The Impact of Natural Hedging on a Life Insurer’s Risk Situation

Longevity 7
September 2011

Nadine Gatzert and Hannah Wesker
Friedrich-Alexander-University of Erlangen-Nürnberg
Introduction
Motivation

- Demographic risk can significantly impact a life insurer’s solvency level
  - Increase in life expectancy poses serious problems to life insurers selling annuities
  - However, risk of unexpected high mortality (e.g. due to pandemics) has increased as well; problem for term life

- But: Hedging instruments are still scarce
  - “Natural Hedge” between term life insurance (death benefit) and annuities (lifelong survival benefits) is effective alternative
  - Use opposed reaction of term life insurance and annuities towards shocks to mortality
  - Hedge shocks to mortality internally through portfolio composition
Introduction

Aim of paper

- Previous literature:

- Aim of this paper:
  1. Quantify impact of natural hedging on a life insurance company’s insolvency risk
     - Holistic model, take into account dynamic interaction between assets and liabilities for a two-product life insurer
  2. Simultaneously *immunize* an insurer’s solvency situation against changes in mortality and *fix the absolute level of risk*
     - Use investment strategy
Model framework
Modeling and forecasting mortality


\[ D_{x,t} \sim \text{Poisson}(E_{x,t} \cdot \mu_x(t)) \quad \mu_x(t) = \exp(a_x + b_x \cdot k_t) \quad q_x(t) = 1 - \exp(-\mu_x(t)) \]

- \( D_{x,t} \): Poisson-distributed number of deaths, \( E_{x,t} \): exposure at risk
- \( a_x \) and \( b_x \): indicating the general shape of mortality over age
- \( k_t \): indicating the general level of mortality in the population (with negative drift)
- Forecasting of \( k_t \) (and \( \mu_x(t) \)) by ARIMA process for estimated time series of \( k_t \)
Model framework
Modeling systematic mortality risk

- Analyze systematic mortality risk in two ways:
  1. Shock to (decreasing) mortality time trend: $e^*k_t$
     - Leads to an unexpected change in the level and future development of mortality
     - Shocks $e > 1$: mortality rates decrease (longevity scenario)
     - Shocks $e < 1$: mortality rates increase (pandemic scenario)
     - How to compose a portfolio of term life and annuities in order to immunize the portfolio against shocks to mortality?
  2. Use empirically observed changes in mortality
     - Analyze usefulness of natural hedging under realized changes in mortality
     - Similar results
Model framework

Model of a life insurance company

- Simplified balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(t)$</td>
<td>$E(t)$</td>
</tr>
<tr>
<td>$B_A(t)$</td>
<td></td>
</tr>
<tr>
<td>$B_L(t)$</td>
<td>$L(t)$</td>
</tr>
</tbody>
</table>

- $A(t)$: market value of assets at time $t$
- $B_A(t)$: book value of liabilities for annuities at time $t$
- $B_L(t)$: book value of liabilities for term life insurance at time $t$
- $E(t)$: equity at time $t$

- Default of the insurance company, if $L(t) = B_L(t) + B_A(t) > A(t)$
Model framework
Liabilities – Premium and benefit calculation

• Premiums and benefits: use actuarial equivalence principle
  ➢ Term life insurance

\[
\sum_{t=0}^{T-1} P \cdot p_x \cdot (1 + r)^{-t} = \sum_{t=0}^{T-1} DB \cdot p_x \cdot q_{x+t} \cdot (1 + r)^{-(t+1)}
\]

➢ Life-long immediate annuity

\[
SP = \sum_{t=0}^{T-1} a_t \cdot p_x \cdot (1 + r)^{-t+1}
\]

• Improve comparability and isolate effect of natural hedging:
  ➢ Calibrate input parameters such that **volume** of both contract types is identical at inception
  ➢ Fix the **number** of contracts sold
Model framework

Liabilities – Book value of liabilities

- Use actuarial reserve to determine book value of liabilities
- Value of one term life insurance contract:

\[ B_L(t) = \sum_{s=0}^{T-t-1} \left[ DB \cdot p_{x+t}(e) \cdot q_{s+x+t}(e) \cdot (1+i)^{-(s+1)} - P \cdot p_{x+t}(e) \cdot (1+i)^{-s} \right] \]

- Value of one annuity:

\[ B_A(t) = \sum_{s=0}^{T-t-1} a \cdot p_{x+t}(e) \cdot (1+i)^{-(s+1)} \]

- Mortality rates are subject to shock
- Value of liabilities \( L(t) \):

\[ L(t) = n_A(t) \cdot B_A(t) + n_L(t) \cdot B_L(t) \]
Model framework

Assets

- Assets follow a geometric Brownian motion:

\[ dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t) \]

- Development of asset base depends on cash-flows of insurance portfolio

\[
\begin{align*}
  t = 0^+ & \quad t = 1^- & \quad t = 1^+ & \quad t = 2^- & \quad \ldots \\
  + E_0 & \quad - n_A(1) \cdot a & \quad + n_L(1) \cdot P & \quad - n_A(2) \cdot a & \quad \ldots \\
  + n_A(0) \cdot SP & \quad - d_L(0) \cdot DB & \quad & \quad & \quad & \quad \\
  + n_L(0) \cdot P & \quad - div & \quad & \quad & \quad & \quad
\end{align*}
\]

Number of annuity contracts active in \( t = 1 \)

Number of life insurance contracts active in \( t = 1 \)

Constant dividend to shareholders

Number of life insurance policyholders who died during \( t = 0 \)

Constant dividend to shareholders

Number of life insurance policyholders who died during \( t = 1 \)
Model framework
Risk measurement

• Probability of default (PD): \[ PD = P(T_d \leq T) \]
  with \[ T_d = (T + 1) \vee \inf \{ t : A(t) < L(t) \}, t = 1, \ldots, T. \]

• Mean Loss (ML): \[ ML = E\left( \max\left( (L(T_d) - A(T_d)) \cdot (1 + r)^{-T_d}, 0 \right) \cdot 1\{T_d \leq T\} \right) \]

• Expected Shortfall (ES) \[ ES = \frac{ML}{PD} \]

• Contractual Payment Obligations (CP)
  \[ CP = n_L(0) \cdot \sum_{t=0}^{T-1} DB \cdot p_x(e) \cdot q_{x+t}(e) \cdot (1 + r)^{-(t+1)} + n_A(0) \cdot \sum_{t=0}^{T-1} a \cdot p_x(e) \cdot (1 + r)^{-(t+1)} \]
  - Only liability side
  - Linear in portfolio composition
Numerical results
Input parameters

- **Liabilities**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age at inception of term life</td>
<td>30</td>
</tr>
<tr>
<td>Max. duration of term life</td>
<td>35</td>
</tr>
<tr>
<td>Age at inception of annuity</td>
<td>65</td>
</tr>
<tr>
<td>Premium for life insurance ($P$)</td>
<td>417</td>
</tr>
<tr>
<td>Single premium for annuity ($SP$)</td>
<td>10,000</td>
</tr>
<tr>
<td>Yearly annuity ($a$)</td>
<td>725</td>
</tr>
<tr>
<td>Death benefit ($DB$)</td>
<td>88,724</td>
</tr>
<tr>
<td>Total number of contracts sold</td>
<td>10,000</td>
</tr>
</tbody>
</table>

- **Assets**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift of assets ($\mu$)</td>
<td>6%</td>
</tr>
<tr>
<td>Volatility of assets ($\sigma$)</td>
<td>10%</td>
</tr>
<tr>
<td>Risk-free interest rate ($r$)</td>
<td>3%</td>
</tr>
</tbody>
</table>
Numerical results
Risk under different shocks to mortality

Expected Shortfall (ES)

<table>
<thead>
<tr>
<th>Change:</th>
<th>-7%</th>
<th>+21%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio of annuities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio of term life</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Change in mortality rates: -7% decrease, +21% increase
- Mortality rates decrease (longevity scenario)
- Mortality rates increase (pandemic scenario)

Gatzert/Wesker “The Impact of Natural Hedging on a Life Insurer’s Risk Situation”
Numerical results
Varying the investment strategy

1) Fixing absolute level of risk (e.g. PD=0.38%)

2) Find immunizing portfolio: 17.0% life insurance

Optimal hedge ratio for different investment strategies

Corresponding level of insurer’s default risk for optimal hedge ratio

Here: for a shock to mortality of $e = 1.1$ (longevity scenario)
Summary

- Results show: Natural hedging can considerably reduce absolute risk level of an insurer and immunize it against shocks to mortality
  - Optimal portfolio composition depends on risk measure
  - Holistic consideration of mortality risk with respect to insurer’s overall risk level is vital (focus on liability side only underestimates risk)
- Investment strategy can have substantial impact on the effectiveness of natural hedging
  - Use investment strategy to simultaneously fix a risk level and immunize the portfolio against shocks to mortality
  - Changing the investment strategy requires adjustment of portfolio mix to immunize portfolio against changes in mortality
The Impact of Natural Hedging on a Life Insurer’s Risk Situation

Thank you very much for your attention!

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