Modelling the cross-border use of collateral in payment and settlement systems

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Abstract

Banks often rely on collateralised intra-day liquidity from the central bank in order to be able to effect payments in payment and settlement systems. If a bank is holding insufficient eligible collateral in a particular country, and therefore cannot obtain credit from the local central bank, it may have to delay the settlement of trades. This constitutes a liquidity risk to the system. But a bank operating in multiple systems may face a mismatch between the location of its collateral holdings and liquidity needs. In this paper, we examine the extent to which the liquidity risk arising from such a mismatch may be mitigated by allowing cross-border use of collateral. We develop a two-country-two-bank model in which risk-neutral banks minimise expected costs with respect to their collateral choice in each country. In our baseline model, in which each banks faces a liquidity need in only one country, we find that liquidity risk is indeed reduced by cross-border use of collateral. Furthermore, banks are able to economise on their holdings of collateral and so efficiency is improved. However, because banks substitute collateral that is fungible across borders for collateral that is not, a central bank may have an incentive to free-ride on the other central banks' acceptance of foreign collateral. In extensions to our model, we show that although, routine cross-border use of collateral will lower the expected level of liquidity risk, there are certain states of the world in which liquidity risk is actually higher than in the absence of cross-border use of collateral, precisely because banks economise on their collateral holdings.

Key words: Payment and settlement systems, collateral, liquidity risk

1. Introduction

Over the past decade, there has been a decided shift towards real-time gross settlement (RTGS) of clean payments and delivery-versus-payment arrangements in securities settlement, all in central bank money (Bank for International Settlements, 2005a,b). With these developments, the focus has shifted away from credit risk and towards liquidity risk: given the asynchronous timing of settlements, each settlement can only be effected if the paying bank has sufficient funds in its account with the central bank. Recognising that high (and hence costly) liquidity requirements could encourage banks to delay settlement, central banks typically make intra-day liquidity available to settlement members on favourable terms. Indeed, access to such credit is often limited only by a bank's holdings of eligible collateral assets.

Intuitively then, liquidity risk in payment and settlement systems might be mitigated if central banks facilitated the efficient management of commercial banks' collateral portfolios and took steps to ensure that the opportunity cost of posting collateral was sufficiently low. This argues in favour of central banks accepting a wide variety of assets as collateral. Typically, however, eligible collateral lists are relatively narrow, with central banks cognisant of potential difficulties in valuing, and even more importantly, should the need arise, in liquidating certain assets. As such, many central banks' eligible lists are restricted to high-quality domestic marketable debt securities. These assets will not necessarily be held by banks in the ordinary course of business and posting them as collateral could entail an opportunity cost.

Restricted eligible collateral lists are a particular issue for banks operating in multiple countries and hence facing settlement obligations in a number of settlement systems. If in each country the central bank accepts domestic securities only, a bank must hold sufficient collateral assets to meet its expected liquidity needs in each fragmented centre in which it is active. Once liquidity demands have been realised, a bank may, given imperfectly correlated liquidity demands, find itself with a shortage of collateral in one market and abundant collateral in another. This collateral will then lay idle while the bank obtains additional assets in the country in which it experiences a shortfall. This mismatch is not only inefficient, but also constitutes a liquidity risk for the settlement system: there will likely be some disruption to the bank's settlement activity while it enters the market to acquire the necessary eligible assets.

Such issues have become increasingly prominent in policy circles. Indeed, the Basle Committee on Payment and Settlement Systems (CPSS) recently initiated a workstream on this topic in response to a call by the Payments Risk Committee, a New York-based consortium of internationally active banks, for increased cross-border use of collateral within G10 countries. A number of central banks already accept foreign collateral on a routine basis, employing a variety of institutional and infrastructural arrangements to mobilise assets (BIS, 2005b). For example, within the European System of Central Banks (ESCB), the Correspondent Central Banking Model (CCBM) was introduced in 1999 to facilitate the cross-border use of collateral to support Eurosystem credit operations, or to obtain liquidity in TARGET (ECB, 2003).[1] Under this arrangement, central banks act as custodians for one another, allowing the use throughout the

(1) The TARGET system is the Trans-European Automated Real-time Gross settlement Express Transfer system, linking national euro real-time gross settlement systems within the ESCB.

system of all eligible securities issued and located in any Eurosystem country. Around 35% of all collateral posted within the Eurosystem is delivered via this arrangement.(2) The European Central Bank has also assessed a number of links between national securities settlement systems (SSSs), which serve as an alternative, but less regularly used, vehicle for mobilising eligible securities across borders. Some other EU central banks, such as the Bank of England and Swedish Riksbank, also allow cross-border use of collateral. The Bank of England, for example, accesses the CCBM to receive euro-denominated securities; indeed, more than 70% of the collateral used to support banks' (non-self-collateralised) intra-day liquidity needs in this way. Outside of the ESCB, the Swiss National Bank employs collateral management services offered by the local SSS to receive euro-denominated securities from several EU countries in support of its repo operations. Again, this is a widely used arrangement, used to deliver some 60% of total collateral posted. Finally, the US Federal Reserve receives a host of foreign collateral assets into its accounts at Euroclear and Clearstream to support discount window lending.

This paper seeks to establish the implications for liquidity risk and efficiency in payment and settlement systems of such cross-border use of collateral, taking into account how it affects banks' choices over how much collateral to hold and where to hold it. We develop a stylised two-country-two-bank model in which risk-neutral banks minimise expected costs with respect to their collateral choice in each country. Banks are active in both countries' settlement systems and face uncertainty as to the location and size of liquidity demands. Although motivated by the question of cross-border use of collateral in intra-day credit operations, the framework described in this paper can readily be applied to other collateralised credit operations; the obvious example here would be longer term collateralised lending by central banks.

The fundamental structure of the model we present can be traced back to early work on the precautionary demand for reserves; e.g. the work by Baltensperger (1974) and Olivera (1971). In these models, the agent chooses a level of reserves so as to minimise a total expected cost function with two key components: the cost of accumulating a reserve inventory at the start of the period, and the cost of adjusting the reserve inventory in the face of a deficiency. The basic form of the problem is given by **(1)**, in which α is the marginal cost of the choice variable, X (in these models, the choice variable is reserves; in ours, it is collateral); the term under the integral represents the expected shortfall in the choice variable (Z being the draw from a distribution of required amounts), conditional on the size of the holding at the start of the period; and p represents the cost associated with facing such a shortfall:

$$E[C] = \alpha X + p\int_X^\infty (Z - X)\, f(Z)dZ \qquad \textbf{(1)}$$

In the context of our problem, each bank faces a similar cost function, comprising the cost of collateral accumulated at the start of the day, and the cost of experiencing a collateral shortfall. The cost of experiencing a shortfall reflects the cost of acquiring additional collateral and any

(2) This and the other statistics in this paragraph on the extent of cross-border use of collateral in G10 countries have been drawn from Bank for International Settlements (2005b), 'Report on cross-border collateral arrangements', Forthcoming.

costs associated with delaying the settlement of trades while additional collateral is sought in the market.

Our model extends the structure in **(1)** to incorporate expected collateral and shortfall costs in two countries, and we solve for collateral choices in each country. Banks make collateral choices under uncertainty as to both the size *and* the location of their liquidity needs. In our baseline model, we assume that each bank only realises a liquidity demand in one country at any one time; in other words, banks' liquidity demands are perfectly negatively correlated across countries. Using our baseline model, we compare outcomes for liquidity risk and efficiency for cases in which: (i) there is no cross-border use of collateral; (ii) one central bank accepts foreign assets as collateral, but the other does not; and (iii) both central banks permit cross-border use of collateral. A number of key results emerge.

First we show that, when both countries permit cross-border use of collateral (which we refer to as the 'symmetric' case), banks will concentrate their holdings in the country with the lowest collateral costs and, given sufficiently high costs of experiencing a shortfall, will reduce precautionary collateral holdings in each country. Importantly, notwithstanding a decline in total collateral held, we find that liquidity risk, as measured by expected collateral shortfalls, will fall in both countries. This reflects the fact that it will always be optimal for a bank to hold a larger quantum of collateral across two connected countries than in a single unconnected location. Hence, there will always be a larger pool from which to draw to meet a given liquidity need in a single country.

When only one country permits cross-border use of collateral, the so-called asymmetric case, we show that banks' collateral choices will be driven by two potentially offsetting factors. On the one hand, banks will shift collateral holdings towards the fungible collateral (i.e. that which can be used across borders); on the other, banks will still be inclined to accumulate larger holdings of the cheaper collateral. When it is the cheaper collateral that can be used across borders, these two factors are mutually reinforcing and the outcome will be as in the case with symmetric cross-border use of collateral. When the fungible collateral is only slightly more expensive, banks will still hold fungible collateral only, but slightly less will be held overall. Again, liquidity risk will decline in both countries. Finally, when the fungible collateral is *significantly* more expensive, collateral will be held in both countries and the expected shortfall in the country accepting foreign collateral will be the same as in the case with no cross-border use. But the expected shortfall in the country *not* accepting foreign collateral will still fall, because banks will always find it optimal to substitute at least some fungible foreign collateral for domestic collateral. The fact that the country not accepting foreign collateral always experiences a decline in liquidity risk suggests a free-rider benefit; this might argue in favour of policy coordination across central banks on implementing cross-border collateral arrangements.

Given lower collateral holdings, and lower expected shortfalls (and hence shortfall costs), when cross-border use of collateral is permitted, banks' total expected costs decline. Hence, efficiency is improved.

We make a number of extensions to the model, so as to enrich the analysis and overcome some of the simplifications in the baseline model. We first consider how banks' decisions are affected by the possibility that operational problems disrupt the links between central banks and foreign CSDs and temporarily prevent the mobilisation of collateral across borders. We find that, if the probability of a disruption is sufficiently high, banks' incentives to gravitate towards the cheapest collateral in the symmetric case will be tempered.

Recognising that central banks may perceive certain costs to routine acceptance of foreign collateral, we also consider an extension in which central banks have the option of accepting collateral in *emergencies* only. Under such a regime, and with a sufficiently low probability that the emergency facility will be triggered, banks' reductions in collateral holdings will be more muted than those if cross-border use of collateral were allowed routinely. As a result, should an emergency situation arise in one country, banks will have a larger pool of collateral to draw upon than they would in the case of routine cross-border use of collateral. Hence, expected shortfalls in the event of an emergency will be lower. If central banks place a higher weight on liquidity risk mitigation in emergency scenarios, in which it may be more difficult to access additional collateral in the home market, than liquidity risk mitigation in normal circumstances, such a policy may be attractive.

Our final extension allows some probability that a bank experiences a liquidity need in both countries simultaneously. Although banks will adjust collateral holdings to take account of this possibility, the fact that there remains a probability that banks experience asynchronous liquidity needs may still encourage banks to reduce total collateral holdings relative to the case with no cross-border use. This implies, therefore, that expected shortfalls will be higher in those states of the world in which a bank realises simultaneous liquidity needs, although the distribution of these shortfalls across the two countries will depend, in our model, on where the collateral is actually being held (i.e. on relative start-of-day collateral costs). Nevertheless, with the probabilities considered in our analysis, the probability-weighted expected shortfall across all possible states will still decline when both countries allow cross-border use of collateral.

The paper is structured, as follows. In Section 2, we introduce the baseline model, in which a bank faces uncertainty as to the size and location of liquidity needs in each country. At this stage we do not allow for the possibility that a bank faces liquidity demands in both countries simultaneously; or indeed in neither country. We outline the timeline of events and decisions, and in Section 3 solve the model for the three cases described above. In Section 4, we compare outcomes for expected collateral shortfalls (our measure of liquidity risk) and total expected costs (our measure of efficiency). Section 5 introduces the extensions to the baseline model. Section 6 introduces some policy issues and considerations, which might explain why more central banks do not currently accept foreign collateral to support routine credit operations. Section 7 concludes.

2. A model of the cross-border use of collateral in payment and settlement systems

In this section, we present our baseline model, sketching the environment and introducing the time-line for banks' actions, before going on in Section 3 to solve for banks' optimal collateral choices.

Our model is necessarily stylised, but is designed to capture the essential features of the commercial bank's collateral decision: (i) decision-making under uncertainty; (ii) cost-minimisation; (iii) imperfectly correlated liquidity needs in multiple centres.

We assume that there are two countries, indexed by $i \in \{1,2\}$. Each is home to a settlement system, a central securities depository (CSD)[(3)], a (deep and liquid) securities market and a central bank. We also assume that there are just two banks, indexed by $j = \{A, B\}$, [(4)] each active in both countries and each is a member of both settlement systems and CSDs. Both banks are risk neutral.

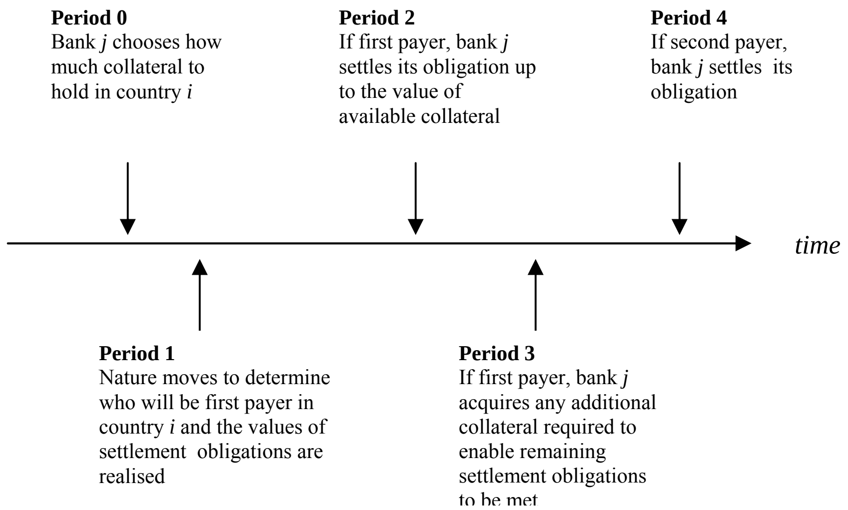
Events take place over the course of a single day, with each bank has one settlement obligation with the other bank in each country. Trades between banks in country *i* are of equivalent value and are settled across accounts at the local central bank on a real-time gross basis. Thus, a bank can only fulfil a settlement obligation when its account balance at the central bank is at least the value of the obligation. In each country, settlement obligations are satisfied sequentially; thus the first payer in each country has to obtain credit from the local central bank to ensure that it has sufficient funds to settle. Because trades have the same value, the second payer is able to recycle the liquidity received from the first payer. The first payer is then able to repay the local central bank.

For simplicity, we assume that a bank will only be first payer in one country; this captures the likelihood that a bank operating in multiple countries will experience a lack of correlation in its settlement obligations (and liquidity needs) across these countries. With this assumption, we restrict attention in the baseline model to bank-specific local liquidity needs. Another interpretation might be that there is a *certain* and an *uncertain* component to banks' routine liquidity needs. Hence, our framework might be thought of as modelling uncertain needs (liquidity *surprises*) only, with the certain component normalised to zero. For simplicity, we model such surprises for each bank as arising in one country only. However, we do extend the framework to allow for a bank to face simultaneous liquidity demands across the two countries in Section 5. Credit will only be granted on the basis of adequate and eligible collateral and bank *j* makes its collateral choice under uncertainty as to the country in which it will be first payer, and the value of the trade.[(5)]

[(3)] It can equally be assumed that the settlement system and CSD are, as is often the case, part of the same entity.
[(4)] We assume no strategic interaction between the two banks in this model. While this assumption may appear implausible with only two banks in the model, it may be strengthened by interpreting the two banks as two 'types': i.e. *n* identical *A*-type, and *n* identical *B*-type, banks.
[(5)] A central bank will normally apply a haircut to protect itself against fluctuations in the value of collateral posted. For simplicity, we assume that the haircut is zero for all collateral (either domestic or foreign) received in both countries.

Assume also that the two countries lie in the same time-zone, so that the settlement systems, CSDs and securities markets operate concurrently. The day comprises five periods. Chart 1, below, illustrates the sequence of events.

Chart 1. A time-line for actions in country *i*



In period 0 banks minimise expected costs to establish their optimal collateral holdings in each country; bank *j* chooses collateral c_i^j in country *i*. The per-unit cost of collateral posted in this period in country *i* is γ_i. [(6)] This decision is made under uncertainty; a bank simply knows the probability that it will be first payer in country *i*, and the distribution from which the value of its settlement obligations, v_i, will be drawn. In period 1 nature moves to determine in which state of the world the banks find themselves. There are two possible states: θ' and θ''. With probability 0.5, bank *A* will be first payer in country 1 and bank *B* first payer in country 2, a state we denote by θ'. With probability 0.5, bank *B* will be first payer in country 1 and bank *A* first payer in country 2, a state denoted by θ''. Also in this period an independent draw is made from an exponential distribution in each country *i* to determine the value of settlement obligations. This distribution is characterised by the probability density function and cumulative distribution function in **(2)**. This distribution has the property that low-value draws will occur with a high probability, and given the long tail, high-value settlement obligations can also occur, but with low probability.

$$f(v_i) = \lambda \exp(-\lambda v_i)$$

$$F(v_i) = 1 - \exp(-\lambda v_i) \qquad \textbf{(2)}$$

Assume bank *j* is the first payer. In period 2 there are three possibilities. Either:

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(6) The cost, or opportunity cost, of obtaining collateral may differ across markets for a number of reasons. For example, there may be differences in credit worthiness or liquidity; or countries may have differing prudential liquidity regimes, with implications for the stock of eligible assets a bank actually holds. James (2003) considers the cost of posting collateral in CHAPS Sterling, the large value payment system of the United Kingdom. The author suggests that a reasonable cost measure might be the reverse-repo spread (i.e. the spread between secured and unsecured borrowing costs), and argues that the existence of a quantitative prudential liquidity requirement in the United Kingdom significantly lowers the opportunity cost of posting collateral for domestic banks that are subject to the regime.

(i) Bank *j* has sufficient collateral in country *i* to be able to raise the necessary funds to satisfy the full value of its settlement obligation;

(ii) Bank *j* has insufficient collateral in country *i*, but central bank *i* accepts foreign collateral in support of its credit operations and bank *j* is holding sufficient idle collateral in the other country which can be brought across (we assume at no cost) to satisfy its collateral requirements;

(iii) Bank *j* has insufficient collateral in country *i* and either foreign collateral is not accepted, or there is insufficient idle collateral in the other country, leaving bank *j* with a shortfall s_i^j.

In event (iii), bank *j* satisfies its settlement obligation up to the value of collateral available. φ is an indicator variable for central bank acceptance of foreign collateral, taking the value 1 if foreign collateral is accepted and 0 if not. Bank *j*'s shortfall in country 1, s_1^j is then:

$$s_1^j = \max\left\{0, v_1 - c_1^j - \varphi c_2^j\right\} \qquad \textbf{(3)}$$

Facing a shortfall in period 3, bank *j* has to obtain additional collateral before it can generate sufficient funds to met its remaining settlement obligation. If foreign collateral is ineligible, these additional assets will have to be obtained in the domestic market; if foreign collateral is eligible, bank *j* will obtain this additional collateral in whichever is the cheaper location.[(7)] We assume that the per-unit cost of acquiring extra collateral in period 3 in country *i* is $\alpha\gamma_i$ where collateral is at least as costly in period 3, as in period 1; i.e. $\alpha \geq 1$.[(8)] One might model α as an increasing function of the size of the shortfall – i.e. the larger the shortfall, the more difficult and so expensive it will be to obtain the necessary collateral. For analytical simplicity, however, α is constant in our model. A bank facing a shortfall faces a further *delay* cost, for while it seeks collateral it must delay the settlement of a portion of its obligation. The delay cost per unit of a shortfall is equal to *d* in both countries and captures, for example, penalties for missing time-critical settlement obligations and reputational costs.

Taking the collateral and delay costs together, the per-unit cost of obtaining additional collateral in period three in country *i* when foreign collateral is not eligible is $\omega_i = d + \alpha\gamma_i$. When foreign collateral is eligible, bank *j* can source it in the cheapest location; hence, the per-unit cost is $\omega_i = d + \alpha \min\{\gamma_1, \gamma_2\}$. We assume that the penalty faced by banks for having inadequate collateral to meet demands is large relative to start of day cost of collateral: we assume $\omega_i / \gamma_i > 4$ for both *i*. As a consequence of this assumption, we do not consider solutions in which banks find it optimal to hold little or no collateral at the beginning of day and instead wait until liquidity needs have been realised before obtaining further collateral. In period 4 the second payer in country *i* settles its obligation, recycling funds received from the first payer. The first payer is then able to repay the central bank.

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(7) It may also be that, by increasing the eligible collateral pool, cross-border use of collateral reduces the time taken to acquire additional assets in the event of a shortfall. We do not explicitly model this effect, although in arguing that any additional collateral acquired late in the day is obtained in the cheapest market, we make the implicit assumption that the cheapest market is also the most accessible.

(8) $\alpha > 1$ is plausible if liquidity conditions are tighter later in the day, or if counterparties raise prices in the knowledge that bank *j* must obtain the assets in order to be able to meet its obligations.

3. Banks' collateral choices

Given the environment and the sequence of events detailed above, banks *A* and *B* minimise the expected cost functions in **(4)**. Expected shortfalls are conditional on the state of the world, as described in Section 2:

$$EC^A = \gamma_1 c_1^A + \gamma_2 c_2^A + \omega_1 \frac{E[s_1^A \mid \theta']}{2} + \omega_2 \frac{E[s_2^A \mid \theta'']}{2}$$

(4)

$$EC^B = \gamma_1 c_1^B + \gamma_2 c_2^B + \omega_1 \frac{E[s_1^B \mid \theta'']}{2} + \omega_2 \frac{E[s_2^B \mid \theta']}{2}$$

The first two terms in a bank's expected cost function are the collateral costs incurred in each country at the start of the day. The remaining terms are the expected shortfall costs incurred in period 3 if bank *j* is first payer, which are based on the expected value of settlement obligations, conditional on such value exceeding collateral obtained in period 0 (either in country *i* only, or in both countries if cross-border use of collateral is permitted). The value of these terms depends on whether collateral can be used across borders. In this section, we will derive banks' collateral choices in period 0 for three cases: (i) no cross-border use of collateral; (ii) asymmetric cross-border use of collateral; (iii) symmetric cross-border use of collateral.

3.1. No cross-border use of collateral

We first assume that neither central bank allows banks to use collateral held abroad to support intra-day credit requirements. Thus, a bank will always have to obtain further collateral domestically if it is first payer and the value of its settlement obligation exceeds the amount of collateral it chooses to hold in that country in period 0. As collateral cannot be used across borders, a bank's conditional expectation of its shortfall in country *i* is independent of its collateral choice in the other country.

We first illustrate the calculation of the expected shortfall when bank *A* is first-payer in country 1. Clearly, the expected shortfall is zero for all realisations of obligation values, v_1, below the amount of collateral posted, c_1^A. The expected shortfall thus simplifies to the expression in **(5)**, which captures the expected excess liquidity need, conditional on the value of the settlement obligation being greater than the amount of collateral posted.

$$E[s_1^A \mid \theta'] = \int_{c_1^A}^{\infty} (v_1 - c_1^A) f(v_1) dv_1 \qquad \textbf{(5)}$$

Integrating **(5)** by parts and substituting in for the exponential distribution (from **(2)**) and solving, yields **(6)**:

$$E[s_1^A \mid \theta'] = \frac{\exp(-\lambda c_1^A)}{\lambda} \qquad \textbf{(6)}$$

And when $\theta = \theta''$:

$$E[s_2^A \mid \theta''] = \frac{\exp(-\lambda c_2^A)}{\lambda} \quad \textbf{(7)}$$

Expected shortfalls for bank *B* are analogous. Plugging these shortfall expressions into the expected cost functions in **(4)**, we obtain:

$$EC^j = \gamma_1 c_1^j + \gamma_2 c_2^j + \omega_1 \frac{\exp(-\lambda c_1^j)}{2\lambda} + \omega_2 \frac{\exp(-\lambda c_2^j)}{2\lambda} \quad \text{for } j = A, B \quad \textbf{(8)}$$

Bank *j* minimises EC^j with respect to its collateral choice in each country, subject to the constraints $c_1^j, c_2^j \geq 0$. The cost-minimising collateral choices are

$$c_i^j{*} = \frac{1}{\lambda} \ln\left(\frac{\omega_i}{2\gamma_i}\right) \qquad i = 1,2 \quad j = A, B \quad \textbf{(9)}$$

Since $\omega_i / \gamma_i > 4$ by assumption, the expected per-unit cost of experiencing a shortfall in period 3 (taking into account the probability of being first payer) is greater than the start-of-day cost. Hence, bank *j* chooses to hold a strictly positive amount of collateral in country *i* in period 0. As one might expect, bank *j*'s collateral choice in a given country is decreasing in the unit cost of obtaining collateral at the start of the day (γ_i) and increasing in the cost of meeting a shortfall later in the day (ω_i). It is also increasing in the mean value of settlement obligations ($1/\lambda$). Given these comparative statics, it can be shown that, when the perceived cost of delay, *d*, is high, a bank will accumulate large precautionary holdings of collateral in each unconnected centre.

Because collateral cannot be used across borders, a bank's collateral choice in one country is independent of its choice in the other. Clearly, if collateral costs are the same in the two countries banks will hold the same amount of collateral in each country at the start of the day. If a cost-differential exists, a greater quantity of collateral will be held in the cheaper country.

3.2. Asymmetric cross-border use of collateral (partial fungibility)

Now let us assume that country 2 collateral can be used to support liquidity needs in country 1, but not vice versa. In this case, the expected shortfall in country 1 is dependent upon the collateral choice in country 2: if first payer in country 1, bank *j* can draw upon on any idle collateral holdings in country 2 to support its liquidity needs. **(10)** formalises the expression for $E[s_1^A \mid \theta']$. Note that c_2^A now appears under, and in the lower bound of, the integral, reflecting the fact that a shortfall will only now arise if the value of a settlement obligation exceeds total collateral available in the two countries:

$$E[s_1^A \mid \theta'] = \int_{c_1^A + c_2^A}^{\infty} (v_1 - c_1^A - c_2^A) f(v_1) dv_1 = \frac{1}{\lambda} \exp(-\lambda(c_1^A + c_2^A)) \quad \textbf{(10)}$$

The expression for the expected shortfall in country 2, on the other hand, is equivalent to that in the case of no cross-border use of collateral (equation **(7))** because country 1 collateral cannot be drawn upon should settlement obligations exceed domestic collateral available in period 2. Bank *B*'s expected shortfalls are, again, analogous.

The expected cost functions for banks *A* and *B* are thus given by **(11)**:

$$EC^j = \gamma_1 c_1^j + \gamma_2 c_2^j + \omega_1 \frac{\exp(-\lambda(c_1^j + c_2^j))}{2\lambda} + \omega_2 \frac{\exp(-\lambda c_2^j)}{2\lambda} \quad \text{for } j = A, B \qquad \textbf{(11)}$$

Recall that with foreign collateral eligible in country 1, $\omega_1 = d + \alpha \min\{\gamma_1, \gamma_2\}$. In country 2, however, any additional collateral required can only be sourced in the domestic market and hence $\omega_2 = d + \alpha\gamma_2$.

The ultimate outcome will be dependent upon relative collateral costs in the two countries. Let us first consider the case in which $\gamma_1 \geq \gamma_2$:

3.2.1 $\gamma_1 \geq \gamma_2$

Lemma 1: If $\gamma_1 \geq \gamma_2$, there is no cost to mobilising collateral across borders, and haircuts for domestic and foreign collateral are identical, it will always be optimal for the bank to hold all of its collateral in country 2.

The intuition for Lemma 1 is as follows. A bank facing uncertainty as to where it will experience a liquidity need, will attach a higher value to collateral which is eligible in both countries: holding fungible collateral makes it less likely that collateral will lay idle once liquidity demands have been realised. Country 2 collateral becomes more attractive still if it can be obtained more cheaply.

In this case, with $\gamma_1 \geq \gamma_2$ and $\omega_1 = \omega_2 = \alpha\gamma_2 + d$, each bank minimises its costs with respect to c_2^j only. For bank *i*, this yields the following optimal collateral choice:

$$c_2^j * = \frac{1}{\lambda} \ln\left(\frac{\omega_2}{\gamma_2}\right) \quad \text{for } j = A, B \qquad \textbf{(12)}$$

$c_2^j * = c^j * > 0$ since $\omega_2 > \gamma_2$.

3.2.2 $\gamma_1 < \gamma_2$

With these relative costs, country 2 collateral remains attractive by virtue of its fungibility but there may be an incentive to hold a proportion of the collateral portfolio in country 1 because it is cheaper. In this case, then, bank j minimises EC^j with respect to its collateral choice in each country, subject to the constraints $c_i^j \geq 0$ for both i. The optimal collateral choice in each country is now dependent upon the choice in the other. Hence, the first-order conditions initially yield two simultaneous equations for each bank. These may be expressed as *best responses* (i.e.each bank's cost minimising collateral choice in country 1(2) given its own collateral choice in country 2(1) and vice versa).[9] Each bank's optimal collateral choice (best response) in country 1, $BR_1^j(c_2^j)$, to its own choice in country 2, and vice versa, is given in **(13)**:

$$BR_1^j(c_2^j) = \frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right) - c_2^j \qquad \text{for } 0 \le c_2^j \le \frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right)$$

$$= 0 \qquad \text{for } c_2^j > \frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right)$$

(13)

$$BR_2^j(c_1^j) = \frac{1}{\lambda}\ln\left(\frac{\omega_1\exp(-\lambda c_1^j) + \omega_2}{2\gamma_2}\right) \quad \text{for } 0 \le c_1^j \qquad \text{for } j = A, B$$

$BR_1^j(c_2^j)$ has a derivative of -1 if c_2^j is sufficiently low since in country 1 collateral from either country are perfect substitutes and a bank only realises a liquidity demand in one of the countries. $BR_2^j(c_1^j)$ has a flatter slope: given that country 1 collateral cannot be used in country 2, the cost-minimising collateral choice in country 2 is less responsive to the choice in country 1. These derivatives are reflected in the slopes of the best response functions in Charts 2 and 3 (located at the end of Section 3.3).

The charts reveal that in certain instances a bank's best responses will lead to identical collateral choices to those in the absence of cross-border use of collateral. For example, when a bank chooses to hold no collateral in country 2, it will be optimal to hold the same amount of collateral in country 1 as when cross-border use of collateral is not permitted. This reflects the fact that if a bank is holding no collateral in country 2, it will respond in country 1 as though foreign collateral is not eligible. And when a bank's collateral choice in country 1 tends to infinity, its best response in country 2 will tend towards its optimal choice with no cross-border use of collateral. The intuition is clear: with infinite collateral available in country 1, a bank can satisfy any liquidity demand in country 1 without the need to draw on idle collateral held in country 2; hence it will make its c_2^j choice on the basis of expected country 2 needs only.

In the charts, a bank's optimal collateral choices are given by the point at which the best response functions cross. These charts are drawn for alternative relative cost scenarios, and may be used to illustrate the conditions under which collateral will be held either in country 2 only, or in both countries. They may also be used to determine how much collateral will be held overall.

If the cost-differential between country 1 and country 2 is sufficiently great, a bank may have an incentive to hold some cheaper country 1 collateral as well as some of the more expensive, but fungible, country 2 collateral. Chart 2 illustrates such a case. Given the slopes of the best response functions, a necessary and sufficient condition for a bank to hold collateral in both countries is that $BR_1^j(c_2^j)$ meets the country 2 collateral axis at a value of c_2^j greater than $BR_2^j(0)$. This may be expressed as a condition on the cost-differential between the two countries:

[9] The use of this terminology does not imply that there is any strategic behaviour by the banks.

$$BR_1^j(c_2^j) = 0 \Leftrightarrow c_2^j = \frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right) > BR_2^j(0) = \frac{1}{\lambda}\ln\left(\frac{\omega_1+\omega_2}{2\gamma_2}\right) \qquad \text{for } j=A,B$$

$$\Leftrightarrow \gamma_2 - \gamma_1 > \gamma_1 \frac{\omega_2}{\omega_1} \qquad \textbf{(14)}$$

If we take the first-order condition with respect to c_2^j (from which we derive $BR_2^j(c_1^j)$) and substitute $BR_1^j(c_2^j)$ for c_1^j we obtain a bank's optimal collateral choices:

$$c_1^j{}^* = \frac{1}{\lambda}\ln\left(\frac{\omega_1(\gamma_2-\gamma_1)}{\omega_2\gamma_1}\right)$$

$$\textbf{(15)}$$

$$c_2^j{}^* = \frac{1}{\lambda}\ln\left(\frac{\omega_2}{2(\gamma_2-\gamma_1)}\right) \qquad \text{for } j=A,B$$

A bank holds country 2 collateral only when the difference between collateral costs is sufficiently small such that **(14)** does not hold. In this case the benefits of fungibility outweigh the additional cost of accumulating country 2, rather than country 1, collateral. This is shown in Chart 3 (where $\gamma_2 - \gamma_1 < \gamma_1(\omega_2/\omega_1)$). A bank's optimal collateral choice under this scenario is:

$$c_2^j{}^* = \frac{1}{\lambda}\ln\left(\frac{\omega_1+\omega_2}{2\gamma_2}\right) \qquad \text{for } j=A,B \qquad \textbf{(16)}$$

3.3 Symmetric cross-border use of collateral (full fungibility)

Finally, we turn to the case in which both central banks accept foreign collateral. With all collateral fungible, bank *A*'s expected shortfall in each country will be determined as in **(10)**. Bank *B*'s expected shortfalls are analogous. Plugging expected shortfalls into the expected cost function, we obtain:

$$EC^j = \gamma_1 c_1^j + \gamma_2 c_2^j + \omega_1 \frac{\exp(-\lambda(c_1^j + c_2^j))}{2\lambda} + \omega_2 \frac{\exp(-\lambda(c_1^j + c_2^j))}{2\lambda} \qquad \text{for } j=A,B \qquad \textbf{(17)}$$

With collateral fully fungible, the location in which a bank chooses to hold collateral is entirely determined by relative costs. (10) Both banks will choose to hold all start-of-day collateral in country 1 and will also obtain any additional collateral during the day in country 1 if $\gamma_1 < \gamma_2$. With these relative costs, bank *j* minimises equation **(17)** with respect to the collateral choice in country 1 only, yielding an optimal choice of $(1/\lambda)\ln(\omega_1/\gamma_1)$. This choice is illustrated in Charts 2 and 3, above, which are drawn for examples with $\gamma_1 < \gamma_2$. Analogously, when $\gamma_1 > \gamma_2$, bank *j* minimises **(17)** with respect to country 2 collateral only, yielding an optimal choice of

(10) This is consistent with evidence from the Eurosystem, where some 63% of the collateral used across borders (as at December 2003) was issued by non-AAA, and hence 'cheaper', issuers (Italy -18.4%; Belgium - 15.6%; Greece - 4.4%; Portugal – 2.4%; and international non-sovereign (allocated to Clearstream Luxembourg, and predominantly non-AAA) – 22.2%). These data are sourced from the European Central Bank, and may be found at: http://www.ecb.int/stats/payments/securities/html/coll2.en.html.

$(1/\lambda)\ln(\omega_2/\gamma_2)$. This is equivalent to the collateral choice for this relative cost scenario in the asymmetric case. Banks are indifferent as to where collateral is held if $\gamma_1 = \gamma_2$. In this case, expected costs can be minimised with respect to total collateral, yielding an optimal start-of-day choice equal to $(1/\lambda)\ln(\omega/\gamma)$ (where $\gamma = \gamma_1 = \gamma_2$ and $\omega = \omega_1 = \omega_2$). The distribution of collateral holdings across the two countries is indeterminate.

Chart 2.
Bank A's optimal collateral choices and best responses when $\gamma_1 < \gamma_2$ **and** $\gamma_2 - \gamma_1 > \gamma_1(\omega_2/\omega_1)$.

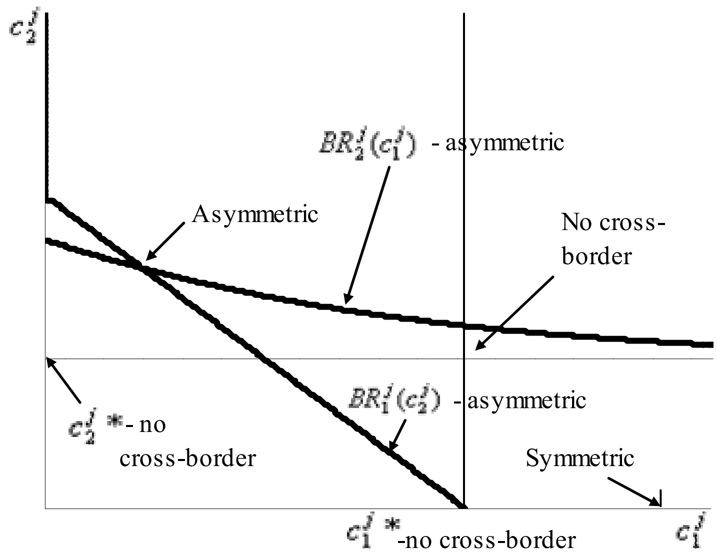
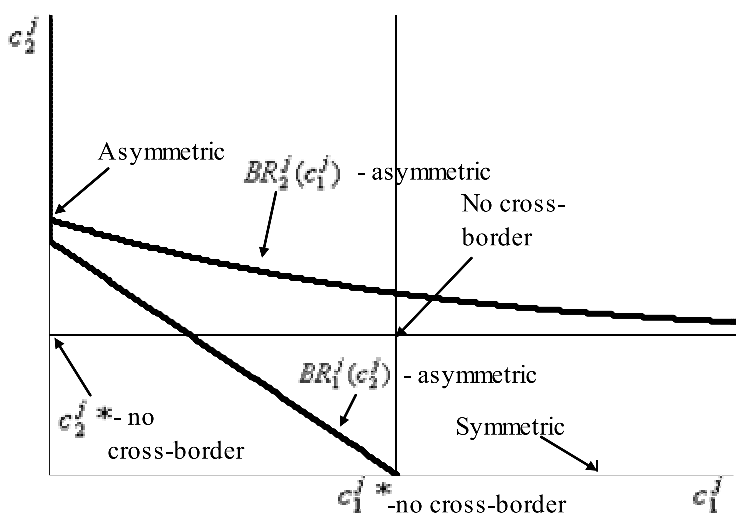


Chart 3.
Bank A's optimal collateral choices and best responses when
$\gamma_1 < \gamma_2$ **and** $\gamma_2 - \gamma_1 < \gamma_1(\omega_2/\omega_1)$.



3.4 Key drivers of collateral choice

It is instructive to examine the underlying drivers of banks' collateral choices once cross-border use of collateral is permitted. The use of collateral across borders impacts on both the total amount of collateral held by banks and also the distribution of collateral holdings across the two countries.

The distributional implication of a central bank accepting foreign collateral is that banks will shift collateral holdings towards the fungible collateral. When cross-border use of collateral is permitted, there is, for any given collateral choice, a higher expected marginal benefit from each incremental unit of fungible collateral purchased at the start of the day. With no cross-border use of collateral, each unit of collateral in either country will only be used if the bank is first payer in this country; i.e. with a probability of 0.5. With collateral fungible across the two centres, on the other hand, the probability that each incremental unit of collateral purchased will be used (in either one location or the other) is strictly greater than 0.5. But the extent to which holdings will gravitate towards the fungible collateral depends on relative start-of-day collateral costs. If fungible collateral is also cheaper, banks will shift their collateral portfolios entirely towards this country. Should the fungible collateral be more expensive, however, there will be a more limited shift. This was illustrated in Chart 2, in which banks substituted fungible country 2 collateral for country 1 collateral but still held a proportion of their portfolios in the location with the cheaper, but non-fungible, collateral.

The total quantum of collateral held will also be affected by the introduction of cross-border use of collateral. The higher expected marginal benefit from accumulating fungible collateral and the ability to shift holdings to the cheaper location will tend to boost total collateral holdings. However, fungibility implies that banks need not accumulate sizeable precautionary reserves of collateral in each country. With banks facing asynchronous liquidity needs across the two countries, there will always be idle collateral in one country when cross-border use of collateral is not permitted. Fungibility enables a bank to draw on idle collateral in one country to meet liquidity needs in the other and the incentive to hold precautionary holdings in both locations is reduced. Our assumption that the costs of incurring a liquidity or collateral shortfall during the day are significantly larger than the start-of-day costs of collateral (recall our assumption that $\omega_i / \gamma_i > 4$, for both *i*) is necessary and sufficient for the second effect to dominate and hence to ensure that the total quantum of collateral accumulated at the start of the day across the two countries will be lower if cross-border use of collateral is permitted.

We can see this if we compare total collateral held by bank *j* when no cross-border use of collateral is permitted, with that when there is either asymmetric or symmetric cross-border use of collateral. **(18)** shows that a bank's total collateral holdings are always lower when there is partial fungibility and the fungible collateral is cheaper, and when there is full fungibility.

$$\frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right)+\frac{1}{\lambda}\ln\left(\frac{\omega_2}{2\gamma_2}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)>\frac{1}{\lambda}\ln\left(\frac{\omega_1}{\gamma_1}\right),\frac{1}{\lambda}\ln\left(\frac{\omega_2}{\gamma_2}\right)$$
$$\Leftrightarrow\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}>\frac{\omega_1}{\gamma_1},\frac{\omega_2}{\gamma_2}\Leftrightarrow\frac{\omega_1}{\gamma_1},\frac{\omega_2}{\gamma_2}>4 \qquad \textbf{(18)}$$

A bank's total collateral held will also be lower when there is partial fungibility and it is the more expensive collateral that is fungible. This is true both for the case in which the cost differential is such that it is optimal for the bank to hold collateral in both countries (equation **(19)**), and that in which it remains optimal to hold only the fungible, but more expensive, collateral (equation **(20)**).

$$\frac{1}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)>\frac{1}{\lambda}\ln\left(\frac{\omega_1(\gamma_2-\gamma_1)}{\omega_2\gamma_1}\right)+\frac{1}{\lambda}\ln\left(\frac{\omega_2}{2(\gamma_2-\gamma_1)}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right)$$
$$\Leftrightarrow\frac{\omega_2}{\gamma_2}>2 \qquad \textbf{(19)}$$

$$\frac{1}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)>\frac{1}{\lambda}\ln\left(\frac{\omega_1+\omega_2}{2\gamma_2}\right)$$
$$\Leftrightarrow\frac{\omega_1}{\gamma_1}\cdot\frac{\omega_2}{\omega_1+\omega_2}>2\Leftarrow\frac{\omega_1}{\gamma_1}>4,\frac{\omega_2}{\omega_1+\omega_2}>\frac{1}{2} \qquad \textbf{(20)}$$

As can be seen in **(19)**, when the fungible collateral is more expensive, and the cost differential is large enough that it is optimal to hold collateral in both locations, the total quantum of collateral held will be equal to the optimal holding in country 1 in the unconnected case ($(1/\lambda)\ln(\omega_1/2\gamma_1)$). This will also be the total quantum of collateral held if $\gamma_2-\gamma_1=\gamma_1(\omega_2/\omega_1)$; though in this case all collateral will be held in country 2. Under these costs the bank will have the same collateral available in country 1 as when cross-border use of collateral is not permitted, but a strictly larger amount in country 2.

When fungible collateral is more expensive, but the cost differential is sufficiently small that it is optimal to hold fungible collateral only, it can readily be shown that the total quantum held in the asymmetric case is strictly lower than that in the fully fungible case. This is simply because $\gamma_2>\gamma_1$. This result is confirmed in **(21)**.

$$\frac{1}{\lambda}\ln\left(\frac{\omega_1}{\gamma_1}\right)>\frac{1}{\lambda}\ln\left(\frac{\omega_1+\omega_2}{2\gamma_2}\right)$$
$$\Leftrightarrow\frac{\omega_1}{\gamma_1}>\frac{\omega_1+\omega_2}{2\gamma_2}\Leftarrow\frac{\omega_1}{\gamma_1}>\frac{\omega_1}{\gamma_2},\frac{\omega_2}{\gamma_2} \qquad \textbf{(21)}$$

In this section we have described the effect of introducing cross-border use of collateral on banks' total collateral holdings at the beginning of the day. With parameter values that capture the notion that the cost of a experiencing a liquidity shortfall during the day is significantly larger than the cost of obtaining collateral at the start of the day, we have demonstrated that introducing cross-border use of collateral will reduce banks' total collateral at the beginning of the day. Changes in banks' collateral holdings can have implications for liquidity risk and efficiency in each settlement system. This will be discussed in the following section.

4. Comparison of outcomes for liquidity risk and efficiency

Drawing on the results presented above, we are able to analyse the implications of cross-border use of collateral for liquidity risk and efficiency in the settlement system. The metrics we apply in this regard are banks' expected shortfalls and expected total costs, respectively.

4.1 Expected shortfalls

We begin with an analysis of expected shortfalls under each of the three cross-border collateral regimes. A bank's expected shortfall is decreasing in the total amount of eligible collateral that it has available when it realises its liquidity demands. In our framework, a single bank will only need liquidity in one country at any one time, so when countries are connected, the amount of eligible collateral available when liquidity demands are realised will equal the total quantum held across the two countries.

We demonstrate that it is always optimal for a bank to hold at least as much collateral in total, across two connected countries, than in a single unconnected country. Hence, a bank is always able to draw on at least as large a pool of collateral, and its shortfall in the event that it experiences a liquidity need is consequently either lower or no larger than in the unconnected case.[11] This will be true even though, as we have seen, a bank will find it optimal to hold less collateral in total across the two countries at the start of the day. It follows that because the two banks are identical, and hence make identical collateral choices, a fall in each bank's expected shortfall in each country implies a fall in the expected shortfall at the country level also.

Proposition 1 confirms that expected shortfalls will decline or remain unchanged in a country whose central bank chooses to accept foreign collateral. Indeed, there will be a decline in all cases other than that in which there is asymmetric cross-border use of collateral and cost-differentials are such that condition **(14)** is satisfied (i.e. fungible collateral is significantly more expensive). In this latter case, expected shortfalls remain as in the unconnected case in the country allowing cross-border use of collateral.

Proposition 1: Expected shortfalls, at both the bank and country level, will either decrease or remain unchanged when a country's central bank accepts foreign collateral.

[11] The difference between expected shortfalls when there is no cross-border use of collateral and expected shortfalls when collateral is either partially or fully fungible are decreasing in the delay cost *d*. Banks have a greater incentive to accumulate precautionary holdings of collateral when the delay cost is higher. This incentive to increase collateral holdings is stronger when cross-border use of collateral is not permissible since idle collateral in one country cannot be employed to meet a liquidity demand in the other country. It follows that when *d* tends to infinity that the differences between expected shortfalls tend to zero.

Proof: Suppose the central bank in country 1 accepts foreign collateral but the central bank in country 2 does not. If $\gamma_1 \geq \gamma_2$, the collateral available to bank *j* when it is first payer in country 1, $(1/\lambda)\ln(\omega_2/\gamma_2)$, is strictly greater than that held in country 1 when there is no cross-border use of collateral, $(1/\lambda)\ln(\omega_1/2\gamma_1)$. If $\gamma_1 < \gamma_2$ and $(\gamma_2 - \gamma_1) < \gamma_1(\omega_2/\omega_1)$, the collateral available to bank *j* in country 1, $(1/\lambda)\ln((\omega_1+\omega_2)/2\gamma_2)$, is greater than $(1/\lambda)\ln(\omega_1/2\gamma_1)$. If $\gamma_1 < \gamma_2$ and $(\gamma_2 - \gamma_1) \geq \gamma_1(\omega_2/\omega_1)$, bank *j* has collateral $(1/\lambda)\ln(\omega_1/2\gamma_1)$ available when it is first payer in country 1, which is equivalent to that available in the unconnected case.

Suppose both countries accept foreign collateral. Expected shortfalls will be strictly lower since $(\omega_1/\gamma_1) > (\omega_1/2\gamma_1),(\omega_2/2\gamma_2)$ when $\gamma_1 < \gamma_2$ and $(\omega_2/\gamma_2) > (\omega_1/2\gamma_1),(\omega_2/2\gamma_2)$ when $\gamma_1 > \gamma_2$. ■

In the asymmetric case, expected shortfalls will also decline in the country that does not accept foreign collateral because banks substitute its collateral for holdings of the other country's collateral. Suppose again that country 2 collateral is accepted by the central bank in country 1, but not vice versa. If $\gamma_1 \geq \gamma_2$, the total collateral available in country 2 is $(1/\lambda)\ln(\omega_2/\gamma_2)$, which is larger than the $(1/\lambda)\ln(\omega_2/2\gamma_2)$ of collateral that is available in the unconnected case. When $\gamma_2 > \gamma_1$, the situation depicted in Charts 2 and 3, the amount of collateral held in country 2 is strictly greater than that in the absence of cross-border use of collateral. This can be seen in the charts, where the function $BR_2^j(c_1^j)$ always lies above the unconnected choice of c_2^j (when $c_1^j < \infty$). Therefore, expected shortfalls decrease in country 2 by virtue of its collateral becoming eligible in country 1.

However, the total amount of collateral available in country 2 will either be the same or lower than would be available if the central bank in country 2 also accepted foreign collateral. Thus, expected shortfalls in country 2 will be greater than or equal to those when collateral is fully fungible. Expected shortfalls will be equivalent when $\gamma_1 \geq \gamma_2$. When $\gamma_2 > \gamma_1$, available collateral is strictly lower, and expected shortfalls consequently higher, than when country 1 collateral is also eligible in country 2. These results are summarised in Proposition 2.

Proposition 2: When only one country allows cross-border use of collateral, expected shortfalls will fall unambiguously in the country that does not accept foreign collateral. If this country then begins to accept foreign collateral, expected shortfalls will fall further if and only if its collateral is more expensive.

Proof. Tables detailing expected shortfalls, at both the country and bank level, for each of the three cases of cross-border use of collateral can be found in the Appendix A1.

4.1.1 Free-riding incentives and coordination of central bank policy

The fact that a country that does not accept foreign collateral enjoys a reduction in liquidity risk when its collateral becomes eligible in the other country has an important policy implication: each country might have an incentive to attempt to free-ride on its neighbour's decision to allow cross-border use of collateral, ultimately driving the system to an equilibrium with no cross-border use of collateral at all. This could occur if the acceptance of foreign collateral entails an operational cost to the central bank, or if there is a perceived a policy-impediment to doing so. [12] This is shown in Proposition 3.

Proposition 3. Under certain relative collateral cost scenarios, and for certain operational and policy costs faced by central banks associated with the acceptance of foreign collateral, the incentive to free-ride could result in no cross-border use of collateral, even if symmetric cross-border use would be optimal for both central banks.

Proof. Consider the case in which $\gamma_1 < \gamma_2$ and $\gamma_2 - \gamma_1 > \gamma_1(\omega_2 / \omega_1)$, and suppose that central bank i incurs a cost ζ_i from accepting foreign collateral. Drawing on the expressions for expected shortfalls in each country detailed in Appendix A1, both countries would be better off with symmetric cross-border use of collateral as long as:

$$\text{Country 1: } \frac{2\gamma_1}{\lambda\omega_1} > \frac{\gamma_1}{\lambda\omega_1} + \zeta_1 \Rightarrow \frac{\gamma_1}{\lambda\omega_1} > \zeta_1$$

$$\text{Country 2: } \frac{2\gamma_2}{\lambda\omega_2} > \frac{\gamma_1}{\lambda\omega_1} + \zeta_2 \Rightarrow \frac{1}{\lambda}\left(\frac{2\gamma_2}{\omega_2} - \frac{\gamma_1}{\omega_1}\right) > \zeta_2 \tag{22}$$

When country 1 accepts country 2 collateral, country 2 is better off if it does not also accept country 1 collateral, as long as:

$$\frac{1}{\lambda}\left(\frac{2(\gamma_2 - \gamma_1)}{\omega_2} - \frac{\gamma_1}{\omega_1}\right) < \zeta_2 \tag{23}$$

If **(23)** holds, country 1 will find it optimal to remain unconnected for all $\varsigma_1 > 0$.
It can be shown that **(23)** is consistent with **(22)** for values of $\varsigma_2 > \varsigma_1$. ■

The incentive to free-ride may lead to neither central bank accepting foreign collateral, despite the fact that both countries would be better off under full fungibility. These benefits could be realised if the countries coordinated their collateral policies to ensure both begin accepting foreign collateral.

4.2 Total expected costs

Cross-border use of collateral will reduce total expected costs. When cross-border use of collateral is permitted, banks could choose to hold the same level of collateral in each country as they would with no cross-border use of collateral. The results above indicate that they do not. Thus, banks' expected costs must fall under partial and full fungibility by revealed preference.

5. Extensions to the model

In this section, we introduce four extensions to the model. First, we adapt the baseline model to allow for the possibility that operational incidents occur which disrupt the cross-border use of collateral. Second, we introduce a new policy option for central banks, allowing the acceptance of foreign collateral in the event of emergencies only. This is important from a policy

[12] Operational costs could be those associated with establishing and maintaining a link between a central bank and a foreign CSD. Some potential policy issues in this regard are discussed in Section 6.

perspective. As will be discussed in Section 6, a central bank may perceive a cost to accepting foreign collateral on a routine basis, but may still consider it beneficial to offer banks greater flexibility in the event of an emergency. In this section we show that there could be a further benefit from accepting foreign collateral in emergencies only: namely, relative to the case with routine cross-border use of collateral, banks will have less of an incentive to reduce their collateral holdings and hence will have a larger quantum available across the two countries in the event of a particularly high liquidity need. In Section 5.3, we address a potential limitation of our baseline model: the assumption that a bank's liquidity needs at any one time arise in one country only. Here we consider the implications of allowing for the possibility that a single bank experiences simultaneous liquidity demands in the two countries.

5.1. Incorporating operational risk

In our baseline model we assumed that there were no additional costs or risks associated with the use of foreign collateral relative to domestic collateral, beyond any differential between start-of-day collateral costs. Hence, in the fully fungible case, banks shifted their holdings entirely to the cheaper collateral. In this section we relax this assumption and introduce the possibility of operational disruption to the links between central banks and CSDs that allow foreign collateral to be used across borders. We restrict attention to the fully fungible case and assume that the link between country 1(2) and country 2(1) fails to operate on any particular day with probability ρ. Banks realise whether or not a link is operational between periods 0 and 1; i.e. after they have obtained collateral, but before liquidity demands have been realised.

The possibility of operational disruption tempers banks' incentives to shift collateral holdings towards cheaper collateral, and if the probability of an operational disruption is sufficiently high, they may have an incentive to continue to hold some of the more expensive collateral. Proposition 4 illustrates this for the case in which country 1 collateral is more expensive than country 2 collateral; the converse case is analogous.

Proposition 4: Banks will hold a positive amount of collateral in both countries when $\gamma_1 > \gamma_2$ if ρ is sufficiently high.

Proof: A bank's expected costs when $\gamma_1 > \gamma_2$ are:

$$E\pi^j = \gamma_1 c_1^j + \gamma_2 c_2^j + \omega_2 (1-\rho) \frac{\exp(-\lambda(c_1^j + c_2^j))}{\lambda} + \omega_1 \frac{\rho}{2} \frac{\exp(-\lambda c_1^j)}{\lambda} + \omega_2 \frac{\rho}{2} \frac{\exp(-\lambda c_2^j)}{\lambda} \quad \textbf{(24)}$$

for $j = A,B$

Minimising with respect to c_1^j and c_2^j gives the following best response functions:

$$BR_1^j(c_2^j) = \frac{1}{\lambda} \ln\left(\frac{\omega_2 (1-\rho) \exp(-\lambda c_2^j) + \omega_1 (\rho/2)}{\gamma_1} \right)$$

(25)

$$BR_2^j(c_1^j) = \frac{1}{\lambda}\ln\left(\frac{\omega_2(1-\rho)\exp(-\lambda c_2^j) + \omega_2(\rho/2)}{\gamma_2}\right)$$

for $j = A,B$

$\partial BR_1^j(c_2^j)/\partial c_2^j < \partial BR_2^j(c_1^j)/\partial c_1^j$ because $\gamma_1 > \gamma_2$. It follows that the necessary and sufficient condition for a bank to hold collateral in both countries is c_2^j such that $BR_1^j(c_2^j) = 0$ is strictly greater than $BR_2^j(0)$:

$$BR_1^j(c_2^j) = 0 \Leftrightarrow c_2^j = \frac{1}{\lambda}\ln\left(\frac{\omega_2(1-\rho)}{\gamma_1 - \frac{\omega_1\rho}{2}}\right) > BR_2^j(0) = \frac{1}{\lambda}\ln\left(\frac{\omega_2(1-\rho) + \frac{\omega_2\rho}{2}}{\gamma_2}\right)$$

(26)

$$\Leftrightarrow \frac{\gamma_2(1-\rho)}{\gamma_1 - \frac{\omega_1\rho}{2}} - (1 - \frac{\rho}{2}) > 0 \qquad \text{for } j=A,B$$

The first derivative of the LHS of this expression is positive, confirming that the condition is more likely to hold, the higher the value of ρ. It is straightforward to show that this expression can hold for $\rho \in (0,1)$. ■

5.2 Cross-border use of collateral in emergencies only

We have so far restricted attention to cases in which central banks face a policy choice between accepting foreign collateral routinely and not accepting foreign collateral at all. An additional policy option, however, is for the central bank to extend the eligible collateral pool in extreme situations only.[13] Such a policy option may offer certain benefits to a central bank.

First, such a policy may allow a central bank to relax the constraints banks may face in acquiring eligible securities in times of crisis, while avoiding any costs it may perceive in accepting foreign collateral on a routine basis. If the central bank faces costs from accepting foreign collateral it can avoid these costs routinely but still help to mitigate severe liquidity strains by accepting foreign collateral only in emergencies. We discuss some of these costs in Section 6.

But there could be a second benefit from accepting foreign collateral in emergencies only. Banks' adjustments to their collateral portfolios in response to the introduction of cross-border use of collateral in emergencies only will be far more muted than in the case of routine cross-border use of collateral. Hence there will be a much smaller reduction in the total quantum of collateral held. This reflects the fact that, in normal states of the world, banks must continue to rely on domestically held collateral only, and so have a much lower incentive to economise on collateral holdings. In the event of a stressed scenario, characterised by particularly high liquidity

[13] This policy option has indeed been exercised by the Bank of England, who announced in November 2004 that US Treasuries would be eligible in 'exceptional circumstances' only.

needs, a larger pool of collateral will be available in total than the case with routine cross-border use. Although expected shortfalls in the normal state of the world would still be lower with routine cross-border use of collateral (and hence, given a sufficiently low probability of a crisis occurring, overall expected shortfalls would be lower), central banks might still perceive a benefit from emergency-only cross-border use if crisis outcomes receive a higher weight in the central bank loss-function.[14] Indeed, for risk averse central banks, the weight placed on such outcomes might be particularly high. It is this second benefit that we investigate in this section.

We modify the model in the following ways in order to be able to distinguish between normal and emergency states of the world. The value of each bank's settlement obligation in country *i* in the normal state is drawn from the same exponential distribution as before. The normal state arises in each country with probability δ. The value of each bank's settlement obligation in country *i* in the emergency state equals $\bar{v}$. The emergency state occurs with probability $1-\delta$. In keeping with the use of the term emergency, we assume that δ and $\bar{v}$ are both *high* in the sense that they satisfy the following inequalities, respectively:

$$\delta > \max\left\{\frac{\omega_1-\gamma_1}{\omega_1},\frac{\omega_2-\gamma_1}{\omega_2}\frac{\omega_2-\gamma_2}{\omega_2},\frac{\omega_1-\gamma_2}{\omega_1}\right\}$$

$$\bar{v} > \frac{1}{\lambda}\ln\left(\frac{\delta^2\omega_1\omega_2}{4(\gamma_1-(1-\delta)\omega_2)(\gamma_2-(1-\delta)\omega_2)}\right) \quad \textbf{(27)}$$

We focus on the case in which $\gamma_1 > \gamma_2$. We also assume that the delay cost d is sufficiently high that $\omega_2 > \gamma_1$. The converse case is analogous.

If both central banks accept foreign collateral in both normal and emergency states of the world, bank *j*'s expected costs become (*R* denotes *routine use*):

$$EC^{j,R} = \gamma_1 c_1^{j,R} + \gamma_2 c_2^{j,R} + \omega_2\delta\frac{\exp(-\lambda(c_1^{j,R}+c_2^{j,R}))}{\lambda} +$$
$$(1-\delta)\omega_2\max\{0,\bar{v}-c_1^{j,R}-c_2^{j,R}\}$$

for $j = A, B$ **(28)**

And hence banks choose the following collateral levels:

$$c_1^{j,R*} = 0$$

$$c_2^{j,R*} = \frac{1}{\lambda}\ln\left(\frac{\delta\omega_2}{\gamma_2-(1-\delta)\omega_2}\right) \quad \text{for } j = A, B \quad \textbf{(29)}$$

(14) Indeed, as suggested in Section 2, in a richer, but less tractable, specification, the late-in-the-day shortfall cost might be modelled as a function of the size of the shortfall, itself a function of the size of a bank's liquidity needs. In such a specification, the shortfall costs associated with crisis outcomes might be higher, and hence this state of the world would be given a higher weight than its simple probability might suggest.

When both central banks only accept foreign collateral in emergencies, the expected cost function becomes (E denotes *emergency use only*):

$$EC^{j,E} = \gamma_1 c_1^{j,E} + \gamma_2 c_2^{j,E} + \frac{\delta}{2}\left[\omega_1 \frac{\exp(-\lambda c_1^{j,E})}{\lambda} + \omega_2 \frac{\exp(-\lambda c_2^{j,E})}{\lambda}\right] + (1-\delta)\omega_2 \max\{0, \bar{v} - c_1^{j,E} - c_2^{j,E}\} \quad \textbf{(30)}$$

for $j = A, B$

And banks choose collateral levels:

$$c_1^{j,E}* = \frac{1}{\lambda}\ln\left(\frac{\delta\omega_1}{2(\gamma_1 - (1-\delta)\omega_2)}\right)$$

$$c_2^{j,E}* = \frac{1}{\lambda}\ln\left(\frac{\delta\omega_2}{2(\gamma_2 - (1-\delta)\omega_2)}\right) \qquad \text{for } j = A, B \quad \textbf{(31)}$$

(32) contains the necessary condition for banks to hold more collateral in total when foreign collateral is accepted in emergencies only than when foreign collateral is accepted routinely.

$$c_1^{j,E} + c_2^{j,E} > c_2^{j,R} \Leftrightarrow \frac{\delta\omega_1}{\gamma_1 - (1-\delta)\omega_2} > 4 \qquad \text{for } j = A, B \quad \textbf{(32)}$$

The inequality in **(32)** always holds under our assumptions that $\omega_1 / \gamma_1 > 4$ and $\omega_2 > \gamma_1$. Hence, in the emergency state of the world, the pool of collateral available to bank *j* to meet a shortfall will be larger when cross-border use of collateral is permitted in emergencies only. And therefore, the shortfall conditional on this state occurring will consequently be lower.

The benefit to a central bank of allowing cross-border use of collateral in emergencies only rather than routinely will be driven by the weight placed upon the mitigation of liquidity risk in the event of a crisis relative to in normal circumstances. To the extent that shortfalls in extreme states of the world will tend to be larger, with potentially greater systemic consequences, it may be reasonable for a central bank to place greater weight on shortfalls in emergency situations and, if sufficiently high, only accept foreign collateral in emergencies.

5.3. Allowing for simultaneous liquidity needs in the two countries

Each bank faces a liquidity demand in only one country in the baseline model. But it seems perfectly reasonable to suppose that a bank could face liquidity needs in both countries simultaneously. In this section we extend the baseline model to show that routine cross-border use of collateral may actually increase expected shortfalls (and hence liquidity risk) in states of the world in which simultaneous needs arise. This follows from the fact that, to the extent that banks retain an incentive to economise on collateral holdings when cross-border use of collateral is permitted, a smaller pool of collateral will be available when liquidity needs are realised in both countries at the same time.

To address this possibility, we introduce two more states of the world, leaving four possible states:

- $\theta = \theta_1$: Bank *A* will be first payer in both countries;
- $\theta = \theta_2$: Bank *A* will be first payer in country 1 and bank *B* will be first payer in country 2;
- $\theta = \theta_3$: Bank *A* will be first payer in country 2 and bank *B* will be first payer in country 1;
- $\theta = \theta_4$: Bank *B* will be first payer in both countries.

We assume that each state of the world arises with probability 0.25.

We focus on bank *A*'s collateral choices. When cross-border use of collateral is not permitted, bank *A*'s collateral choices in each country are identical to those in **(9)**. When collateral is fully fungible, bank *A*'s expected shortfall in country 1 when state θ_1 occurs takes the form of expression **(33)**. [15] The shortfall in country 2 in this state is analogous.

$$E[s_1^A \mid \theta_1)] = \int_0^{c_2^A} \int_{c_1^A + c_2^A - v_2}^{\infty} (v_1 - c_1^A + v_2 - c_2^A) f(v_1) f(v_2) dv_1 dv_2 + \int_{c_2^A}^{\infty} \int_{c_1^A}^{\infty} (v_1 - c_1^A) f(v_1) f(v_2) dv_1 dv_2$$

$$= \left(\frac{1}{\lambda} + c_2^A \right) \exp(-\lambda (c_1^A + c_2^A)) \qquad \textbf{(33)}$$

The structure of expression **(33)** implies that collateral held in country 2 will only be brought across to meet a shortfall in country 1 once all country 2 needs have been satisfied. This might, therefore, lead to asymmetric outcomes across countries. For example, if country 2 collateral was cheaper, and hence all collateral was held in country 2, it might be that country 2 needs could be fully satisfied, while a large shortfall was realised in country 1. For the other scenarios in which bank *A* is first payer, $\theta = \theta_2, \theta_3$, the expected shortfall takes the form of equation **(10)** in the baseline model. Clearly, the expected shortfall is zero in state θ_4.

We shall focus on the case in which $\gamma_1 > \gamma_2$. Neither bank has an incentive to hold any country 1 collateral and will minimise expected costs with respect to the collateral choice in country 2. Bank *A*'s expected cost function is:

$$EC^A = \gamma_2 c_2^A + \omega_2 \left(\frac{1}{\lambda} + \frac{c_2^A}{4} \right) \exp(-\lambda c_2^A) \qquad \textbf{(34)}$$

Minimising **(34)** with respect to c_2^j yields:

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[15] Another possibility might have been to assume that, in the event of collateral needs in both countries, bank A allocated collateral pro-rata. The approach taken here is reasonable, however, given our assumption that delay costs are equivalent in both countries and that additional collateral can always be sourced in the cheapest market when collateral is fully fungible. Hence, if total collateral is insufficient to meet needs in both markets, bank A will be indifferent as to the country in which it ultimately realises the shortfall, for the cost implications are equivalent. Of course, central banks will not be indifferent.

$$c_2^A * = \frac{1}{\lambda}\ln\left(\frac{\omega_2}{4\gamma_2}(3+\lambda c_2^A *)\right) \quad \textbf{(35)}$$

Comparing **(35)** with **(9)** leads to the result in Lemma 2.

As in our baseline model, when both central banks begin accepting foreign collateral, banks could be induced to reduce the total amount of collateral that they hold (in particular, if they hold large precautionary reserves when cross-border use of collateral is not available). Lemma 2 establishes the condition necessary for banks to reduce the total quantum of collateral held once collateral becomes fully fungible.

Lemma 2: Both banks reduce the total amount of collateral that they hold when use of cross-border collateral is permitted if and only if:

$$\frac{\omega_1}{\gamma_1} - \ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right) > 3 \quad \textbf{(36)}$$

Proof: See appendix A2.

The implication of reduced total collateral holdings when **(36)** holds is that liquidity risk must be higher in at least one of the countries in the state of the world in which simultaneous needs arise. Recall that in our model we assume that a bank facing liquidity demands in two countries will first meet, to the extent possible, its demand in the country in which it is holding collateral. For banks, the distribution of shortfalls between countries is immaterial; its costs will be the same. However, the way in which banks allocate collateral across countries does have important implications for country-level liquidity risk and hence is relevant to central banks.

Indeed, the country with the cheaper collateral enjoys a fall in liquidity risk from the introduction of cross-border collateral because banks choose to hold collateral in this country only. This result is shown in Lemma 3 in the context of our example.

Lemma 3: Bank *A* holds more country 2 collateral when collateral is fully fungible than it does when there is no cross-border use of collateral. It follows that the expected shortfall in country 2 must be lower when collateral is fully fungible.

Proof: Let $c_2^A * = (\varepsilon / \lambda)\ln(\omega_2 / 2\gamma_2)$. It is straightforward to show that **(35)** is satisfied if and only if $\varepsilon > 1$. ■

In summary, the introduction of symmetric cross-border use of collateral will reduce expected shortfalls in the country with cheaper collateral, in all states of the world. However, in the country with more expensive collateral costs, expected shortfalls will be higher in those states of the world in which one of the banks faces simultaneous liquidity needs. In the other states of the world (ie. those in which banks face asynchronous liquidity demands) expected shortfalls will remain lower, as in the baseline model, and solving numerically for reasonable parameter values it can be shown that on a probability-weighted basis, across all states of the world, expected shortfalls in this country will still fall with symmetric cross-border use of collateral.

6. Policy considerations

In this section we will highlight some of the key policy messages arising from the analysis above. We consider these messages in the same order in which we encountered them in our analysis.

6.1 Concentration in banks' collateral holdings and central bank preferences

We established in Section 3 that, in the absence of transfer costs and other frictions, it would always be optimal for a bank to hold all of its collateral in the location in which collateral costs were lower. In the fully fungible case, this led to polar outcomes depending on relative costs. But central banks may perceive costs from banks shifting their collateral portfolios in this way since it could have implications for balance sheet or operational risk. We consider three specific sources of risk in this context: (i) risk concentration in the central bank's collateral portfolio; (ii) enforceability of the collateral mechanism in a different legal jurisdiction; and (iii) concentration on a particular link or infrastructure for mobilising collateral across border.

A central bank will always favour a diversified collateral portfolio and hence could be unwilling to accommodate commercial banks' preferences to hold all their collateral in one country. If a country's collateral is cheaper to obtain or post due to lower credit quality, this may be a particular concern for the receiving central bank. Clearly, instruments such as haircuts or concentration limits could be used by a central bank to mitigate this risk.

Legal risks, custody risks or uncertainty over the ability to realise or liquidate a foreign collateral asset in the event of counterparty failure may also discourage a central bank from extending its eligible list. In the European Union, the Settlement Finality Directive and the Collateral Directive have helped to shape a common framework within which the cross-border enforceability of the collateral mechanism can be assured, and implementation of the Hague Convention should introduce an element of comfort more globally (BIS, 2005b). Nevertheless, the legal underpinning of some cross-border linkages remains less secure.

Finally, should a large proportion of a central bank's collateral portfolio be sourced overseas, there would be a dependence on the particular infrastructure used to mobilise the assets. Even if operational risk were taken into account by banks, as in the extension to our baseline model, only the internal cost consequences of an operational failure would be taken into account in the individual bank's optimisation problem; the systemic implications would not be internalised.

In summary, our analysis suggests that the introduction of cross-border use of collateral could induce banks to reallocate their collateral portfolios significantly. This could give rise to balance sheet and operational risks for the central bank. The central bank thus faces a trade-off between these risks and the mitigation of liquidity risk.

6.2 International coordination of policy

We showed in the baseline model that when one country accepts foreign collateral and the other does not, the latter will always experience a decline in liquidity risk. This result follows from the fact that banks have an incentive to shift collateral holdings towards the fungible collateral, even when this is more expensive, because this collateral is less likely to lay idle once liquidity demands have been realised.

The implication of this is that each country could have incentives to try to free-ride on the other country's decision to accept foreign collateral, given real and perceived costs associated with cross-border use of collateral.(16) Under certain cost scenarios, when both countries attempt to free-ride, the equilibrium outcome may involve no cross-border use of collateral, even though both countries might be better off if collateral were fully fungible. This free-riding problem might be overcome if cross-border use of collateral was implemented via coordinated policy among central banks.

6.3 Cross-border use of collateral in an emergency only

One reason why a central bank may be willing only to accept foreign collateral in emergency situations is precisely because of the costs that central banks may face from accepting foreign collateral (e.g. balance sheet risk costs). A central bank might be willing to face these costs in situations of extreme liquidity strains where the systemic implications of a bank's failure to meet large liquidity needs might be greater. We showed in Section 5.2 that there could be another justification for a central bank only accepting foreign collateral in an emergency. While banks have an incentive to economise on precautionary holdings of collateral when cross-border use of collateral is allowed routinely, this incentive is more muted when a cross-border facility is triggered only in response to low-probability extreme events. As a consequence, when an emergency situation arises, banks will have more collateral available. Hence, liquidity risk in the event of a crisis will be lower under an emergency-only regime than under routine acceptance of foreign collateral.

6.4 Banks may face simultaneous liquidity needs

The introduction of cross-border use of collateral does not lead to reduced liquidity risk in all states of the world. It was shown in Section 5.3 that, to the extent that banks economise on their collateral holdings, expected collateral shortfalls may be higher in those states of the world in which a bank realises simultaneous liquidity needs.

Under the assumption that a bank will first allocate collateral to meet a shortfall in the country in which collateral is actually held, the distribution of shortfalls across the two countries will depend on relative collateral costs. Indeed, it can be shown that, in a low collateral-cost country, expected shortfalls will still be lower in all states of the world. In the high-cost country expected shortfalls will be higher in states in which one bank realises liquidity needs in both countries.

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(16) These costs would include those outlined in Section 6.1 as well as other costs associated with establishing and maintaining links between central banks and foreign CSDs.

However, liquidity risk will remain lower in the other states of the world and we find that, with the probabilities considered in our analysis, probability-weighted expected shortfalls across all possible states will still decline.

7. Conclusions

This paper has introduced a framework for the consideration of issues associated with the cross-border use of collateral. We have developed and described a two-country-two-bank model in which risk-neutral banks are members of both countries' settlement systems and minimise expected costs with respect to their collateral choices in each country. Decisions are made under uncertainty as to the value of settlement obligations and the location in which liquidity (and hence collateral) will be required. In our baseline model a bank can experience a liquidity demand in only one country at any one time. We then consider several extensions: allowing for operational disruptions to the infrastructure supporting cross-border use of collateral; allowing foreign collateral to be delivered in emergencies only; and then allowing for simultaneous liquidity needs in the two countries.

Solving the baseline model, we establish some key results. We show that expected collateral shortfalls in each country's system are always lower when collateral is fully fungible across countries, and that even under only partial fungibility expected shortfalls are either lower or unchanged in both countries. Allowing cross-border use of collateral on a routine basis thus appears to help to mitigate liquidity risk. By relaxing the constraints in a bank's cost-minimisation problem, cross-border use of collateral also serves to lower banks' total expected costs (the combination of collateral costs and the costs associated with delaying settlement in response to a shortfall). To the extent that there are benefits to cross-border use of collateral, the observation that free-rider incentives may, under certain circumstances, lead to a sub-optimal equilibrium in which neither bank accepts foreign collateral, raises important policy coordination issues.

We find that under our assumption that the cost of meeting a shortfall is sufficiently higher than the start-of-day collateral cost, there will be a decline in the total quantum of collateral held by banks: banks are able to reduce precautionary collateral holdings in each country in the knowledge that they will be able to draw on a global pool should a liquidity need arise. While, in the baseline case, liquidity risk falls in spite of the decline in total collateral holdings, due to the asynchronicity of liquidity demands, a decline in total collateral held across the two countries can increase liquidity risk when we allow for simultaneous liquidity needs. Indeed, we find that in the states of the world in which simultaneous needs arise, expected shortfalls increase at least for the country with more expensive collateral.

In another extension, we introduce an alternative policy option in which central banks permit cross-border use of collateral in emergency situations only. To the extent that emergencies arise with a low probability, such a regime induces a more muted reduction in banks' collateral holdings compared with routine cross-border use of collateral. This leaves banks with a larger pool on which to draw should an emergency (characterised by a high liquidity demand) arise, and hence such a policy delivers lower expected shortfalls in a stressed state of the world than both the unconnected and fully fungible models. The latter continues to do better in routine states. Hence, a central bank's policy choice will rest on whether it perceives any costs to accepting foreign collateral on a routine basis, and the extent to which it places greater weight on the mitigation of liquidity risk in emergencies. A fuller account of central bank risk preferences in this context, ideally integrated with this model framework, could be a fruitful avenue for future research.

We leave for future work further development of our models, to allow analysis of a richer range of possible scenarios. Potential extensions that might be considered in future work include incorporation of costs associated with the mobilisation of assets across borders, or the application of haircuts, and the modification of late-in-the-day shortfall costs to make them a function of the size of the shortfall.

References

Baltensperger, E (1974), 'The precautionary demand for reserves', *American Economic Review*, Vol. 64, No. 1, pages 205-210.

Bank of England (2004), 'Bank of England Operations in the Sterling Money Markets: Operational Notice', November 2004

Bank for International Settlements (2005a), 'Developments in large-value payment systems', Forthcoming

Bank for International Settlements (2005b), 'Report on cross-border collateral arrangements', Forthcoming

European Central Bank (2003), 'Correspondent central banking model (CCBM): procedures for Eurosystem counterparties.'

James, K (2003), 'A statistical overview of CHAPS Sterling', *Bank of England Financial Stability Review,* Issue 14, June 2003

James, K and Willison, M (2004), 'Collateral Posting Decisions in CHAPS Sterling', *Bank of England Financial Stability Review,* Issue 17, December 2004

Olivera, J H G (1971), 'The square-root law of precautionary reserves', Journal of Political Economy, Vol. 79, No. 5, pages 1095-1104.

Payments Risk Committee (2003), 'Managing payment liquidity in global markets: risk issues and solutions.'

Appendix

A1. Expected shortfalls in the baseline model

Table A1
Expected shortfalls for Bank $j = A, B$

	Unconnected	Asymmetric†	Symmetric	Comments
$\gamma_1 \geq \gamma_2$	$\frac{1}{\lambda}\left(\frac{\gamma_1}{\omega_1}+\frac{\gamma_2}{\omega_2}\right)$	$\frac{\gamma_2}{\lambda\omega_2}$	$\frac{\gamma_2}{\lambda\omega_2}$	Unconnected > Asymmetric= Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 > \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{1}{\lambda}\left(\frac{\gamma_1}{\omega_1}+\frac{\gamma_2}{\omega_2}\right)$	$\frac{1}{\lambda}\left(\frac{\gamma_1}{\omega_1}+\frac{\gamma_2-\gamma_1}{\omega_2}\right)$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected > Asymmetric > Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 \leq \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{1}{\lambda}\left(\frac{\gamma_1}{\omega_1}+\frac{\gamma_2}{\omega_2}\right)$	$\frac{1}{\lambda}\left(\frac{2\gamma_2}{\omega_1+\omega_2}\right)$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected > Asymmetric > Symmetric

Table A2(a)
Expected shortfalls for Country 1

	Unconnected	Asymmetric†	Symmetric	Comments
$\gamma_1 \geq \gamma_2$	$\frac{2\gamma_1}{\lambda\omega_1}$	$\frac{\gamma_2}{\lambda\omega_2}$	$\frac{\gamma_2}{\lambda\omega_2}$	Unconnected > Asymmetric= Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 > \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{2\gamma_1}{\lambda\omega_1}$	$\frac{2\gamma_1}{\lambda\omega_1}$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected = Asymmetric > Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 \leq \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{2\gamma_1}{\lambda\omega_1}$	$\frac{1}{\lambda}\left(\frac{2\gamma_2}{\omega_1+\omega_2}\right)$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected > Asymmetric > Symmetric

Table A2(b).
Expected shortfalls for Country 2

	Unconnected	Asymmetric†	Symmetric	Comments
$\gamma_1 \geq \gamma_2$	$\frac{2\gamma_2}{\lambda\omega_2}$	$\frac{\gamma_2}{\lambda\omega_2}$	$\frac{\gamma_2}{\lambda\omega_2}$	Unconnected > Asymmetric= Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 > \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{2\gamma_2}{\lambda\omega_2}$	$\frac{1}{\lambda}\left(\frac{2(\gamma_2-\gamma_1)}{\omega_2}\right)$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected > Asymmetric > Symmetric
$\gamma_1 < \gamma_2$ $\gamma_2 - \gamma_1 \leq \gamma_1 \frac{\omega_2}{\omega_1}$	$\frac{2\gamma_2}{\lambda\omega_2}$	$\frac{1}{\lambda}\left(\frac{2\gamma_2}{\omega_1+\omega_2}\right)$	$\frac{\gamma_1}{\lambda\omega_1}$	Unconnected > Asymmetric > Symmetric

† The results are shown for the case in which country 2 collateral is fully fungible but country 1 collateral is not.

A2. Proof of Lemma 2

The total amount of collateral that bank *A* holds when collateral is fully fungible equals the level of collateral $c_2^A *$ that satisfies **(35)**. The total amount of collateral held by bank *A* when collateral cannot be used across borders equals

$$\frac{1}{\lambda}\ln\left(\frac{\omega_1}{2\gamma_1}\right)+\frac{1}{\lambda}\ln\left(\frac{\omega_2}{2\gamma_2}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right) \qquad \textbf{(A1)}$$

Let $c_2^A * = \frac{\varepsilon}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)$, where $\varepsilon \in \Re_+$. To prove the lemma we will show that $\varepsilon \in (0,1)$ in order for **(35)** to be satisfied. If we plug $c_2^A * = \frac{\varepsilon}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)$ into **(35)** we get

$$\frac{\varepsilon}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_2}{4\gamma_4}\left(3+\varepsilon\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)\right)\right) \qquad \textbf{(A2)}$$

Both sides of **(35)** are increasing in ε. It will suffice to show $\varepsilon \in (0,1)$ by demonstrating that the RHS of **(35)** strictly exceeds the LHS for $\varepsilon = 0$ and that the LHS strictly exceeds the RHS for $\varepsilon = 1$:

$$\varepsilon = 0: \ \frac{\varepsilon}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)=0<\frac{1}{\lambda}\ln\left(\frac{3\omega_2}{4\gamma_2}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_2}{4\gamma_4}\left(3+\varepsilon\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)\right)\right)$$

$\varepsilon = 1$: **(A3)**

$$\frac{\varepsilon}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)<\frac{1}{\lambda}\ln\left(\frac{\omega_2}{4\gamma_2}\left(3+\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)\right)\right)=\frac{1}{\lambda}\ln\left(\frac{\omega_2}{4\gamma_4}\left(3+\varepsilon\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)\right)\right)$$

if $\frac{\omega_1}{\gamma_1}-\ln\left(\frac{\omega_1\omega_2}{4\gamma_1\gamma_2}\right)>3$. ■