Coherent Pricing of Life Settlements Under Asymmetric Information

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Background & Literature Review

The life settlement market:

- Abiding investment opportunity
- Senior insureds w/ below average health ⇐ "Viatical settlements" (1980s)
- Securitization in the capital market (Chen et al. (2011), Stone and Zissu (2006))
- Limited number of contracts ⇒ idiosyncratic risk factors

Recent market investigations:

- Expected returns 8-12% from a policy-by-policy basis (Gatzert (2010))
- Open-end life settlement funds returned ≈ 4.8% (Braun et al. (2011))
- **Bad quality** of underlying life expectancy estimates?
  - Systematic biases – should be swiftly corrected
  - Unsystematic errors – cannot explain aggregate underperformance
- Rating agencies declined rating these "death bonds" due to "unique risks"
Main Findings

Different view points based on adverse selection

- One-period expected utility model ⇒ offer price in competitive market
  - With symmetric information on health condition
  - With asymmetric information on health condition
  - Adjustment of pricing scheme ⇔ clientele effects (Hoy and Polborn (2000), Villeneuve (2003))

- Extended framework ⇒ applicable pricing formulas
  - Frailty model ⇒ heterogeneity in life tables
  - Life-time utility evaluation ⇒ threshold set for settling
  - Generalizations ⇒ option to settle in later periods

- Numerical examples
  - Impact of asymmetric information varies
  - Extreme cases: no effect or market breakdown
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Symmetric Information

Simple one-period expected utility model

Representative policyholder

- One-period term-life insurance: $F$
- No future contingent premiums, zero cash surrender value
- Condition: $p$ (survival probability to the end of the period)
- $u(\cdot)$ and $v(\cdot)$: utilities from life insurance benefits

Under a competitive secondary life market:

- $OP_{sym}^* (p) = \frac{(1-p) \times F}{1+R}$
  - $U^r = p \times u(0) + (1 - p) \times v(F)$
  - $U^s = p \times u((1 - p)F) + (1 - p) \times v((1 - p)F)$
- Settle if and only if $U^s \geq U^r$
Asymmetric Information

\( \bar{p} \): estimate of \( p \) from third party \( \Rightarrow f(p|\bar{p}) \)

Without considering policyholder’s behavior:

- \( OP^a(\bar{p}) = \frac{\mathbb{E}[(1-p)|\bar{p}] \times F}{1+R} \)
- Not economically rational!

With considering policyholder’s behavior:

- \( U^r = p \times u(0) + (1 - p) \times v(F) \)
- \( U^s(p, OP) = p \times u(OP \times (1+R)) + (1 - p) \times v(OP \times (1+R)) \)
- \( U^s(p, OP) - U^r(p) \geq 0 \) \( \iff p \geq \frac{v(F) - v(OP \times (1+R))}{u(OP \times (1+R)) - u(0) + v(F) - v(OP \times (1+R))} \triangleq p^*(OP) \)
- \( OP^e(\bar{p}) \triangleq \arg \max_x \left \{ \int_{p^*(x)}^1 ((1 - p)F - x(1 + R)) f(p|\bar{p}) dp = 0 \right \} \)
- Average Clientele Risk?
  - Time point: settling vs. purchasing the policy
  - Derived price: independent settlement vs. level premium
Implication

Proposition

With asymmetric information with respect to \( p \), the rational expectation offer price, \( OP^e(\bar{p}) \), will be smaller than \( OP^a(\bar{p}) \), for all estimates \( \bar{p} \).

Proof. It is sufficient to show that

\[
\int_{p^*(OP^a)}^{1} ((1 - p)F - (1 + R) \times OP^a)f(p|\bar{p})dp \leq 0
\]

\( \Leftrightarrow \)

\[
F \times \int_{p^*(OP^a)}^{1} ((1 - p) - \mathbb{E}[(1 - p)|\bar{p}])f(p|\bar{p})dp \leq 0
\]

\( \Leftrightarrow \)

\[
\mathbb{E}[p|\bar{p}, p \geq p^*(OP^a)] - \mathbb{E}[p|\bar{p}] \geq 0
\]

- Value of the policy is far underestimated \( \Rightarrow \) reject
- Offer price exceeds intrinsic value \( \Rightarrow \) settle
- Explanation for the discrepancy between expected and realized returns!
- Coherent pricing should take policyholder’s decision into account
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Heterogenous Life Tables

Extended framework:

- Multi-period environment
- Whole-life policy, annual premium $P$, death benefit $F$

Heterogeneity w.r.t. individual mortality rates (Vaupel et al. (1979), Hoermann and Russ (2008))

- Current frailty models: fail to connect average of (heterogeneous) individual tables to population table

\[ \mathbb{E}^j[\tau p_x^j(T)] = \tau p_x(T), \forall \tau, \text{ and } \tau p_x^j(T) \in [0, 1], \forall \tau, j \]

- We propose:

\[ \tau p_x^j(T) = \tau p_x(T) + A_j \times \min\{\tau p_x(T), 1 - \tau p_x(T)\} e^{-\gamma(\tau-1)}, \]

s.t. $A_j \in [-1, 1]$, and $\mathbb{E}[A_j] = 0$. 
Policyholders’ Decision Making

Value function when retaining:

\[ V^r_T(W_0, j) = \max_{c_T} \sum_{\tau = 1}^{\omega - x} (\tau - 1)p_\tau^j(T) \times \beta^{\tau - 1} \times u(c_T - P) + \sum_{\tau = 1}^{\omega - x} (\tau - 1)p_\tau^j(T) - \tau p_\tau^j(T)) \times \beta^\tau \times v(W_\tau + F), \]

s.t.

\[ W_\tau = (W_{\tau - 1} - c_\tau) \times \frac{1}{p(\tau - 1, 1)}, \tau = 1, \ldots, \omega - x. \]

Value function when settling:

\[ V^s_T(W_0, OP, j) = \max_{c_T} \sum_{\tau = 1}^{\omega - x} (\tau - 1)p_\tau^j(T) \times \beta^{\tau - 1} \times u(c_T) + \sum_{\tau = 1}^{\omega - x} (\tau - 1)p_\tau^j(T) - \tau p_\tau^j(T)) \times \beta^\tau \times v(W_\tau), \]

s.t.

\[ W_1 = (W_0 - c_1 + OP) \times \frac{1}{p(0, 1)}, \]

and

\[ W_\tau = (W_{\tau - 1} - c_\tau) \times \frac{1}{p(\tau - 1, 1)}, \tau = 2, \ldots, \omega - x. \]

Threshold set (settling preferred to retaining):

\[ \Omega(OP) = \{ A_j : V^s_T(W_0, OP, j) \geq V^r_T(W_0, j) \}. \]
Pricing Formula & Generalization

With symmetric information:

$$OP^{sym}(j) = \sum_{\tau=1}^{\omega-x} \left[ \left( \tau - 1 \right) p_x^j(T) - \tau p_x^j(T) \right] \times \frac{F}{(1 + R)^\tau} - \tau - 1 \left( \tau - 1 \right) p_x^j(T) \times \frac{P}{(1 + R)^{\tau-1}}$$

With asymmetric information:

$$OP^a(\bar{A}) \equiv \arg \max_z \left\{ \int_{\Omega(z)} \left( \sum_{\tau=1}^{\omega-x} \left[ \left( \tau - 1 \right) p_x^j(T) - \tau p_x^j(T) \right] \times \frac{F}{(1 + R)^\tau} ight. \\
- \tau - 1 \left( \tau - 1 \right) p_x^j(T) \times \frac{P}{(1 + R)^{\tau-1}} - z \right) f(A_j|\bar{A})dA_j = 0 \right\}$$

If allowing settling in future periods:

- $V_T^e(W_0, j)$ increases $\Rightarrow$ truncate $\Omega(OP)$ $\Rightarrow$ more significant adverse selection effects
- Systematic mortality risk at population level matters
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Life Table Projections

- Population: year 1978 (age 50) ⇔ year 2008 (age 80)
  - U.S. (female) mortality data from Human Mortality Database
  - Lee-Carter model
  - Year 1978: mortality forecasts for premium setting (data from 1958-1977) ⇒ $16.245 per $1,000 ($r = 4\%$)
- Individual: $\gamma = 0.1$, $A_{y+1}^{2}$ follows a Beta distribution with parameters $\alpha = \beta = 2$

![Empirical CDF](image1.png)

$1\ P_{80}$

![Empirical CDF](image2.png)

$11\ P_{80}$

![Empirical CDF](image3.png)

$21\ P_{80}$
Symmetric Case

- \( u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \ \gamma = 1.584 \) (cf. Hall and Jones (2007))
- \( v(W) = \frac{1+r}{r} \times (\frac{r}{1+r} W)^{1-\gamma} \)
- \( W_0 = 500,000, \ F = 1,000,000,000, \ r = 4\%, \ \beta = 1/1.04 \)

Symmetric Information:

Settle only when \( OP^{sym} \geq 323,370 \iff R \leq 9.95\% \)
Settlement Decision

- By comparing value functions ⇒ reservation price for each type $A_j$
- Calculate $OP^{sym}$ with hurdle rates $R$ at 4%, 8%, and 12%
- When $OP^{sym}$ crosses with the reservation price curve ⇒ asymmetric choice from policyholders
- Threshold set: $\Omega(\text{OP}) = [A^*(\text{OP}), 1]$

![Reservation and Actuarially Fair Prices](chart1)

$A^*(\text{OP})$
**Equilibrium Offer Prices**

Beta Distribution, $R = 0.07$

Beta Distribution, $R = 0.08$

- $R = 0.07$: $OPE = OP^{sym} = $412,680 (no adverse selection)
- $R = 0.08$: $OPE = $367,930 < OP^{sym} = $378,810 (modest effect of asymmetric information)
Equilibrium Offer Prices

- $R = 0.09$: fatal impact from adverse selection $\Rightarrow$ market breaks down
- $R = 0.08$ (uniform $A_j$): $OP^e = $339,100 (stronger adverse selection)
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Contributions:
- Effect of asymmetric information on the profit structure of life settlement company
- Applicable pricing formulas for life settlement transactions
- Explanation of the discrepancy between estimated and realized returns
- New angle on the financial analysis of life settlements
- Promote the mortality-linked capital market as a whole

Future projects:
- Calibrate the model parameters
- Sensitivity tests
- Including option to settle later ⇒ more severe impact of adverse selection
Thank you!