StMoMo: An R Package for Stochastic Mortality Modelling

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Agenda

- Motivation and Literature Review
- Generalised Age-Period-Cohort mortality models
- StMoMo package
- Conclusions
StMoMo: Stochastic Mortality Modelling

Who is MoMo?
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Who is MoMo?
Advances in mortality modelling

- **Lee-Carter model** (Lee and Carter 1992)
  - Add more bilinear age-period components (Renshaw and Haberman 2003)
  - Add a cohort effect (Renshaw and Haberman 2006)

- **Two factor CBD model** (Cairns, Blake, and Dowd 2006)
  - Add cohort effect, quadratic age term (Cairns et al. 2009)
  - Combine with features of the Lee-Carter (Plat 2009)

- **Many more models** proposed in the literature (e.g. Aro and Pennanen (2011), O’Hare and Li (2012), Börger, Fleischer, and Kuksin (2013), Alai and Sherris (2014))
Mortality modelling in R

- **Demography** (Hyndman 2014)
  - Lee-Carter model and several of its variants
- **ilc** (Butt, Haberman, and Shang 2014)
  - Lee-Carter with cohorts and Lee-Carter under a Poisson framework
- **Lifemetrics**
  (http://www.macs.hw.ac.uk/~andrewc/lifemetrics/)
  - CBD and extensions
  - Lee-Carter with cohorts and Lee-Carter under a Poisson framework
Limitation of existing R packages

- Not easily expandable to include new models
- Limited forecasting and simulation capabilities
- Limited tools for goodness-of-fit analysis
- Do not allow for parameter uncertainty
Limitation of existing R packages

- Not easily expandable to include new models
- Limited forecasting and simulation capabilities
- Limited tools for goodness-of-fit analysis
- Do not allow for parameter uncertainty

- **StMoMo** seeks to overcome these limitations
StMoMo: An R package for Stochastic Mortality Modelling

- On CRAN:  
  http://cran.r-project.org/web/packages/StMoMo/
- Development version on Github:  
  https://github.com/amvillegas/StMoMo
- To install the stable version on R CRAN:

  `install.packages("StMoMo")`

- To load within R:

  `library(StMoMo)`
Overview of the structure of **StMoMo**

```
Generalised Age-Period-Cohort model

Define abstract GAPC

lc          rh          apc          StMoMo          cbd          m6          m7

Fit the model to data

fit

- plot
- residuals
- forecast
- simulate
- bootstrap

plot
plot
plot
plot
simulate
```
Generalised Age-Period-Cohort stochastic mortality models

**StMoMo** is based on the unifying framework of the family of Generalised Age-Period-Cohort stochastic mortality models

- General Age-Period-Cohort model structure (Hunt and Blake 2015)
- Generalised (non-)linear model (Currie 2014)
General Age-Period-Cohort model structure

EW: male death rates (1961)

\[ \log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i \kappa_t(i) + \beta_0 x + \gamma_t - x \]
General Age-Period-Cohort model structure

EW: male death rates (1965)

\[ \log \mu_{xt} = \alpha x + \sum_{i=1}^{N} \beta_i x \kappa_{t(i)} + \beta_0 x \gamma_t - x \]
General Age-Period-Cohort model structure

EW: male death rates (1970)

\[ \log \mu_{xt} = \alpha x + \sum_{i=1}^{N} \beta_i x \kappa_{it} \]

log death rates

age

EW: male death rates (1970)
General Age-Period-Cohort model structure

EW: male death rates (1975)

log death rates

age
General Age-Period-Cohort model structure

EW: male death rates (1980)

\[ \log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i (i) x \kappa_t (i) + \beta_0 x \gamma_t - x \]
General Age-Period-Cohort model structure

EW: male death rates (1985)

\[
\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i \kappa_t(i) + \beta_0 \gamma_t - x
\]
General Age-Period-Cohort model structure

EW: male death rates (1990)

\[
\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i x_{\kappa_t(i)} + \beta_0 x_{\gamma_t} - x
\]
General Age-Period-Cohort model structure

General Age-Period-Cohort model structure

EW: male death rates (2000)

\[
\log \mu_{xt} = \alpha_x + \sum_{i=1}^N \beta_i(x) \kappa_t(i) + \beta_0(x) \gamma_t - x
\]
General Age-Period-Cohort model structure

$\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i (i) x \kappa_t (i) + \beta_0 x \gamma_t - x$
General Age-Period-Cohort model structure

EW: male death rates (2010)

\[ \log \mu_{xt} = \alpha_x + N \sum_{i=1}^{N} \beta_i x \kappa_t(i) + \beta(0) x \gamma_t - x \]
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

\[ \log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_i \kappa(t_i) + \beta_0 \gamma_t - \chi \]
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

\[
\log \mu_{xt} = \alpha_x
\]
General Age-Period-Cohort model structure

\[
\log \mu_{xt} = \alpha_x + \kappa_t
\]
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

\[
\log \mu_{xt} = \alpha_x + \beta_x \kappa_t
\]
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

\[
\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)}
\]
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_{x(i)}^{(i)} \kappa_{t(i)} + \gamma_{t-x}
General Age-Period-Cohort model structure

EW: male death rates (1961–2011)

\[
\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}
\]
Generalised Age-Period-Cohort stochastic mortality models

1. Random Component:

\[ D_{xt} \sim \text{Poisson}(E^c_{xt} \mu_{xt}) \quad \text{or} \quad D_{xt} \sim \text{Binomial}(E^0_{xt}, q_{xt}) \]
Generalised Age-Period-Cohort stochastic mortality models

1. Random Component:

\[ D_{xt} \sim \text{Poisson}(E_{xt}^c \mu_{xt}) \text{ or } D_{xt} \sim \text{Binomial}(E_{xt}^0, q_{xt}) \]

2. Systematic Component:

\[ \eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta^{(i)} x \kappa^{(i)} t + \beta^{(0)} x \gamma_{t-x} \]

- Lee-Carter type: \( \beta^{(i)} x \), non-parametric
- CBD type: \( \beta^{(i)} x \equiv f^{(i)}(x) \), pre-specified parametric function
Generalised Age-Period-Cohort stochastic mortality models

1. Random Component:

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- Lee-Carter type: \( \beta_x^{(i)} \), non-parametric
- CBD type: \( \beta_x^{(i)} \equiv f(i)(x) \), pre-specified parametric function

3. Link Function:

\[ g \left( \mathbb{E} \left( \frac{D_{xt}}{E_{xt}} \right) \right) = \eta_{xt} \]

- log-Poisson: \( \eta_{xt} = \log \mu_{xt} \)
- logit-Binomial: \( \eta_{xt} = \logit q_{xt} \)
4. **Set of parameter constraints:**

- Most mortality models are only identifiable up to a transformation
- Need parameters constraints to ensure identifiability
- **Constraint function** $\nu$ mapping an arbitrary vector of parameters

\[
\theta := \left( \alpha_x, \beta_x^{(1)}, \ldots, \beta_x^{(N)}, \kappa_t^{(1)}, \ldots, \kappa_t^{(N)}, \beta_x^{(0)}, \gamma_{t-x} \right)
\]

into a vector of transformed parameters

\[
\nu(\theta) = \tilde{\theta} = \left( \tilde{\alpha}_x, \tilde{\beta}_x^{(1)}, \ldots, \tilde{\beta}_x^{(N)}, \tilde{\kappa}_t^{(1)}, \ldots, \tilde{\kappa}_t^{(N)}, \tilde{\beta}_x^{(0)}, \tilde{\gamma}_{t-x} \right)
\]

satisfying the model constraints with no effect on the predictor $\eta_{xt}$ (i.e. $\theta$ and $\tilde{\theta}$ result in the same $\eta_{xt}$)
GAPC stochastic mortality models with StMoMo

GAPC model are constructed using the function

\[ \text{StMoMo}(\text{link, staticAgeFun, periodAgeFun, cohortAgeFun, constFun}) \]
GAPC stochastic mortality models with **StMoMo**

GAPC model are constructed using the function

\[
\text{StMoMo}(\text{link, staticAgeFun, periodAgeFun, cohortAgeFun, constFun})
\]

- **link**: defines the link and random component.

\[
\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}
\]
GAPC stochastic mortality models with StMoMo

GAPC model are constructed using the function

\texttt{StMoMo(link, staticAgeFun, periodAgeFun, cohortAgeFun, constFun)}

- \texttt{link}: defines the \textbf{link} and \textbf{random component}.
- The \textbf{predictor} is defined via:

\[ \eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x} \]
GAPC stochastic mortality models with \textbf{StMoMo}

GAPC model are constructed using the function

\texttt{StMoMo}(\texttt{link, staticAgeFun, periodAgeFun, cohortAgeFun, constFun})

- \texttt{link}: defines the \texttt{link} and \texttt{random component}.
- The \texttt{predictor} is defined via:
  - \texttt{staticAgeFun}: logical indicating if $\alpha_x$ is present.

\[
\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}
\]
GAPC stochastic mortality models with **StMoMo**

GAPC model are constructed using the function

\[
\text{StMoMo}(\text{link}, \text{staticAgeFun}, \text{periodAgeFun}, \text{cohortAgeFun}, \text{constFun})
\]

- **link**: defines the **link** and **random component**.
- The **predictor** is defined via:
  - **staticAgeFun**: logical indicating if \( \alpha_x \) is present.
  - **periodAgeFun**: list of length \( N \) defining \( \beta_x(i) \), \( i = 1, \ldots, N \).

\[
\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t(i) + \beta_x^{(0)} \gamma_{t-x}
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- The **predictor** is defined via:
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  - **cohortAgeFun**: defines parameter \( \beta_x^{(0)} \)

\[
\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}
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GAPC stochastic mortality models with StMoMo

GAPC model are constructed using the function

\[
\text{StMoMo}(\text{link, staticAgeFun, periodAgeFun, cohortAgeFun, constFun})
\]

- **link**: defines the link and random component.
- The **predictor** is defined via:
  - staticAgeFun: logical indicating if \( \alpha_x \) is present.
  - periodAgeFun: list of length \( N \) defining \( \beta_x^{(i)} \), \( i = 1, \ldots, N \).
  - cohortAgeFun: defines parameter \( \beta_x^{(0)} \)
- constFun: Implementation of constraint function \( v(\theta) = \tilde{\theta} \) which defines the set of parameter constraints

\[
\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}
\]
GAPC stochastic mortality models with **StMoMo**

<table>
<thead>
<tr>
<th>Model</th>
<th>Predictor ((\eta_{xt}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>(\alpha_x + \beta_x^{(1)} \kappa_t^{(1)})</td>
</tr>
<tr>
<td>CBD</td>
<td>(\kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)})</td>
</tr>
<tr>
<td>APC</td>
<td>(\alpha_x + \kappa_t^{(1)} + \gamma_{t-x})</td>
</tr>
<tr>
<td>M7</td>
<td>(\kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + ((x - \bar{x})^2 - \hat{\sigma}<em>x^2) \kappa_t^{(3)} + \gamma</em>{t-x})</td>
</tr>
</tbody>
</table>

For consistency, all under a log-Poisson setting:

\[
D_{xt} \sim \text{Poisson}(E_{xt}^c \mu_{xt})
\]

\[
\log \mu_{xt} = \eta_{xt}
\]
Lee-Carter model (Lee and Carter 1992)

\[ \eta_{xt} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} \]

**Predictor:** \[ \eta_{xt} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} \]

**Constraints:** \[ \sum_x \beta_x^{(1)} = 1, \quad \sum_t \kappa_t^{(1)} = 0 \]

\[ \nu(\theta) = \tilde{\theta}: (\alpha_x, \beta_x^{(1)}, \kappa_t^{(1)}) \rightarrow (\alpha_x + c_1 \beta_x^{(1)}, \frac{1}{c_2} \beta_x^{(1)}, c_2 (\kappa_t^{(1)} - c_1)) \]

with \[ c_1 = \frac{1}{n} \sum_t \kappa_t^{(1)} \quad c_2 = \sum_x \beta_x^{(1)} \]
Lee-Carter model (Lee and Carter 1992)

Predictor: $\eta_{xt} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)}$

Constraints: $\sum_x \beta_x^{(1)} = 1, \quad \sum_t \kappa_t^{(1)} = 0$

$v(\theta) = \tilde{\theta}: (\alpha_x, \beta_x^{(1)}, \kappa_t^{(1)}) \to (\alpha_x + c_1 \beta_x^{(1)}, \frac{1}{c_2} \beta_x^{(1)}, c_2 (\kappa_t^{(1)} - c_1))$

with $c_1 = \frac{1}{n} \sum_t \kappa_t^{(1)} \quad c_2 = \sum_x \beta_x^{(1)}$

#Define constraint function
constLC <- function(ax, bx, kt, b0x, gc, wxt, ages){
c1 <- mean(kt[1, ], na.rm = TRUE)
c2 <- sum(bx[, 1], na.rm = TRUE)
list(ax = ax + c1 * bx[, 1], bx[, 1] = bx[, 1] / c2, 
     kt[1,] = c2 * (kt[1, ] - c1))}

#Define model
LC <- StMoMo(link = "log", staticAgeFun = TRUE, 
              periodAgeFun = "NP", constFun = constLC)
CBD model (Cairns, Blake, and Dowd 2006)

**Predictor:** \( \eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} \)

**Constraints:** No constraints necessary
**CBD model (Cairns, Blake, and Dowd 2006)**

**Predictor:** \( \eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} \)

**Constraints:** No constraints necessary

---

### R Code

```r
#B2: x - \bar{x}
f2 <- function(x, ages) x - mean(ages)

#Define model
CBD <- StMoMo(link = "log", staticAgeFun = FALSE,
              periodAgeFun = c("1", f2))
```
Model definition: Predefined functions for common models

LC <- lc

CBD <- cbd(link = "log")

APC <- apc()

M7 <- m7(link = "log")

## Poisson model with predictor:
\[ \log m_{x,t} = a_{x} + b_{1x} k_{1t} \]

## Poisson model with predictor:
\[ \log m_{x,t} = k_{1t} + f_{2x} k_{2t} \]

## Poisson model with predictor:
\[ \log m_{x,t} = a_{x} + k_{1t} + g_{t-x} \]

## Poisson model with predictor:
\[ \log m_{x,t} = k_{1t} + f_{2x} k_{2t} + f_{3x} k_{3t} + g_{t-x} \]
Model definition: Predefined functions for common models

LC <- lc()
CBD <- cbd(link = "log")
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M7 <- m7(link = "log")
Model definition: Predefined functions for common models

```r
LC <- lc()
CBD <- cbd(link = "log")
APC <- apc()
M7 <- m7(link = "log")
```

## Poisson model with predictor:
\[ \log m_{x,t} = a[x] + b1[x] \cdot k1[t] \]

## Poisson model with predictor:
\[ \log m_{x,t} = k1[t] + f2[x] \cdot k2[t] \]

## Poisson model with predictor:
\[ \log m_{x,t} = a[x] + k1[t] + g[t-x] \]

## Poisson model with predictor:
\[ \log m_{x,t} = k1[t] + f2[x] \cdot k2[t] + f3[x] \cdot k3[t] + g[t-x] \]
Model fitting: Data

Sample data for England & Wales males aged 0-100 for the period 1961-2011

Dxt <- EWMaleData$Dxt
Ext <- EWMaleData$Ext
ages <- EWMaleData$ages  #0-100
years <- EWMaleData$years  #1961-2011
Model fitting: Data

Sample data for England & Wales males aged 0-100 for the period 1961-2011

Dxt <- EWMaleData$Dxt
Ext <- EWMaleData$Ext
ages <- EWMaleData$ages  #0-100
years <- EWMaleData$years #1961-2011

Dxt

```
# 0 9988 10573 10401 10011 9518 9357 8673 8705 8331
# 1 665 598 665 588 571 616 549 552 567
# 2 398 353 378 354 354 389 374 381 381
# 3 249 259 261 254 292 301 281 316 275
```
Model fitting

#Ages for fitting
ages.fit <- 55:89

#Fit other models
LCfit <- fit(LC, Dxt = Dxt, Ext = Ext, ages = ages, years = years, ages.fit = ages.fit)
APCfit <- fit(APC, Dxt = Dxt, Ext = Ext, ages = ages, years = years, ages.fit = ages.fit)
CBDfit <- fit(CBD, Dxt = Dxt, Ext = Ext, ages = ages, years = years, ages.fit = ages.fit)
M7fit <- fit(M7, Dxt = Dxt, Ext = Ext, ages = ages, years = years, ages.fit = ages.fit)
Parameter estimates

\[ \text{plot}(\text{LCfit}) \]
Goodness-of-fit: Residuals
Goodness-of-fit: Residuals

#Compute residuals
LCres <- residuals(LCfit)
CBDres <- residuals(CBDfit)
APCres <- residuals(APCfit)
M7res <- residuals(M7fit)
Goodness-of-fit: Residual heatmaps

plot(LCres, type = "colourmap", reslim = c(-3.5, 3.5))
Forecasting and simulation

- **Period indexes:** Multivariate random walk with drift

\[
\kappa_t = \delta + \kappa_{t-1} + \xi_t^\kappa, \quad \kappa_t = \begin{pmatrix}
\kappa_{t}^{(1)} \\
\vdots \\
\kappa_{t}^{(N)}
\end{pmatrix}, \quad \xi_t^\kappa \sim N(0, \Sigma),
\]

- **Cohort effect:** ARIMA\((p, q, d)\) with drift

\[
\Delta^d \gamma_c = \delta_0 + \phi_1 \Delta^d \gamma_{c-1} + \cdots + \phi_p \Delta^d \gamma_{c-p} + \epsilon_c + \delta_1 \epsilon_{c-1} + \cdots + \delta_q \epsilon_{c-q}
\]
### Forecasting

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50-year ahead ($h = 50$) central projections: period indexes, cohort index, and one-year death probabilities:

```r
LCfor <- forecast(LCfit, h=50)
CBDfor <- forecast(CBDfit, h=50)
APCfor <- forecast(APCfit, h=50, gc.order = c(1,1,0))
M7for <- forecast(M7fit, h=50, gc.order = c(2,0,0))
```
Forecasted period and cohort indexes

\texttt{plot(M7for, parametricbx = FALSE)}
Simulation

LCsim <- simulate(LCfit, nsim=500, h=50)
CBDsim <- simulate(CBDfit, nsim=500, h=50)
APCsim <- simulate(APCfit, nsim=500, h=50,
                   gc.order=c(1,1,0))
M7sim <- simulate(M7fit, nsim=500, h=50,
                   gc.order=c(2,0,0))
Simulation trajectories

# Plot period index trajectories for the LC model
plot(LCfit$years, LCfit$kt[1,],
     xlim=c(1960,2061), ylim=c(-65,15),
     type="l", xlab="year", ylab="kt",
     main="Period index (LC)"
matlines(LCsim$kt.s$years, LCsim$kt.s$sim[1,,1:20],
         type="l", lty=1)
library(fanplot)
plot(LCfit$years, (Dxt/Ext)["65",], xlab="year", ylab="q(65,t) (log scale)"
fan(t(LCsim$rates["65",,]), start=2012,
  probs=c(2.5,10,25,50,75,90,97.5), n.fan=4, ln=NULL,
  fan.col=colorRampPalette(c("black","white")))
Fancharts

LC

CBD

APC

M7
Parameter uncertainty and bootstrapping

**StMoMo** implements:

- Semiparametric bootstrapping (Brouhns et al., 2005)
- Residuals bootstrapping (Koissi et al., 2006)
Parameter uncertainty and bootstrapping

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- Residuals bootstrapping (Koissi et al., 2006)

```r
LCboot <- bootstrap(LCfit, nBoot=500,
                     type="semiparametric")
plot(LCboot, nCol = 3)
```

![Graphs showing α_x vs. x, β_x(1) vs. x, and κ_t(1) vs. t](image)
Conclusion

- Use the framework of GLMs to define the GAPC family of models
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- **StMoMo** uses this unifying framework to implement the vast majority of stochastic mortality models in the literature
  - Model fitting
  - Analysis of goodness-of-fit
  - Projection and simulations
  - Bootstrapping and parameter uncertainty
Conclusion

- Use the framework of GLMs to define the GAPC family of models

- **StMoMo** uses this unifying framework to implement the vast majority of stochastic mortality models in the literature
  - Model fitting
  - Analysis of goodness-of-fit
  - Projection and simulations
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- Easy implementation and comparison of a wide range of models making it useful for:
  - Actuaries analysing longevity risk
  - Use in the classroom
Future work

- New models for forecasting time indexes (e.g. VAR models)

- Allow for $\beta_x^{(i)} = f^{(i)}(x; \theta_i)$ (see Hunt and Blake (2014))

- Multipopulation models

- Shiny web app
http://cran.r-project.org/web/packages/StMoMo/
https://github.com/amvillegas/StMoMo

Thank you!

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References


References II


References III


