Bargaining for Over-The-Counter Risk Redistributions: The Case of Longevity Risk

Tim Boonen, Anja De Waegenaere and Henk Norde
Netspar, CentER, Tilburg University
Introduction
Goal: redistributing stochastic variables (risk) Over-The-Counter in “fairest way”

Setting:
- Cooperative game-theoretic model
- Redistribution obtained via swap-contracts

Allow for all forms of redistributions

Key issue:
- No liquid market
- Trade Over-The-Counter
Focus: Longevity risk; Why?

- Illiquid market, where there are no equilibrium prices
- Redistributions between annuities and death benefits (cf. Wang et al. (2010))
- Literature shows that longevity risk is prominent for pension funds and life insurers. See e.g. Hári et al. (2008) and Coughlan et al. (2007)
- Prices are heavily debatable (see Bauer et al (2010)). Two focusses:
  - equivalent utility pricing principle (Cui (2008) and Cox, Lin and Pedersen (2010))
  - Prices obtained directly from (scarce) longevity-linked bonds in the market (Lin and Cox (2005))
- We model the OTC bargaining problem as a Non-Transferable Utility (NTU) game.
- We allow for heterogeneous beliefs regarding the underlying probability distribution. Very relevant for applications with longevity risk.
- Calibrated example shows hedge benefit is large.
Longevity risk
Longevity risk: Risk that individuals live longer or shorter than expected

- Micro longevity risk diminishes if pool size is sufficiently large (see Oliveiri and Pitacco (2001), Milevsky, Promislow and Young (2006) and Hári et al. (2008))
- Macro longevity risk: Risk that the population as a whole lives longer or shorter
  - Systematic part of longevity risk

We focus on macro longevity risk.
Key issue: large variety of longevity risk models

Prominent examples:
- Lee-Carter model (1992)
- P-spline model (Currie, Durban and Eilers (2004))

Different data used for obtaining longevity distribution
- For instance, different horizon of data
The model
Firms redistribute risk in order to increase expected utility of the present value of the Net Asset Value at a future evaluation date $T$:

$$X_i(T) \equiv \frac{NAV_i(T)}{(1 + r)^T} = \frac{A_i(T) - L_i(T)}{(1 + r)^T},$$

where

- $A_i(T)$ is the asset value at time $T$
- $L_i(T)$ the value of the liabilities. Typically:

$$L_i(T) = BEL_i(T) + MVM_i(T),$$

where $BEL_i(T)$ is the best estimate of future liability payments and $MVM_i(T)$ the market value margin (e.g. according to Solvency II) (a risk loading)

- $r$ is the risk-free rate
Solvency II: set financial return equal to risk-free rate.

We have

\[ A_i(t) = (1 + r)A_i(t - 1) - \tilde{L}_{i,t}, \]

where \( \tilde{L}_{i,t} \) is the liability payment at time \( t \).

Hence, we obtain

\[ X_i(T) = A_i(0) - \sum_{\tau=1}^{T} \frac{\tilde{L}_{i,\tau}}{(1 + r)^\tau} - \frac{L_i(T)}{(1 + r)^T}. \]
Important to note:

- In the current literature, redistributions have longer maturity and intermediate payment dates
- Then, every year there is a payment
- In our model, we allow for this, namely as $T = T^{\text{max}}$:

$$X_i(T^{\text{max}}) = A_i(0) - \sum_{\tau=1}^{T^{\text{max}}} \frac{\tilde{L}_{i,\tau}}{(1 + r)^\tau}$$
• Rolling contract every year more dynamic as we can take into account that
  - The mortality model can be updated
  - There has been attrition
  - New participants have entered the fund
  - New regulations have been introduced
  - Poor asset returns increase need for hedging longevity

• Moreover, we obtain in a calibrated example that the standard deviation of $X_i(1)$ is approximately 50% of standard deviation of $X_i(T_{max})$
The Game
Firms use a Von-Neuman-Morgenstern utility function \( u_i \) such that \( u_i' > 0, \ u_i'' < 0 \)

Let the risk profiles be given by \( (X_i(T))_{i \in N} \) and the (heterogeneous) probability measures by \( (\Omega, (\mathbb{P}_i)_{i \in N}) \), where \( \Omega \) finite

There is complete information about the risk profiles, utility functions and beliefs regarding the underlying probability measures of all firms
What is a Non-Transferable Utility game \((N, V)\)?

Nash-Bargaining problem (Nash (1950)) in case of 2 firms:

\[ V(\{1, 2\}) \]
Bargain for \((X_{i}^{\text{post}})_{i \in N}\) such that \(\sum_{i \in N} X_{i}^{\text{post}} = \sum_{i \in N} X_{i}\)

- Firms valuate a risk using

\[
\Delta U_{i}(X_{i}^{\text{post}}) = E^{\mathbb{P}_{i}}[u_{i}(X_{i}^{\text{post}}) - u_{i}(X_{i})]
\]

- Than, the we define the game:

\[
V(S) = \left\{ a \in \mathbb{R}^{S} \bigg| \exists (X_{i}^{\text{post}})_{i \in S} \in \mathbb{R}^{\Omega \times S} : \sum_{j \in S} X_{j}^{\text{post}} = \sum_{i \in S} X_{i}, a \leq (\Delta U_{i}(X_{i}^{\text{post}}))_{i \in S} \right\},
\]

for all \(S \subset N\).
Pareto optimal

- Every $a \in V(N)$ such that there does not exist a redistribution $(X_{i}^{\text{post}})_{i \in N}$ such that
  $$(\Delta U_i(X_{i}^{\text{post}}))_{i \in N} \geq a$$

Individually Rational

- $\Delta U_i(X_{i}^{\text{post}}) \geq 0$ for all firms in $N$

Core-element

- For every element for the core, there does not exist a subset of firms that can form a redistribution that is weakly beneficial for all members of this set and strict for at least one firm
- Core is non-empty
Numerical implementation
Interest rate is given by $r = 0.03$

$MVM_i(T) = 0$ for all $i \in N$ and for all $T \geq 1$

All firms use same Lee-Carter model and same data-set: $P_i = \mathbb{P}$ (will be relaxed)

We use data about a “realistic” liability portfolio of a pension fund

We assume that the pension fund has 50,000 participants; each receive 1 unit a year after retirement;

For the death benefit insurer, we assume:

- Fixed pay-off of 10 units in case of death before retirement
- young participants
- varying size
Let there be an average age pension fund and a death benefit insurer. Value liabilities $X^\ell_i(1) = A_i(0) - X_i(1)$; prior and posterior:
Pay-off of swap only time $T = 1$: 

![Histogram of frequency distribution](chart.png)
$X_i(T)$ as function of $T$, mean, 2.5%-quantile and 97.5-quantile:
zero-utility premium $= p : \Delta U_i (X_i^{post} - \mu_i) = 0 \, \forall i \in N$,
buffer $= \frac{Q_{0.975}(X_i^\ell(T)) - E[X_i^\ell(T)]}{E[X_i^\ell(T)]}$.

- We obtain:
  - Risk redistribution has worth approximately 375 for both firms (zero-utility principle), in case of an average age pension fund and a death benefit insurer
  - buffer reduces from 1.98% to 0.54% for pension fund and from 9.25% to 1.34% for the death benefit insurer:

<table>
<thead>
<tr>
<th>$T$</th>
<th>Zero-Utility premium</th>
<th>% reduction buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pension fund</td>
<td>Insurer</td>
</tr>
<tr>
<td>1</td>
<td>377</td>
<td>374</td>
</tr>
<tr>
<td>5</td>
<td>1285</td>
<td>1246</td>
</tr>
<tr>
<td>10</td>
<td>1717</td>
<td>1650</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>2322</td>
<td>2201</td>
</tr>
</tbody>
</table>
Gains as function of size $\gamma$ of death benefit insurer:
The case of two death benefit insurers and one pension fund.

Let two death benefit insurers have size $\frac{\gamma}{2}$, so that total risk equals two-firm problem previously.

Then, for $T = 1$:

<table>
<thead>
<tr>
<th></th>
<th>Pension fund</th>
<th>Insurers ($i = 2, 3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>zero-utility premium</td>
<td>168</td>
<td>168</td>
</tr>
<tr>
<td>% reduction buffer</td>
<td>80%</td>
<td>78%</td>
</tr>
</tbody>
</table>
According to Borch (1962) and homogeneous probability measures, all Pareto optimal outcomes are obtained using \( \sum_{j \in N} X_j(T) \) only.

Here, heterogeneous probability measures \((P_i)_{i \in N}\) on \( \sum_{j \in N} X_j(T) \) only are relevant for determining Pareto set.

Therefore, we discretize \( \sum_{j \in N} X_j(T) \) by a partition of the interval.

Every probability measure will result in different probabilities on “attaining” a part of the partition.