Bayesian Model Averaging in the Presence of Structural Breaks

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Objective

1) Model specification:

Derivation of a new model specification for forecasting in the presence of model uncertainty and structural breaks

2) Application:

Application to US stock data and evaluation of the economic gain

Outline

Literature

Model

Application to data and results

Conclusion

Previous literature

Marquering and Verbeek (2004)

Pesaran and Timmermann (2002)

Pesaran, Pettenuzzo and Timmermann (2006)

Avramov (2002) and Cremers (2002)

Pettenuzzo and Timmermann (2005)

Our contribution

Algorithm:

- Difficult to derive the marginal likelihood for non-linear specifications
- Computational problems when K (number of explanatory variables) is big

Economic gain of the strategy

The model [1]

Benchmark model

$$y_t = \beta_0 + \sum_{j=1}^{k} \beta_j x_{jt} + \varepsilon_t, \qquad (1)$$

Extensions

$\swarrow \qquad \searrow$

Breaks $\qquad$ Averaging

$\downarrow \qquad \downarrow$

$$y_t = \beta_{0t} + \sum_{j=1}^{k} \beta_{jt} x_{jt} + \varepsilon_t \qquad y_t = \beta_0 + \sum_{j=1}^{k} s_j \beta_j x_j + \varepsilon_t$$

$\searrow \qquad \swarrow$

$$(s_j, \beta_{jt})$$

The model [2]

$$y_t = \beta_{0t} + \sum_{j=1}^{k} s_j \beta_{jt} x_{jt} + \varepsilon_t, \tag{2}$$

$$\beta_{jt} = \beta_{j,t-1} + \kappa_{jt} \eta_{jt}, \qquad j = 0, \ldots, k, \tag{3}$$

The inclusion of x_{jt} in the model is described by a latent binary random variable $s_j = 0, 1$ with $\Pr[s_j = 1] = \lambda_j$

Structural breaks are described by an unobserved uncorrelated 0/1 process κ_{jt} with $\Pr[\kappa_{jt} = 1] = \pi_j$ for $j = 0, \ldots, k$

Estimation

Define $\theta = (\sigma^2, \lambda_1, \ldots, \lambda_k, q_0^2, \ldots, q_k^2, \pi_0, \ldots, \pi_k)$, $S = (s_1, \ldots, s_k)$, $B = \{\beta_t\}_{t=1}^T$ and $K = \{\kappa_t\}_{t=1}^T$

- Draw S conditional on B, K, θ, r and x similarly to Kuo and Mallick (1998)
- Draw K conditional on S, θ, r and x as in Gerlach, Carter and Kohn (2000)
- Draw B conditional on S, K, θ, r and x as in Carter and Kohn (1994)
- Draw θ conditional S, B, K, r and x

Predictive density

$$p(r_{T+1}|r,x,x_{T+1}) = \iint \sum_S \sum_K \sum_{K_{T+1}} p(r_{T+1}|S,x_{T+1},\beta_{T+1},\sigma^2)$$

$$p(\beta_{T+1}|\beta_T,\kappa_{T+1},\theta) \prod_{j=0}^{k} \pi_j^{\kappa_{j,T+1}}(1-\pi_j)^{1-\kappa_{j,T+1}} p(B,K,S,\theta|r,x)dBd\theta, \tag{4}$$

By averaging over the posterior distribution of S we implicitly take a weighted average over all possible model specifications.
By applying the posterior density we reflect the uncertainty about the in-sample structural breaks K.
By averaging with respect to the unknown K_{T+1} we account for uncertainty about future breaks.

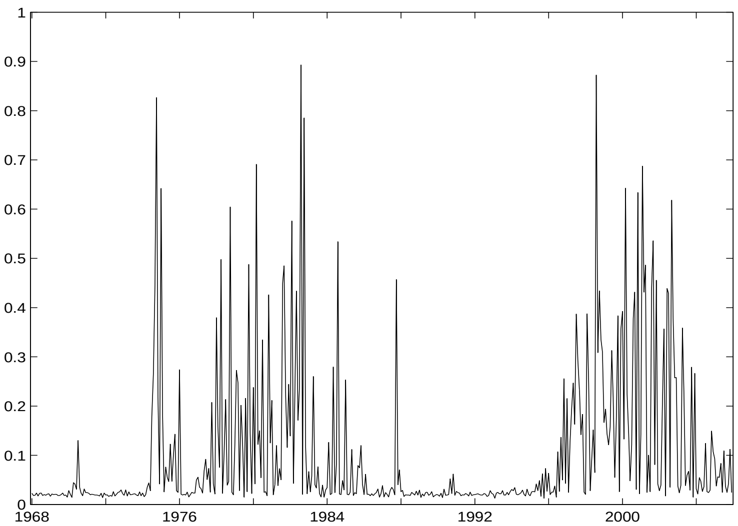
Application

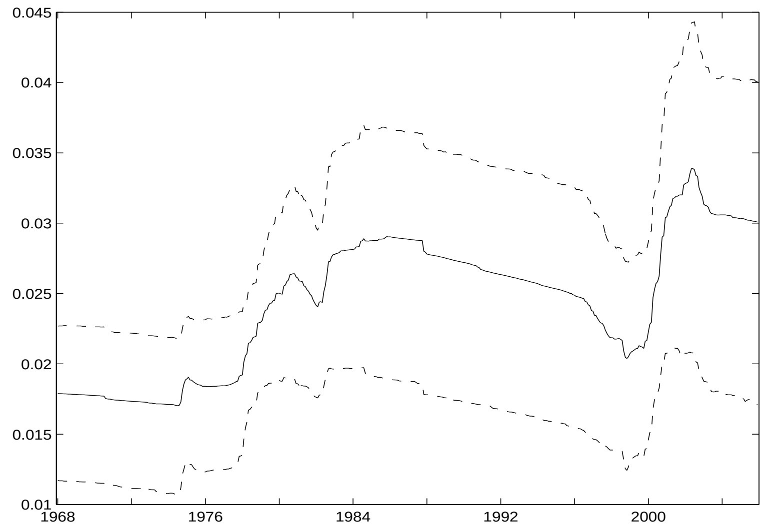
- Monthly excess return on the S&P 500 index over the sample 1966(1):2005(9)
- 12 explanatory variables
- Out-of-sample: 1976:1-2005:12 (360 observations)
- Expanding window
- Bayesian inference for all the models and informative priors
- 1-step ahead forecast horizon
- Active investment exercise
- Power utility-based performance measure

In sample results [1]

Variable	Mean posterior inclusion probability
PE_{-1}	0.336
DY_{-1}	0.728
$I3_{-1}$	0.900
$DI3_{-1}$	0.606
TS_{-1}	0.220
CS_{-1}	0.621
YS_{-1}	0.283
INF_{-2}	0.122
IP_{-2}	0.388
MB_{-2}	0.601
$LVOL_{-1}$	0.140

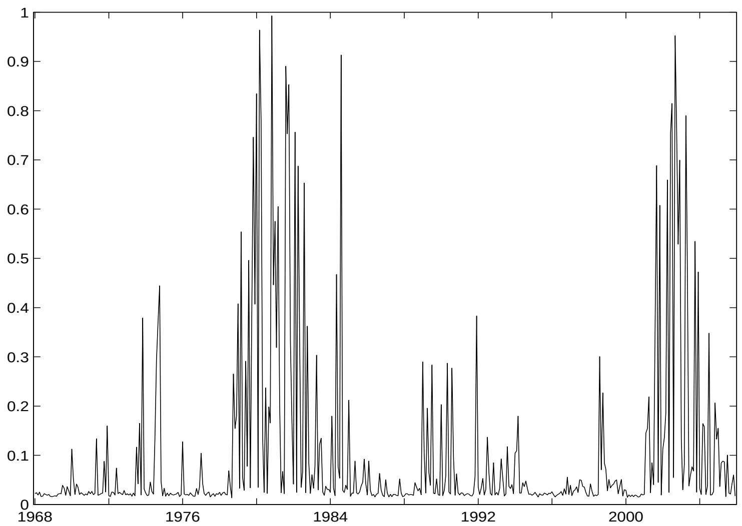
In sample results [2]: DY

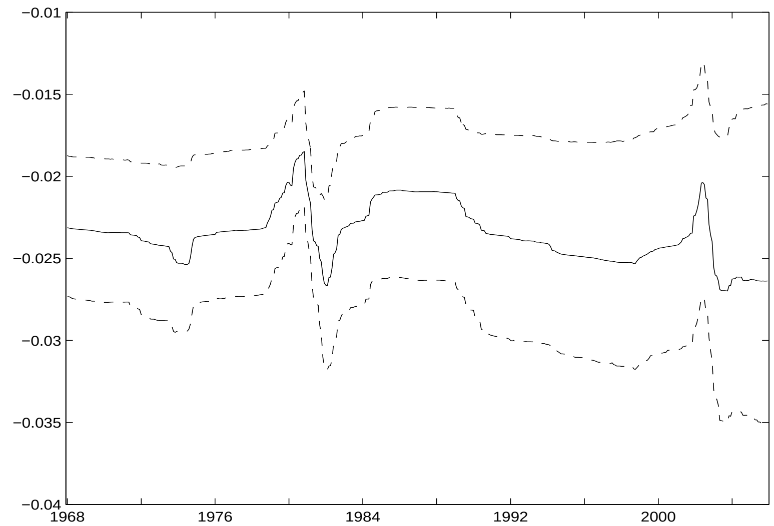




Note: The figure presents the posterior means (solid line) of $\kappa_{j,t}$ on the left side $\beta_{j,t}$ on the right side, conditional upon inclusion of the jth variable ($s_j = 1$). The dashed lines are the 25th and 75th percentiles of the posterior densities.

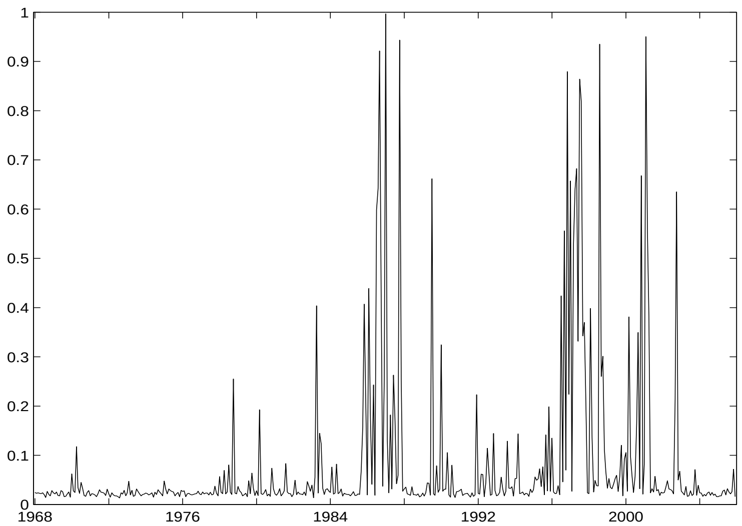
In sample results [3]: I3

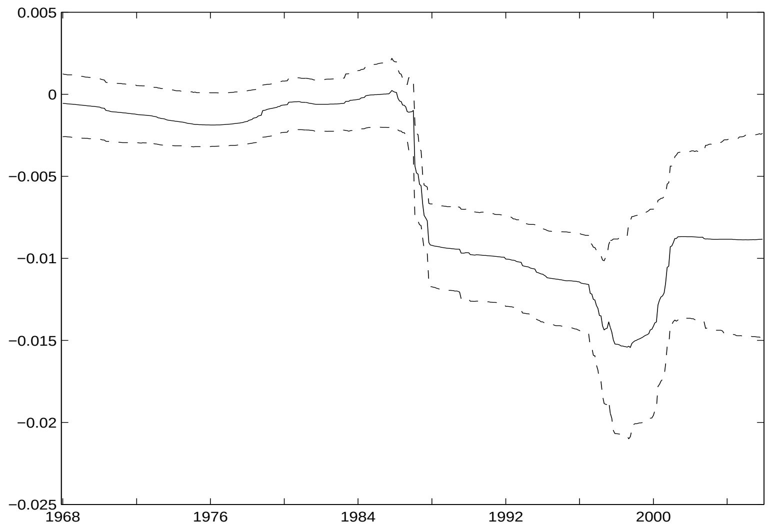




Note: The figure presents the posterior means (solid line) of $\kappa_{j,t}$ on the left side $\beta_{j,t}$ on the right side, conditional upon inclusion of the jth variable ($s_j = 1$). The dashed lines are the 25th and 75th percentiles of the posterior densities.

In sample results [4]: MB





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Active strategy: $\gamma = 5$

Strategy	Mean	St dev	SR	Δ_m	$\Delta_{0.5m}$	Δ_b
I: 100% market	11.98	14.99	11.72			
II: 50% market	8.94	7.51	11.70			
III: 0% market	5.89	0.89	0.00			
IV: BMASB (0,1)	11.05	8.31	17.92	249.50	165.65	386.65
V: SB (0,1)	9.45	8.06	12.74	97.70	13.85	234.85
VI: BMA (0,1)	6.50	1.69	10.36	-70.73	-154.57	66.42
VII: Linear (0,1)	7.35	4.57	9.16	-19.48	-103.33	117.67

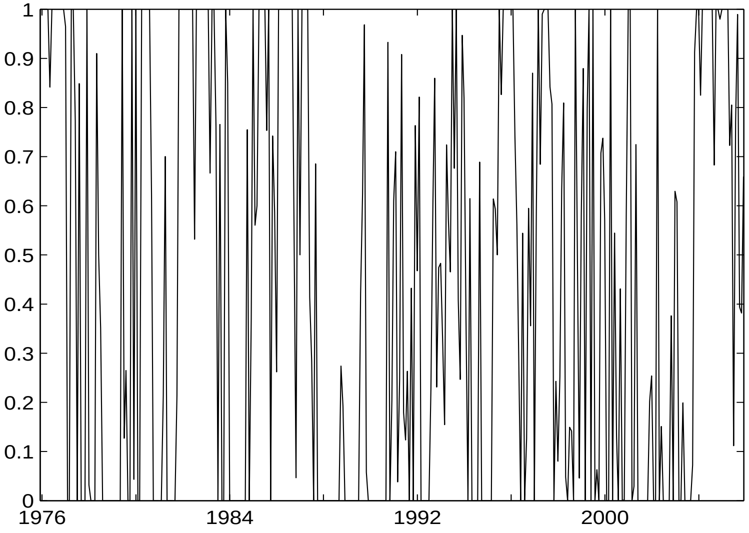
Strategy BMASB gives the highest performance fees

Active strategy: 0.1% transaction costs

Strategy	Mean	St dev	SR	Δ_m	$\Delta_{0.5m}$	Δ_b
I: 100% market	11.98	14.99	11.71			
II: 50% market	8.93	7.51	11.69			
III: 0% market	5.89	0.89	0.00			
IV: BMASB (0,1)	10.42	8.30	15.72	217.94	134.02	355.21
V: SB (0,1)	8.82	8.06	10.49	67.80	-16.12	205.07
VI: BMA (0,1)	6.45	1.69	9.53	-72.83	-156.75	64.44
VII: Linear (0,1)	7.12	4.56	7.75	-30.51	-114.43	106.76

The fees are still positive!

Stock portfolio weights



Note: The figure presents the portfolio weight for stocks in the portfolios based on excess stock return forecasts from the general model, allowing for model uncertainty and structural breaks in the regression parameters.

Conclusion

New framework that combines parameter uncertainty, model averaging and structural breaks

Over the period 1966-2005 several structural breaks occurred in the relationship between US stock returns and predictor variables

Incorporating all three sources of uncertainty simultaneously has economic value