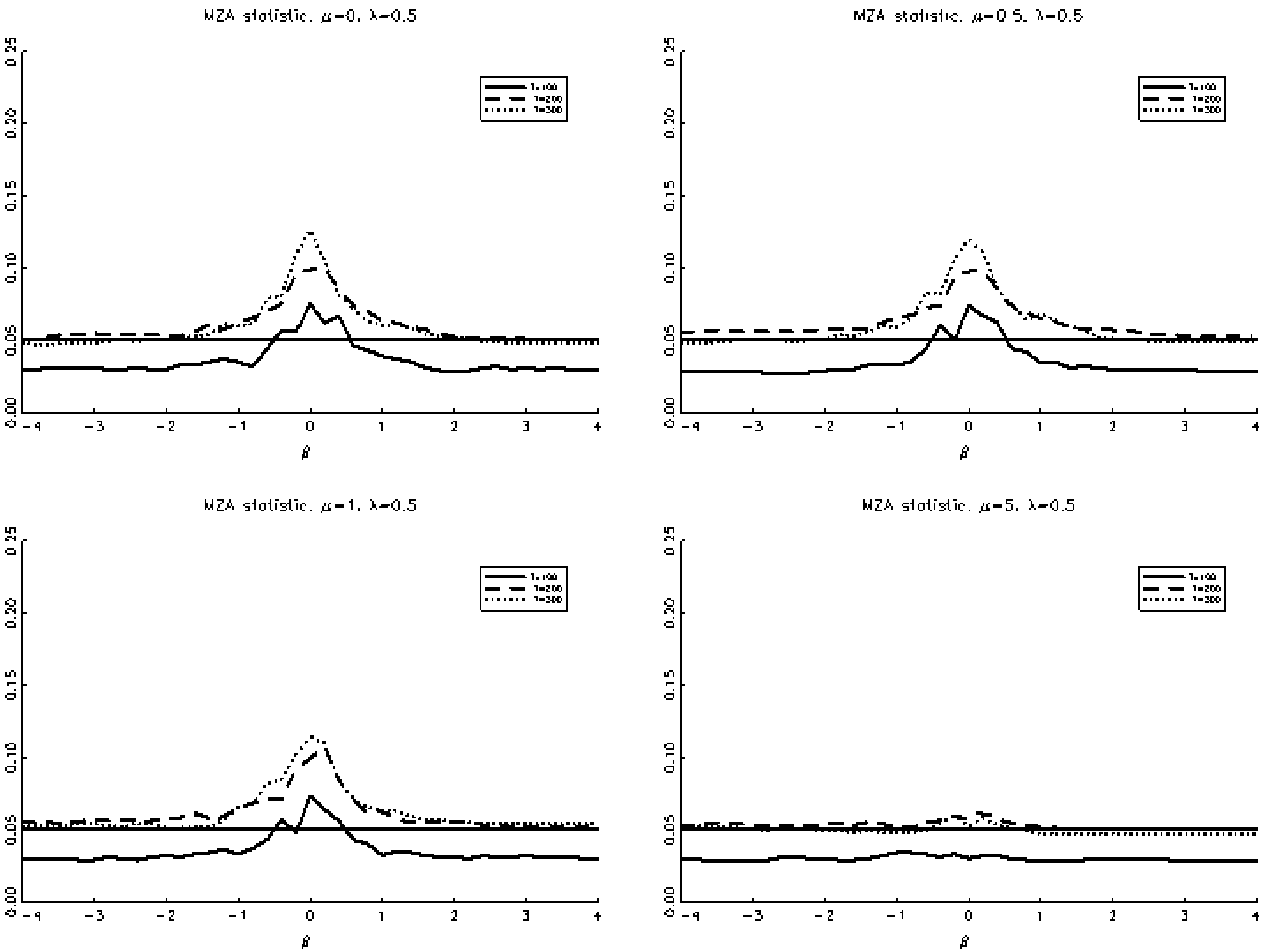


Figure 13: Empirical size for the MSB test, $\lambda = 0.5$

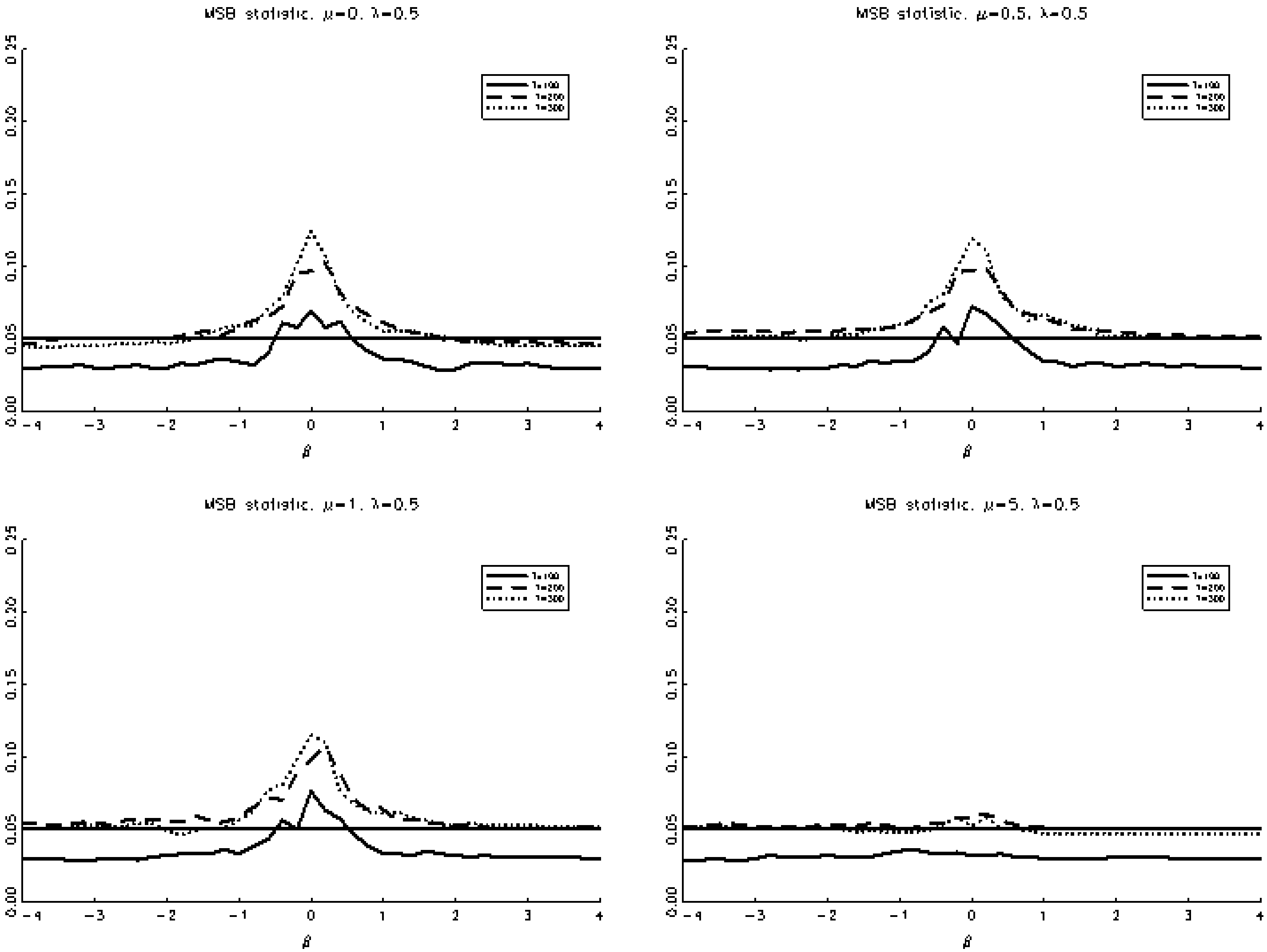
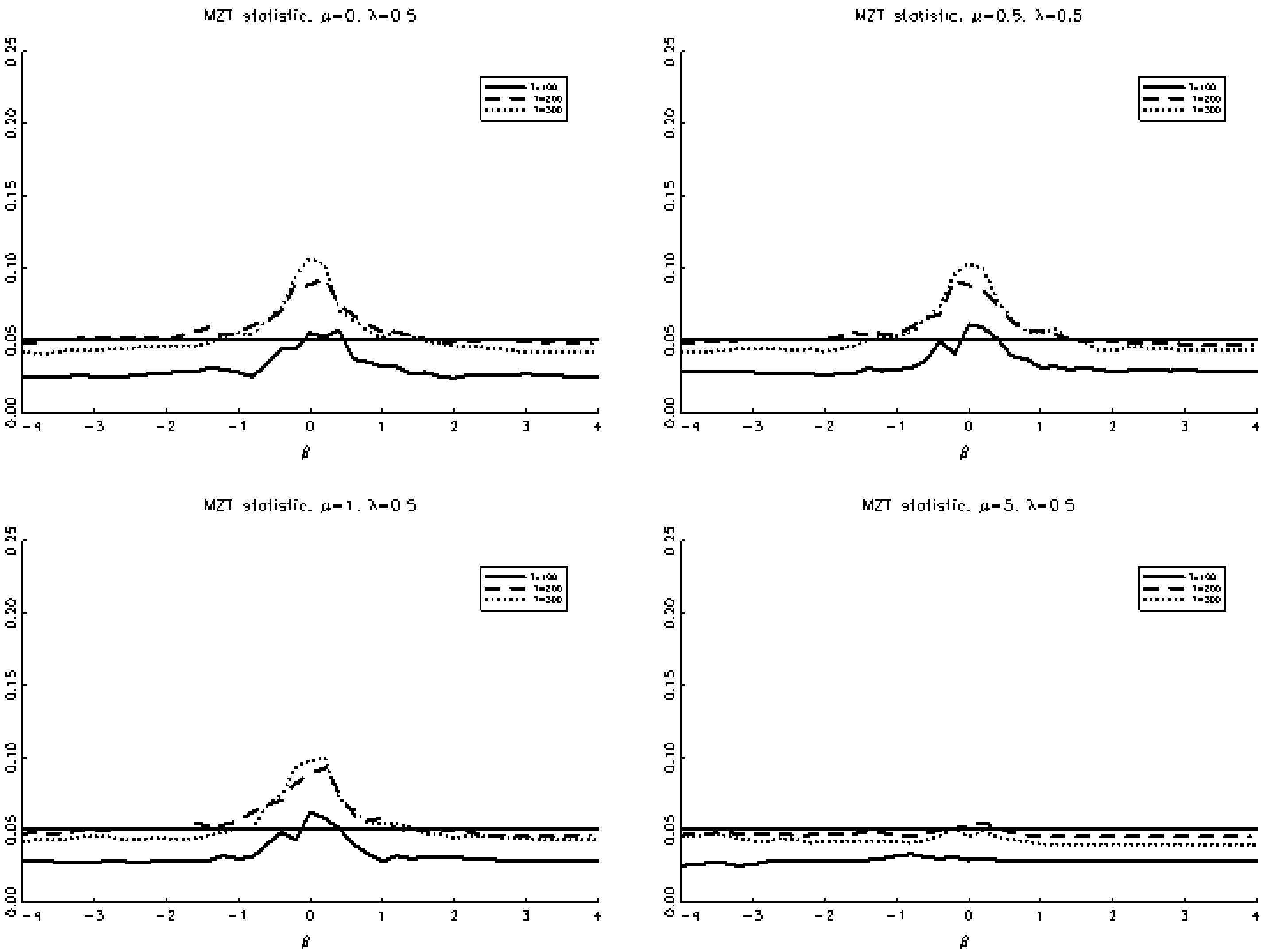


Figure 14: Empirical size for the MZT test, $\lambda = 0.5$



Finite sample performance: one structural break

Graphs for the empirical size of the statistics with $\lambda^0 = 0.5$ (pre-testing)

Figure 1: Empirical size for the P_T test (with pretesting), $\lambda = 0.5$

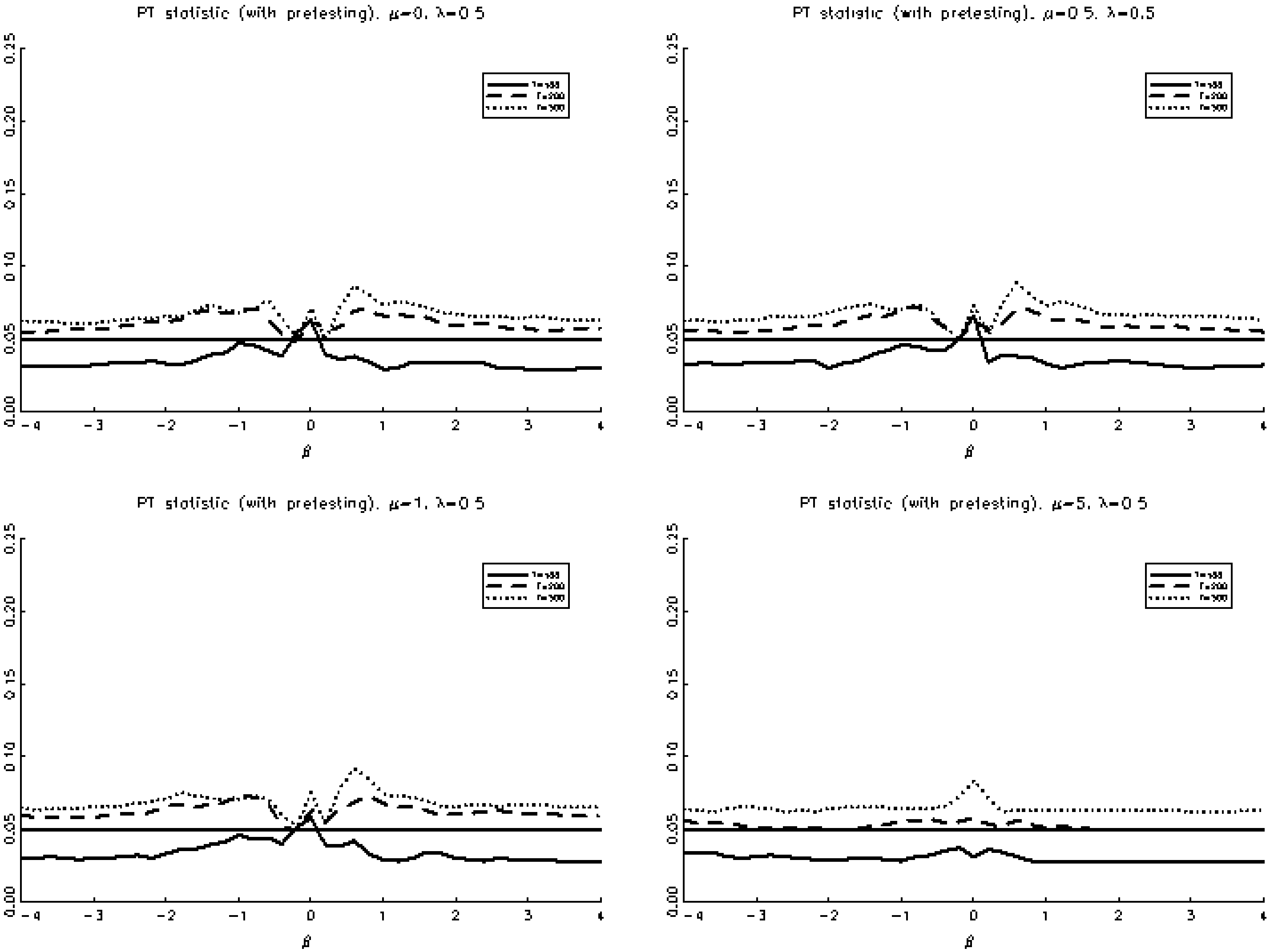


Figure 2: Empirical size for the MP_T test (with pretesting), $\lambda = 0.5$

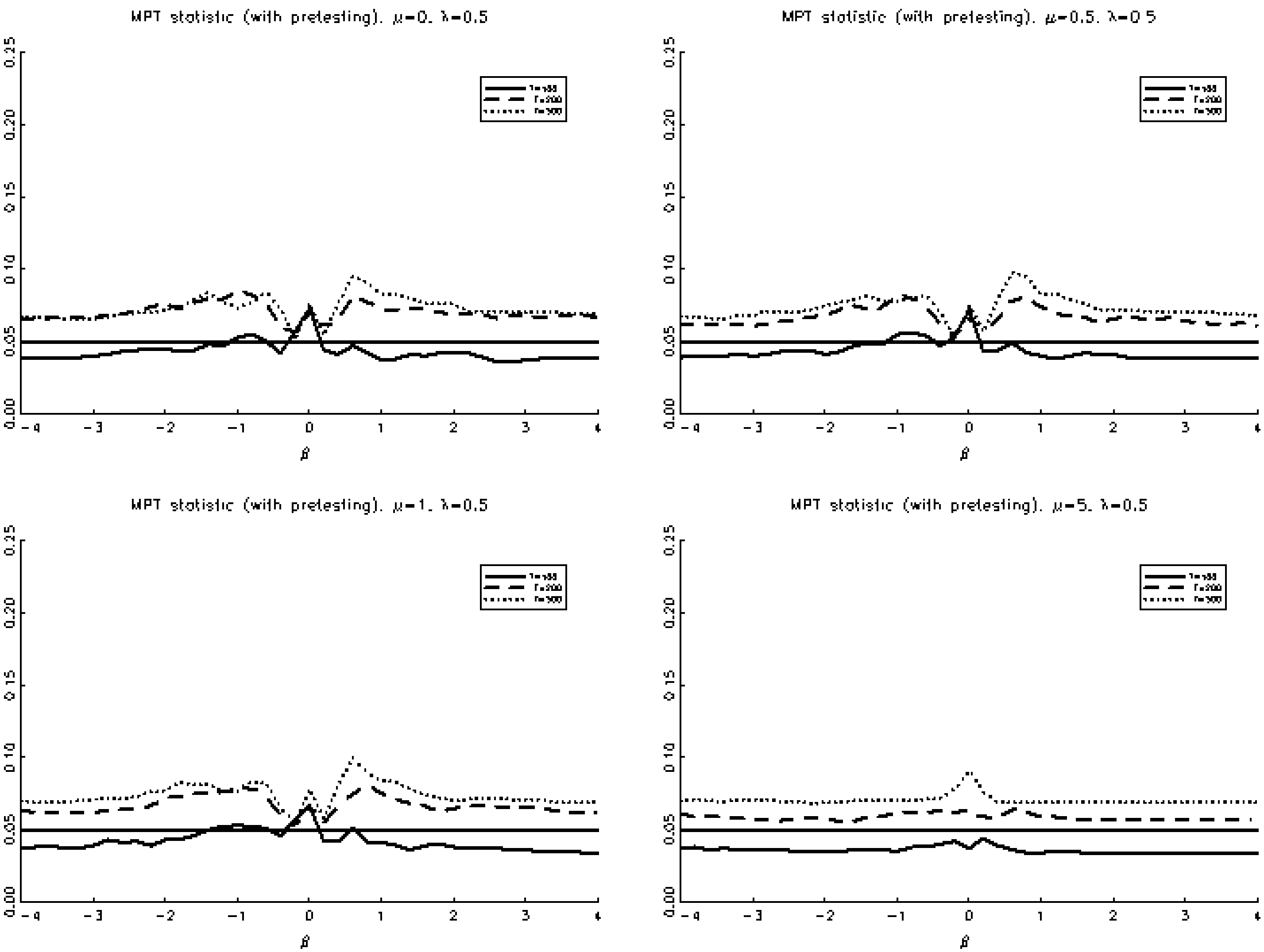


Figure 3: Empirical size for the ADF test (with pretesting), $\lambda = 0.5$

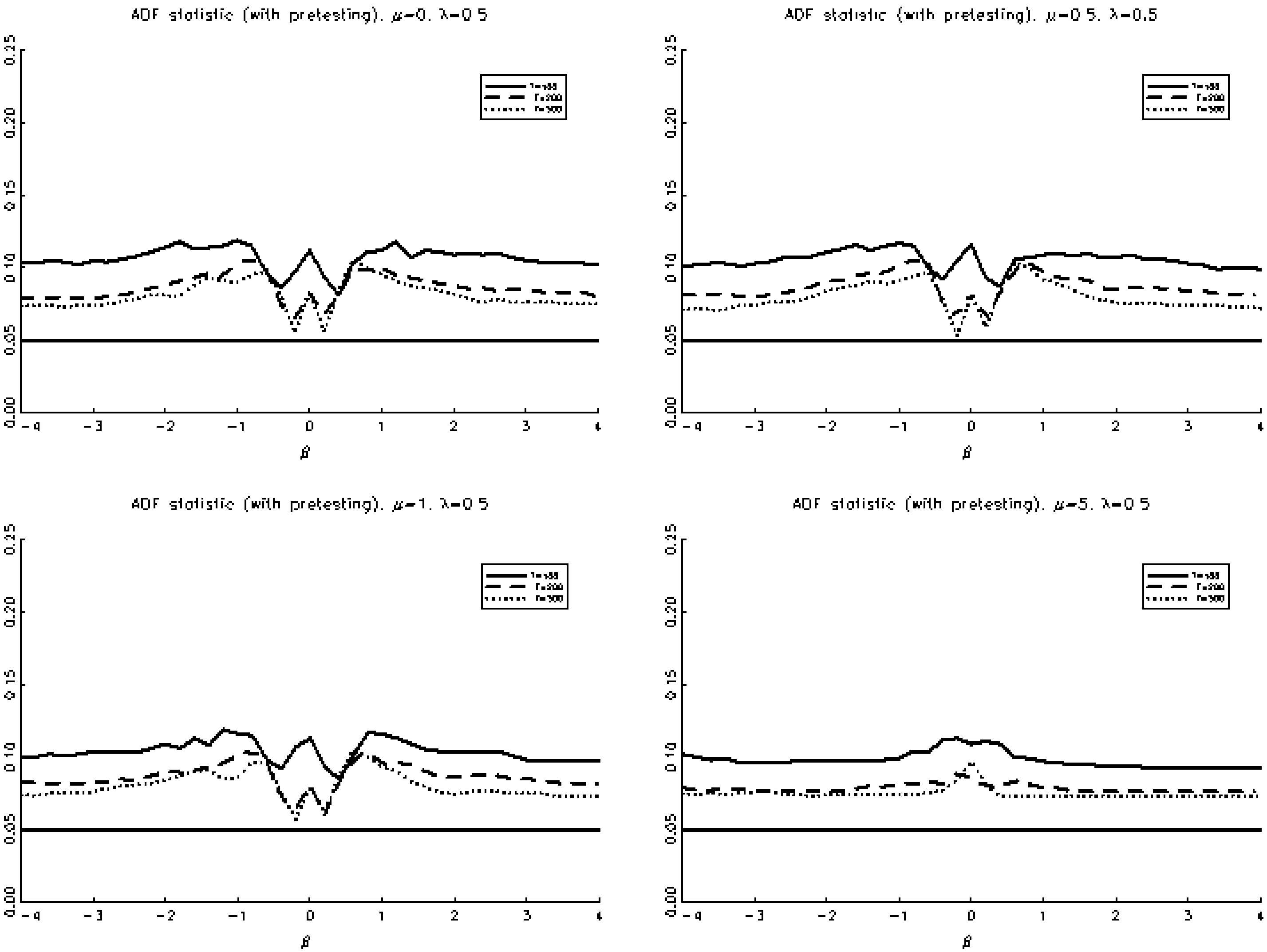


Figure 4: Empirical size for the ZA test (with pretesting), $\lambda = 0.5$

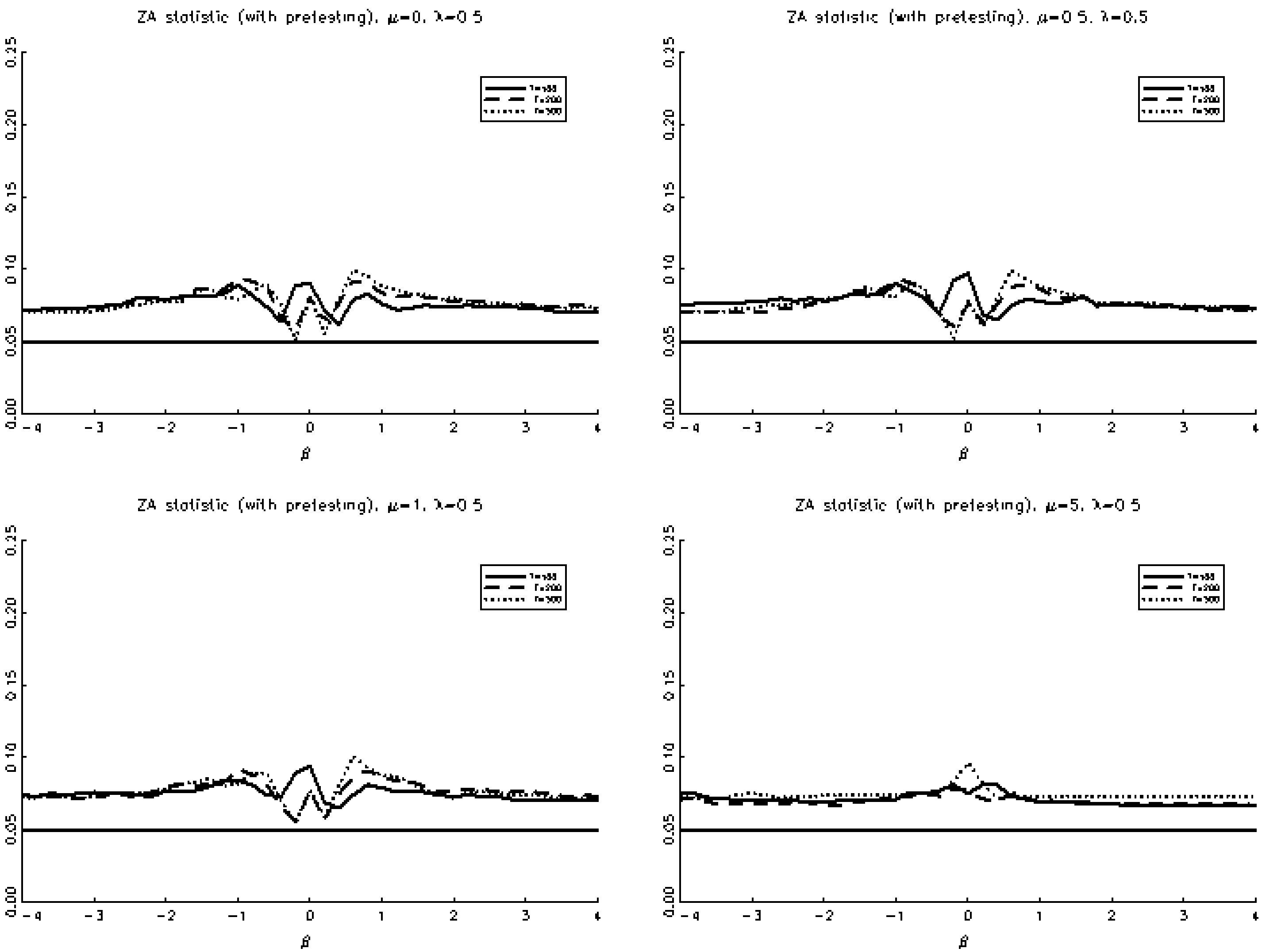


Figure 5: Empirical size for the MZA test (with pretesting), $\lambda = 0.5$

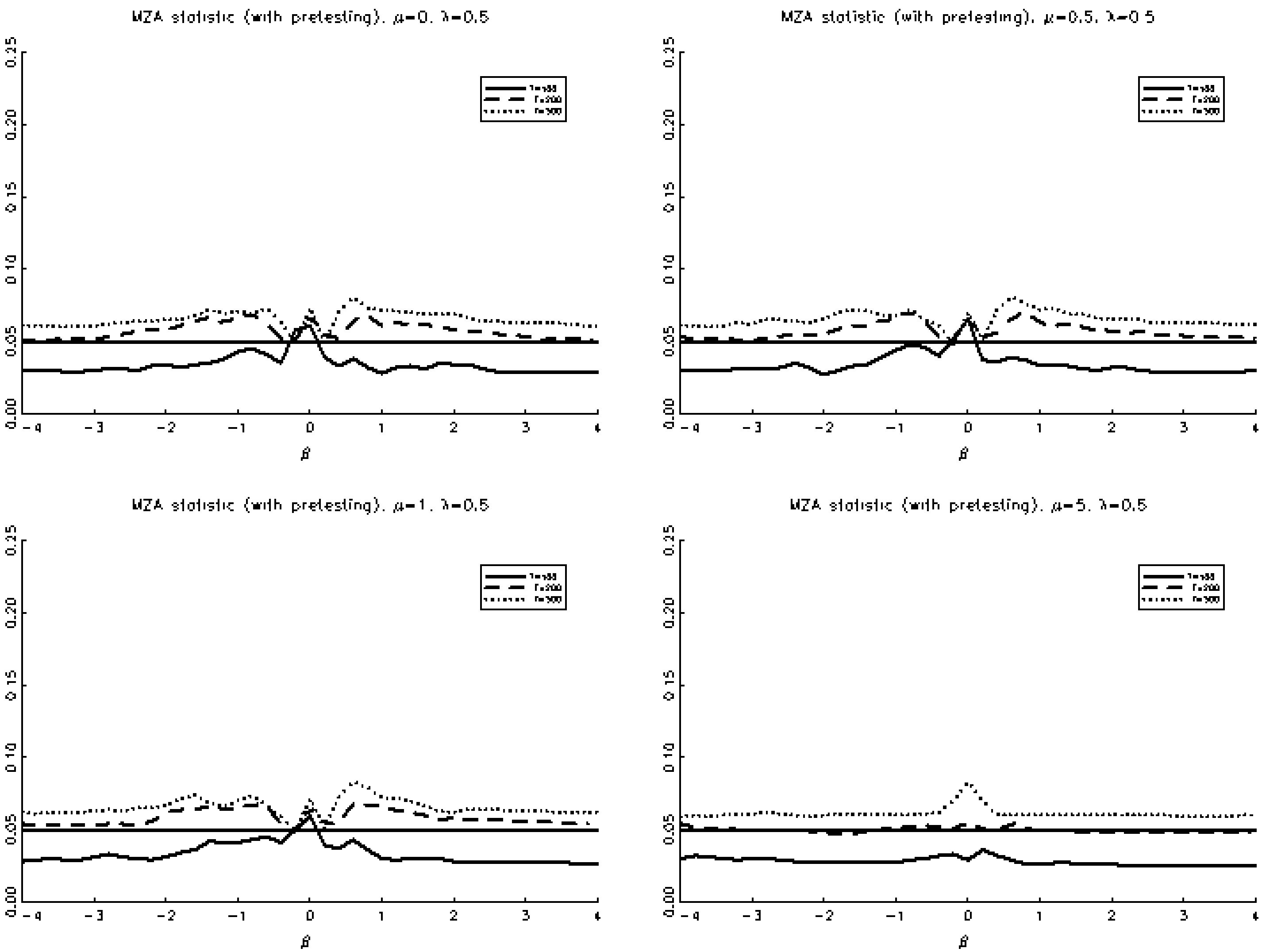


Figure 6: Empirical size for the MSB test (with pretesting), $\lambda = 0.5$

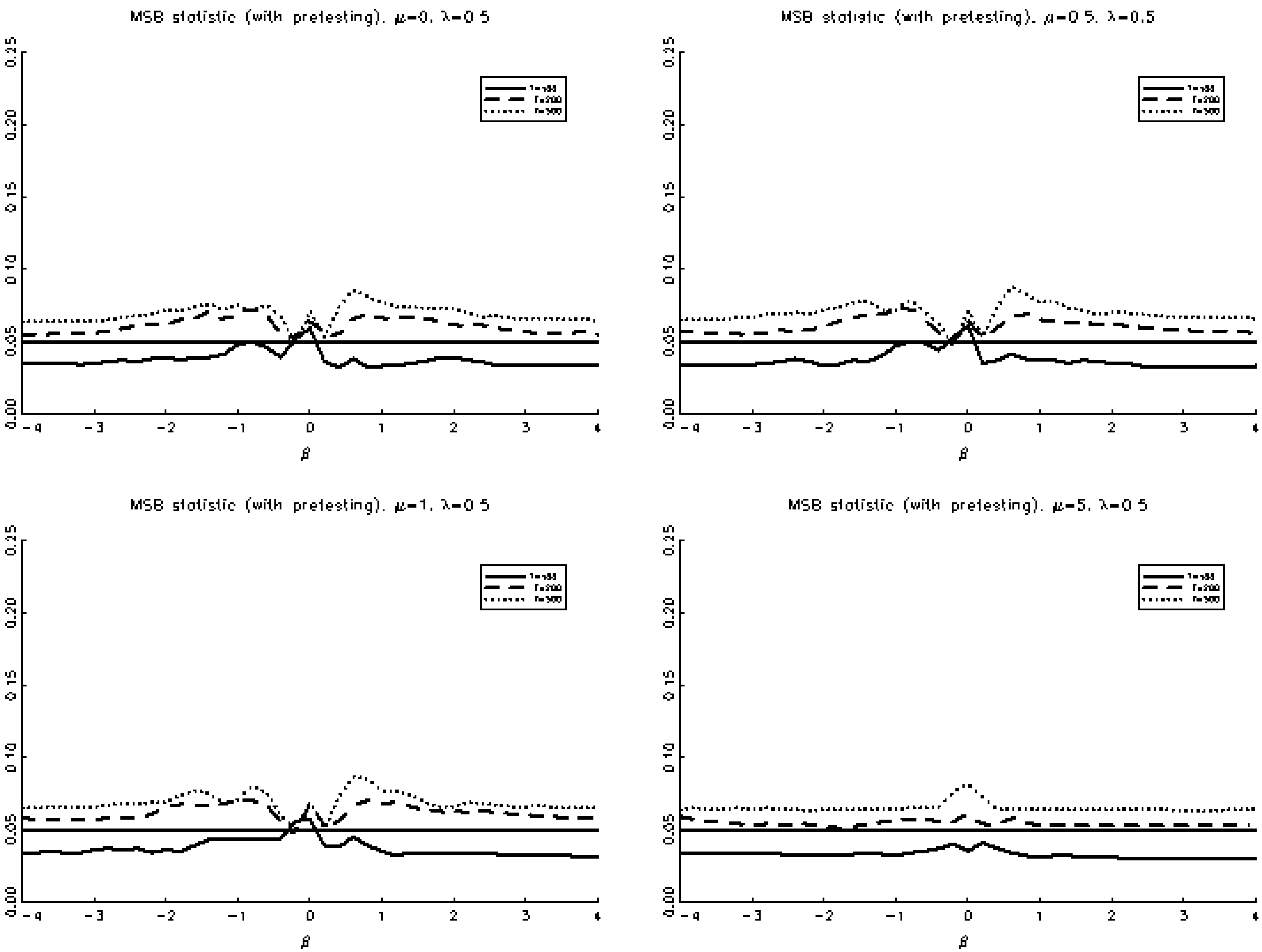
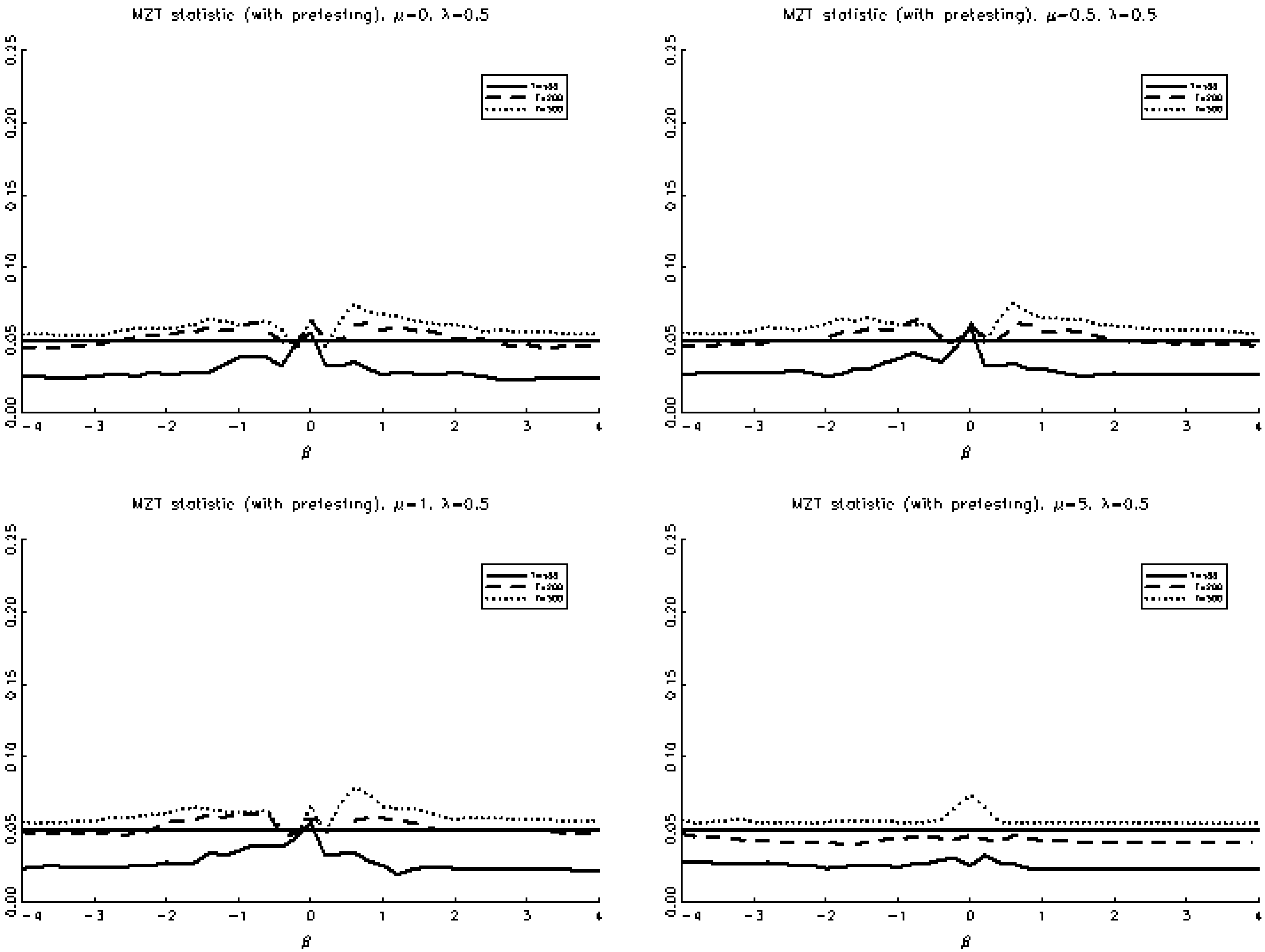


Figure 7: Empirical size for the MZT test (with pretesting), $\lambda = 0.5$



Graphs for the empirical power of the statistics with $\lambda^0 = 0.5$ (no pre-testing)

Figure 29: Empirical power for the P_T test, $\lambda = 0.5$

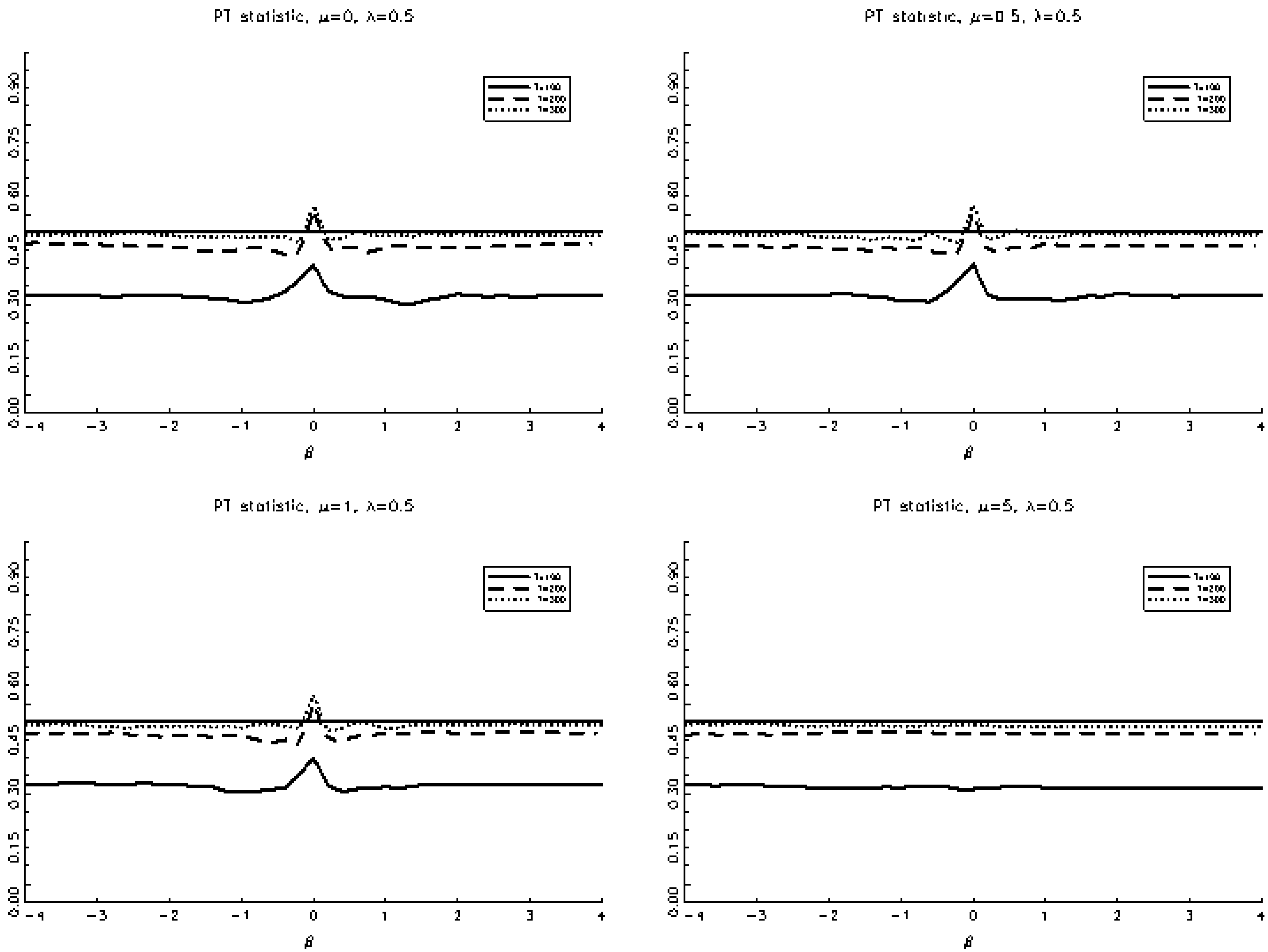


Figure 30: Empirical power for the MP_T test, $\lambda = 0.5$

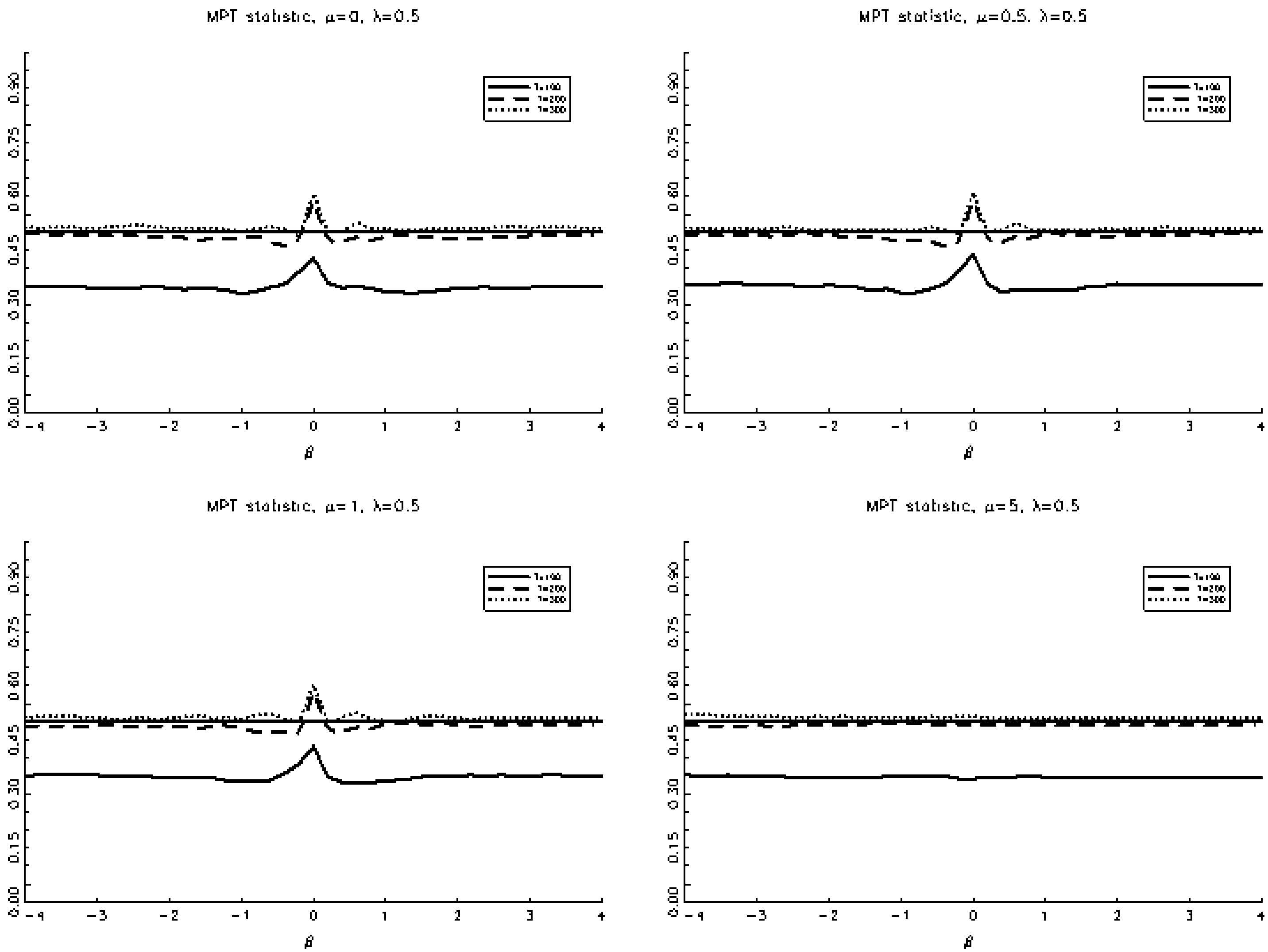


Figure 31: Empirical power for the ADF test, $\lambda = 0.5$

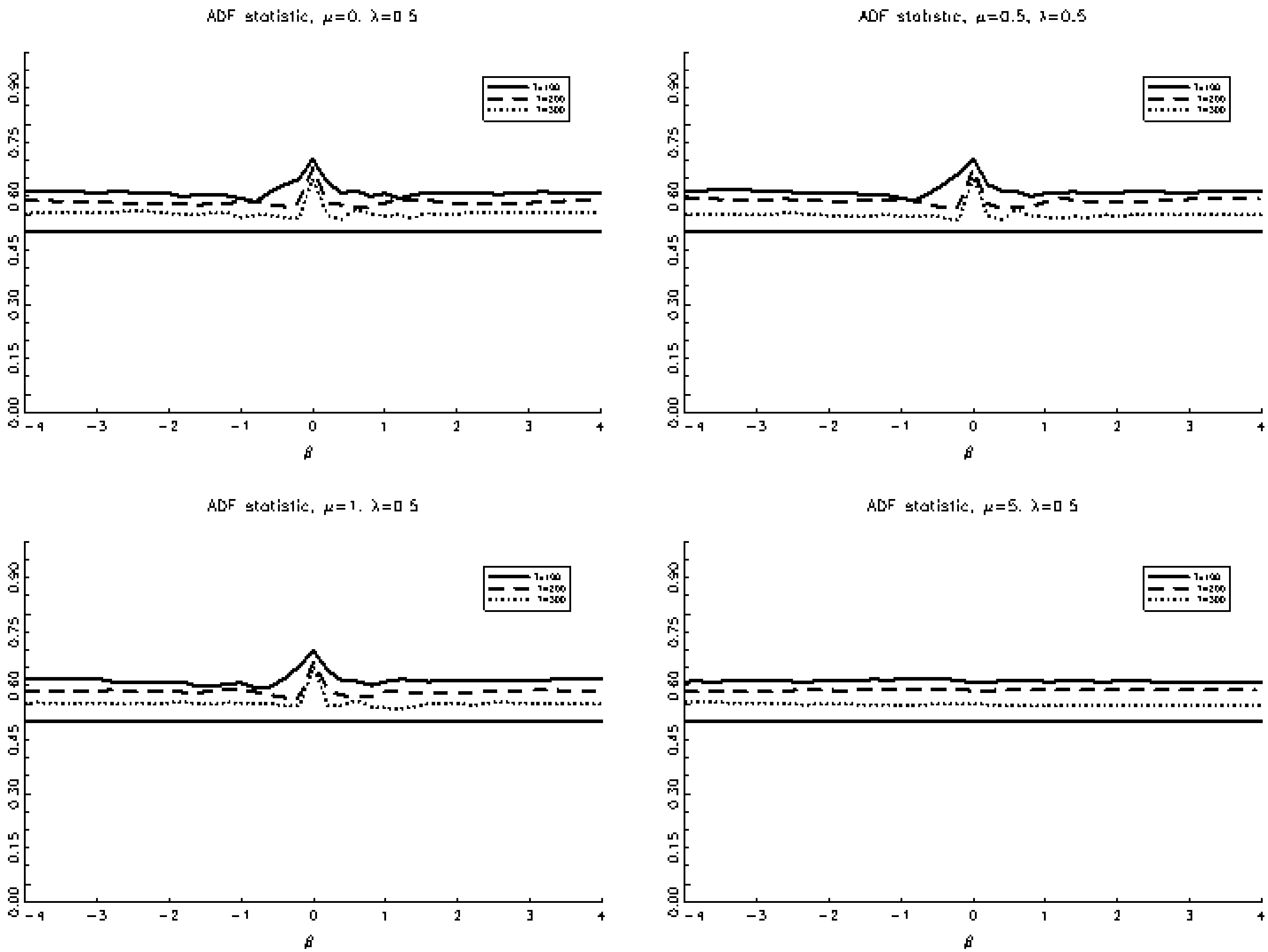


Figure 32: Empirical power for the ZA test, $\lambda = 0.5$

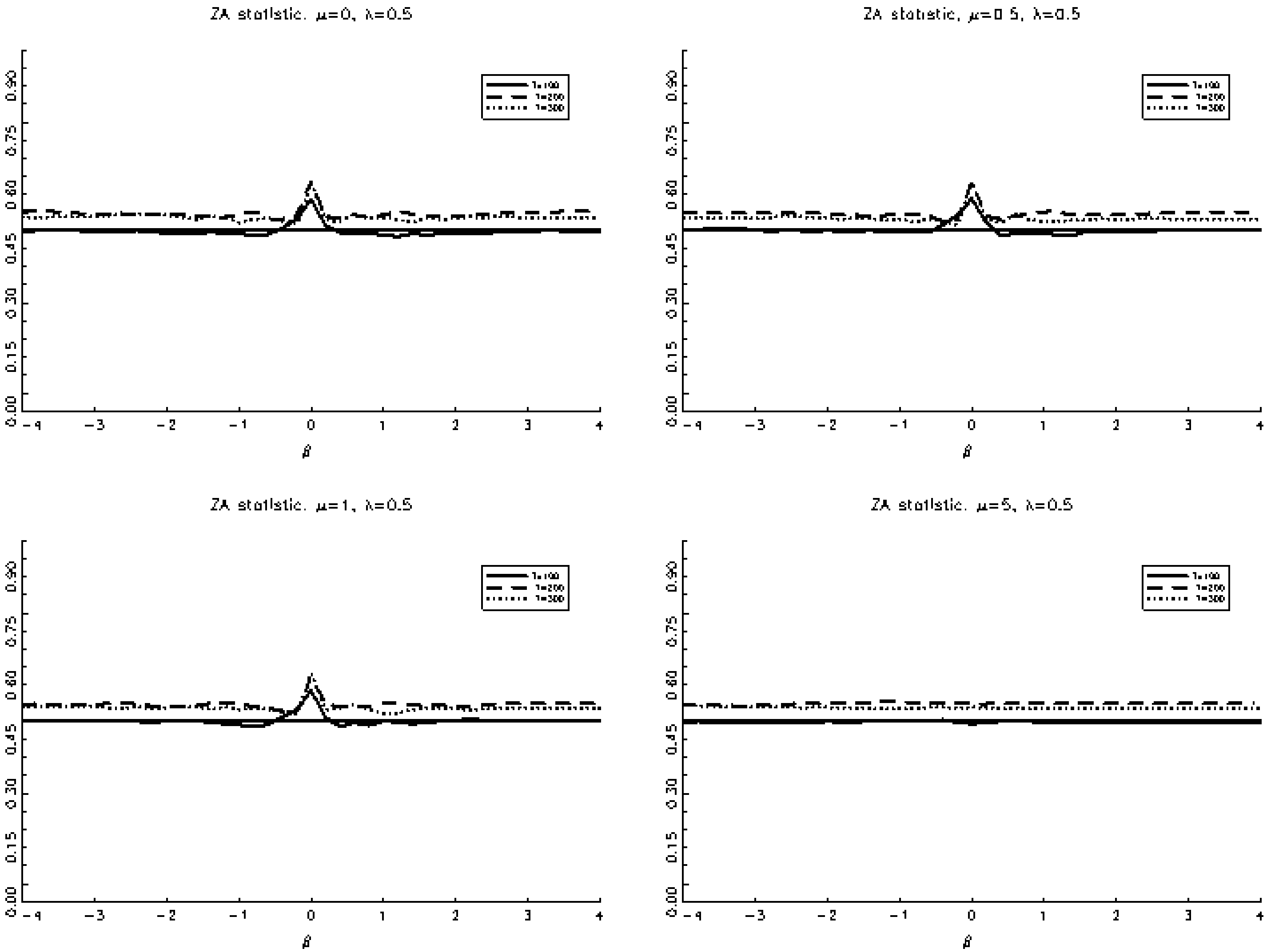


Figure 33: Empirical power for the MZA test, $\lambda = 0.5$

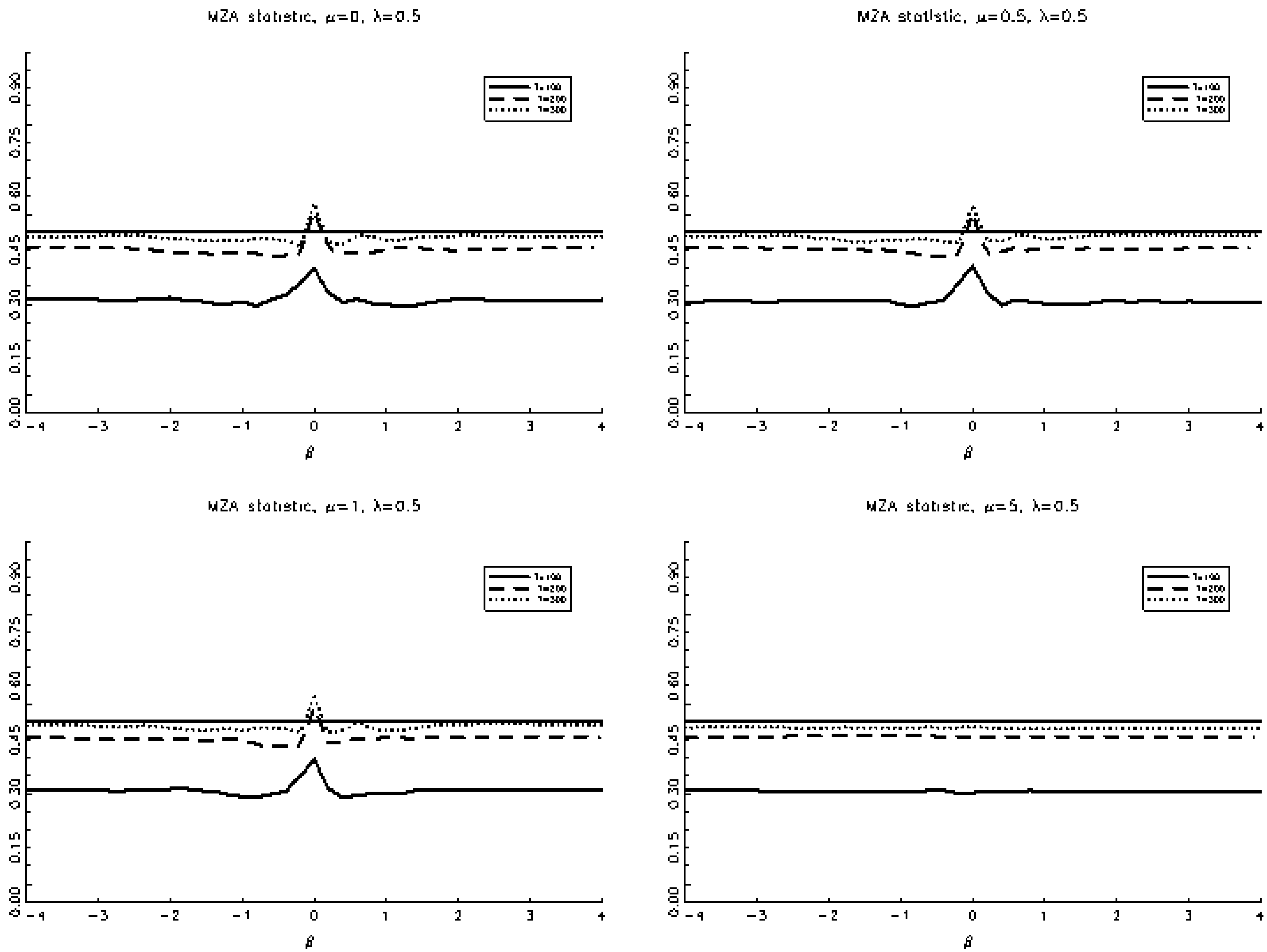


Figure 34: Empirical power for the MSB test, $\lambda = 0.5$

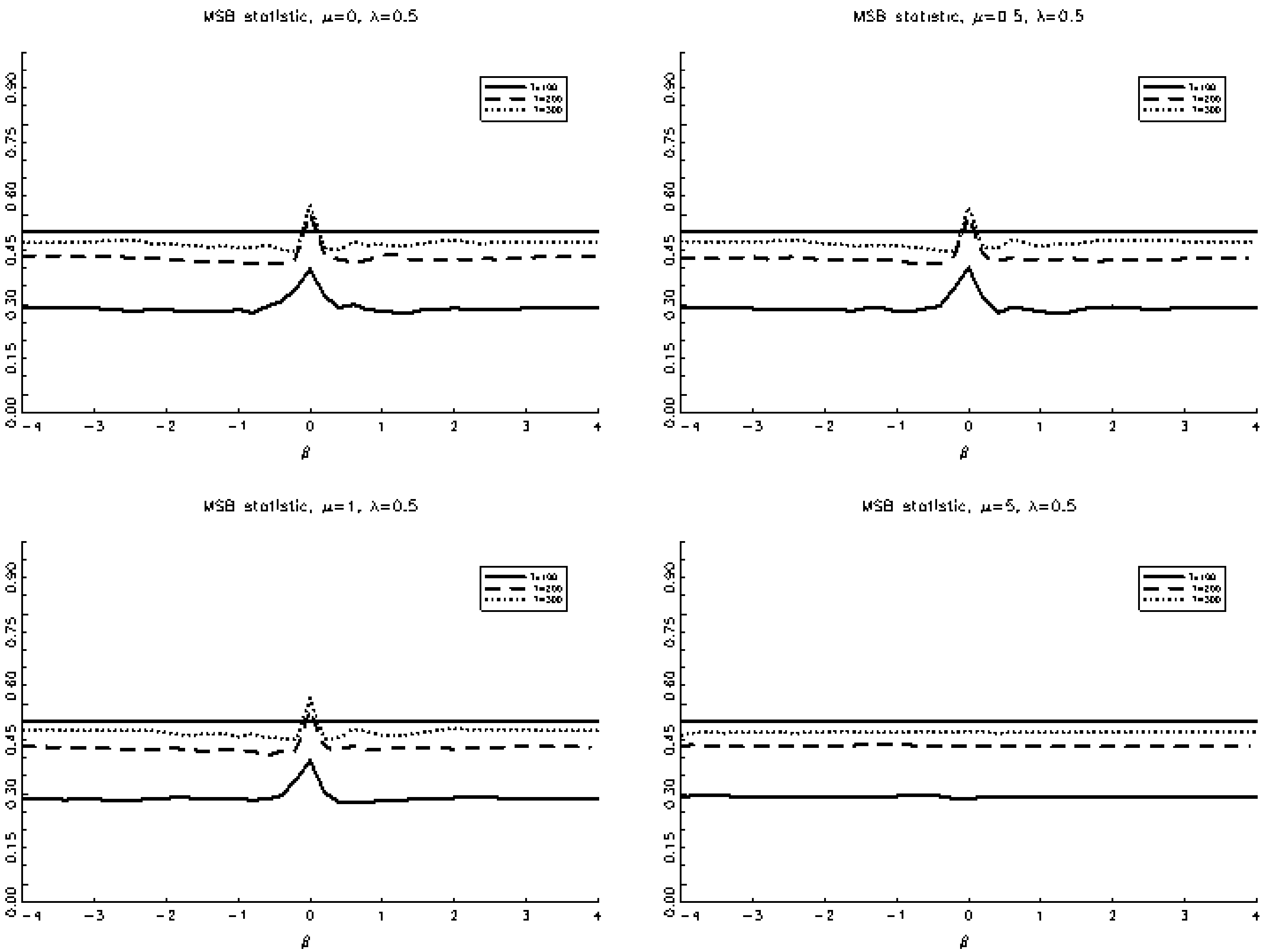
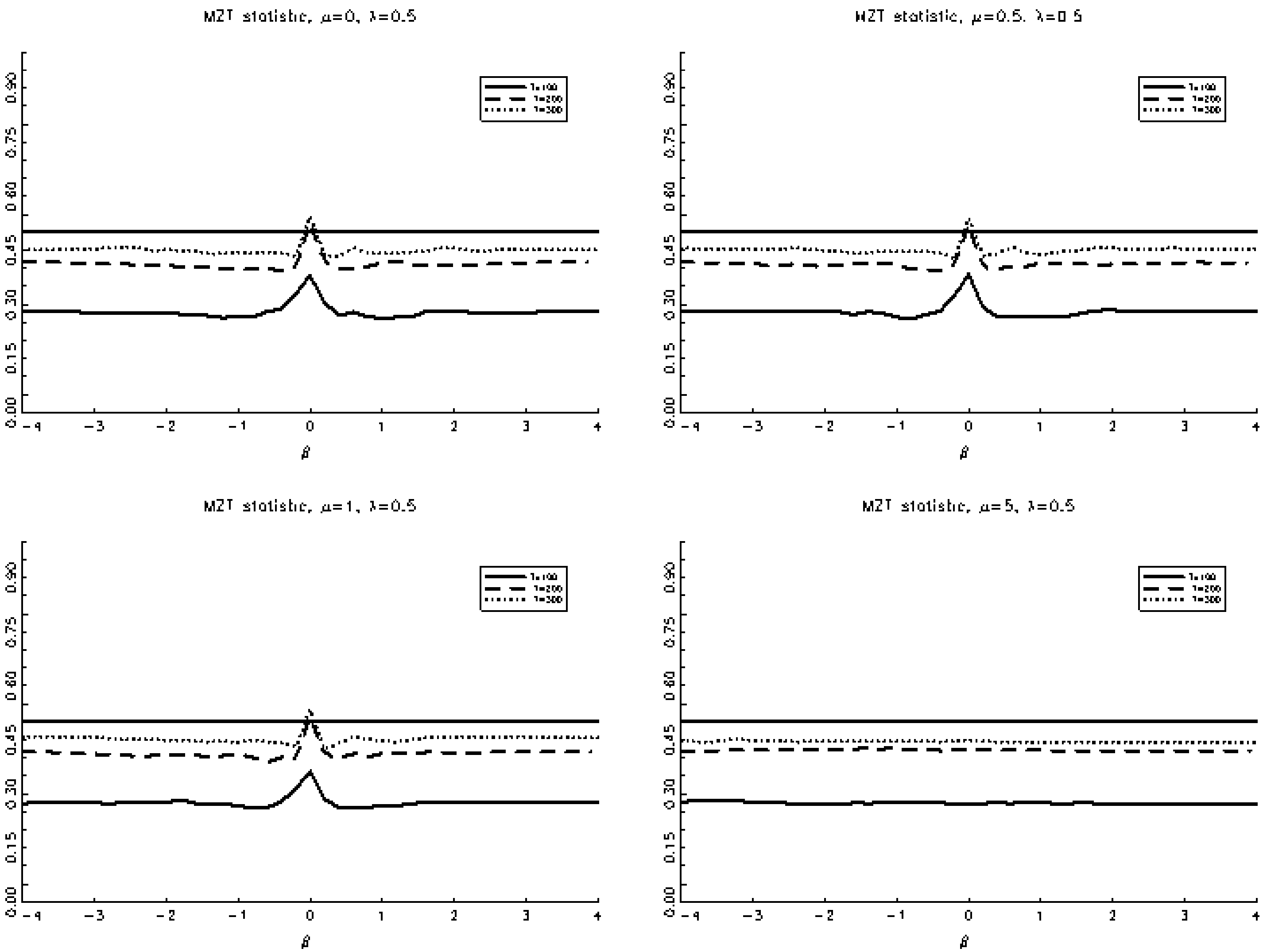


Figure 35: Empirical power for the MZT test, $\lambda = 0.5$



Finite sample performance: one structural break

- For the P_T and MP_T statistics, the size distortions are not large, although the theoretical derivations indicates that, unless there is a large level shift, $\hat{\lambda} = \arg\min_{\lambda \in \Lambda(\varepsilon)} S(\bar{\alpha}, \lambda)$ does not converge at a fast enough ratio to warrant that the statistics have the same limiting distribution as for the known break case
- The ADF and ZA statistics show an over-rejection tendency
- The MZA and MSB statistics have the right size as T increases
- The tests have good power

Finite sample performance: two structural breaks

Features of the Monte Carlo experiment:

- Empirical size $(\alpha = 1)$ and power $(\alpha = \bar{\alpha})$ is investigated using the following DGP

$$
\begin{aligned}
y_t &= d_t + u_t \\
d_t &= \mu_1 DU_t(T_1^0) + \beta_1 DT_t^*(T_1^0) + \mu_2 DU_t(T_1^0) + \beta_2 DT_t^*(T_2^0) \\
u_t &= \alpha u_{t-1} + v_t,
\end{aligned}
$$

1. The magnitude of the level shifts $\mu_1 = \mu_2 = \mu_b = \{0, 0.5, 1, 5\}$
2. β_1 ranging from -4 to 4 in increments of 0.2
3. β_2 ranging from -5 to 5 in increments of 0.25
4. Three vectors of break fractions $\lambda^0 = (0.3, 0.5)$, $(0.3, 0.7)$, and $(0.5, 0.7)$
5. The sample size is set at $T = \{100, 200, 300\}$
6. $v_t \sim iid\ N(0, 1)$, $u_0 = 0$.

Graphs for the empirical size of the statistics with $\lambda_3^0 = 0.5$ and $\lambda_2^0 = 0.5$ (no pre-testing)

Figure 43: Empirical size for the P_T test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

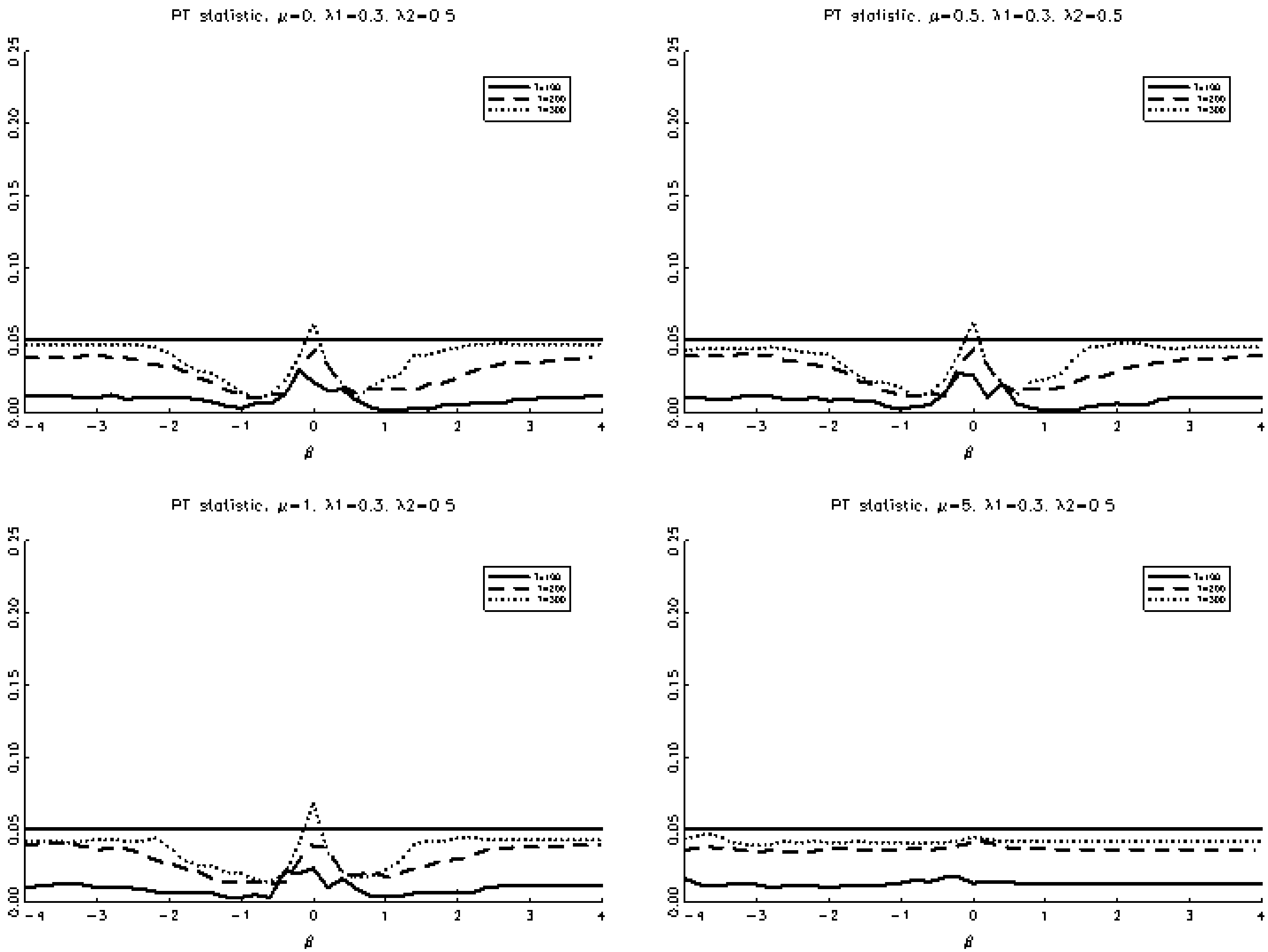


Figure 44: Empirical size for the MP_T test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

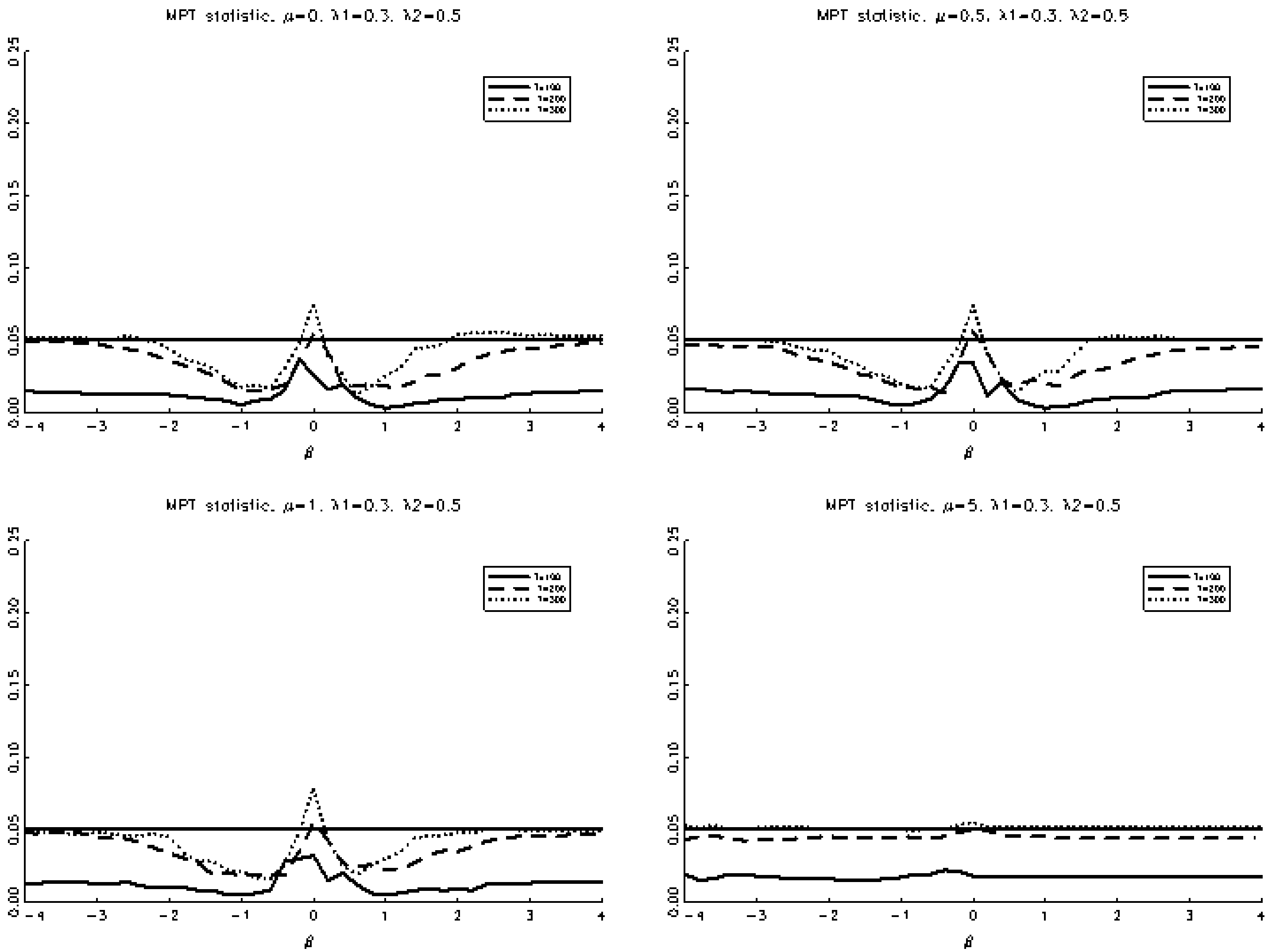


Figure 45: Empirical size for the ADF test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

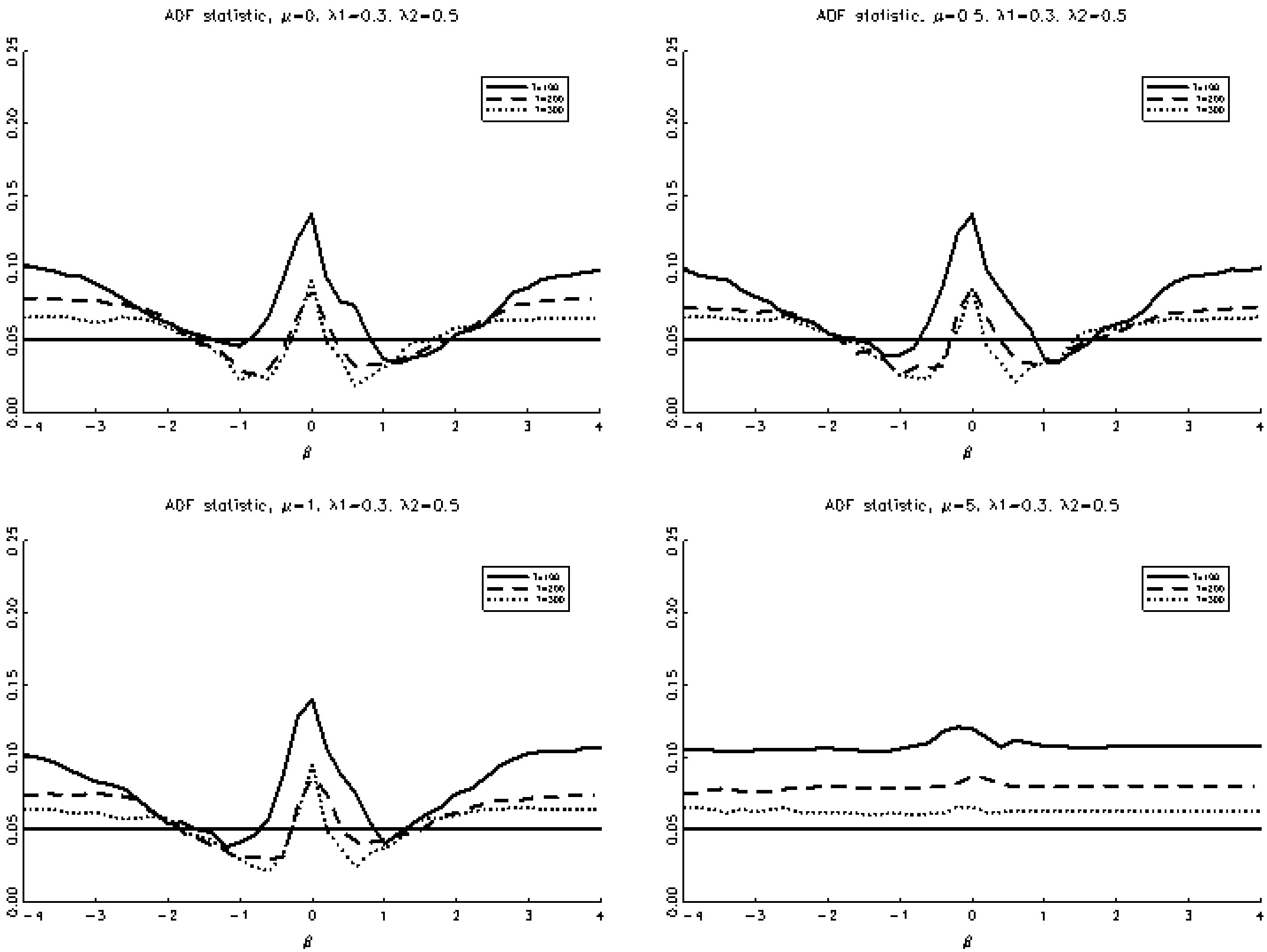


Figure 46: Empirical size for the ZA test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

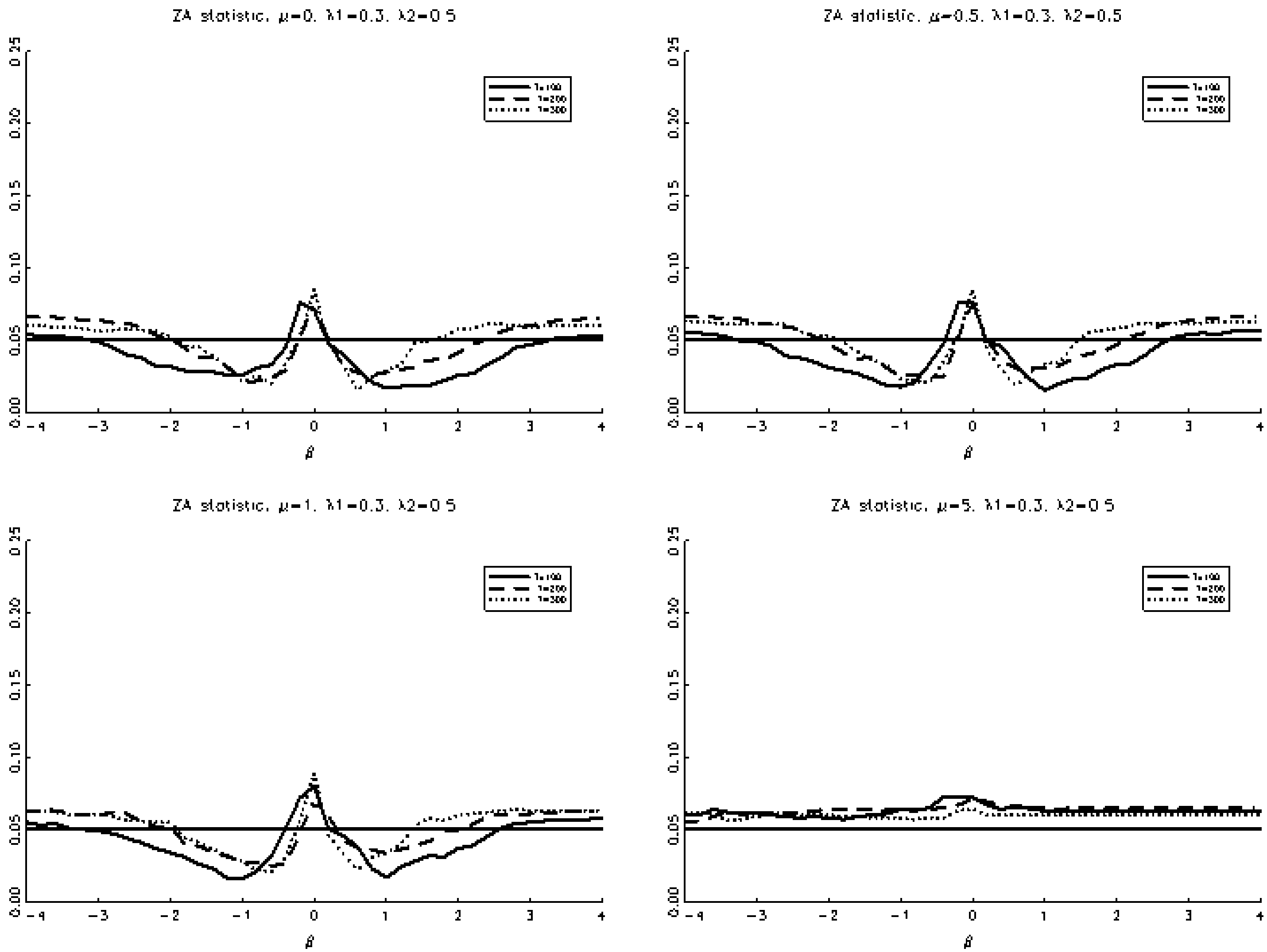


Figure 47: Empirical size for the MZA test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

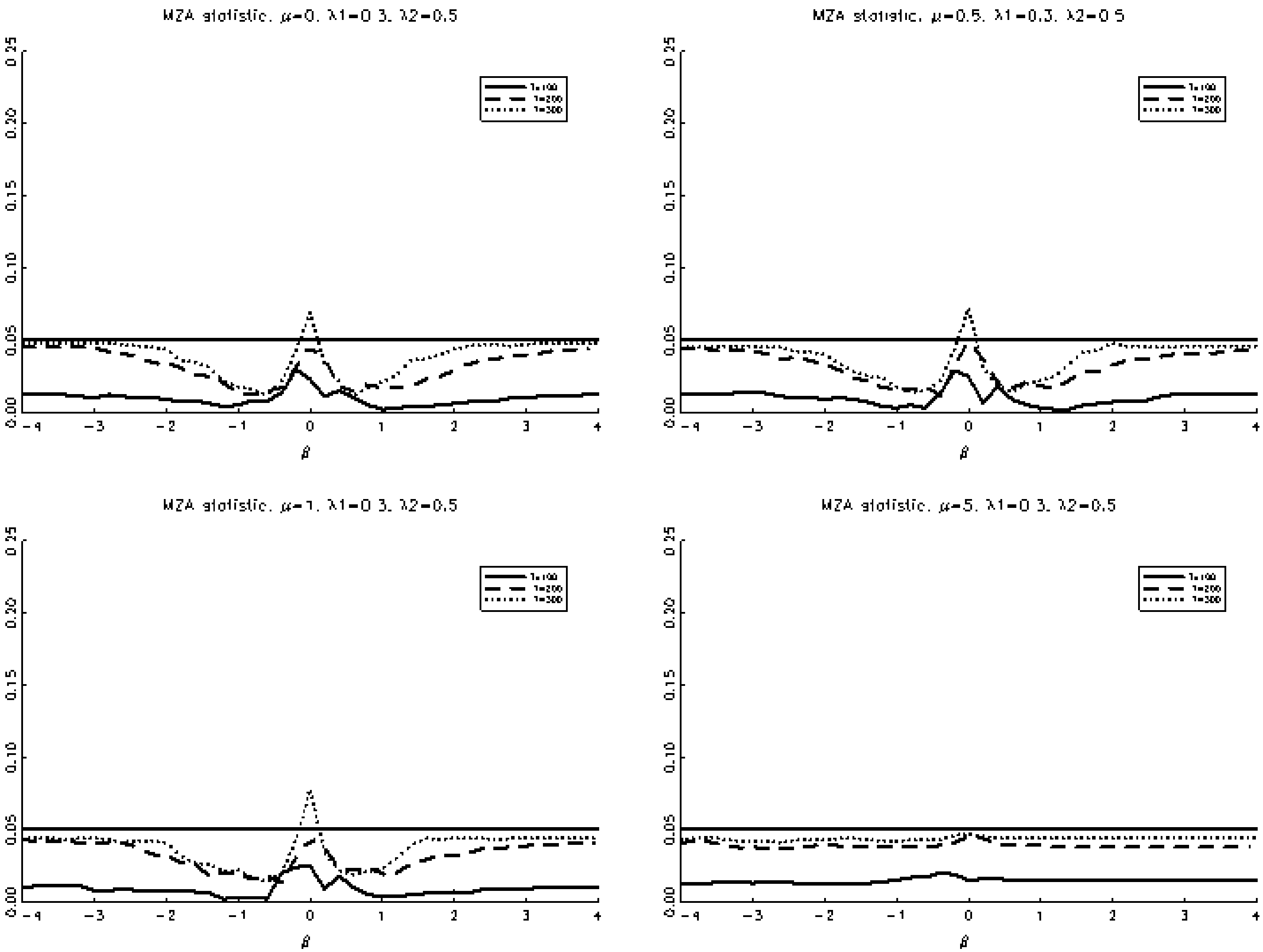


Figure 48: Empirical size for the MSB test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$

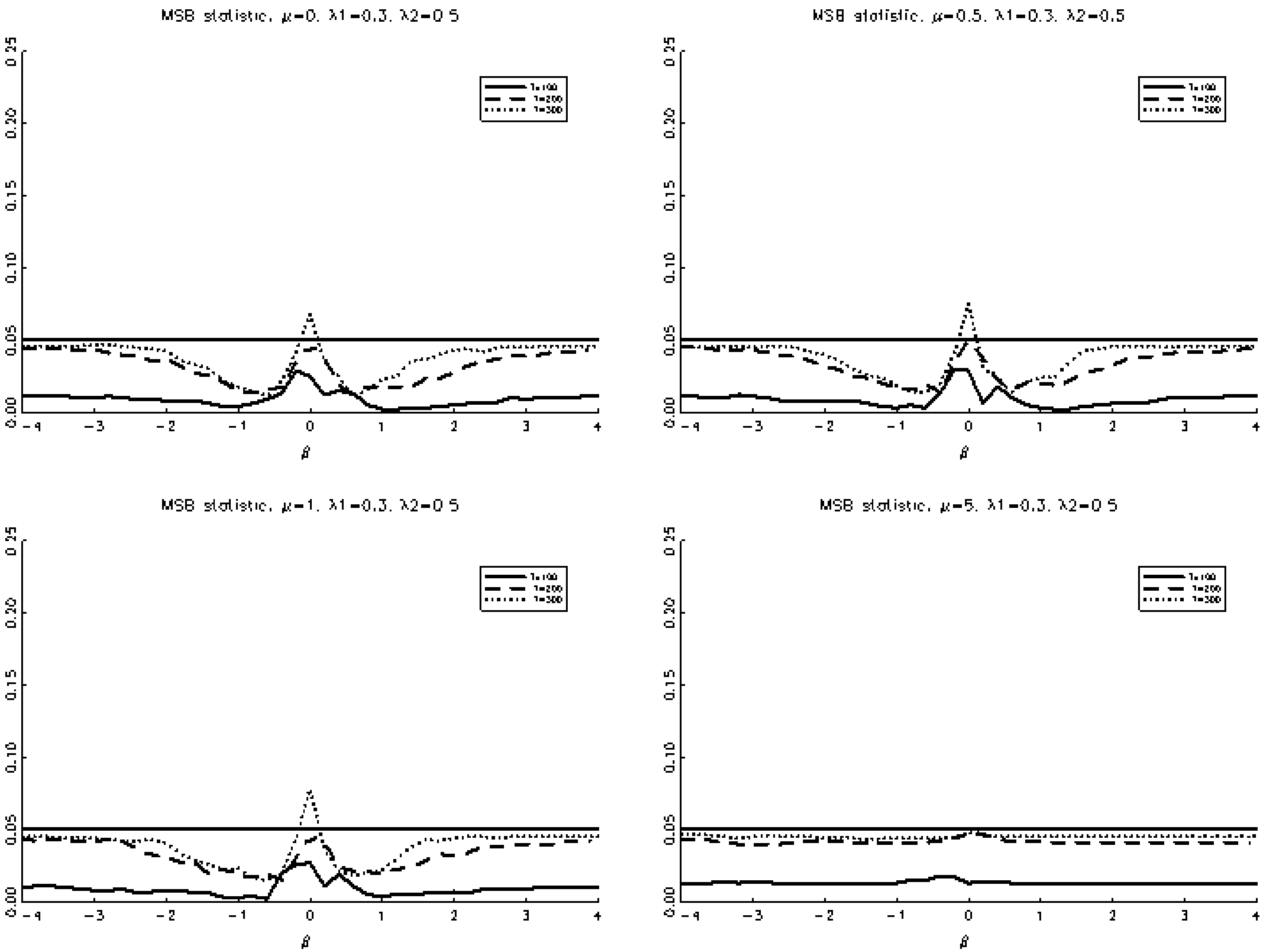
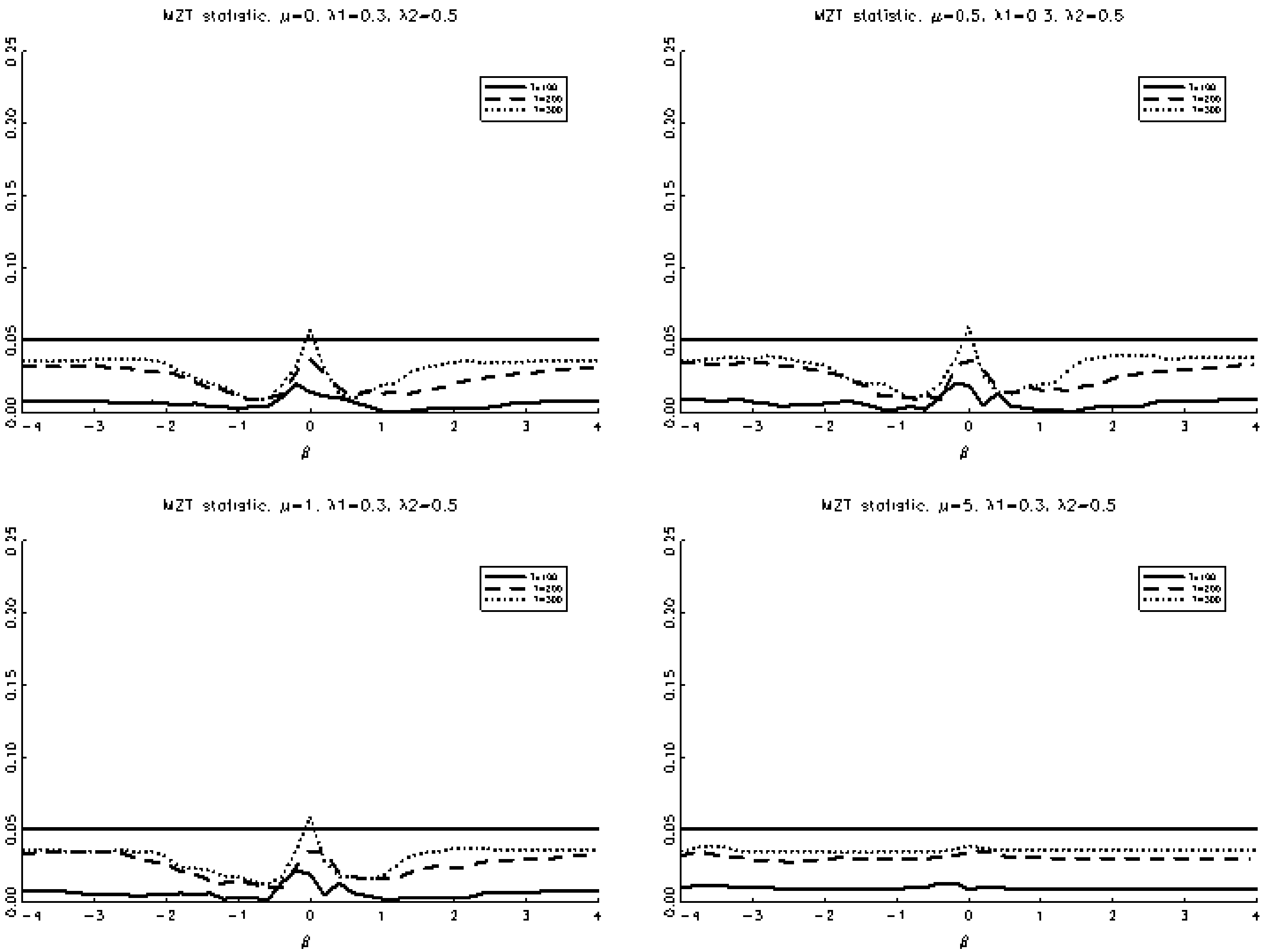


Figure 49: Empirical size for the MZT test, $\lambda_1 = 0.3$ and $\lambda_2 = 0.5$



Conclusions

- In this paper we have proposed up to seven test statistics to test the null hypothesis of unit root allowing for the presence of multiple structural breaks
- The structural breaks can affect either the level and/or the slope
- Minimization of the GLS-detrended-based sum of the squared residuals produces consistent estimates of the break fraction vector
  1. For the P_T and MP_T statistics the rate of convergence of these estimates is not fast enough to warrant that the statistics have the same limiting distribution as for the known break case
  2. For the other statistics, the use of these estimated break fractions leads to test statistics with the same limiting distribution as for the known breaks case

Conclusions

- Response surfaces have been estimated to approximate, for up to $m = 5$ breaks:
  1. the parameter that is used in the GLS estimation $(\bar{c})$
  2. the asymptotic critical values
- An efficient dynamic algorithm is designed to obtain the estimates when there are more than one structural break (less time consuming)
- Monte Carlo simulations indicate that the statistics, when combined with pre-testing, has good statistical properties in terms of empirical size and power
  1. Pre-testing using the approach in Perron and Yabu (2005) is highly recommended to avoid size distortions when testing the order of integration