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Nonparametric cointegration analysis of real exchange rates

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This study indirectly addresses the issue of potential nonlinearities in real exchange rate adjustment for 18 OECD economies 1973–1998 using recent developments in the theory of nonparametric cointegration. While the standard Johansen tests yield mixed evidence, the results from a new nonparametric approach are clearly supportive of real exchange rate stationarity. Since the latter approach allows for a relatively general data-generating process, the findings are consistent with nonlinear mean reversion.

I. INTRODUCTION

The recent literature on testing for long-run purchasing power parity (PPP) for the post-1973 era, while mostly favourable, still contains dissenting voices. Rogoff's (1996) verdict that the PPP puzzle is unresolved remains valid. One of the main issues addressed in this paper is how to overcome the low power of unit root tests especially against near stationary alternatives. Panel unit root tests had promised to overcome this problem.[1] However, they have not produced clear cut results on the stationarity of real exchange rates or the validity of long-run PPP.[2] Resolving the issue of mean reversion is quite crucial given that the assumption of long-run purchasing power parity (PPP) is one of the cornerstones of many open economy and nominal exchange rate models. The main motivation for this study is to test for real exchange rate mean reversion by allowing for nonlinearities in the data generating process (DGP).

The question of nonlinearities in mean reversion can be addressed in several ways. One consists of applying linearity tests and directly estimating nonlinear models. An analytical basis for such models has been provided by Dumas (1992) and Uppal (1993) who show that, in the presence of transaction costs, real exchange rates adjust towards equilibrium in a nonlinear fashion. Both O'Connell (1998a) and Coakley and Fuertes (1997) suggest that nonlinearities due to transaction costs seem consistent with the main features of the PPP puzzle outlined in Rogoff (1996). The main implication is that the speed of adjustment to restore long-run equilibrium depends on the magnitude of disequilibrium. In this context a growing literature has emerged on the estimation of threshold autoregressive (TAR) models and smooth transition autoregressive (STAR) models of real exchange rates.[3] A related but distinct approach invokes the idea that real exchange rates can be represented as stationary fluctuations around a deterministic trend function with possible shifts in slope and intercept by means of the breaking trend functions of Perron (1989, 1997).[4]

This study takes a different approach by indirectly addressing the issue of nonlinearities in real exchange

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[1] Panel unit root or cointegration tests, such as those developed by Abuaf and Jorion (1990), Im *et al.* (1997), Levin and Lin (1993), Pedroni (1995), Sarno and Taylor (1998) and Taylor and Sarno (1997), *inter alios*, have been extensively applied to PPP.

[2] See, for example, Papell (1997) and O'Connell (1998b).

[3] See, for example, Baum *et al.* (1998), Coakley and Fuertes (1997), Michael *et al.* (1997), Michael *et al.* (1997), Obstfeld and Taylor (1997), O'Connell (1998a), O'Connell and Wei (1997), Taylor *et al.* (2000).

[4] For PPP applications, see Papell (1998) and Perron and Vogelsang (1992).

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rate adjustment. It is motivated by the potential superiority of nonparametric methods at detecting cointegration when the error correction mechanism (ECM) is nonlinear.[5] In the latter case, mean reversion might not be adequately detected by standard linear methods, which assume a constant speed of adjustment. Recent simulation results in Pippenger and Goering (1993) and Balke and Fomby (1997) suggest a potential loss of power in standard unit root and cointegration tests under threshold autoregressive DGPs. This motivates testing for real exchange rate mean reversion in a panel of OECD economies over the 1973–1998 period applying the novel multivariate cointegration method of Bierens (1997a), which allows for a relatively general DGP. The Bierens test results are supportive of real exchange rate mean reversion while those from the Johansen (1988) full maximum likelihood approach are mixed. These findings are tentatively interpreted as adding to the recent evidence on nonlinearities in real exchange rates.

The paper unfolds as follows. Section II presents Bierens' nonparameteric cointegration tests and their asymptotic and small sample properties. The data description and results of empirical tests follow in Section III. A final section concludes.

II. NONPARAMETRIC COINTEGRATION ANALYSIS

The real exchange rate q_t is defined as the nominal exchange rate deflated by relative price indices:

$$q_t = s_t - (p_t - p_t^*) = s_t - \pi_t \qquad (1)$$

where s_t is the natural logarithm of the nominal exchange rate defined as the domestic price of foreign currency, p_t and p_t^* represent the logarithms of the domestic and foreign price levels, respectively, and π_t is relative prices. The stochastic process q_t can also be viewed as deviations from long-run PPP which implies stationary real exchange rates or that shocks have only transitory effects. One way of testing for long-run PPP is by means of cointegration tests. The crucial issue here is whether the nominal exchange rate (s_t) and relative prices (π_t) form a cointegrated system with parameters $[1 \quad -1]$. If real exchange rates are stationary, s_t and π_t should move together one-for-one in the long run.

This paper focuses on nonparametric cointegration analysis of long-run PPP. More specifically, the Bierens (1997a) model-free multivariate approach is employed which is in the same spirit as Johansen's method. On one hand, the test statistics involved in both approaches are obtained from the solutions of a generalized eigenvalue problem and, on the other, the hypotheses tested are the same. The main difference is that, in the nonparametric approach, the generalized eigenvalue problem is formulated on the basis of two random matrices which are constructed independently of the DGP. These matrices consist of weighted means of the system variables in levels and first differences and are constructed such that their generalized eigenvalues share similar properties to those in the Johansen approach.

Bierens' approach provides two alternative statistics for empirically determining the cointegration rank r or cointegration space dimension. Both statistics are calculated from the ordered solutions $\hat{\lambda}_{1,m} \geqslant \cdots \geqslant \hat{\lambda}_{n,m}$ of a generalized eigenvalue problem. First, Bierens proposes the lambda-min statistic, $\hat{\lambda}_{n-r_0,m}$, which is analogous to Johansen's maximum eigenvalue statistic, for testing the hypotheses:

$$H_0(r_0): r = r_0; \quad H_1(r_0): r = r_0 + 1 \qquad (2)$$

The asymptotic null distribution of this test is nonstandard and critical values can be found in Bierens (1997a). Second, he proposes the $g_m(r_0)$ statistic to estimate r consistently. This statistic converges in probability to infinity if the true number of cointegrating vectors r is not equal to r_0 and it is at most of order in probability one, if the true number of cointegrating vectors is indeed r_0.

The parameter m is a natural number such that $m \geqslant n$, where n is the dimension of the underlying system. The choice of m is made an integral part of these testing and estimation procedures by providing optimal values, tabulated in Bierens (1997a), for different significance levels and values of r_0 and n. Finally, to test for linear restrictions on the cointegrating vectors, Bierens proposes the trace and lamba-max statistics and recommends $m = 2n$ as a rule of thumb for both. He also derives their asymptotic distributions and corresponding critical values.[6]

The small sample performance of the nonparametric cointegration approach is examined and compared with Johansen's approach by means of a limited Monte Carlo simulation in Bierens (1997a) which is conducted on the basis of a linear nonstationary DGP. In particular a VAR(8) model is estimated from real economic data (wages, GNP) and used to generate samples of size $T = 80$. The simulations suggest similar small sample properties for both approaches with this particular DGP and illustrate the importance for correctly choosing the lag order of the underlying VAR model in the parametric

[5] In a different but related context, Coakley and Fuertes (2000) use persistence profiles to demonstrate that real exchange rates seem to follow a nonlinear adjustment path in the wake of system-wide shocks.
[6] The derivation of Bierens' cointegration statistics is briefly outlined in the Appendix.

Table 1. *Unit root tests*

	Nominal exchange rate		Relative prices			
			CPI		WPI	
	PP[b]	BG3[c]	PP	BG3	PP	BG3
AU[a]	−7.10 (0.301)[d]	7.02 (0.090)[d]	−1.83 (0.760)	203.79 (0.003)	−2.54 (0.727)	74.99 (0.008)
BG	−7.72 (0.267)	0.70 (0.613)	−1.93 (0.790)	92.62 (0.007)	–	–
CN	−2.20 (0.779)	25.84 (0.025)	−3.54 (0.590)	8.41 (0.075)	−0.98 (0.889)	63.30 (0.010)
DK	−6.59 (0.338)	4.46 (0.140)	−4.32 (0.509)	17.23 (0.037)	−3.41 (0.624)	12.77 (0.050)
FN	−7.69 (0.239)	5.44 (0.116)	−5.06 (0.433)	107.59 (0.006)	−4.54 (0.529)	35.58 (0.018)
FR	−5.74 (0.417)	3.91 (0.159)	−2.86 (0.638)	140.05 (0.005)	−6.42 (0.316)	6.17 (0.102)
GE	−7.36 (0.300)	5.83 (0.108)	−1.93 (0.727)	772.64 (0.001)	−2.87 (0.668)	152.45 (0.006)
GR	−0.13 (0.946)	202.63 (0.003)	−0.19 (0.927)	1128.33 (0.001)	−0.24 (0.915)	1305.8 (0.000)
IT	−3.22 (0.632)	37.98 (0.017)	−1.80 (0.729)	2018.34 (0.000)	–	–
JP	−2.31 (0.735)	33.28 (0.019)	0.44 (0.968)	213.73 (0.003)	−0.27 (0.935)	274.82 (0.002)
LX	−7.67 (0.274)	0.82 (0.564)	−2.39 (0.692)	118.12 (0.005)	–	–
NH	−7.38 (0.290)	3.53 (0.176)	−0.62 (0.907)	379.72 (0.002)	−6.29 (0.391)	54.46 (0.012)
NW	−6.07 (0.371)	4.86 (0.129)	−1.70 (0.810)	49.61 (0.013)	−0.53 (0.936)	24.21 (0.026)
PT	−1.42 (0.833)	152.77 (0.004)	−1.29 (0.840)	1220.98 (0.001)	–	–
SD	−3.25 (0.616)	17.89 (0.035)	−1.09 (0.858)	185.99 (0.003)	−0.82 (0.876)	229.37 (0.003)
SP	−2.57 (0.693)	33.43 (0.019)	−0.65 (0.906)	204.49 (0.003)	−1.31 (0.828)	384.10 (0.002)
SW	−6.50 (0.356)	14.90 (0.043)	−1.97 (0.763)	527.05 (0.001)	−3.33 (0.615)	231.53 (0.003)
UK	−7.87 (0.210)	6.70 (0.094)	−3.46 (0.549)	271.64 (0.002)	−1.63 (0.789)	357.56 (0.002)

Notes: [a] AU (Austria), BG (Belgium, CN (Canada), DK (Denmark), GE (Germany), GR (Greece), FN (Finland), FR (France), IT (Italy), JP (Japan), LX (Luxembourg), NH (The Netherlands), NW (Norway), PT (Portugal), SD (Sweden), SP (Spain), SW (Switzerland), UK (United Kingdom).
[b] Phillips–Perron ρ-test. Critical values: −14 (5%) and −11.2 (10%).
[c] Bierens–Guo test #3. Critical values: 12.7 (5%) and 6.3 (10%).
[d] Simulated and observed p-values for the PP and BG3 tests, respectively, in parentheses.

method.[7] Bierens acknowledges that Johansen's method may provide additional information on the presence of linear trends in the cointegrating relation(s) and can also be employed in innovation response analysis and forecasting.

III. DATA AND RESULTS

Monthly data for 18 OECD economies were taken from *Datastream* and span the 25-year period 1973:1–1998:1.[8] End of month spot bilateral exchange rates *vis-à-vis* the US dollar and both the consumer price index (CPI) and wholesale price index (WPI) with 1990 as base year are employed.[9]

Unit root tests are initially conducted on the nominal exchange rate and relative price series.[10] Two nonparametric methods are applied, the Phillips–Perron (PP) (1988) ρ-test where the null is nonstationarity and the Bierens and Guo (1993) Cauchy test #3 where the null is stationarity. The PP statistic is computed with a truncation parameter $p = [cT]^k$ in the Newey–West variance estimator, where $c = 5$ and $k = 0.2$ are adopted following Bierens (1997b). Since the PP tests may be subject to size distortion in finite samples, its p-values are simulated on the basis of 1000 replications of a Gaussian AR(1) process for the underlying variables in first differences. As Table 1 reports, the test results validate modelling nominal exchange rates and relative prices as $I(1)$ processes[11] in virtually all cases and are consistent with those of Hassler

[7] Note however that the Hubrich *et al.* (1998) unconditional test simulation results, on the basis of a Gaussian linear VAR(1) model, indicate that Bierens' tests have conservative size in some cases and therefore lower power than Johansen's tests. This may imply a bias towards underestimation of the true cointegrating rank.
[8] Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, The Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the UK.
[9] Belgium, Italy, Luxembourg and Portugal are excluded from the WPI analysis due to data unavailability.
[10] Bieren's (1998) EasyReg package and GAUSS 3.2 were used for the empirical analysis.
[11] In addition, the same unit root tests applied to the variables in first differences indicate that these can be regarded as $I(0)$ processes. Unreported results are available on request.

Table 2. *Johansen cointegration test results*

	CPI				WPI			
	L^{a}	λmax $r=0/r=1$ $r=0/r=2$	$\lambda trace$ $r=0/r>0$ $r\leqslant 1/r>1$	β'^{b} $H_0: \beta' = (1 \quad -1)^{c}$	L	λmax $r=0/r=1$ $r=1/r=2$	$\lambda trace$ $r=0/r>0$ $r\leqslant 1/r>1$	β' $H_0: \beta' = (1 \quad -1)$
AU	13	5.502 4.253	9.755 4.253	–	1	12.947 4.681	17.628 4.681	–
BG	13	6.295 2.685	8.990 2.685	–	–	–	–	–
CN	1	3.643 0.089	3.732 0.089	–	1	28.105* 1.399	29.504* 1.399	(1, −1.188) 2.552 [0.110]
DK	7	16.585* 3.748	20.333* 3.748	(1, −3.514) 8.839 [0.003]	6	10.201 3.716	13.916 3.716	–
FN	3	16.020* 3.149	19.169* 3.149	(1, −5.303) 10.386 [0.001]	1	17.468* 1.812	19.280* 1.812	(1, −2.186) 7.002 [0.008]
FR	5	33.106* 3.697	36.803* 3.697	(1, −2.389) 20.716 [0.000]	1	13.279 4.357	17.637 4.357	–
GE	3	12.098 3.882	15.979 3.882	–	2	14.780 3.290	18.089* 3.289	(1, −1.724) 4.970 [0.026]
GR	13	7.581 0.003	7.585 0.003	–	2	12.216 0.080	12.296 0.080	–
IT	2	27.257* 3.726	30.983* 3.726	(1, −1.537] 12.204 [0.000]	–	–	–	–
JP	13	10.255 1.123	11.379 1.123	–	2	6.338 0.142	6.480 0.142	–
LX	4	13.072 3.931	17.003 3.931	–	–	–	–	–
NH	13	5.167 1.799	6.967 1.799	–	2	28.107* 3.275	31.381* 3.275	(1, −2.916) 16.428 [0.000]
NW	13	10.435 2.594	13.029 2.594	–	2	22.106* 0.564	22.670* 0.564	(1, −0.740) 1.644 [0.200]
PT	1	36.029* 1.400	37.429* 1.400	(1, −1.229) 12.518 [0.000]	–	–	–	–
SD	2	5.974 0.915	6.888 0.915	–	2	14.787 0.731	15.518 0.731	–
SP	13	24.030* 4.765	28.796* 4.765	(1, 15.786) 4.925 [0.026]	6	20.152* 2.162	22.313* 2.162	(1, −1.196) 3.262 [0.071]
SW	3	13.868 4.807	18.675* 4.807	(1, −2.348) 7.338 [0.007]	2	29.379* 3.734	33.112* 3.734	(1, −2.173) 19.916 [0.000]
UK	13	14.707 7.412	22.118* 7.412	(1, −23.628) 6.408 [0.011]	2	13.739 4.916	18.655* 4.916	(1, −0.811) 0.510 [0.475]

Notes: [a] Lag order selected using AIC and SBC criteria and sequence of LR statistics.
[b] Estimated cointegrating vector normalized with respect to nominal exchange rate.
[c] LR test for linear over-identifying restrictions, *p*-values in brackets.
* Significant at the 5% level.

and Wolters (1995), Steigerwald (1996) and Baum *et al.* (1998), *inter alios*.[12]

Next mean reversion or whether real exchange rates and relative prices are cointegrated with vector $\beta' = [1 \quad -1]$ are tested. As Table 2 reports, the Johansen procedure provides evidence of cointegration in eight out of 18 countries in the CPI-based series. However, the cointegrating null $[1 \quad -1]$ is rejected for all eight countries at even the 1% level in most cases. In the WPI-based analysis, cointegration in eight out of 14 countries is found and the

[12] Some attention has focused recently on whether prices are $I(2)$. See Haldrup (1997) for a survey of the literature dealing with $I(2)$ variables. Granger *et al.* (1997) alternatively propose a class of nonlinear growth processes which might characterize some price series.

[1 −1] over-identifying restriction is rejected in a further four at better than the 5% level. This leaves support for real WPI exchange rate mean reversion in only four cases: Canada, Norway, Spain and the UK.

Bierens' test results provide evidence of cointegration in 15 out of 18 CPI-based series and the [1 −1] co-integrating null is rejected only in five of them at the 5% level. Thus CPI real exchange rate mean reversion is supported in a total of ten countries. The WPI-based analysis indicates cointegration in 12 out of 14 countries (Switzerland and the UK are exceptions) and fails to reject the cointegrating null [1 −1] in them all.[13] These results are reported in Table 3. The clear evidence of mean reversion in dollar denominated real exchange rates provided by the Bierens nonparametric approach contrasts sharply with most of the existing evidence obtained from standard (non-panel) unit root and cointegration methods.[14]

How does one account for this apparent discrepancy between the findings from the Bierens and Johansen approaches?[15] In principle, one should expect similar results from both methods on the basis of Bierens' (1997a) Monte Carlo simulations using a linear ECM.[16] However, the standard (linear) cointegration framework presents a misspecification problem when the true nature of the adjustment process is nonlinear and the speed of adjustment varies with the magnitude of the disequilibrium. In this context one could speak of regime-sensitive adjustment mechanisms where the transition between regimes could be either smooth as in STAR models or discontinuous as in TAR models.

Nonlinearities in real exchange rate adjustment may arise due to market frictions such as transaction costs leading to persistent behaviour when PPP deviations are within no-arbitrage bands but mean reversion beyond them. In this context, these findings indirectly corroborate those of a number of recent studies where the hypothesized nonlinear dynamics of real exchange rates has been successfully calibrated and tested for different samples for the post-1973 period.[17] Though using different nonlinear models, these recent contributions to the literature all produce substantial evidence of mean reversion for sizeable deviations from PPP. One important implication of these studies is that failure to establish real exchange rate stationarity on the basis of standard linear methods does not necessarily invalidate long-run PPP. This may help to reconcile the conflicting evidence obtained from the Johansen approach and Bierens' nonparametric method.

The estimates of the linear VAR model upon which Johansen's tests are based may thus reflect an average from the different regimes, yielding a biased measure of the true varying speed of adjustment. Nevertheless, linear cointegration methods may still perform reasonably well in the presence of nonlinearities, as the Balke and Fomby (1997) results illustrate, when the bias in the cointegration tests and estimators is sufficiently small to give the correct output on average.[18] If, despite overall stationarity, the process spends a long time in the no-arbitrage band, nonlinearities might adversely affect the performance of standard cointegration tests. The latter might well be the case with real exchange rates and this constitutes a possible rationale for these results.

IV. CONCLUSIONS

This paper indirectly addresses the issue of nonlinear real exchange rate adjustment for 18 OECD economies in the 1973–1998 period. It does so by analysing and contrasting the results from the nonparametric cointegration framework of Bierens (1997a) and from the standard Johansen (1988) maximum likelihood approach. Although the mean reversion hypothesis receives limited support from the Johansen method, the results from the Bierens approach are largely favourable, especially for the WPI real exchange rate series. This apparent discrepancy is interpreted as a consequence of significant nonlinearities in real exchange rate adjustment to PPP. In this sense, the findings indirectly corroborate those of a number of recent studies which investigate and adduce support for such nonlinearities. Further empirical research is warranted to establish the robustness of this interpretation.

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[13] The somewhat stronger evidence in favour of stationarity for the WPI series may be explained by their higher proportion of tradables.
[14] See for example Papell (1997) for a recent summary of the results from standard approaches.
[15] In other applications they have yielded similar results. For instance, Kanas (1998) obtains the same results from the Johansen and Bierens methods applied to testing for linkages between the US and European equity markets with different span periods.
[16] Note that the Hubrich *et al.* (1998) simulation results, which suggest a tendency for Bierens' tests to underestimate the true cointegrating rank, cannot explain the empirical findings.
[17] See footnote 3.
[18] Van Dijk and Franses (1997) provide simulation results on the adequacy of standard cointegration methods in a nonlinear context.

Table 3. *Bierens cointegration test results*

	CPI			WPI		
	λmin[a] $r=0/r=1$ $r=1/r=2$	$g_m(r_0)$ $r_0=0,1,2$	β'[b] $H_0: \beta' = (1 \quad -1)$[c]	λmin $r=0/r=1$ $r=1/r=2$	$g_m(r_0)$ $r_0=0,1,2$	β' $H_0: \beta' = (1 \quad -1)$
AU	0.019	21.52e + 0.01	(1, −7.087)	0.011*	46.81e + 0.01	(1, −3.659)
	1.787	**13.09e + 0.01**[d]	2.99	1.896	**53.53e + 0.00**	2.91
		37.64e + 0.06			17.30e + 0.06	
BG	0.033	38.05e + 0.05	(1, −5.834)			
	5.199	**87.49e - 0.05**	4.99*	–	–	–
		21.29e + 0.02				
CN	0.001*	65.96e + 0.03	(1, −4.065)	0.000*	10.72e + 0.07	(1, −2.131)
	0.029*	**15.70e + 0.02**	1.08	0.063	**21.21e - 0.02**	1.06
		12.28e + 0.04			75.57e + 0.00	
DK	0.005*	82.16e + 0.01	(1, −7.479)	0.000*	58.73e + 0.01	(1, −2.989)
	2.932	**12.74e + 0.00**	4.88*	11.001	**12.66e - 0.01**	3.70
		98.59e + 0.06			13.79e + 0.06	
FN	0.004*	14.17e + 0.03	(1, −1.727)	0.001*	83.75e + 0.02	(1, −3.897)
	0.922	**74.70e - 0.01**	1.67	0.470	**48.70e + 0.00**	4.28
		57.16e + 0.04			96.71e + 0.04	
FR	0.002*	68.54e + 0.02	(1, −2.061)	0.001*	20.65e + 0.01	(1, −3.121)
	0.861	**17.72e + 0.00**	1.65	4.469	**21.82e + 0.00**	4.61
		11.82e + 0.05			39.23e + 0.06	
GE	0.016*	**10.89e + 0.01**	(1, −6.297)	0.006*	**98.48e + 0.00**	(1, −3.625)
	1.032	77.64e + 0.01	4.25	2.159	19.59e + 0.01	3.19
		74.38e + 0.06			82.25e + 0.06	
GR	0.009*	21.78e + 0.02	(1, −5.313)	0.005*	28.43e + 0.05	(1, −2.556)
	1.113	**33.37e + 0.00**	5.04*	1.160	**23.55e - 0.03**	3.49
		37.19e + 0.05			28.49e + 0.02	
IT	0.040	**12.01e + 0.00**				
	2.127	16.56e + 0.02	–	–	–	–
		67.42e + 0.07				
JP	0.032	14.79e + 0.04	(1, −2.704)	0.003*	18.02e + 0.05	(1, −11.947)
	0.187	**17.32e + 0.00**	1.48	0.131	**29.21e - 0.01**	1.38
		54.76e + 0.03			44.94e + 0.02	
LX	0.037	33.76e + 0.02	(1, −5.336)			
	5.061	**10.41e - 0.01**	4.57	–	–	–
		23.99e + 0.05				
NH	0.014*	30.87e + 0.02	(1, −7.440)	0.013*	19.94e + 0.01	(1, −3,885)
	2.081	**67.33e** − 0.01	3.39	1.515	**19.67e + 0.01**	3.53
		26.24e + 0.05			40.61e + 0.06	
NW	0.001*	48.62e + 0.02	(1, −11.001)	0.000*	31.47e + 0.05	(1, −6.684)
	0.839	**26.28e + 0.00**	2.76	0.845	**40.09e - 0.03**	2.99
		16.66e + 0.05			25.74e + 0.02	
PT	0.016*	**62.70e + 0.00**	(1, −2.035)			
	1.948	37.81e + 0.01	3.33	–	–	–
		12.92e + 0.07				
SD	0.012*	52.64e + 0.02	(1, −13.298)	0.001*	38.12e + 0.03	(1, −3.421)
	1.722	**57.65e - 0.01**	5.60*	4.687	**10.75e − 0.02**	4.39
		15.39e + 0.05			21.25e + 0.04	
SP	0.010*	**24.53e + 0.00**	(1, −36.765)	0.016*	**10.23e + 0.00**	(1, −2.724)
	4.653	16.95e + 0.01	13.03*	6.247	22.54e + 0.01	4.60
		33.02e + 0.07			79.15e + 0.07	
SW	0.054	**19.13e + 0.00**		0.039	**18.16e + 0.00**	
	1.730	15.71e + 0.02	–	1.850	14.49e + 0.02	–
		42.35e + 0.07			44.61e + 0.07	
UK	0.132	**45.94e + 0.00**		0.040	**15.85e + 0.01**	
	0.317	19.44e + 0.03	–	0.254	87.83e + 0.02	–
		17.63e + 0.07			51.10e + 0.06	

Notes: [a] Lambda-min test statistic; parameter m chosen from optimal values tabulated in Bierens (1997a).
[b] Estimated cointegrating vector normalized with respect to nominal exchange rate.
[c] Trace test for linear restrictions, $m = 2n$ with n the dimension of the system.
[d] Estimated number of cointegrating vectors is: $\hat{r} = argmin_{r_0 \leqslant 2} g_m(r_0)$, $m = 2$. Bold indicates minimum value of statistic.
* Significant at the 5% level.

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APPENDIX

Bierens' (1997a) nonparametric cointegration method

The general framework within which the Bierens nonparametric tests and estimators are considered assumes an observable n-variate process z_t, $t = 1, \ldots, T$, generated as:

$$z_t = \pi_0 + \pi_1 t + y_t \qquad \text{(A1)}$$

where $\pi_0(n \times 1)$ and $\pi_1(n \times 1)$ are optional mean and trend terms, and y_t is a zero-mean unobservable process such that Δy_t is stationary and ergodic. Apart from these regularity conditions, the method does not require further specification of the DGP for z_t, and in this sense, it is completely nonparametric. Bierens takes advantage of the contrasting asymptotic behaviour of z_t and Δz_t by defining the following matrices:

$$A_m = \frac{8\pi^2}{T}\sum_{k=1}^{m} k^2 \left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(2k\pi(t-0.5)/T\right)z_t\right) \times \left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(2k\pi(t-0.5)/T\right)z_t\right)'$$

$$B_m = 2T\sum_{k=1}^{m}\left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(2k\pi(t-0.5)/T\right)\Delta z_t\right) \times \left(\frac{1}{T}\sum_{t=1}^{T}\cos\left(2k\pi(t-0.5)/T\right)\Delta z_t\right)' \qquad \text{(A2)}$$

which are based on weighted means of z_t and Δz_t. Though there are many possible choices for the weight functions, $\cos\left(2k\pi(t-0.5)/T\right)$ is recommended since it ensures invariance of the test statistics to drift terms.

By defining the pair of random matrices $P_T = A_m$ and $Q_T = (B_m + T^{-2}A_m^{-1})$, the ordered generalized eigenvalues $\hat{\lambda}_{1,m} \geqslant \cdots \geqslant \hat{\lambda}_{n,m}$ obtained as solutions of the problem $\det\left[P_T - \lambda Q_T\right] = 0$ have similar properties to those in the Johansen approach and therefore can be used for testing hypotheses about the cointegration rank r. First, Bierens proposes the lambda-min test statistic, $\hat{\lambda}_{n-r_0,m}$, to test for the null hypothesis $r = r_0$ against the alternative $r = r_0 + 1$. The corresponding asymptotic null distribution is non-standard and he tabulates the critical values. The null is rejected if the test statistic is too small. The test parameter m is also tabulated for different significance levels as a function of r_0 and n, such that the lower bound of the power of the test is maximized.

Second, Bierens' approach provides the $g_m(r_0)$ statistic for estimating r consistently which again is calculated from the Bierens' generalized eigenvalues:

$$\hat{g}_m(r_0) = \begin{cases} \left(\prod_{k=1}^{n}\hat{\lambda}_{k,m}\right)^{-1} & \text{if } r_0 = 0 \\ \left(\prod_{k=1}^{n-r_0}\hat{\lambda}_{k,m}\right)^{-1}\left(T^{2r_0}\prod_{k=n-r_0+1}^{n}\hat{\lambda}_{k,m}\right) & \text{if } r_0 = 1,\ldots,n-1 \\ T^{2n}\prod_{k=1}^{n}\hat{\lambda}_{k,m} & \text{if } r_0 = n \end{cases} \qquad \text{(A3)}$$

This statistic employs the tabulated optimal values for m when $r_0 < n$ while $m = n$ is chosen for $r_0 = n$. It verifies $g_m(r_0) = O_p(1)$ for $r = r_0$ and converges in probability to infinity if $r \neq r_0$. A consistent estimate of r is thus given by $\hat{r} = argmin_{r_0 \leqslant n} g_m(r_0)$.

Once the dimension of the cointegration space is determined, one can deploy tests for linear restrictions on the cointegrating vectors. Bierens proposes nonparametric tests on the basis of the ordered solutions of the following eigenvalue problem:

$$\det\left[H^T A_m H - \lambda H^T (A_m + T^{-2}A_m^{-1})^{-1} H\right] = 0 \qquad \text{(A4)}$$

which involves the random matrix A_m and the matrix of hypothesized restrictions H. Critical values for the trace and lambda-max test statistics with $m = 2n$ are given in Bierens (1997a).