



# Currency spillovers and tri-polarity: a simultaneous model of the US dollar, German mark and Japanese yen<sup>☆</sup>

Ronald MacDonald<sup>a,\*</sup>, Ian W. Marsh<sup>b</sup>

<sup>a</sup> *Department of Economics, University of Strathclyde and CESifo, 100 Cathedral Street,  
Glasgow G4 0LN, UK*

<sup>b</sup> *Cass Business School, 106 Bunhill Row, London EC1Y 8TZ, UK*

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## Abstract

This paper presents a simultaneous model of exchange rates between the US dollar, German mark and Japanese yen. In addition to incorporating long-run equilibria and short-run dynamics, the model is designed to capture complex interaction between currencies not normally considered in exchange rate models. The model is demonstrated to be an economically and statistically superior forecasting tool over relatively short horizons, thereby demonstrating that the random walk paradigm no longer rules the roost.

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## 1. Introduction

The advent of the European monetary integration means that the international monetary system is now characterized by a tri-polar currency structure, consisting of the US dollar, Japanese yen and euro. Recently, there has been considerable interest in policy circles regarding the operation and design of this system. Should it continue to be effectively a freely floating system, a managed float (based perhaps

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\* Corresponding author. Tel.: +44-141-548-3861; fax: +44-141-552-5589.

*E-mail address:* [r.r.macdonald@strath.ac.uk](mailto:r.r.macdonald@strath.ac.uk) (R. MacDonald).

on a target zone arrangement) or one of complete fixity? In this paper, we seek to shed some light on the operation of the new system by considering a tri-polar exchange rate model consisting of the US dollar, Japanese yen and German mark, which may be regarded as the antecedent to the new system. Because of the different operational objectives of the European Central Bank relative to the Bundesbank, the euro may, of course, have different properties to the mark. Nevertheless, we believe that the study of the dollar–mark–yen system is useful, both in terms of indicating how a tri-polar network may be modelled, and also in suggesting how such a system may behave in terms of its long- and short-run properties.

Our tri-polar exchange rate system represents an extension of the joint modelling of exchange rates, interest rates and prices by Johansen and Juselius (1992), Juselius (1995), Juselius and MacDonald (2000), and MacDonald and Marsh (1997, 1999). We extend this literature in two main ways. First, we seek to interpret all the significant cointegrating vectors in an economic and statistical sense (MacDonald and Marsh, 1997, for example, only focus on the first significant vector). Second, and as we have noted, we use a tri-polar structure, rather than a single currency analysis, which has been the norm in this literature, in order to capture currency spillovers. Although little research has been undertaken on modelling exchange rate spillovers (other than by accounting for contemporaneous correlation between exchange markets), it seems a natural component of an exchange rate model since it is widely accepted that currency markets are not independent.

One of the main contributions of this paper is the emphasis laid on the complex dynamic interactions among the variables in an exchange rate system. One of the deficiencies of many fundamental-based models is that they often neglect the dynamics inherent in the exchange rate process. Even if exchange rate models do incorporate dynamics, they are often very limited in scope. The econometric methods proposed here ensure a very rich dynamic specification for the exchange rate relationship and also for the equations of the other fundamental variables driving the exchange rate. Our exchange rate model is assessed using standard statistical criteria and also by what has become the benchmark by which an exchange rate model is judged, namely its ability to outperform a random walk model in an out-of-sample forecasting exercise.

The outline of the remainder of this paper is as follows. In the next section, we present a motivational discussion for the modelling approach adopted. In Section 3, the econometric methods are briefly outlined and our estimated long-run cointegration relationships discussed. The out-of-sample forecasting results are presented in Section 4. The paper ends with a concluding section.

## **2. Motivation and related research**

Long-run purchasing power parity (PPP) forms the basis of our modelling exercise. Since, in one form or another, it is an assumed condition for many exchange rate models, PPP has received vast attention in the academic literature. While there is now general agreement that PPP does not hold in the short-run, except for case

of (near) hyperinflation, as a long-run proposition, the evidence is much more favourable (see, for example, Froot and Rogoff, 1995; MacDonald, 1995). However, the fact that the majority of empirical papers on PPP reject the restrictions of proportionality and symmetry is rather worrying for a PPP-based study. While measurement errors may partly account for the difference between estimated and theoretical values, the divergence is sufficiently large for us to conclude that something more than measurement errors is at hand. In line with Cassel's synthesis of PPP and asset demand approaches to the exchange rate, we shall augment the traditional PPP relationship (see Holmes, 1967, for further discussion).

The particular approach used here to augment PPP is discussed in some detail by Johansen and Juselius (1992), Juselius (1995), MacDonald and Marsh (1997), and Juselius and MacDonald (2000) and essentially entails supplementing the PPP relationship with an interest differential. Johansen and Juselius (1992) and Juselius (1995) motivate such a model in terms of combining PPP with a UIP condition, while MacDonald and Marsh (1997) have labelled this approach to exchange rate modelling as Casselian PPP. The latter follows from Cassel's emphasis on the role of capital flows in preventing an exchange rate from always being at its PPP determined level. In the work of Johansen and Juselius (1992), Juselius (1995), MacDonald and Marsh (1997) and Juselius and MacDonald (2000), the Casselian approach is shown to produce stationary combinations of exchange rates, relative prices and interest differentials. However, in all these papers, currency markets are analysed separately. The main novelty in the present paper is to note that currency markets are closely linked and so any statistical analysis should explicitly recognize this in our econometric modelling. For the US dollar bilateral exchange rates of the Japanese yen and German mark, our tri-polar approach amounts to analysing the following data vector:

$$x_t = (s_t^{\text{GER}}, s_t^{\text{JAP}}, p_t^{\text{GER}}, p_t^{\text{JAP}}, p_t^{\text{US}}, i_t^{\text{GER}}, i_t^{\text{JAP}}, i_t^{\text{US}})', \quad (1)$$

where  $s$  denotes the (log) bilateral spot exchange rate against the US dollar,  $p$  denotes the (log) price level and  $i$  the short-term interest rate and the superscripts have an obvious interpretation. We then propose testing hypotheses on this vector which allow us to partition the long-run cointegrating space into two long-run stationary relationships of the form:

$$\begin{aligned} \beta^{\text{GER}} x_t &= \omega_1^{\text{GER}} (i_t^{\text{GER}} - i_t^{\text{US}}) - \omega_2^{\text{GER}} (p_t^{\text{GER}} - p_t^{\text{US}}) + s_t^{\text{GER}}, \\ \beta^{\text{GER}} x_t &= \omega_1^{\text{JAP}} (i_t^{\text{JAP}} - i_t^{\text{US}}) - \omega_2^{\text{JAP}} (p_t^{\text{JAP}} - p_t^{\text{US}}) + s_t^{\text{JAP}}. \end{aligned}$$

Although there are no spillovers from one currency market to the other in these long-run relationships, there may nevertheless be information contained in the joint process from the two markets which is important in defining the long-run and would be ignored by simply conditioning on the information set from one market. However, direct spillovers do appear in the short-run dynamic equations. For example, the dynamic equation for the Japanese yen exchange rate would have the

following form:

$$\begin{aligned} \Delta s_t^{\text{GER/JAP}} = & \gamma_0 + \sum_{i=1}^l \gamma_{1i} \Delta s_{t-i}^{\text{GER}} + \sum_{i=1}^l \gamma_{2i} \Delta s_{t-i}^{\text{JAP}} + \sum_{i=1}^l \gamma_{3i} \Delta p_{t-i}^{\text{GER}} \\ & + \sum_{i=1}^l \gamma_{4i} \Delta p_{t-i}^{\text{JAP}} + \sum_{i=1}^l \gamma_{5i} \Delta p_{t-i}^{\text{US}} + \sum_{i=1}^l \gamma_{6i} \Delta i_{t-i}^{\text{GER}} + \sum_{i=1}^l \gamma_{7i} \Delta i_{t-i}^{\text{JAP}} \\ & + \sum_{i=1}^l \gamma_{8i} \Delta i_{t-i}^{\text{US}} + \alpha_1 \beta^{\text{GER}} x_{t-1} + \alpha_2 \beta^{\text{JAP}} x_{t-1} \end{aligned} \tag{2}$$

where  $l$  represents the lag length of the differenced VAR. Hence, spillovers arise both through dynamic interactions and also through the significance of the cointegrating relationships (the alphas). As we shall see in our empirical section, both these effects turn out to be important in our analysis.

### 3. Econometric methods and results

#### 3.1. Econometric modelling

In this section, we briefly outline the econometric methods used to estimate our exchange rate models. In short, we use a vector autoregressive (VAR) model to determine the number of cointegrating relationships and to define the cointegrating vectors. This part of our analysis is based on the methods of Johansen (1995). The familiar VAR representation is:

$$x_t = \sum_{i=1}^{l+1} A_i x_{t-i} + \psi D_t + v_t, \tag{3}$$

which may be reparameterized into the VECM representation as:

$$\Delta x_t = \sum_{i=1}^l \Gamma_i \Delta x_{t-i} + \Pi x_{t-1} + \psi D_t + v_t, \tag{4}$$

where  $x$  is a vector of variables entering the system,  $D_t$  contains deterministic components (constant, trend, centred seasonal dummies and event dummies) and  $v_t$  is assumed to have mean 0, be homoscedastic and serially uncorrelated. The order of the VAR is assumed finite to exclude moving average components, and the parameters  $A_i$ ,  $\Gamma_i$ ,  $\psi$ , and  $\Sigma$  (the covariance matrix of  $v$ ) are assumed constant.  $\Pi$  is interpreted as the matrix of long-run responses. If the data cointegrate,  $\Pi$  must be of reduced rank,  $r < n$ , where  $n$  is the number of variables in  $x$  and it may be factored as (see Johansen, 1995):

$$\Pi = \alpha \beta'$$

where  $\beta$  and  $\alpha$  are  $n \times r$  matrices, which give the cointegrating vectors (empirical long-run relationships) and associated adjustment matrix, respectively. Due to the

reduced form nature of the cointegrating vectors, there is an increasing trend towards performing transformations and imposing restrictions on the estimated cointegrating space to reveal ‘meaningful economic relationships’.

The imposition of restrictions on the cointegrating vector or adjustment matrix will change the estimated short-run dynamics of Eq. (4) and the coefficients of the deterministic variables. These new coefficients are denoted by a tilde. If we also denote the restricted cointegration space by  $\bar{\Pi} = \bar{\alpha}\bar{\beta}'$ , the constrained VAR (CVAR) can be written as:

$$\Delta x_t = \sum_{i=1}^l \tilde{\Gamma}_i \Delta x_{t-i} + \bar{\Pi} x_{t-1} + \psi D_t + w_t, t \quad (5)$$

This is the full vector form of the dynamic equation (4) above.

### 3.1.1. Long-run equilibria in the tri-polar system

Our eight variable system comprises two bilateral exchange rates (dollar–mark and dollar–yen), three price indices and three short-term interest rates. The expected exchange rate changes are not included in the vector of variables as these will be  $I(0)$  under the weakest assumptions of rationality. The data run from January 1983 through September 1998 and are extracted from the IMF’s International Financial Statistics CD-ROM. The start date for estimation is set to exclude turbulence in the interest rate markets in the early 1980s caused by changes in the Federal Reserves’ operating procedures, and which necessitated the use of many intervention dummies in our previous work (MacDonald and Marsh, 1997). The final data point in our sample was chosen to be September 1998, rather than December 1998, in order to exclude some market turbulence in the run up to European Monetary Union in January 1999. Just two dummies are needed in this system (1985:11 and 1986:01), primarily to remove two outliers in the Japanese interest rate series. Four lagged levels in the VAR, chosen after performing a battery of tests, centred seasonal dummies and an unconstrained constant are included in the system. Some of the tests indicated a shorter lag length, but we chose to include the maximum amount of relevant information albeit at the possible cost of estimation accuracy.

The interest rates used in this study are short-term (3 months) eurocurrency interest rates rather than the long-term bond yields used in our previous work (MacDonald and Marsh, 1997). There are two justifications for this. First, short-term rates have been used successfully by other authors who exploit similar systems (Fisher et al., 1990; Johansen and Juselius, 1992; Lee et al., 1994). Second, factors such as taxation, duration and liquidity effects can cause bond yields to differ from the pure interest rate which we would like to observe. These factors are not only less prevalent in the short-term euromarkets, but are also more similar between currencies leading to, we hope, a better proxy for capital flows.

Consumer prices are our preferred price measure for Germany and the US, while for Japan, we use the wholesale price index. This change is motivated primarily by the marked differences in behaviour of the Japanese consumer and wholesale price

Table 1  
Johansen cointegration analysis—tri-polar system

Null hypothesis	Trace	Trace critical (95%)
$r = 0$	186.92	165.58
$r \leq 1$	133.87	131.70
$r \leq 2$	91.66	102.14
$r \leq 3$	60.93	76.07
$r \leq 4$	37.62	53.12
$r \leq 5$	19.00	34.91
$r \leq 6$	7.90	19.96
$r \leq 7$	2.52	9.24

Note: Trace critical values from Osterwald-Lenum (1992).

indices. In many countries, these proxies for ‘the’ price level follow similar trends. For Japan over the period studied they do not, and our use of the wholesale index produces results which are more in conformity with the basic model. In particular, we are able to impose the correct coefficient restrictions on the Japanese systems when the Japanese WPI is used, but not with the CPI. We have no good explanation for this finding, but note that there is a controversy in the PPP literature as to the appropriate price series to use in a PPP calculation (see, for example, Frenkel, 1981), and our tests seem to indicate that for Japan, the appropriate series is the wholesale price index. It could be that distribution costs are much important for Japan than for the other countries in our sample and that this drives an important wedge between the CPI and WPI, rendering the CPI an inappropriate price measure for Japan (see MacDonald and Ricci, 2001 for a further discussion of the importance of distribution costs in PPP calculations).

Diagnostic tests (not reported here to save space but available from the authors on request) indicate that the residuals are random, but that there are normality problems in our system. Reassuringly, this is due to kurtosis, rather than to skewness, and therefore should not affect our results (see Paruolo, 1995). Incorporating further dummy variables into the VAR models proved unsuccessful in removing this kurtosis.

The Johansen cointegration Trace test results are detailed in Table 1. These indicate two significant relationships, a finding supported by the presence of two noticeably larger eigenvalues and an analysis of the roots of the companion matrix (not reported).<sup>1</sup>

Although a number of the coefficients are correctly signed in the unrestricted cointegrating relationships reported in Table 2, the relationship does not conform closely to the hypothesized relationships. However, it may be that coefficients which appear quite large in absolute terms in the two equations (say, the Japanese price coefficient in the first vector and the coefficient on the German price series in

<sup>1</sup> In particular, the number of roots close to unity in the companion matrix was the same as the number of common stochastic trends in the cointegrated VAR, namely 6.

Table 2  
The normalized cointegration relationships and the adjustment matrices

	Standardized beta vector		Standardized alpha vector	
	Vector 1	Vector 2	Vector 1	Vector 2
$s^{ger}$	1	0.034	-0.019 (1.57)	0.006 (1.28)
$s^{jap}$	-0.235	1	-0.000 (0.05)	0.014 (2.77)
$p^{ger}$	5.691	16.073	-0.000 (0.34)	-0.001 (4.53)
$p^{jap}$	-1.285	-15.230	-0.000 (0.34)	0.002 (3.02)
$p^{US}$	-3.292	-9.885	-0.002 (4.36)	-0.000 (2.04)
$i^{ger}$	0.043	-0.204	-0.395 (4.86)	0.090 (2.70)
$i^{jap}$	-0.001	0.227	-0.050 (0.69)	-0.80 (0.28)
$i^{US}$	-0.083	0.130	0.135 (1.21)	0.044 (0.98)

Note: Numbers in parenthesis are numerical *t*-values.

the second) are in fact insignificantly different from 0. Exclusion and homogeneity restrictions are formally tested below. The normalized alpha terms indicate clear adjustment to the two disequilibrium errors. For example, both US prices and the German interest rate adjust significantly to the first disequilibrium error. The German exchange rate also adjusts (negatively) to this error, although the significance is quite weak. The pattern of adjustment is much richer with respect to the second disequilibrium error, with all the prices adjusting in addition to the German interest rate and Japanese yen. The evident spillovers we are picking up here would seem to reinforce the use of the tri-polar framework.

Tests of the restrictions implied by the theory are summarized in Table 3. Simple PPP (with symmetry and proportionality imposed) is rejected for both exchange rates (see hypothesis A1–A3). Casselian PPP cannot be rejected for either exchange rate (hypothesis B1 and B2), and the tests would seem to indicate that PPP augmented by the relevant interest differential is an acceptable description of both cointegrating relationships (hypothesis B3).

Table 3  
Restrictions on cointegration space—tri-polar system

Hypothesis	Economic interpretation	Implied restrictions	Distributions	Test statistic
A1	PPP for Germany	$\beta_1 = (1, 0, -1, 0, 1, 0, 0, 0)'$	$\chi^2(6)$	32.44 (0.00)
A2	PPP for Japan	$\beta_2 = (0, 1, 0, -1, 1, 0, 0, 0)'$	$\chi^2(6)$	31.60 (0.00)
A3	PPP holds for both	$\beta_1 = (1, 0, -1, 0, 1, 0, 0, 0)'$ , $\beta_2 = (0, 1, 0, -1, 1, 0, 0, 0)'$	$\chi^2(12)$	74.99 (0.00)
B1	PPP plus interest differential for Germany	$\beta_1 = (1, 0, -1, 0, 1, m, 0, -m)'$	$\chi^2(5)$	2.65 (0.75)
B2	PPP plus interest differential for Japan	$\beta_2 = (0, 1, 0, -1, 1, 0, n, -n)'$	$\chi^2(5)$	8.29 (0.14)
B3	PPP plus interest differential for both	$\beta_1 = (1, 0, -1, 0, 1, m, 0, -m)'$ , $\beta_2 = (0, 1, 0, -1, 1, 0, n, -n)'$	$\chi^2(10)$	18.48 (0.05)

Note: Numbers in parenthesis in the final column are *p*-values.

Table 4  
Loadings associated with restricted cointegrating vectors

Variable	CV <sub>1</sub>	CV <sub>2</sub>
$s_t^{\text{ger}}$	-0.016 (1.08)	-0.004 (0.67)
$s_t^{\text{jap}}$	0.013 (0.88)	-0.021 (3.18)
$p_t^{\text{ger}}$	0.002 (2.38)	0 (0.34)
$p_t^{\text{jap}}$	0 (0.11)	-0.001 (1.89)
$p_t^{\text{US}}$	-0.001 (1.74)	0.001 (2.01)
$i_t^{\text{ger}}$	-0.429 (4.31)	0.110 (2.57)
$i_t^{\text{jap}}$	0.104 (1.23)	-0.071 (1.88)
$i_t^{\text{US}}$	0.019 (0.14)	-0.023 (0.38)

Note: Numbers in parenthesis are *t*-values.

The final restricted cointegrating vectors are as follows:

$$CV_1 = s_t^{\text{GER}} - p_t^{\text{GER}} + p_t^{\text{US}} + 0.135(i_t^{\text{GER}} - i_t^{\text{US}}),$$

$$CV_2 = s_t^{\text{JAP}} - p_t^{\text{JAP}} + p_t^{\text{US}} + 0.375(i_t^{\text{JAP}} - i_t^{\text{US}}).$$

These vectors closely accord with the view discussed in Section 2 that interest differentials reflect capital mobility. It may seem surprising that the unconstrained estimates reported in Table 2, which seem to be quite far from the hypothesized priors, can be constrained to such tightly defined relationships. However, the constrained estimates take account of the variance of the coefficients, something that cannot be gleaned from an inspection of the point estimates themselves.<sup>2</sup> The loadings matrix associated with the two cointegrating vectors, reported in Table 4, shows the importance of the tri-polar nature of the system. In particular, the fact that German interest rates, and to a lesser extent US prices, significantly adjust to remove disequilibrium in both cointegrating vectors implies that disturbances will be transmitted around the whole system, even if a shock occurs to just one leg. For example, if there is a shock to Japanese prices which pushes CV<sub>2</sub> out of equilibrium, not only will Japanese variables act to restore equilibrium, but so will US prices and even German interest rates. Since both these latter terms help to form CV<sub>1</sub>, disequilibrium spills from one exchange rate to another. This seems to confirm the conventional wisdom on how foreign exchange markets work, although it has not to our knowledge been demonstrated formally before in the context of this kind of model.

The fact that US interest rates appear to be weakly exogenous in the above systems is perhaps not surprising given that US interest rates are usually seen as leading (i.e. pushing) world financial markets. Indeed, a formal test that the two alpha terms in the US interest rate equation are insignificant (plus hypothesis B3) produces a chi-squared statistic (with 12 degrees of freedom) of 18.69 and an associa-

<sup>2</sup> Indeed, one good reason for explicitly testing the constraints is that the unconstrained estimates may simply reflect collinearity amongst the variables entering our VAR.



ted  $p$ -value of 0.10. That US prices appear to be adjusting in the above systems is probably intuitive enough given the large US current account imbalances necessitating relative price adjustments between the US and its trading partners. The formal test that the alpha terms on the two error correction terms in the US price equation produces a chi-squared statistic of 21.91 and a  $p$ -value of 0.04.

One issue which we have not addressed in the above modelling is the potential I(2) ness of our price series. For example, Juselius and MacDonald (2000) have demonstrated that in the kind of modelling framework used here that prices are likely to be I(2). However, they also note that, despite this, the real exchange rate is an I(1) process. Since in our constrained model we work with the real exchange rate, we do not pursue the I(1) vs. I(2) issue in this paper although we believe it is an interesting topic for future research, particularly in instances when the degree one homogeneity of the nominal exchange rate in relative prices is not satisfied.<sup>3</sup>

#### 4. Forecast accuracy

While the equilibrium estimates for the tri-polar model look plausible, ever since Meese and Rogoff (1983), the real test of the validity of an exchange rate model lies in its forecasting power.

The above analysis has been performed on the system estimated up to September 1998. To allow the computation of out-of-sample forecasts, the entire model was reestimated over the period January 1983–December 1995. The long-run cointegrating relationships were reimposed, namely PPP augmented by interest differentials, but the coefficients on the interest differentials were reestimated. Out-of-sample forecasts for 1–12 months ahead were then made. One observation was then added to the end of the sample and the entire process repeated, including estimation of the model. The final estimation is for the period January 1983–September 1998. The free coefficients in the cointegrating relationships and the short-run dynamics were therefore allowed to change over time. The coefficient on the interest differential in the German relationship varies between 0.107 and 0.134, while the coefficient on the Japanese differential in the second vector lies in the range 0.172–0.361. The relative stability of the German vector strengthens our belief that we have captured a meaningful relationship. However, the movements in the Japanese coefficient may indicate that we have excluded an important variable from our simple model, and we question the appropriateness of (short-term) interest rates as a proxy for capital flows in the Japanese case. Further refinements of the model might usefully employ some measure of stock market performance which may be qualitatively more important for Japanese capital flows.

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<sup>3</sup> An anonymous referee has pointed out that the existence of I(2) trends will affect the critical values of the Trace statistics. However, and as we have noted, the estimated eigenvalues and the roots of the characteristic polynomial from our system seem to support the existence of two stationary cointegrating vectors.

Table 5  
Out-of-sample forecasting performance of tri-polar model

Horizon	Deutsche mark				Japanese yen			
	RMSE ratio (model)	Direction (model)	RMSE ratio (VAR)	Direction (VAR)	RMSE ratio (model)	Direction (model)	RMSE ratio (VAR)	Direction (VAR)
1	0.952	60.61	0.970	60.61	1.137	45.45	1.226	45.45
2	0.927	65.63	1.010	65.63	1.100	59.38	1.286	31.25
3	0.946	61.29	1.073	61.29	0.984	67.74	1.327	29.03
4	0.927 <sup>1</sup>	70.00	1.145	50.00	0.911	70.00	1.370	20.00
5	0.902 <sup>1</sup>	65.52	1.190	34.48	0.848 <sup>2</sup>	79.31	1.385	10.34
6	0.890 <sup>3</sup>	71.43	1.218	42.86	0.793 <sup>3</sup>	71.43	1.410	14.29
7	0.885 <sup>3</sup>	62.96	1.240	25.93	0.699 <sup>3</sup>	85.19	1.436	11.11
8	0.925 <sup>2</sup>	61.54	1.272	19.23	0.641 <sup>3</sup>	92.31	1.443	7.69
9	0.956	56.00	1.305	8.00	0.621 <sup>3</sup>	100.00	1.437	4.00
10	0.945	70.83	1.319	4.17	0.582 <sup>3</sup>	95.83	1.442	0.00
11	0.930 <sup>3</sup>	65.22	1.318	13.04	0.568 <sup>3</sup>	100.0	1.443	4.35
12	0.935 <sup>2</sup>	81.82	1.313	13.64	0.555 <sup>3</sup>	100.0	1.444	4.55

Notes: The superscripted numbers 1, 2 and 3 in the columns labelled RMSE ratio are 10%, 5% and 1% significance levels calculated using the Diebold–Mariano test statistic with a correction for serial correlation imparted by the overlapping forecasts.

As in the original Meese and Rogoff study, and in many articles since, we take the driftless random walk as the benchmark against which we judge our model. The forecast performance is detailed in [Table 5](#). For each horizon, it gives the ratio of the RMSE of the exchange rate forecasts from our preferred model with the weak exogeneity of US interest rates imposed,<sup>4</sup> to those from a random walk model, the proportion of correct directional forecasts, and the RMSE ratio of a VAR set up in differences but with no cointegrating vectors. Good performance would be indicated by an RMSE ratio less than unity and directional ability in excess of 50%. The significance of the relative forecast accuracy of the model compared to that of the random walk alternative is tested using the [Diebold–Mariano \(1995\)](#) procedure.

The results are clearly impressive. For the deutsche mark, the forecasts from the tri-polar model are both more accurate than the random walk and more likely to correctly suggest the direction of change than chance would suggest at all forecast horizons. The relative accuracy becomes statistically significant at the 4-month forecast horizon. The yen forecasts become significantly more accurate than the random walk at the 5-month horizon. In terms of directional forecast ability, the yen forecasts again prove better than pure chance would suggest at all horizons over 1-month. Without seeking to denigrate the results, we would point out that over the longer forecast intervals, the yen consistently depreciated over the forecast period which explains the exceptional directional ability shown by the model from the 8-month forecast horizon. However, the model can only be asked to forecast reality, and we feel that the impressive predictions of the more volatile German unit validate its performance.

The RMSE ratios for the differenced VAR (i.e. no cointegrating relationships) are clearly inferior to both the VECM formulation and a random walk. The importance of including the cointegrating vectors is made apparent by this forecasting exercise. Furthermore, the importance of imposing restrictions to reveal meaningful economic relationships, as noted in [Section 3.1](#) above, is made clear by considering the forecast performance of the model with unrestricted cointegrating vectors. Although not reported in [Table 5](#) to save space, the RMSE ratio of the unconstrained system was greater than unity for both exchange rates at all horizons, and the directional ability noticeably inferior to that of the constrained model for the both rates at all horizons in excess of 1 month. These results confirm the finding in the literature that the use of restrictions on the cointegrating relationships improves the long-run forecast performance of error correction models and also extends it to encompass short horizons.

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<sup>4</sup> The results are in fact very similar when the alpha terms on the US interest rates are left unrestricted.

## 5. Conclusions

In this paper, we have sought to model the mark, dollar and yen using a simultaneous tri-polar structure, which allows for spillovers between the currencies. The modelling strategy may be judged successful since we produced an interpretable exchange rate model with desirable economic properties. Most significantly, perhaps, when confronted with what is taken to be the most severe test of an exchange rate model—its out-of-sample forecasting ability—the model is capable of producing predictions which compare extremely well to two industry standard benchmarks. This performance is statistically significant and, we believe, of practical relevance—forecasts from models closely related to those presented here are currently used by corporate entities as part of their financial decision-making processes.

Given the current interest in academic and policy circles in the behaviour of the tri-polar relationship between the euro, dollar and yen, we believe our results are suggestive of the functioning of such a system. Our approach suggests, for example, that such a system will exhibit both long- and short-run predictability (and the interaction amongst variables accords with economic intuition) and this may be useful from the perspective of setting target zone bands and monitoring their evolution over time. Furthermore, our analysis suggests that shocks to the system, originating in the system itself, do not create excess exchange rate volatility. This in turn may give comfort to those who favour a freely floating structure for the tri-polar grouping since it seems to imply that the system will be more stable than a system based on multiple bilateral relationships.

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