Comparison of Pricing Approaches for Longevity Markets

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Background

- Longevity Risk in Pensions and annuity portfolios.
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- LAGIC in Australia; Solvency II in U.K.
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- Longevity Risk in Pensions and annuity portfolios.
- LAGIC in Australia; Solvency II in U.K.
- Longevity-linked securities: Bonds, swaps, options.
Motivation

- Capital markets want to diversify their portfolio; annuity providers/pension funds want to hedge their longevity risk. Win-Win.
Capital markets want to diversify their portfolio; annuity providers/pension funds want to hedge their longevity risk. Win-Win.

Create longevity instruments allowing for capital markets to buy into.
Capital markets want to diversify their portfolio; annuity providers/pension funds want to hedge their longevity risk. Win-Win.

Create longevity instruments allowing for capital markets to buy into.

Challenge: find "the" fair price for longevity risk.
Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.
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- Investigate four approaches used to price $S$-forwards
Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.

- Investigate four approaches used to price $S$-forwards

- Comment on the results based on the four approaches.
Cairns Blake and Dowd Model

Denote a 1-year death probability for a person currently aged $x$ at time $t$ by $q_{x,t}$, this is modeled via,

$$
\ln \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}).
$$

(1)

Where $\bar{x}$ is the average of the ages. We adapt this model to incorporate an error component in the measurement equation so that a state-space approach can be applied.

$$
\ln \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}) + \epsilon_t
$$

(2)

Cairns et al. (2006) suggests that $\kappa_{1,t}$ and $\kappa_{2,t}$ can be modeled by a 2-dimension random walk with drift,

$$
\begin{bmatrix}
\kappa_{1,t} \\
\kappa_{2,t}
\end{bmatrix} =
\begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix} +
\begin{bmatrix}
\kappa_{1,t-1} \\
\kappa_{2,t-1}
\end{bmatrix} + \omega_t
$$
State-space framework

Our framework is as follows:

\[
    y_t = \begin{bmatrix}
        \ln \left( \frac{q_{x_1,t}}{1-q_{x_1,t}} \right) \\
        \vdots \\
        \ln \left( \frac{q_{x_n,t}}{1-q_{x_n,t}} \right)
    \end{bmatrix}
    = \begin{bmatrix}
        1 & (x_1 - \bar{x}) \\
        1 & (x_2 - \bar{x}) \\
        \vdots & \vdots \\
        1 & (x_n - \bar{x})
    \end{bmatrix}
    \begin{bmatrix}
        \kappa_{1,t} \\
        \kappa_{2,t}
    \end{bmatrix}
    + \begin{bmatrix}
        \varepsilon_{x_1,t} \\
        \vdots \\
        \varepsilon_{x_n,t}
    \end{bmatrix},
\]

\[
    \begin{bmatrix}
        \kappa_{1,t} \\
        \kappa_{2,t}
    \end{bmatrix}
    = \begin{bmatrix}
        \theta_1 \\
        \theta_2
    \end{bmatrix}
    + \begin{bmatrix}
        \kappa_{1,t-1} \\
        \kappa_{2,t-1}
    \end{bmatrix}
    + \omega_t.
\]

Where \( \varepsilon_t \sim \text{i.i.d } \mathcal{N}(0, \sigma^2_{\varepsilon}) \) and \( \omega_t \sim \mathcal{N}(0, \Sigma_\omega) \).
Prior Choices

Parameters are estimated Markov Chain Monte Carlo method\(^1\),
\[
\pi(\sigma_{\varepsilon}^2) \sim I.G(a_{\varepsilon}, b_{\varepsilon})
\]
\[
\pi(\theta) \sim N(\mu_\theta, \Sigma_\theta)
\]
\[
\pi(\Sigma_\omega | \Sigma_{11}, \Sigma_{22}) \sim I.W\left(\nu + 2 - 1, 2\nu \text{ diag}\left(\frac{1}{\Sigma_{11}}, \frac{1}{\Sigma_{22}}\right)\right)
\]
\[
\pi(\Sigma_{kk}) \overset{i.i.d}{\sim} I.G\left(\frac{1}{2}, \frac{1}{A_k}\right) \quad \text{for} \quad k \in (1, 2)
\]

Priors were chosen such that they had conjugate forms to their respective likelihoods. A hierarchical structure was chosen for \(\Sigma_\omega\), to avoid a biased estimation from a regular Inverse-Wishart prior (Huang et al., 2013; Gelman et al., 2006). The hyper parameters were chosen such that the priors were non-informative.

\(^1\)I.G Inverse.Gamma(\(\alpha, \beta\)), N is Normal(\(\mu, \sigma^2\)), I.W is Inverse.Wishart(\(\nu, \phi\))
Summary statistics

N=10000 draws, 3000 burn-in period, using joint mortality of Australian dataset 1961-2011 taken from the Human Mortality Database (HMD)

Table 1: Summary Statistics for the estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Mean</th>
<th>95% HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>$4.79325 \times 10^{-2}$</td>
<td>($-2.48523 \times 10^{-2}$, $-5.81384 \times 10^{-2}$)</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$4.22871 \times 10^{-3}$</td>
<td>($7.29511 \times 10^{-5}$, $7.69795 \times 10^{-4}$)</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>$2.63008 \times 10^{-2}$</td>
<td>($2.45841 \times 10^{-3}$, $2.8157 \times 10^{-3}$)</td>
</tr>
<tr>
<td>$\Sigma_{11}$</td>
<td>$1.15681 \times 10^{-3}$</td>
<td>($7.11692 \times 10^{-4}$, $1.82368 \times 10^{-3}$)</td>
</tr>
<tr>
<td>$\Sigma_{12}$</td>
<td>$2.42572 \times 10^{-5}$</td>
<td>($1.08446 \times 10^{-5}$, $4.249324 \times 10^{-5}$)</td>
</tr>
<tr>
<td>$\Sigma_{22}$</td>
<td>$1.56558 \times 10^{-6}$</td>
<td>($9.0352 \times 10^{-7}$, $2.516597 \times 10^{-6}$)</td>
</tr>
</tbody>
</table>
Fitted curves $\kappa_t$
Consider the $k^{th}$ step, Let $N$ be the draws after the burn in period and $n = 1, \ldots, N$. Then, $\kappa_{T+k}^{(n)} \sim N(\kappa_{T+k-1}^{(n)} + \theta^n, (\Sigma_{\omega})^{(n)})$, $y_{T+k}^{(n)} \sim N(\kappa_{T+k,1}^{(n)} + (x - \bar{x})\kappa_{T+k,2}^{(n)}, (\sigma^2_{\epsilon})^{(n)}\mathbb{I})$. 

![Survival curve at age 65](image)
We studied at 4 different pricing approaches:

1) Risk-neutral method (Cairns et al., 2006)
2) The 2-factor Wang transform (Wang, 2002)
3) Canonical valuation/ Maximum entropy method (Li and Ng, 2011)
4) An economic approach/ Tatonnement economics (Zhou et al., 2015)

The first two of these methods require data to find the risk-premium $\lambda$. Hence, we will use the issued but not sold EIB-bond to calibrate.
Using the setup for the EIB-bond Cairns et al. (2006), we apply Australian mortality projections to males aged 65 with a longevity spread of $\delta = 0.002$ over a $T = 25$ year period.

1) The price obtained by EIB/BNP was in 2004, we assume that the prices have not been inflated since that time for 2011.

2) The original EIB-bond was setup for England and Welsh males aged 65, we assume the same longevity spread $\delta$ for Australian population.

3) For ease of calculations, we assume a constant interest rate of 3%.

Let $\bar{\Pi}_t(x, T)$ be the bond price at time $t$, and $\hat{S}(x, i)$ is the risk-neutral survival probability then,

$$\bar{\Pi}_t(x, T) = \sum_{i=1}^{T} P(t, i)e^{\delta i} \hat{S}(x, i)$$

Under these assumptions, we find that the bond price $\bar{\Pi}_0 \approx 13.46739$
S-forward

Definition

An S-forward contract is a swap where the fixed rate payer pays an amount \( K \in (0, 1) \) in exchange for the realised survival probability \( Tp_x \). An S-forward contract written for a population aged \( x \) at time \( t \), over a maturity period \( T \), will thus have a pricing formula under risk-neutral density is given by:

\[
SF(x, t, T, K) = P(t, T)E_Q[Tp_x - K|\mathcal{F}_t].
\]

Since an S-Forward contract has $0$ inception cost, we have to find the value of \( K(T) \) such that the there will be no upfront cost.

\[
K(T) = E_Q[Tp_x|\mathcal{F}_t]
\]
Pricing an $S$-forward

Under the 4 different pricing methodologies, if we were to price an $S$-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
   \[ K(T) = E_Q [ T \hat{p}_x | \mathcal{F}_t ] = \tilde{S}(x, T) \]
Pricing an $S$-forward

Under the 4 different pricing methodologies, if we were to price an $S$-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
$$K(T) = E_Q [\tau p_x |\mathcal{F}_t] = \tilde{S}(x, T)$$

2) Under Wang Transform Method,
$$K(T) = E \left[ Q \left( \Phi^{-1}(S(x, t)) + \lambda \right) \right]$$
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2) Under Wang Transform Method,  
   \[ K(T) = E [Q (\Phi^{-1}(S(x, t)) + \lambda)] \]

3) Under Canonical Valuation method,  
   \[ K(T) = \tau p_x^{market} \]
Pricing an $S$-forward

Under the 4 different pricing methodologies, if we were to price an $S$-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
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2) Under Wang Transform Method,
   \[ K(T) = E [ Q ( \Phi^{-1}(S(x, t)) + \lambda ) ] \]

3) Under Canonical Valuation method, $K(T) = Tp^\text{market}_x$

4) Under the Tatonnement Approach, the value $K(T)$ is determined by the market based on supply and demand.
Risk-neutral pricing method

Our 2-D random walk with drift process:

\[ \kappa_t = \theta + \kappa_{t-1} + (\Sigma_\omega)^{\frac{1}{2}} Z \]

Where, \((\Sigma_\omega)^{\frac{1}{2}}(\Sigma_\omega)^{\frac{1}{2}} = \Sigma_\omega\) and \(Z \sim N(0, \mathbb{I})\) is under real-world probability measure \(\mathbb{P}\). Cairns et al. (2006) suggests that similar to the continuous time case, we can convert to the risk-neutral density (equivalent martingale measure) by,

\[ \tilde{Z} = \lambda + Z, \]

Where, \(\lambda\) is the market price of longevity risk and \(\tilde{Z} \sim N(0, \mathbb{I})\) under \(\mathbb{Q}\). Then,

\[ \kappa_t = \kappa_{t-1} + (\theta - (\Sigma_\omega)^{\frac{1}{2}} \lambda) + (\Sigma_\omega)^{\frac{1}{2}} \tilde{Z} \]
Risk-neutral pricing method

Under risk-neutral assumption the EIB-bond price is given by:

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) E_{Q(\lambda)} \left[ e^{-\int_{t}^{i} \mu_x(u) du} | \mathcal{F}_t \right]. \quad (5)$$

Matching the price at initial time $t = 0,$

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, i, \lambda)$$

<table>
<thead>
<tr>
<th>Market Price of Risk</th>
<th>Value</th>
<th>$\tilde{\Pi}_0(65, 25)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda_1, \lambda_2)$</td>
<td>$(0.27307, 0.27307)$</td>
<td>13.46739</td>
</tr>
<tr>
<td>$(\lambda_1, \lambda_2)$</td>
<td>$(0.24505, 0)$</td>
<td>13.46739</td>
</tr>
</tbody>
</table>
2-factor Wang Transform

Wang (2002) proposes a universal pricing method, such that, assuming we have a liability $X$ over a time period $[0, T]$ with $F_X(x) = P(X < x)$, then with a market price of risk $\lambda$, the risk-adjusted (distorted) function of $F(X)$ can be found by,

$$F^*(x) = Q \left( \Phi^{-1}(F(x)) + \lambda \right)$$

Where, $F^*(x)$ is the risk-adjusted function for $F(x)$ and $Q \sim Student - t(\nu)$, Since our aim is to find the risk-neutral survival probability(our underlying), we have,

$$\tilde{S}(x, t) = E \left[ Q \left( \Phi^{-1}(S(x, t)) + \lambda \right) \right] \text{ for } t \in [0, T]$$
2-factor Wang Transform

To find $\lambda$ using the EIB-bond

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) Q (\Phi^{-1}(S(x, T)) + \lambda).$$  \hfill (6)

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, T).$$

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<td>$\lambda$</td>
<td>0.3478043</td>
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</table>
The maximum entropy principle was first proposed by Stutzer (1996) and used by Kogure and Kurachi (2010); Foster and Whiteman (2006) used in longevity context to find market survival probability denoted by $\tilde{T}p^\text{market}_x$. In our case we by using the EIB-Bond in combination with the maximum entropy principle to find $\tilde{T}p^\text{market}_x$. 
The four pricing approaches

S-forward

Canonical Valuation methodology

1. Let \( \mathbf{p}_x^j = (p_{x}^{1j}, p_{x}^{2j}, \ldots, p_{x}^{Tj}) \), for \( j = 1, \ldots, N \), and let \( \pi \) denote the empirical distribution for \( \mathbf{p}_x \).

2. \( \bar{\Pi} \) denotes the market price of the EIB-bond \( \bar{\Pi}(65,25) \).

3. Let \( \pi^* \) be the risk-neutral distribution for \( \pi \), then \( \sum_{j=1}^{N} \pi_j \pi_j^* = \bar{\Pi} \).

4. Then the maximum entropy principle stipulates that, \( \pi^* \) should minimize the Kullback-Leiber Information divergence, \( \sum_{j=1}^{N} \pi_j^* \ln \left( \frac{\pi_j^*}{\pi_j} \right) \), subject to the constraint \( \pi_j^* > 0 \) and \( \sum_{j=1}^{N} \pi_j^* = 1. \)
Kapur and Kesavan (1992) derived the solution to the minimization of $\sum_{j=1}^{N} \pi_j^* \ln \left( \frac{\pi_j^*}{\pi_j} \right)$ subject to $\bar{\Pi}$, which is given by

$$\hat{\pi}_j^* = \frac{\pi_j \exp(\gamma \bar{\Pi}^j)}{\sum_{j=1}^{N} \pi_j \exp(\gamma \bar{\Pi}^j)}$$

2. Find $\gamma$ from, $\bar{\Pi} = \frac{\sum_{j=1}^{N} \bar{\Pi}^j \exp(\gamma \bar{\Pi}^j)}{\sum_{j=1}^{N} \exp(\gamma \bar{\Pi}^j)}$.

3. $\sum_{j=1}^{N} t^j p_x^j \pi_j^* = t^j p_x^{market}$

In our case, we don’t assume there is a risk premium $\lambda$, but there is a $\gamma$ parameter which ”corrects” the real world probability $t^j p_x$ to adjust for the market accepted $T p_x^{market}$.
The four pricing approaches

S-forward

Tatonnement Approach

This approach was first suggested by (Zhou et al., 2015). We price our S-forward instrument based on the equilibrium price that matches market supply and demand.

Assume we have a buyer (investor (B)) of an S-forward and a seller (hedger (A)).
Tatonnement Approach

This approach was first suggested by (Zhou et al., 2015). We price our S-forward instrument based on the equilibrium price that matches market supply and demand.

1. Assume we have a buyer (investor (B)) of an S-forward and a seller (hedger (A)).
2. Definition of the following parameters:
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   - $\theta_A$ is the supply of an S-forward, $\theta_B$ is the demand.
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2. Definition of the following parameters:
   - $\theta_A$ is the supply of an S-forward, $\theta_B$ is the demand.
   - $\omega_t^A$ is the wealth of A, $\omega_t^B$ is the wealth of B at time $t$. 
Tatonnement Approach

This approach was first suggested by (Zhou et al., 2015). We price our S-forward instrument based on the equilibrium price that matches market supply and demand.

1. Assume we have a buyer (investor (B)) of an S-forward and a seller (hedger (A)).
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   - $\theta_A$ is the supply of an S-forward, $\theta_B$ is the demand.
   - $\omega^A_t$ is the wealth of A, $\omega^B_t$ is the wealth of B at time $t$.
   - Denote the function $g(S(x, t))$ denote the gains of the S-forward at time $t$. 
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   - Denote the function $g(S(x, t))$ denote the gains of the S-forward at time $t$.
   - $f(S(x, t))$ represents the payout for the survival probability for the hedger at time $t$.

3. Then,
   $$\theta_A = \sup_{\theta_A} E \left\{ U\{ \omega^A_{t-1} e^r - \theta^A g(S(x, t)) - f(S(x, t)) \} \right\}$$
Tatonnement Approach

This approach was first suggested by (Zhou et al., 2015). We price our $S$-forward instrument based on the equilibrium price that matches market supply and demand.

1. Assume we have a buyer (investor (B)) of an $S$-forward and a seller (hedger (A)).
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   - $\omega^A_t$ is the wealth of A, $\omega^B_t$ is the wealth of B at time $t$.
   - Denote the function $g(S(x, t))$ denote the gains of the $S$-forward at time $t$.
   - $f(S(x, t))$ represents the payout for the survival probability for the hedger at time $t$.
3. Then,
   \[
   \theta_A = \sup_{\theta_A} E \left[ U\{\omega^A_{t-1}e^r - \theta^A g(S(x, t)) - f(S(x, t))\} \right]
   \]
4. \[
   \theta_B = \sup_{\theta_A} E \left[ U\{\omega^B_{t-1}e^r + \theta^B g(S(x, t))\} \right]
   \]
Tatonnement Approach

Since $g$ is an $S$-forward, we have, $g = (S - K)$. Choosing an Exponential Utility function, the following algorithm is used to obtain a price $K$. (Zhou et al., 2015).

for each time period $t \in [1, T]$

1. Guess an initial $K$. 

"The four pricing approaches
$S$-forward"
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1. Guess an initial $K$.
2. Determine $\theta_A$ and $\theta_B$. 
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for each time period \( t \in [1, T] \)

1. Guess an initial \( K \).
2. Determine \( \theta_A \) and \( \theta_B \).
3. if \( \theta_A = \theta_B \) then stop, and set \( K(t) = K \)
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3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
4. else, update $K$ by:
   1) $K^{i+1} = K^i + h^i$
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3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
4. else, update $K$ by:
   1) $K^{i+1} = K^i + h^i$
   2) Where $h^i = \gamma |K^i|(\theta_B - \theta_A)$
Results

For each of the methods, 1000 samples from the MCMC was used after burn-in. A Monte-Carlo average was taken when an expectation was involved. Using a portfolio which consists of people aged 65 at time 0, with a hedging period of $T = 5, 10, 15, 20, 25$ of the $S$-forward. The prices are shown below:

<table>
<thead>
<tr>
<th>Period</th>
<th>real-world</th>
<th>Risk-Neutral</th>
<th>Wang-T</th>
<th>Canonical</th>
<th>Tat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(5)$</td>
<td>0.93233</td>
<td>0.93577</td>
<td>0.96720</td>
<td>0.93219</td>
<td>0.93225</td>
</tr>
<tr>
<td>$K(10)$</td>
<td>0.83398</td>
<td>0.84430</td>
<td>0.90622</td>
<td>0.83378</td>
<td>0.83398</td>
</tr>
<tr>
<td>$K(15)$</td>
<td>0.69621</td>
<td>0.72295</td>
<td>0.80547</td>
<td>0.69648</td>
<td>0.69621</td>
</tr>
<tr>
<td>$K(20)$</td>
<td>0.51740</td>
<td>0.56046</td>
<td>0.65226</td>
<td>0.52053</td>
<td>0.51740</td>
</tr>
<tr>
<td>$K(25)$</td>
<td>0.31560</td>
<td>0.37075</td>
<td>0.44740</td>
<td>0.32598</td>
<td>0.31790</td>
</tr>
</tbody>
</table>
Bayesian inference allows us to have prediction uncertainty in a systematic way via the prior distribution.

The different choices of pricing approaches, produced "similar" results, except for the tatonnement approach. Under economic conditions, it shows that there really isn’t a need for a "premium" if both investor and hedger acts "rationally".

Under the transformation method, the premium is much higher than other approaches. This is because the effect of the distortion operator causes a greater change in mortality directly compared with the risk-neutral method.
Future research

- Investigation is still going on, to finding the "correct’ way to price instruments.
- At the moment this is all theoretical work, this paper introduces these methods, and applies to pricing an S-forward contract.
- Next ...
Thanks for Listening

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Markov-Chain Monte-Carlo (MCMC)

Let $\psi = (\sigma^2, \Sigma, \theta)$. An MCMC method will be used to explore the posterior distribution and parameter states.

- Obtain initial draws denoted by $\psi^0$.
- Conditional on $\psi^0$, find the distribution of latent states via the Kalman Filter.
- Latent variable $\kappa_{1:T}$ drawn recursively from Backward Sampling (Carter and Kohn, 1994).
- Conditional on drawn latent variable, draw model parameters from their respective conditional posterior density.


