Implicit Options and Games between Debtor and Creditor
with Credit Derivatives

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Abstract

In contrast to previous researches on credit derivatives starting from credit risks, this paper examines default risks affected by the credit derivatives. The linkage of default risks and credit derivatives lies in the implicit options existing in the debt relationships. These options can also be changed if credit derivatives are introduced into the debt relationships, e.g., hedge the default exposure with credit derivatives. Firstly we show that it is optimal for creditors with both safety covenant and CDS protection to enforce early bankruptcy under certain conditions. This result suggests that other not fully hedged default risks can be more likely to enforce bankruptcy. The framework to inspect tradeoffs between credit derivative costs and expected payoffs is borrowed to study strategic debt service (SDS). Credit risks associated with SDS games between debtors and creditors with and without credit default swap (CDS) hedge are compared, and strategies are suggested to debtors and creditors respectively when CDS can play an important role in determining default or not. The capital structure arbitrage in the credit market is also discussed.
**Introduction**

Credit risks are associated with every credit activities and risky bonds have been widely traded. Credit markets have recently witnessed exponential growth of credit and related derivatives such as options on debt, total return swaps, default put and credit default swap (CDS). Furthermore, new products like exotic CDS and basket and portfolio credit derivatives are evolving quickly. These fast developing markets and widely applications spurs a lot of original researches. In addition to the traditional methods using historical data to value credit risks, there are structural models based on firm value’s passage to certain barrier and intensity models using an arbitrage-free bankruptcy process, which is related to trigger default. One of the first structural models is by Merton (1974), later models with default barrier are also developed e.g., Longstaff and Schwartz (1995) and Briys and Varenne (1997). In these models, the default happens once the firm’s value reaches the default barrier. They link the firm’s value to the credit risks. Intensity models, in contrast, assume an exogenous default process which can follows its own dynamics. Usually, the default process is assumed to be Poisson process and the intensity is calibrated from existing credit products with the same or close credit risks. Therefore, intensity models can largely prevent arbitrage opportunities between credit products. Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997) are some examples for intensity models. Crouchy, Galai and Mark (2000) provide a good review for various credit models.

Although these papers get very good results about credit derivatives from studying credit risks, e.g. modeling credit risks, pricing and hedging credit products,
exotic and portfolio credit derivatives and derivatives on credit portfolio, they ignore one important aspects: the credit derivatives can also conversely affect the credit risk to some degree, especially in the default case. This is true because, different from the equity and most other derivatives underlying prices of which are determined by general market participants, the default risks underlying CDS are greatly affected by a small group of participants, practically those who has interests directly linked to the default risk and CDS. Firstly, default events are primarily determined by the equity-holders (debtors) to maximize their equity claims by playing capital structure games with debt-holders (creditors). Creditors with safety covenant to protect their debt claims can also enforce the bankruptcy. Unfortunately, usually they can also buy credit derivatives like CDS to hedge the default risks. This paper differentiate previous researches on credit risks in that it examines credit derivatives’ effects to credit risks in a framework of capital structure game in which both parties (equity-holders and debt-holders) try to maximize their claims to the firm.

There are also some papers on relationship between equity and debt products. E.g., Kwan (1996) examines the relationship of stock and bond prices. For the capital structure and their implications to equity and debt prices, Mella-Barral and Perraudin (1997), Mella-Barral(1999) and Collin-Dufresne and Goldstein (2001) are good references. However, they are done without considering both parties’ accessibility to credit derivatives. Other researches have done been on capital structure arbitrage. However, mainly they just try to discover the discrepancies implied by equity and debt prices and
try to take advantage of them. In this aspect, this paper extends capital structure arbitrage into credit default markets.

This paper is organized as follows. Right after introduction part is the background knowledge and basic model section followed by the early exercise with safety covenant part. Then the strategic debt service with credit derivatives is presented. The final section is for conclusion and result. In this paper, we use some acronyms for better representations. SDS refers to Strategic Debt Service and CDS for Credit Default Swap.

**Background Knowledge and Basic Model**

**Default and safety covenant**

Basically, default occurs when the debtors fail to meet the obligations. It includes following forms: bankruptcy (filing for protection), failure to pay coupon or principle, obligation default and accelerations, repudiation and moratorium or even restructuring. The most common forms are bankruptcy, failure to pay and obligation default.

The bankruptcy processes are regulated under some laws in addition to business conventions. The bankruptcy can be either forced by creditors or requested by debtors. Under Chapter 11 of the U.S. Bankruptcy Code, management, who represents the equity-holders, can file for reorganization, which is subject the approval of authorities. In file reorganization, the equity-holders give up some controls to the company and accept restrictions in return for totally and partly debt reductions.
According to Chapter 7 of the U.S. Bankruptcy Code, if debtors do not meet their debt obligations, creditors are entitled to force liquidation sale and be paid first. The payment is up to the initially agreed face value of the debt, out of the proceeds of this open market sale.

If the scrapping value if liquidation is much less than possible values from continued operations, the firm can also choose to operate it themselves or by its representatives or just sell to other parties who are interested in taking over and service the debt payments. The new owners have following possibilities: to financially restructure the firm issuing debt, to renegotiate these new debt contracts as current owners do, to operate the firm and to declare formal bankruptcy. Current asset values depend on the expected post-liquidation value of the firm, i.e., operations under the new owners.

In a lot of debt agreements, there is also safety covenant providing a mechanism for creditor to protect their debt claims from further deterioration by enforcing bankruptcy once the firm’s value is low enough. However, the creditors have only rights, not obligations to do so.

Implicit options and game between equity- and debt-holders

In some circumstances, either party of debtors and creditors has only rights, not obligations for certain contingent claims. This is the basic concept of options. We examine some implicit options in the debt relationship.

Debtors hold the firm and thus have following some options. First, debtors have option not to meet their debt obligations, i.e., to default, and when to default. The
decision to file for Chapter 11 is in debtors’ hands and debtors in possession provision gives them the exclusive right to propose a reorganization plan. Debtors also control the timing of liquidation. That is, as long as they meet their debt obligations, the creditors cannot trigger liquidation.

For debt agreement with safety covenant clauses, the creditors have the option to force bankruptcy for protections. In case of default, they have another options to choose liquidation or continued operation.

As the bankruptcy is costly due to processing costs, liquidation costs and etc., the debtors can play a blackmail game by offering strategic debt service and threatening bankruptcy if creditors do not accept. The value of strategic debt service is less than the debt value and more than firm’s value after default. They expect that creditors would accept the offer because the firm’s scrapping value or value with continued operation is less than the service value. On the other hand, they still control the company without fully servicing the debt.

If creditors do not reject the strategic debt service immediately, debtors and creditors will renegotiate the contracts, taking into account debtors’ option not to meet their debt obligations, and creditors’ option to trigger liquidation and sell the firm. This gives debtors extensive bargaining power in renegotiation.

Safety covenant protects creditors to some extent. They can enforce bankruptcy before the firm’s value down to unfavorable level. It is possible that safety covenant also provides protections from debtors’ strategic debt service blackmailing.
Credit default swap

Among various credit derivatives, CDS is the most commonly one for default risks (there is debate if restructuring can be counted as default) and also the most liquid at the moment. In CDS, sellers agree to pay to buyers the losses that typical lenders would suffer upon a credit event of a certain reference entity. In exchange for the payment upon credit event, the buyers will pay fees regularly at each payment interval up to the default event (if a default occurs between two payment dates, the accrued fraction of the next payment up to default still need to be paid at default time. This paper uses CDS to represent other credit derivatives mainly because, with its good liquidity, CDS is the best product for hedge default risk, which is very closely related to SDS. The linkage between default risk and SDS is just the topic of this paper.

Basic model

We present the basic setup, which is used extensively in later sections of this paper. We use the structural model because we want to examine the equity and liability relationships. The credit dynamics of debt with face value $F$ are based on the firm’s value $V$, which follows Geometric Brownian Motion (GBM) with initial value $V_0$, constant diffusion factor $\mu$ and volatility $\sigma$.

\[
\frac{dV}{V_i} = \mu dt + \sigma dW_i
\]  

(1.1)
In the risk-neutral probability $Q$, without arbitrage implies following dynamics:

$$\frac{dV_t}{V_t} = r dt + \sigma dW_t$$

(1.2)

Assume constant continuously compounding interest rate $r$, constant discretely compounding interest rate $r_p$ for each period $t_r$. $r$ and $r_p$ are consistent interest rates with different compounding methods. Assume CDS with maturity $T$, discrete spread payment $k$ (percentage), and payment interval $t_{cds}$. Further assume $t_{cds} = t_r$. The bond will default once $V_t \leq h_d$ where $h_d$ is the default barrier. Let

$$\tau_{h_d} = \inf \{ t : V_t \leq h_d \}$$

(1.3)

the first passage time that $V_t$ goes below $h_d$. Generally default time is denoted $t_d$. The percentage bankruptcy cost is $\alpha$ and the firm’s value after bankruptcy is $h_d(1 - \alpha)$ if default at barrier $h_d$.

For any $t < T$, the present value of CDS protection buyer’s payment

$$A(t) = kF \sum_{i=0}^{b} (1 + r_p)^i + kF \left( \frac{t}{t_{cds}} - b \right) (1 + r_p)^{\left( t/t_{cds} \right)}$$

(1.4)

where $b = \left\lfloor \frac{t}{t_{cds}} \right\rfloor$ is the integer part of $\frac{t}{t_{cds}}$. The second term is the present value of the payment accrued from last whole payment up to $t$. Protection buyers receive

$$B(t) = (1 + r_p)^{t/t_{cds}} \left( F - V_t(1 - \alpha)I_{t \geq t_{cds}} \right)$$

(1.5)
where $I_{(t \leq \tau_h)}$ is the indicator function. Here we use the general form $V(t(1-\alpha))$ instead of $h_d(1-\alpha)$ because the value after default $V(\tau^*_h)$ can be smaller than $h_d$ because of jump.

Both $A(t)$ and $B(t)$ stop once $t = \tau_{h_d}$.

No arbitrage implies that the Credit Default Swap spread $k$ is the value satisfying

$$E[A - B] = 0, \text{ i.e.}$$

$$\int_0^\infty (A(t) - B(t)) f_{\tau_{h_d}}(t) dt = 0$$

(1.6)

where $f_{\tau_{h_d}}(t)$ is the density function of the stopping time $\tau_{h_d}$ and can be shown as$^1$

$$f_{\tau_{h_d}}(t) = e^{-\eta h_d} - V_0 \left( \frac{e^{-\eta h_d} - V_0}{\sqrt{t/\sigma}} \right) \phi \left( \frac{t h_d - V_0}{\sqrt{t/\sigma}} \right)$$

for $t > 0$ (1.7)

where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

For the early exercise problem with safety covenant, we use $h_s$ to denote safety covenant barrier and $t_s$ for the time when $V_t$ reaches $h_s$. Time between hits $h_s$ and $h_d$ is $\tau_{h_s \to h_d}$:

$$\tau_{h_s \to h_d} = \inf \left\{ t : V_t = h_d \mid V_{t_s} = h_s \right\}$$

$^1$ Generally, for standard Brownian motion $B_t$, let $\tau_a = \inf \{ t : B_t = a \}$, then we have following properties: $f_{\tau_a}(t) = \frac{a}{\sqrt{2\pi}} \phi \left( \frac{ea}{\sqrt{t}} \right)$ for all $a \geq 0$ and $t > 0$, $P(\tau_a < \infty) = 1$, $E(\tau_a) = \infty$, and $P(\tau_a \geq t) \leq \int_t^\infty \frac{e^{-as}}{2\sqrt{s}} ds = \frac{a}{\sqrt{t}}$ for all $a \geq 0$ and $t > 0$ (Steele (2000), p. 69). $f_{\tau_a}(t)$ is the density function of $\tau_a$. All these properties can be useful for this paper by transforming $V_t$ into $B_t$ with time and value transformations (Girsarov’s Theorem).
with relationship

\[ t_d = t_s + \tau_{h_s \rightarrow h_d} \]

For the distribution of \( \tau_{h_s \rightarrow h_d} \), we get the density function for \( t > 0 \)

\[
f_{\tau_{h_s \rightarrow h_d}}(t) = \frac{e^{-rt}h_d - h_s}{t^{3/2}} \phi \left( \frac{e^{-rt}h_d - h_s}{\sqrt{t}} \right)
\]

(1.8)

This answers every question about the distribution of \( \tau_{h_s \rightarrow h_d} \), and we get following results:

\[
P(\tau_{h_s \rightarrow h_d} < \infty) = 1
\]

(1.9)

\[
E(\tau_{h_s \rightarrow h_d}) = \infty
\]

**Early Exercise with Safety Covenant**

In a lot of debt agreements, there is safety covenant providing a mechanism for creditor to protect their debt claims from further deterioration by enforcing bankruptcy once the firm’s value is low enough. However, the creditors have only rights, not obligations to do so. We regard the protections provided with these clauses as a safety covenant option. If the creditors with safety covenant also buy CDS to hedge the default risks (the default risks are double hedged with safety covenant and CDS), we would know if it is still optimal for them to early exercise the safety covenant option, i.e. to enforce bankruptcy.

Though safety covenant clauses vary much, one of the most important factors affecting creditors’ decision to utilize safety covenant clauses or not is the first time that firm’s value reached the stipulated threshold level (safety covenant barrier). Unless the creditors have very strong beliefs that the firm will eventually perform well, the drop to
threshold level negatively affects mostly their impressions about the firm’s futures. Even if the firm may perform somehow better after drop to the threshold level, this performance rebound can still not completely ease creditors’ worries. So we will base our analysis on the first time that the firm value reaches the safety covenant barrier and check what happen after that.

**Optimality analysis**

Although it can be treated as an option with possibility of early exercise, we are still not sure if it is optimal to exercise it. For example, the American put options on equity are never optimal for early exercise if there are no dividend payments associated with the stock. This section will examine if early exercise can be optimal.

Intuitively, we know that after firm’s value $V_i$ reaches the safety covenant barrier $h_s$, the creditors have two choices. The first is to enforce bankruptcy immediately by utilizing safety covenant clauses and to save CDS spreads payments if possible. However, since $h_s \geq h_d$ they will possibly lose some payments from the CDS. The second is to wait until the firm dies naturally- $V_i$ further slide to $h_d$. Surely, creditors can obtain more from the CDS’s payoffs if the firm can go to default. However, they have to pay more CDS spreads. Furthermore, it is possible that the firm will never go default, they will get nothing from CDS and have to pay spreads until the CDS’s maturity $T$. So creditors have
to balance the tradeoffs of payoffs to make early-exercising decisions with the $V_t$ uncertainty.

In the following parts, we would first show that it is possibly optimal to early-exercise if the firm will go bankrupt naturally. Obviously it is always optimal to early-exercise if the firm will not go bankrupt (of course, with the assumed firm value dynamics in this paper and alike, the firm always have an non-zero probability to bankrupt naturally, however small the probability is). We find that it is optimal for early exercise, i.e. the safety covenant option has positive values.

One feature of the CDS is that the spreads are paid discretely at the end of each interval $t_{cb}$. If a default occurs between two fee payments dates, the protection buyer still have to pay the fraction of the next payment that has accrued until the time of default. Since the CDS spreads are discretely paid after interval, following denotations would be helpful. Following denotations are useful:

$$a = \left\lfloor \frac{t_s}{t_{cb}} \right\rfloor, \quad b = \left\lfloor \frac{t_s}{t_{cds}} \right\rfloor, \quad \text{and}$$

$$c = \left\lfloor \frac{\tau_{h_i \rightarrow h_j}}{t_{cds}} \right\rfloor$$

where function $\left\lfloor x \right\rfloor$ takes the integer part of $x$. Since $t_d = t_s + \tau_{h_i \rightarrow h_j}$, we have following inequality

$$c \leq b - a \leq c + 1$$

(2.1)
Our objective function is to maximize the expected value of maximized \( \ell(t_{h \rightarrow h_d}) \) as follows:

\[
P_{\text{Safety-Covenant}} = \mathbb{E}_{\{T_{h \rightarrow h_d} \geq T - t_s\}} \sup \ell(t_{h \rightarrow h_d})
\]  

(2.2)

where

\[
\ell(t_{h \rightarrow h_d}) = F - h_s(1 - \alpha) - kF \left( \frac{t_s}{t_{cd}} - a \right) + kF (1 + r_p)^{t_{st} - a} \left( \sum_{i=1}^{a} \left( 1 + r_p \right)^{a - i} \right) - \\
\left( F - h_s(1 - \alpha) - kF \left( \frac{t_d}{t_{cd}} - b \right) \left( 1 + r_p \right)^{h_s + y_{t_{st} - a}} - kF (1 + r_p)^{t_{st} - a} \left( \sum_{i=1}^{b} \left( 1 + r_p \right)^{b - i} \right) \right)
\]

(2.3)

\( \ell(t_{h \rightarrow h_d}) \) is just the difference of payoffs between exercising at \( t_s \) and \( t_d \). Note that

\[ E_{t_s} \ell(t_{h_d}) = 0 \]

as it is equivalent to eq. (1.6) in pricing CDS. \( P_{\text{Safety-Covenant}} \) can be interpreted as the value of implicit option granted by the safety-covenant.

The payoff if exercise at \( t_s \) constitutes of CDS payoff \( F - h_s(1 - \alpha) \) and cumulative spread payments \( K(t_s) = kF \left( \frac{t_s}{t_{cd}} - a \right) + kF (1 + r_p)^{t_{st} - a} \left( \sum_{i=1}^{a} \left( 1 + r_p \right)^{a - i} \right) \) up to default. Obviously, \( K(t_s) \) is increasing with default time \( t_s \). The payoff if default at \( t_d \) includes similar parts \( K(t_d) \). With these denotations, we can simplifies (2.3) into:

\[
\ell(t_{h \rightarrow h_d}) = F - h_s(1 - \alpha) - K(t_s) - \left( 1 + r_p \right)^{h_s + y_{t_{st} - a}} \left( F - h_s(1 - \alpha) - K(t_s + r_{h \rightarrow h_d}) \right)
\]

(2.4)
It is easy to show \( \ell(\tau_{h_i \to h_j}) \) increases with \( \tau_{h_i \to h_j} \) since \( K(t_d) \) also increases with \( \tau_{h_i \to h_j} \) and discount factor \((1 + r_p)^{-\tau_{h_i \to h_j}}\) decrease with \( \tau_{h_i \to h_j} \). This shows that for a single path of firm value, it is better to exercise at the first time \( t_x \) when \( V \) reaches \( h_x \) than later time when \( V \) reaches \( h_y \) if \( F - h_x(1 - \alpha) - K(t_x) > 0 \). As \\
\[ \ell(0) = (h_x - h_d)(1 - \alpha) < 0 \quad \text{and} \quad \ell(\infty) = F - h_y(1 - \alpha) - K(t_x), \]
\[ \ell(\tau_{h_i \to h_j}) = 0. \]
So for a single path once \( \tau_{h_i \to h_j} > \tau^{*}_{h_i \to h_j} \), it is always optimal to exercise.

As indicated out in (1.8), we can \( E(\tau_{h_i \to h_j}) = \infty \), implying that for natural default happening at \( t_d \), \( P(\tau_{h_i \to h_j} > \tau^{*}_{h_i \to h_j}) > 0 \). So there are some paths that the safety covenant can be early exercisable at \( t_x \) for optimality even with CDS. The optimality to early exercise with these double-hedged risks is of fair significance because it is more likely that not fully hedged risks will be early exercised. Early exercises can have significant to the credit default process.

**Numerical results**

Based on eq. (1.2) through (1.6), we calculate the CDS spread using Monte Carlo simulation with following parameters: \( V_0 = 70 \), \( r_p = 0.015 \), \( \sigma = 0.3 \), \( F = 60 \), \( T = 2 \), \( t_{cdh} = t_r = 0.25 \), \( h_d = 42 \), and \( \alpha = 0.3 \). Eq. (1.5) is used because in reality the safety covenant bankruptcy can be enforced if can default at all the time when \( V_t < h_d \), not only at the first passage time. Therefore, the CDS payoff at debt default is \( V_t(1 - \alpha) \) instead of \( h_x(1 - \alpha) \). \( V_t \) is assumed to follow the process of (1.2) and simulation of 200,000 paths gives the result \( k = 0.105 \).
With this $k$ value, following (2.1) through (2.4) with $h = 66$, we use Least Square Monte Carlo (LSMC) simulation of Longstaff & Schwartz (1998) to get the option value $P_{\text{Safety-Covenant}} = 0.706$, or 0.0117 if measured as spread. In order to improve accuracy, we run the least-square regression and compare the exercise conditions (i.e. if payoff from immediate exercise is more than holding) before check the safety covenant boundary $V_i \leq V_d$. This method gives us more paths for regression than otherwise regressing on paths already satisfying $V_i \leq V_d$. Framework in (2.1) through (2.4) can also be useful when immediate liquidation and take over involved. The only minor changes are to change $V_i (1-\alpha)$ into $\gamma$ or $V_i^N (1-\alpha)$.

Furthermore, $V_i$ is then assumed to follow the jump-diffusion process with jump intensity $\lambda = 4$ and expected log-jump-size 0. The risk neutral condition is also accommodated as in Appendix 1. The intensity of jump is the average of all kinds of jumps including credit jump, market price jump and etc. The Brownian bridge method for barrier options in Metwally & Atiya (2002) is used for fast-simulation. LSMC is also used for $P_{\text{Safety-Covenant}}$ as above. We get increased values of $k = 0.19$, $P_{\text{Safety-Covenant}} = 0.973$ and 0.016 as spread.

All these numerical results show that the safety covenant can contribute to more than 10% to the total CDS spread, suggesting that the implicit option associated with safety
covenant cannot be ignored even with hedge from CDS. Furthermore, it provides us with a framework for examining the other games played between debtors and creditors.

**Strategic Debt Service**

**Model**

As equity-holder and debt-holder’s decisions are mainly based on the firm’s value and cash flow and it is always true that free cash flow is important during the bankruptcy-prone period, we start this section with examining the cash flows and derive the firm’s value from the cash flow. We do not examine the tax advantages associated with debt, as firms close to default are unlikely to generate profits that are subject to capital tax.

Assume the free cash flow follow the following GBM process in the risk-neutral world:

\[
\frac{dp}{p_t} = \mu_p dt + \sigma_p dW_t
\]  

(3.1)

Let \( V_t, Q_t \) and \( L_t \) denote the firm’s value at time \( t \), equity and liabilities (bonds, loans etc.) respectively.

\[
V_t = Q_t + L_t
\]  

(3.2)

\( V_t \) is a function of \( p_t \), i.e. \( V_t = V(p_t) \). By Itô’s formula, we get the capital gain \( dV_t^c \) for short period \( dt \)
\[ dV_i^c = \left( \mu p_i V_i^t + \frac{\sigma^2 p_i^2}{2} V_i^t \right) dt + V_i^t p_i \sigma dW_i \]

However, we should also add the continuous exogenous cash flow \( p_t \) from firm’s operation

\[ dV_i = \left( p_t + \mu p_i V_i^t + \frac{\sigma^2 p_i^2}{2} V_i^t \right) dt + V_i^t p_i \sigma dW_i \quad (3.3) \]

Under the risk-neutral world, financial market equilibrium impose following condition:

\[ dV_i = V_i r dt \]

We get

\[ rV_i = p_t + \mu p_i V_i^t + \frac{\sigma^2 p_i^2}{2} V_i^t \quad (3.4) \]

The dynamics of \( V_i \) we get from cash from is consistent with the assumed in e.q. (1.2) as

\[ \sigma = \frac{V_i^t p_i \sigma_p}{V_i} . \]

Suppose the firm’s payments for debt is \( c \), including coupon payment or average capital costs for loan and bonds’ principal payments. Similarly, we reach the partial differential equation (pde) for \( Q_i \) and \( L_i \):

\[ rQ_i = p_t - b_t + \mu p_i Q_i^t + \frac{\sigma^2 p_i^2}{2} Q_i^t \quad (3.5) \]

\[ rL_i = b_t + \mu p_i L_i^t + \frac{\sigma^2 p_i^2}{2} L_i^t \quad (3.6) \]
When the generated cash flow down to certain level $p_d$ which corresponds to default barrier $h_d$ for firm value of $V_t$, the firm is liable to bankruptcy.

After bankruptcy, the debt holders may liquidate immediately or operate it themselves or by their representatives or just sell to other parties we are interested in taking over and service the debt payments. Let $\gamma$ denote the scrapping value after immediate liquidation and $\xi$, the cash flows generated by new owners. We also suppose that $\xi < 1$, a reasonable assumption since debt holders are not so capable in managing a new company new to them. Let $V_t^N = V_t^N(p_t)$ denote the value of the firm in the hands of the new owners. Then $V_t^N$ satisfies

$$rV_t^N = \xi p_t + \mu p_t V_t^N + \frac{\sigma^2 p_t^2}{2}V_t^N.$$  

(3.7)

If choose to liquidate the firm, they can get the constant scrapping value which is much smaller than $V(p_t)$ (usually only about 50% of $V(p_t)$ according to Hamilton et al. (2001)), reflecting the costs from bankruptcy and liquidation.

At the first reach of bankruptcy trigger price $p_d$, in the absence of arbitrage,

$$\tilde{L}(p_d) = \min\{V_t^N(p_d), c / r\}.$$  

(3.8)

$$\tilde{Q}(p_d) = \max\{0, V(p_d) - c / r\}.$$  

(3.9)

Since equity holders decide the timing of bankruptcy, the trigger price will be selected in order to maximize the value of equity. So $Q'(p_d) = 0$.  

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Assume constant liability costs $c$ and interest rate $r$. To simplify the calculation, further take the liability as perpetual since their maturity is usually much longer than the bankruptcy-prone period of interest to us. The perpetual liability can be treated with principal $c/r$. If $\gamma < c/r$, the liabilities are risky and

$$\hat{Q}(p) = \frac{p}{r - \mu} - \frac{c}{r} \left[ \frac{p_d}{r - \mu} - \frac{c}{r} \right] \lambda$$  \hfill (3.10)$$

$$\hat{L}(p) = \frac{c}{r} + \left[ \nu^N(p_d) - \frac{c}{r} \right] \lambda$$  \hfill (3.11)$$

where $p_d$ is given by

$$p_d = -\frac{\lambda}{1 - \lambda} \frac{c}{r}(r - \mu)$$  \hfill (3.12)$$

and $\lambda$ is negative root of the equation $\lambda(\lambda - 1)\frac{\sigma^2}{2} + \lambda \mu = r$ and

$$\nu^N(p) = \frac{\xi p}{r - \mu} - \left( \gamma - \frac{\xi p_s}{r - \mu} \right) \lambda$$

where $p_s = -\frac{\lambda}{1 - \lambda} \frac{\gamma}{\xi}(r - \mu)$

If $\gamma \geq c/r$, the debt is risk-less and

$$\hat{Q}(p) = \frac{c}{r}$$

$$\hat{L}(p) = \nu^N(p_d) - \frac{c}{r}$$
**Strategic debt service without credit derivatives**

The above results are just for situations without renegotiation, i.e. the stipulated debt clauses are strictly observed. However, it is well known that the debt-holders implicitly sell options to the equity-holders during the debt transaction. One option is that the equity-holders have the rights to decide when to go bankrupt (under safety covenant the bankruptcy can also be forced by the debt-holders as well as decided by the equity-holders). So the equity-holders can try to take more by threatening bankruptcy. The equity-holders’ strategy is to offer to pay an amount $s(p)$ smaller than $c$ which is the minimal acceptable to the debt-holders. Why the debt-holders willing to accept a smaller amount under some conditions? Because the firm’s value $V(p_i)$ is not big enough (with lower $p_i$) and value $V_i^n$ if take over happens, or scrapping value $\lambda$ is not bigger than the $s(p)$. If the debt-holders do not accept $s(p)$, the equity-holders just threat bankruptcy and the debt-holders will mostly be worse off. The equity-holders try to maximize the equity-value.

Since bond-holders have claims on three possible values: $V_i$, $V_i^n$ and $\gamma$. $V(p_i) > V^n(p_i) > \gamma$. These values suggest three payment service possibilities

$$s(p) = \begin{cases} r\lambda & p \in [p_e^*, p_u] \\ \xi p & p \in [p_u^*, p_s] \\ c & p \in [p_s^*, \infty) \end{cases}$$

(3.13)
$p^*_c$ and $p^*_u$ are bankruptcy trigger price for different owners. When $p_i$ goes to below $p^*_c$, bankruptcy happens and no payment. When $p_i$ goes to below $p^*_u$, the new holder will take bankruptcy. Note that they keep constant when the firm’s leverage ratios change if firm’s value $V(p_i)$ follow the same dynamics because when $p_i$ reaches $p^*_c$ or $p^*_u$, the outgoing payment has nothing to do with $c$, the debt. We get them for the pure equity firm:

$$p_c = -\frac{\lambda}{1-\lambda} \gamma(r-\mu)$$

$$p_u = -\frac{\lambda}{1-\lambda} \gamma(r-\mu)$$

We should get $p^*_s$, the boundary price to begin renegotiations. With service strategy $s(p)$ rather than strictly paying $c$, the dynamics for $Q_i$ and $L_i$ then change to:

$$rQ_i = p_i - s(p) + \mu p_i Q_i + \frac{\sigma^2 p_i^2}{2} Q_i$$

Solving these equations with boundary conditions (refer to Appendix 1 for detailed solutions), we can get a unique $p^*_s$. With renegotiation and payment $s(p)$, If $\gamma < c/r$, the liabilities are risky and
\[
L(p) = \begin{cases} 
\frac{c}{r} + \left[V^N(p_s) - \frac{c}{r} \left( \frac{p}{p_s} \right) \right]^{\lambda} & p > p_s \\
V^N(p_s) & p \leq p_s 
\end{cases}
\]

(3.16)

where \( p_s \) is given by

\[
p_s = -\frac{\lambda}{1 - \lambda} \frac{c}{\xi r} (r - \mu)
\]

(3.17)

If \( \gamma \geq c / r \), the debt is risk-less and

\[
L(p) = \frac{c}{r}
\]

(3.18)

The firm is liquidated the first time that \( p \) hits \( p_c \). It is easy to show that \( L(p) < \hat{L}(p) \) since \( s(p) \) is smaller than \( c \) for some prices \( p \).

**SDS with credit derivatives: fully hedged**

This section examines strategic debt service when debt-holders have access to credit derivatives to hedge their default risk exposures. In ISDA (International Swaps and Derivatives Association)’s CDS’s Master Agreement, the industry standard of CDS agreement, there are no restrictions to the trading parties. Actually, using CDS to hedge default risks incurred from lending activities is a very common phenomenon in the market, and those creditors are not treated as insiders as in equity markets. Following are some aspects when all default risk exposures are fully hedged.
First, it is obvious that debt-holders need not concede to equity-holders’ threatening bankruptcy since the loss from bankruptcy can be offset from CDS gains. Debt-holders do not worry about the blackmail anymore. Actually they want to default immediately to avoid further CDS spreads payment. This goes back to the non-negotiation case (3.10) and (3.11).

Second, in face of debtors’ proposed SDS, creditors have following strategies to take advantage of it.

1. Mostly, the creditors have incomplete information about the firm’s value, of which the debtors are much better informed. However, now the creditors can infer from debtors’ proposed SDS that the firm’s value is not far above the default boundary. They can use this message by hedging the exposures if not yet or even take advantage of it by establishing extra CDS positions. So debtors’ SDS cannot financially damage the debtors’ claims, but can benefit them.

2. Complete hedge also provides creditors another option: to attend SDS negotiation without loss. Since the proposed SDS need to be renegotiated and such renegotiations take some time, creditors may initially agree to renegotiate and try to get more information about the firm during negotiation. The obtained information is useful for future strategies. Meanwhile, they can also adjust their CDS positions based on latest information.
Third, debt-holders may somehow hope bankruptcy to happen earlier as long as the recovery rate is not too worse. Then they can take advantage termination of CDS spread payments. This is similar to the case discussed in e.q. (2.4). How can the debt-holders facilitate bankruptcy while bankruptcy is decided by equity-holders to maximize equity value? The creditors can extend credit line (loan more to the firm) and request safety covenant clauses for the new loans.

Debtors should be careful in proposing SDS. It is very possible they achieve nothing but leak information and incur costs during renegotiation. The information associated with SDS proposal and possibly leaked out hinders the firm’s further credit activities. They should check the open interest of CDS in the market to see if the creditors have mostly hedge their risks. The open interest during renegotiation could be indicative of the negotiation result.

**SDS with credit derivatives: partially hedged**

It is more realistic that the creditors can only partially hedge their credit exposures. Let $\beta < 1$ denote the share of debt principal under default hedge. The partially hedged creditors do not accept the SDS $s(p)$ in (3.13) because those $s(p)$ are equivalent to the cash flow after default at $p$ if the debtors are willing to default at $p$. So $s(p)$ are the minimal acceptable to creditors.

Now creditors have to pay CDS spreads but cannot get the payoffs at default if accept SDS, as SDS is not a default event. Since debt-holders have claims on three
possible values: $V_t$, $V_t^N$, and $\gamma$ for un-hedged part and on the hedged $\beta_r^C$. The hedged part requires payment $\left(\frac{\beta_r^C}{r}\right) = \beta c$. The payment for CDS spreads should also be included because creditors have to pay them with SDS and not if default. Here we just assume that CDS spread $\frac{k_c}{r}$ is also continuously paid. The minimal acceptable cash flows to creditors $s_1(p)$ is as follows:

$$
\begin{align*}
    s_1(p) &= \begin{cases} 
    \beta c + \frac{k_c}{r}(1 - \beta) r \lambda & p \in [p_e^*, p_u] \\
    \beta c + \frac{k_c}{r}(1 - \beta) \bar{\lambda} p & p \in [p_u, p_{ts}]
    \end{cases} \\
    &\quad p \in \left[p_{ts}, \infty\right) \tag{3.20}
\end{align*}
$$

All parameters have similar meaning as in (3.13) and $p_e$ and $p_u$ have the same value as before because they are independent of bond payment $c$. We also get $pde$ as in (3.15)

$$
\begin{align*}
    rQ_t &= p_t - s_1(p) + \mu p_t Q_t + \frac{\sigma_t^2 p_t^2}{2} Q_t^* \\
    rL_t &= s_1(p) + \mu p_t L_t + \frac{\sigma_t^2 p_t^2}{2} L_t^* \tag{3.21}
\end{align*}
$$

Solve them with boundary conditions as in Appendix 1 to get an unique

$$
p_{ts} = -\frac{\lambda}{1 - \lambda}(1 - \beta) r \left( \frac{k_c}{r} \right) \left( r - \mu \right), \text{ the boundary price to begin renegotiations. With renegotiation and payment } s(p) \text{, If } \gamma < c/r, \text{ the liabilities are risky and}
$$
\[ L_1(p) = \begin{cases} \frac{c}{r} + \left[ \frac{V^N(p_{ls})}{V^N(p_{ls})} - \frac{c}{r} \right] \left( \frac{p}{p_{ls}} \right) \lambda & p > p_{ls} \\ \frac{c}{r} & p \leq p_{ls} \end{cases} \] (3.22)

If \( \gamma \geq c/r \), the debt is risk-less and

\[ L_1(p) = \frac{c}{r} \]

The firm is liquidated the first time that \( p \) hits \( p_c \). It is easy to show that \( L_1(p) < \hat{L}(p) \) and \( L_1(p) > L(p) \) since \( s_i(p) \) is smaller than \( c \) for some prices \( p \) and bigger than \( s(p) \) for some \( p \).

**Capital structure arbitrage with credit derivatives**

This section examines the relationship between equity and debt prices in the framework default risk hedged with CDS, partially and fully.

Since equity-holders’ option to offer \( s_i(p) \) is implicit and when reach debt agreement debt-holders have not buy CDS. So this option has mostly been priced into the debt price reflected by the higher yield in the form of better coupon, higher seniority and other better clauses to debt-holders than otherwise. However, debt-holders can establish CDS positions after the agreement. Debt-holders get extra benefits because of equity-holders’ unrealized expectation to offer \( s(p) \) and get full concession from debt-holders. Equity-holders can only achieve the concession associated with \( s_i(p) \). The extra benefit
for creditors is the difference of debt under $s(p)$ and $s_1(p)$, i.e. $L_1(p) - L(p)$. For full hedge, the benefit is $\hat{L}(p) - L(p)$.

Furthermore, equity-holders’ optimistic expectation to get concession can also make the firm’s equity over-valued at $Q(p)$. However, the efficient price for the equity is only $Q_1(p)$ for partial hedge and $\hat{Q}(p)$ for full hedge. Debt-holders can buy more liquid put equity options to partly hedge credit risks or even make profits when the value back to efficient price $Q_1(p)$ or $\hat{Q}(p)$. The expected payoffs from the put options are $Q_1(p) - Q(p)$ for partial hedge and $\hat{Q}(p) - Q(p)$ for full hedge.

Debt-holders somehow hope bankruptcy to happen earlier as long as the recovery rate is not too worse. Then they can take advantage of termination of CDS spread payments. This is similar to the case discussed in early sections. How can the debt-holders facilitate bankruptcy while bankruptcy is decided by equity-holders to maximize equity value? One possible way is to offer further loan while keep demand on safety covenant.

**Further extensions**

With credit derivatives, creditors can be strong enough to make take-it-or-leave-it offers which debtors can either accept or reject as they are. Since debtors always prefer coupon reductions, they will simply accept all creditor concessions, as long as they think the equity is not worthless and are willing to operating. If the creditors has only partial hedge
and the payoffs from CDS for hedged part can not fully cover the lost from the un-hedged part, they can provide debtors with the minimum incentives to stay in operations.

Creditors can also optimize over the time they finally take possession of the firm through bankruptcy rather than continuing to reorganize the debt. Creditor’s concessions are made as late as possible as to provide debtors with the minimum incentive to stay in operation, and to avoid bankruptcy costs. The equity value at each date of debt forgiveness was kept just marginally greater than zero, to keep debtors with a marginal incentive not to walk out of the firm.

In those take-it-or-leave-it offers, creditors have great control to the company. Actually, they can decide when to default as compared to normal cases where debtors decided that. Creditors’ existing and planned positions on CDS and put equity options can greatly affect the firm’s bankruptcy process.

**Conclusion**

This paper examines the implicit options associated with safety covenant and strategic debt service (SDS). These options’ effects to the default risks are inspected. I find safety covenant option is early exercisable and SDS games between creditors and debtors affect the dynamics of default process. New changes in default risks can happen when the creditors hedge their default exposure with credit default swap. Credit derivatives have influence on the game of defaulting choices.
Appendix 1: Proof of (3.16) with the method in Dixit (1989).

If $\gamma < c/r$, as in (3.13), for $p \geq p_s$, $L(p)$ satisfies the equation (3.15b). The general solution to (3.15b) is

$$L(p) = \frac{c}{r} + C_1 p^{\lambda_1} + C_2 p^{\lambda_2}$$

where $\lambda_1$ is the negative root of the equation $\lambda(\lambda - 1)\frac{\sigma^2}{2} + \lambda\mu = r$ and $\lambda_2$ the positive one.

Obviously $\lim_{p \to \infty} L(p) = \frac{c}{r}$. However $\lim_{p \to \infty} p^{\lambda_2} = \infty$, implying that $C_2 = 0$.

The no arbitrage at boundary $p_s$ implies that $L(p_s) = V^N(p_s)$ and $L'(p_s) = V^N'(p_s)$.

$L(p_s) = V^N(p_s)$ is the smoothing passing condition discussed in Dixit (1989). For $p < p_s$, there is equality $L(p) = V^N(p)$. These equations can yield the results in (3.17) and (3.16).
References


