LONGEVITY RISK OF PENSIONS
IN CZECH REPUBLIC

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1. INTRODUCTION

The contribution deals with some aspects of longevity risk in the framework of the pension system in the Czech Republic:

(1) **Sustainability of pensions**: one investigates

- which regular annuity spending (spending rate) is admissible
- for a given pension account
- with the given investment efficiency and
- under the given tolerance.
(2) Generation Life Tables (GLT):

- in the pension context one should apply special Life Tables with projections distinguishing among particular generations (cohorts);
- moreover, these Tables should be adjusted in a special way to be suitable for pensions.

(3) Unisex Life Tables (ULT):

- due to various anti-discrimination laws concerning the insurance the ULT should be constructed for the Czech population.
Pension system in Czech Republic:

– after 1989 (the end of the epoch of socialism) in 1996 very up-to-date *state pension system* (the *first pillar* comparable e.g. with Germany):
  – public
  – mandatory
  – PAYG
  – DB (with a small fixed component) and

– insignificant *private pension funds* (the *third pillar*) with the *state support* (more „public saving system“ than pension system)

– unfortunately after 2000 the *demographic situation in the Czech Republic has become critical*:
Comparison of the age distribution of the Czech population in 1950:
Age distribution of the Czech population in 2008:

Source: Eurostat (2008)
Numbers of survivors till particular ages from the Life Tables of the Czech Republic in 2010:

Source: Czech Statistical Office (2010)
the state pension system and its mandatory costs become critical for the state budget, see e.g. the enormous volume of benefits in 2011:

- population in Czech Republic: 10 546 000
- number of registered retirees: 2 873 000
- number of pension annuities: 3 501 000 (plus 70 000 to abroad)
  - number of old-age pensions: 2 340 000
  - number of disability pensions: 445 000
  - number of survivor’s pensions: 716 000
- mean monthly old-age pension: 10 552 CZK
  - male: 11 700 CZK, female: 9 584 CZK
- mean monthly salary: 24 319 CZK
- mean age of retirees: 68 years
- number of contributors: 5,040,000
- number of contributors/1 retiree: 1.75
- number of contributors/1 old-age retiree: 2.15

↓

Pension reform since 2013 is urgent introducing:

- the second pillar with opt-out;
- new progressive solutions involving commercial life insurance (e.g. DC with buyout by private insurance companies);
- actuarial analyses are supposed.
2. SUSTAINABILITY OF PENSIONS

− how much to “save” annually during the accumulation phase and how much to “spend” annually during the decumulation phase;

− many random aspects → the best approach is the one applied in modern finance, namely the \textit{Value-at-Risk (VaR)} = the highest loss which can occur with a given (tolerance);

− in the pension context: \textit{probability of sustainable pension} = the probability that the retired person will not be ‘ruined’ before the moment of death
Assumption:

- DC pension plans;
- the contributions to the system are defined in advance by a percentage of the participant’s salary;
- accumulated capital on the participant’s account in the age of retirement → decumulated by corresponding annual pension payments;
- the investment risk is fully on the side of the participants of the pension plan (not on the side of the pension provider);

↓
- two aspects of random character should be at least considered:
(1) **Randomness of interest rates** \( r(t) \):
- for investment of the capital from the participant’s account;
- originally in the age of retirement the participant holds \( S_0 = w \);
- this type or randomness can be modeled by means of *geometric Brown motion*:

\[
S_t = S_0 \cdot e^{B_t(\mu, \sigma)} = S_0 \cdot e^{\mu \cdot t + \sigma \cdot B_t},
\]

\( \mu \) is the *drift*

\( \sigma \) is the *volatility*
– in particular, $S_t$ is log-normally distributed:

$$\ln S_t \sim N(\ln S_0 + \mu \cdot t, \sigma^2 t);$$

with the mean value:

$$\mathbb{E}(S_t) = S_0 \cdot e^{\left(\mu + \frac{\sigma^2}{2}\right) \cdot t} = S_0 \cdot e^{\nu \cdot t};$$

and the median value:

$$\text{M}(S_t) = S_0 \cdot e^{\mu \cdot t}.$$
(2) **Randomness of the future lifetime** $T_x$ **of an individual aged** $x$:

- can be modeled in the simplest case by the *exponential law of mortality*:

$$
_t p_x = \exp\left\{- \int_x^{x+t} \lambda_x \, ds\right\} = e^{-\lambda_x \cdot t},
$$

$\lambda_x$ is the *instantaneous force of mortality*;

\[\downarrow\]

- the *expected remaining lifetime* (the *life expectancy*) at age $x$:

$$
e_x = E(T_x) = \frac{1}{\lambda_x};
$$
the median remaining lifetime at age $x$:

$$M(T_x) = \frac{\ln(2)}{\lambda_x}.$$
(3) **Combination of randomness of** $r(t)$ **and** $T_x$: 
- **the present value** $PV_x$ **of the standard pension** (which pays unit annual payments in continuous time): 
  \[ PV_x = \int_0^{T_x} e^{-(\mu \cdot t + \sigma \cdot B_t)} \, dt; \]

↓

- **the probability of ruin** (the **probability of unsustainable pension**): 
  \[ P(PV_x > w) = P\left( \int_0^{T_x} e^{-(\mu \cdot t + \sigma \cdot B_t)} \, dt > w \right), \]

$w > 0$ is the sum on the participant’s account at the age of retirement $x$;
it can be approximated as

\[
P(PV_x > w) \sim \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^{1/w} z^{\alpha-1} \exp\left(-\frac{z}{\beta}\right) \, dz =
\]

\[
= 1 - \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^w y^{-(\alpha+1)} \exp\left(-\frac{1}{\beta y}\right) \, dy
\]

where

\[
\alpha = \frac{2\mu + 4\lambda_x}{\sigma^2 + \lambda_x} - 1, \quad \beta = \frac{\sigma^2 + \lambda_x}{2}
\]

and \(\Gamma(\alpha)\) is the gamma function \(\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} \, e^{-z} \, dz\).
Results for the Czech Republic:

(1) **Financial data:**
- *technical interest rate* 2.5 % used for insurance calculations according to the legislative in the Czech Republic in year 2012;
- *more scenarios* both for the drift $\mu$ and the volatility $\sigma$ using the technical interest rate 2.5 % as one of possibilities.

(2) **Longevity data:**
- *Life Tables* (LT) for males and females in the Czech Republic in 2010.
The probability of ruin for various

- *retirement ages* 55, 60, ..., 85;
- *spending rates* 1/w (e.g. the spending rate 0.06 → a pension account of 1 000 000 will pay 60 000 annually, i.e. 5 000 monthly);
- *investment drifts and volatilities*. 
Probability of ruin for various retirement ages and spending rates with fixed $\mu = 1$ % and $\sigma = 5$ % - males:

<table>
<thead>
<tr>
<th>Male $x$</th>
<th>Spending rate $1/w$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>55</td>
<td>0.6 %</td>
</tr>
<tr>
<td>60</td>
<td>0.4 %</td>
</tr>
<tr>
<td>65</td>
<td>0.2 %</td>
</tr>
<tr>
<td>70</td>
<td>0.1 %</td>
</tr>
<tr>
<td>75</td>
<td>0.1 %</td>
</tr>
<tr>
<td>80</td>
<td>0.0 %</td>
</tr>
<tr>
<td>85</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>

↓

- e.g. under a **conservative investment strategy** ($\mu = 1$ % and $\sigma = 5$ %) a male (retirement age of 65, spending rate 0.06): the unsustainable pension with **probability 23.0 %** (a female: 32.1 %).
**Probability of ruin for various retirement ages and spending rates with fixed $\mu = 2.5\%$ and $\sigma = 5\%$ - males:**

<table>
<thead>
<tr>
<th>Male</th>
<th>Spending rate 1 / $w$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>55</td>
<td>0.1 %</td>
</tr>
<tr>
<td>60</td>
<td>0.1 %</td>
</tr>
<tr>
<td>65</td>
<td>0.1 %</td>
</tr>
<tr>
<td>70</td>
<td>0.0 %</td>
</tr>
<tr>
<td>75</td>
<td>0.0 %</td>
</tr>
<tr>
<td>80</td>
<td>0.0 %</td>
</tr>
<tr>
<td>85</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>

\[\downarrow\]

- e.g. under the *legislative investment strategy* ($\mu = 2.5\%$ and $\sigma = 5\%$) a male (retirement age of 65, spending rate 0.06): the unsustainable pension with *probability* 15.4% (a female: 21.5%).
Probability of ruin for various retirement ages and spending rates with fixed $\mu = 5 \%$ and $\sigma = 10 \%$ - males:

<table>
<thead>
<tr>
<th>Male</th>
<th>Spending rate 1 / w:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>55</td>
<td>0.0%</td>
</tr>
<tr>
<td>60</td>
<td>0.0%</td>
</tr>
<tr>
<td>65</td>
<td>0.0%</td>
</tr>
<tr>
<td>70</td>
<td>0.0%</td>
</tr>
<tr>
<td>75</td>
<td>0.0%</td>
</tr>
<tr>
<td>80</td>
<td>0.0%</td>
</tr>
<tr>
<td>85</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

↓

- e.g. under an *effective investment strategy* ($\mu = 5 \%$ and $\sigma = 10 \%$) a male *(retirement age of 65, spending rate 0.06)*: the unsustainable pension with probability *9.7%* (a female: *13.0%*).
The probabilities of unsustainable pension ($\mu = 2.5\%$ and $\sigma = 5\%$) with increasing spending rate:

Males and females CZ 2010
Maximal spending rates $1/w$ admissible with a given tolerance of $1\%$ (i.e. one tolerates the probability of ruin of $1\%$) with fixed $\sigma = 5\%$ - - males:

<table>
<thead>
<tr>
<th>Male $x$</th>
<th>0 %</th>
<th>0.5 %</th>
<th>1 %</th>
<th>1.5 %</th>
<th>2 %</th>
<th>2.5 %</th>
<th>3 %</th>
<th>4 %</th>
<th>5 %</th>
<th>6 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.857 %</td>
<td>1.023 %</td>
<td>1.201 %</td>
<td>1.387 %</td>
<td>1.583 %</td>
<td>1.786 %</td>
<td>1.998 %</td>
<td>2.441 %</td>
<td>2.909 %</td>
<td>3.398 %</td>
</tr>
<tr>
<td>60</td>
<td>1.053 %</td>
<td>1.221 %</td>
<td>1.397 %</td>
<td>1.582 %</td>
<td>1.774 %</td>
<td>1.973 %</td>
<td>2.179 %</td>
<td>2.609 %</td>
<td>3.061 %</td>
<td>3.533 %</td>
</tr>
<tr>
<td>65</td>
<td>1.315 %</td>
<td>1.483 %</td>
<td>1.659 %</td>
<td>1.841 %</td>
<td>2.030 %</td>
<td>2.225 %</td>
<td>2.425 %</td>
<td>2.842 %</td>
<td>3.279 %</td>
<td>3.734 %</td>
</tr>
<tr>
<td>70</td>
<td>1.686 %</td>
<td>1.855 %</td>
<td>2.030 %</td>
<td>2.210 %</td>
<td>2.396 %</td>
<td>2.586 %</td>
<td>2.782 %</td>
<td>3.186 %</td>
<td>3.607 %</td>
<td>4.044 %</td>
</tr>
<tr>
<td>75</td>
<td>2.265 %</td>
<td>2.435 %</td>
<td>2.609 %</td>
<td>2.787 %</td>
<td>2.970 %</td>
<td>3.156 %</td>
<td>3.346 %</td>
<td>3.738 %</td>
<td>4.143 %</td>
<td>4.561 %</td>
</tr>
<tr>
<td>80</td>
<td>3.177 %</td>
<td>3.348 %</td>
<td>3.521 %</td>
<td>3.698 %</td>
<td>3.878 %</td>
<td>4.061 %</td>
<td>4.246 %</td>
<td>4.626 %</td>
<td>5.016 %</td>
<td>5.416 %</td>
</tr>
<tr>
<td>85</td>
<td>4.686 %</td>
<td>4.857 %</td>
<td>5.030 %</td>
<td>5.205 %</td>
<td>5.383 %</td>
<td>5.562 %</td>
<td>5.743 %</td>
<td>6.112 %</td>
<td>6.489 %</td>
<td>6.874 %</td>
</tr>
</tbody>
</table>

\[ \downarrow \]

- e.g. under a *conservative investment strategy* ($\mu = 1\%$ and $\sigma = 5\%$) and a *low tolerance of 1\%* → the maximal spending rate for a male (retirement age of 65): *1.659\%* (i.e. *16,590 annually from 1,000,000*)
Maximal spending rates $1/w$ admissible with a given tolerance of 10 \% (i.e. one tolerates the probability of ruin of 10 \%) with fixed $\sigma = 5$ \%- males:

<table>
<thead>
<tr>
<th>Male x</th>
<th>0 %</th>
<th>0.5 %</th>
<th>1 %</th>
<th>1.5 %</th>
<th>2 %</th>
<th>2.5 %</th>
<th>3 %</th>
<th>4 %</th>
<th>5 %</th>
<th>6 %</th>
</tr>
</thead>
</table>

↓

- e.g. under the legislative investment strategy ($\mu = 2.5$ \% and $\sigma = 5$ \%) and a high tolerance of 10 \% → the maximal spending rate for a male (retirement age of 65): 5.012 \% (i.e. 50 120 annually from 1 000 000)
Minimal capital for unit life annuity with given tolerance ($\mu = 2.5\%$ and $\sigma = 5\%$) – male:

Males CZ 2010:
Minimal capital for unit life annuity with given tolerance ($\mu = 2.5\%$ and $\sigma = 5\%$) – female:
3. GENERATION LIFE TABLES FOR PENSION SYSTEM

– special Life Tables with projections distinguishing among particular generations (cohorts);

– these Tables must be adjusted in a special way to be suitable for pensions.
**Generation LT in Czech Republic** should respect two facts typical just for pensions:

1. *decreasing trend* of the rates of mortality in time;
2. *selection* of pension portfolios.

- in the *Current* (or *Period*) *LT*:
  
  \( q_x \) depends only on the age \( x \) (e.g. \( q_{65} \) from such Tables in 2010 concerns the male generation 1945);

- in the *Generation LT*:

  \( q_{x}^{\tau} \) depends also on the year of birth (therefore the probability discussed above should be denoted as \( q_{65}^{1945} \)).
this principle must be respected also for *multiyear* probabilities, e.g.

\[ 5 p_{65}^{1945} = p_{65}^{1945} \cdot p_{66}^{1945} \cdot p_{67}^{1945} \cdot p_{68}^{1945} \cdot p_{69}^{1945} \]

the application of the Generation LT can be recommended just for the *Czech Republic* with a *high longevity risk in the pension system*.
Numbers of survivors $l_x$ for generations 1935, 1955 a 1975 according to the Generation LT for the Czech Republic - male:

Source: Cipra (1998)
Numbers of survivors $l_x$ for generations 1935, 1955 and 1975 according to the Generation LT for the Czech Republic - female:

Source: Cipra (1998)
### Annual payment for life annuity using Generation and Current LT:

**Monthly payment for life annuity (the pension account of 1 000 000 in age 60)**

<table>
<thead>
<tr>
<th>Pension from year:</th>
<th>Male in age 60 by means of:</th>
<th>Female in age 60 by means of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{60}= \ddot{a}_{60}(0%)$</td>
<td>$\ddot{a}_{60}(4%)$</td>
</tr>
<tr>
<td>1996</td>
<td>3 810</td>
<td>5 859</td>
</tr>
<tr>
<td>2000</td>
<td>3727</td>
<td>5 775</td>
</tr>
<tr>
<td>2005</td>
<td>3 626</td>
<td>5 675</td>
</tr>
<tr>
<td>2010</td>
<td>3 534</td>
<td>5 581</td>
</tr>
<tr>
<td>2015</td>
<td>3 446</td>
<td>5 492</td>
</tr>
<tr>
<td>2020</td>
<td>3 364</td>
<td>5 408</td>
</tr>
<tr>
<td>2025</td>
<td>3 287</td>
<td>5 329</td>
</tr>
<tr>
<td>2030</td>
<td>3 215</td>
<td>5 254</td>
</tr>
<tr>
<td><strong>Current</strong></td>
<td>5 128</td>
<td>7 111</td>
</tr>
</tbody>
</table>

*Source: Cipra (1998)*
Applying the Generation LT the differences are not negligible from the financial point of view:

↑

– the monthly payments for pensions paid off from the age 60 achieved in various years if the pension account in the age 60 is 1 000 000;

– three alternative calculation formulas:
  – \( \frac{1 000 000}{12 \cdot e_{60}} \)
  – \( \frac{1 000 000}{12 \cdot ä_{60}(4 \%) } \)
  – \( \frac{1 000 000}{12 \cdot ä_{60}(6 \%) } \)
– e.g. for the *interest rate 4 % p.a.* and for the *male generation of the age 60 in 2010*:
  
  – *Current LT in 2010*: the monthly annuity payment is *even 7 111*;
  
  – *Generation LT*: the monthly annuity payment is *only 5 581*. 

4. UNISEX LIFE TABLES FOR PENSION SYSTEM

- *Unisex LT* can be looked upon as a reaction to the movement against the discrimination using the gender as a calculation parameter in pension systems;

- the *probabilities of death* $q_x$ according to the *Unisex LT for the Czech Republic in 2003* confirm the expectation that the *unisex mortality is approximately an average of the male and female mortality:*
Probabilities of death $q_x$ according to the Unisex LT for the Czech Republic in 2003

Source: Smetana a Cipra (2005)
Single premium for deferred life annuities with annual payments of 12 000 for the Czech Republic in 2003 (2.4 % p. a.)

<table>
<thead>
<tr>
<th>Age</th>
<th>Deferment</th>
<th>Unisex (U)</th>
<th>Males (M)</th>
<th>Females (F)</th>
<th>Ratio U/M (%)</th>
<th>Ratio U/F (%)</th>
<th>Ratio M/F (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>40</td>
<td>58 629</td>
<td>50 595</td>
<td>71 864</td>
<td>115.88 %</td>
<td>81.58 %</td>
<td>70.40 %</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>74 821</td>
<td>64 791</td>
<td>91 382</td>
<td>115.48 %</td>
<td>81.88 %</td>
<td>70.90 %</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>95 880</td>
<td>83 397</td>
<td>116 564</td>
<td>114.97 %</td>
<td>82.26 %</td>
<td>71.55 %</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>125 114</td>
<td>110 017</td>
<td>150 445</td>
<td>113.72 %</td>
<td>83.16 %</td>
<td>73.13 %</td>
</tr>
</tbody>
</table>

Source: Smetana a Cipra (2005)

- **longevity risk must be modified if respecting the gender effect**
- e.g. single premium in the **age of 20 with deferment of 40** (i.e. retirement age of 60): the **ratio M/F** is **70.40 %**
  - **ratio U/M** is **115.88 %**
  - **ratio U/F** is **81.58 %**
Level annual premium for deferred life annuities with annual payments of 12 000 CZK for the Czech Republic in 2003 (2.4 % p. a.)

<table>
<thead>
<tr>
<th>Age</th>
<th>Deferment</th>
<th>Unisex (U)</th>
<th>Males (M)</th>
<th>Females (F)</th>
<th>Ratio U/M (%)</th>
<th>Ratio U/F (%)</th>
<th>Ratio M/F (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>40</td>
<td>2 293</td>
<td>1 997</td>
<td>2 784</td>
<td>114.81 %</td>
<td>82.36 %</td>
<td>71.74 %</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>3 534</td>
<td>3 091</td>
<td>4 272</td>
<td>114.33 %</td>
<td>82.72 %</td>
<td>72.35 %</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>6 134</td>
<td>5 398</td>
<td>7 370</td>
<td>113.62 %</td>
<td>83.23 %</td>
<td>73.25 %</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>14 287</td>
<td>12 712</td>
<td>16 986</td>
<td>112.40 %</td>
<td>84.12 %</td>
<td>74.84 %</td>
</tr>
</tbody>
</table>

Source: Smetana a Cipra (2005)

- longevity risk must be modified if respecting the gender effect
- e.g. level premium in the age of 20 with deferment of 40 (i.e. retirement age of 60): the ratio M/F is 71.74 %
  ratio U/M is 114.81 %
  ratio U/F is 82.36 %
5. CONCLUSIONS

Three important aspects that must be taken into account when constructing products burdened by longevity risk:

(1) sustainability of pensions;
(2) the generation longevity;
(3) the gender longevity.

It has been shown numerically that these aspects really play a very important role for the pensions in the Czech Republic.

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REFERENCES


