

# **DOLLAR COST AVERAGING - THE ROLE OF COGNITIVE ERROR**

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## **Abstract**

Dollar Cost Averaging (DCA) has long been shown to be mean-variance inefficient, yet it remains a very popular investment strategy. Recent research has attempted to explain this popularity by assuming more complex investor risk preferences. However, this paper demonstrates that DCA is sub-optimal regardless of investor preferences over terminal wealth. Instead this paper offers a simpler explanation: That DCA's continued popularity is due to a specific and demonstrable cognitive error in the argument that is normally put forward in favor of the strategy. Demonstrating this error should help investors make better-informed decisions about whether to use DCA.

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## **I. Introduction**

Dollar cost averaging (DCA) is the practice of building up investments gradually over time in equal dollar amounts, rather than investing the desired total immediately in one lump sum. This strategy is widely recommended in popular investment guides and the media, but its continued popularity presents a challenge to academics, since it has long been shown that DCA is mean-variance inefficient.

Recent research has proposed a range of alternative explanations for DCA's popularity. These are reviewed briefly in the section II, but they are unsatisfactory for three key reasons. First, explanations based on non-variance forms of investor risk preference should be rejected because DCA can be shown to be a sub-optimal strategy regardless of preferences over terminal wealth (this is demonstrated in section V). Second, other explanations rely on additional – and often unverifiable – assumptions about investors or the markets involved. This is particularly problematic given that proponents of DCA generally recommend it without any consideration of investors' objectives, expectations or preferences. Finally, most of the theories which have been advanced to explain DCA's popularity make no reference to the factor which is usually central to the case made by its proponents: That DCA automatically purchases more securities when the price is lower, and so achieves an average purchase cost which is below the average market price.

The empirical evidence clearly shows that DCA's lower costs do not lead to higher returns, but it has never been made clear why this is so. Previous papers have argued that the fact that

DCA buys at an average purchase cost which is below the average price is irrelevant because investors cannot subsequently sell at this average price (Thorley, 1994; Milevsky and Posner, 2003), but this misrepresents the case put forward for DCA. If investors could sell at the average price then DCA would generate guaranteed short-term profits. This is not what its proponents are claiming. Instead DCA is proposed as a better way to enter longer-term trades. Investors do not know the price at which they will eventually sell these assets, so DCA remains risky. But whatever exit price is subsequently achieved, it seems obvious that buying at a lower average cost must result in higher profits than buying at a higher average cost would have, so DCA appears to generate higher returns. Indeed, this argument appears to be so attractive that investors are happy to ignore contrary research which has shown that DCA is an inefficient strategy. Addressing this argument is central to explaining why DCA remains so popular.

Section III identifies the hidden flaw in this argument, and shows that comparing DCA's average cost with the average market price is systematically misleading. This gives us a much simpler explanation of DCA's continuing popularity: That investors are making a cognitive error in failing to recognize the flaw in the argument presented by DCA's proponents. Previous research has put forward more complex explanations for DCA's popularity because this cognitive error has not hitherto been properly identified.

This paper makes a contribution on two fronts. First it makes a positive contribution by offering a better explanation of DCA's popularity – one which for the first time fully addresses the key argument put forward by DCA's proponents. Second, identifying this cognitive error

should improve welfare by helping investors make better-informed decisions about whether to use DCA. Given DCA's wide popularity the benefits could be substantial.

The rest of this paper is structured as follows: The next section reviews previous explanations for DCA's popularity. Section III analyzes the counterfactuals that are being compared when we note that DCA purchases shares at below the average price, and gives an intuitive demonstration of why this does not imply that DCA will generate higher profits. Section IV demonstrates this result more formally. Section V shows that alternative strategies can generate identical outturns to DCA with less initial capital, and hence that DCA is a sub-optimal strategy regardless of investor preferences. Wider welfare implications are considered in section VI. Conclusions are drawn in the final section.

## **II. Explanations for the popularity of DCA**

Earlier research has clearly demonstrated that DCA is mean-variance inefficient. Constantinides (1979) showed that as DCA commits the investor to continue making equal periodic investments, it must be dominated by more flexible strategies which allow the investor to make use of the additional information which is available in later periods. A large number of empirical studies have subsequently found that investing the whole desired amount in one lump sum generally gives better mean-variance performance than DCA. These include Knight and Mandell (1992/93), Williams and Bacon (1993), Rozeff (1994) and Thorley (1994).

Proponents sometimes claim that DCA improves diversification by making many small purchases but, as Rozeff (1994) notes, by investing gradually DCA leaves overall profits most

sensitive to returns in later periods, when the investor is nearly fully invested. Earlier returns have less effect because the investor then holds mainly cash. Better diversification is achieved by investing immediately in one lump sum, and thus being fully exposed to the returns in each sub-period. Milevsky and Posner (2003) extend the analysis into continuous time, and show that it is always possible to construct a constant proportions continuously rebalanced portfolio which will stochastically dominate DCA in a mean-variance framework. They also show that for typical levels of volatility and drift there is a static buy and hold strategy which dominates DCA.

As the evidence became overwhelming that DCA is mean-variance inefficient, research turned to attempts to explain why investors nevertheless persist with it. Some investigated whether DCA outperforms on non-variance measures of risk. Leggio and Lien (2003) consider the Sortino ratio and upside potential ratio. Their results vary between asset classes, but overall they reject claims that DCA is superior. DCA substantially reduces shortfall risk (the risk of falling below a target level of terminal wealth) compared to a lump sum investment (Dubil, 2005; Trainor, 2005), but even if investors consider this to be worth the associated reduction in expected return, Constantinides' critique remains potent: Less rigid strategies should be expected to dominate, for example by allowing investors to increase their exposures if their portfolios are safely above the required minimum value.

Statman (1995) argues that behavioral finance can explain DCA's continued popularity, with investors framing outturns in line with prospect theory, showing different risk preferences over gains and losses. Furthermore, by committing investors to continue investing at a constant

rate, DCA limits choice in the short term, which may reduce regret and protect investors from their tendency to base investment decisions on naïve extrapolation of recent price trends.

However, subsequent papers have found that prospect theory is not a satisfactory explanation. Leggio and Lien (2001) find that DCA remains an inferior strategy even when investors have prospect theory preferences and loss aversion. Fruhwirth and Mikula (2008) show that DCA dominates a buy-and-hold strategy when investors also show excess sensitivity to outliers, but this cannot explain DCA's popularity since both strategies are dominated by simply investing half the available funds immediately and keeping the rest in risk-free assets. The intuition behind this result is that - as Rozeff showed - DCA is an inefficient means of reducing overall risk levels.

Section V below demonstrates a much more general result: That DCA is sub-optimal regardless of investors' risk preferences, since an alternative strategy can be constructed which generates the same distribution of terminal wealth but requires less capital. Thus hypothesising such alternative investor preferences is a sterile area of research which cannot explain DCA's continued popularity.

Other papers have sought to justify the use of DCA by making alternative assumptions about the forecasting of market returns. Milevsky and Posner (2003) show that if an investor has a firm forecast of the value of a security at the end of the horizon, then as long as volatility is sufficiently high the expected return from DCA conditional on this forecast will exceed the corresponding expected return from investing in this security in one lump sum. However, this explanation assumes that this expected terminal value remains fixed throughout the horizon, and

does not change as market prices shift. Thus, for example, a fall in market prices increases the expected future return and so makes DCA's purchase of more shares at this lower price attractive. If investor expectations instead tend to shift in line with market prices (either in response to the same underlying news that shifts market prices, or because investor sentiment is directly affected by market price movements), then this property is removed, and DCA becomes unattractive.

Brennan et al. (2005) investigate whether DCA's use can be explained by weak-form inefficiency in equity returns. They find that the degree of mean reversion in US equity returns (1926-2003) was too small to offset the underlying inefficiency of DCA as a strategy for building up a new portfolio, but that it was large enough to make DCA a beneficial strategy for adding a new stock to an already well-diversified portfolio. However, this is a new result which required detailed econometric study. For this to explain DCA's popularity the authors are forced to assume that this property was already known to investors as part of inherited "folk finance" wisdom.

In sum, ever since it was shown that DCA is mean-variance inefficient, research in this field has developed more and more complex theories to explain its continued popularity. Non-variance investor risk preferences have been suggested, but are not satisfactory explanations. Other more complex explanations depend upon additional – and often unverifiable – assumptions such as "folk finance" or constant investor price expectations. Occam's razor tells us that theories which do not require such assumptions should be preferred. Indeed, we observe that in practice DCA tends to be recommended to investors without any detailed consideration of their

goals, expectations or risk preferences, or the properties of the market involved. DCA is widely recommended by commentators in the financial press, in books and online. Some of these claim that DCA reduces risk, whilst many do not. But proponents almost invariably stress the fact that DCA buys at below the average price, suggesting that it increases expected returns. Addressing this argument – and showing why a lower average purchase price does not increase expected returns – is thus central to explaining DCA’s popularity.

### **III. Identifying the Cognitive Error**

Table 1 shows a numerical example typical of those used by proponents of DCA (the alternative ESA strategies are not normally made explicit and will be explained later). A fixed \$60 each period is invested in a specific equity. The price is initially \$3, allowing 20 units to be purchased. The sharp fall to \$1 allows 60 units to be purchased for the same dollar outlay in period 2, whilst the rebound to \$2 allows 30 units to be bought in the final period. The argument made in favor of DCA is that it buys shares at an average cost ( $\$180/110 = \$1.64$ ) which is lower than the average market price of the shares over the period during which they were accumulated (\$2). This is achieved because DCA automatically buys more shares during periods when they are relatively cheap and fewer when they are more expensive.

[Table 1 here]

Greenhut (2006) takes issue with the particular return assumptions which are often used in such “demonstrations” of the superiority of DCA. However, there is a much more general issue here. The average purchase cost for DCA investors gives greater weight to periods when the price is relatively low, so price fluctuations will always mean that DCA investors buy at less than the average price, regardless of the particular path taken by prices. The difference is particularly large in the example above due to the large price movements, but any price volatility favors DCA. Only when the share price remains unchanged in all periods will the average cost equal the average price. Rather than challenging the particular numbers used, we need to examine why a strategy which buys assets at a lower average cost does not in fact lead to higher expected profits.

Previous studies have found that DCA is mean-variance inefficient compared to investing the whole desired amount immediately in one lump sum, but proponents of DCA are making a different comparison. In noting that the average cost achieved by DCA is less than the average price they are implicitly comparing DCA with a strategy which invests the same amount by buying a constant number of shares each period (thus achieving an average purchase cost equal to the unweighted average price). This is the comparison that we must make here in order to understand why the case in favor of DCA is misleading.

Table 1 compares the cashflows under DCA with two alternative strategies which buy a constant number of shares in each period (equal share amounts: ESA1 and ESA2). The difference between these two alternatives may appear to be a trivial matter of scale, but it is in this difference that the false comparison lies.

ESA1 is an attempt to invest the same total amount as DCA over these three periods, but to do so in equal share amounts. With the share price initially at \$3, a reasonable approach would be to buy 20 shares, since if prices remain at this level in periods 2 and 3 we will end up investing exactly the \$180 total that we desire. But our strategy then requires that we buy 20 shares in each of the following periods, and when prices in periods 2 and 3 turn out to be substantially lower, we end up investing only \$120. It is only with perfect foreknowledge of future share prices that we could have known that the only way of investing \$180 in equal share amounts is to buy 30 shares each period, as shown in ESA2.

The fact that DCA buys shares at an average cost which is below the unweighted average price during this period effectively compares the DCA strategy with the ESA2 strategy which invests the same dollar total in equal share amounts. But ESA2 can only achieve this if we know future share prices – otherwise we will generally end up investing the wrong amount. Moreover, this foresight is used in a way which systematically reduces profitability. In this example, the ESA2 strategy reacts to the knowledge that prices are about to fall by investing more than it otherwise would in period 1. Conversely, it would invest less in period 1 if prices in subsequent periods were going to be higher. This is the only way to invest the correct amount but, of course, it systematically reduces profits.

Table 2 shows the same strategies, but with the share price rising rather than falling. The DCA strategy again invests \$60 each period, but as prices rise fewer shares are purchased in the later periods. Once again DCA achieves an average cost ( $\$180/47=\$3.83$ ) below the average price (\$4) by buying more shares when they are relatively cheap. This effectively compares the

DCA strategy with the ESA2 strategy, which invests the same total amount, but buys only 45 shares compared to 47 using DCA.

[Table 2 here]

However, as we saw earlier, the real alternative to DCA is ESA1. In practice our best guess would again be to invest one third of our total budget in the first period, since if prices were to stay at this level we would invest the correct amount. But when prices subsequently rise we end up spending substantially more than this (\$240). ESA2 invests the correct amount, but it achieves this only by knowing that prices are about to rise and responding to this knowledge by buying fewer shares than ESA1. Again, profits are reduced.

Comparing DCA's average cost with the average price effectively compares DCA with a strategy which uses perfect foresight to systematically reduce profits and increase losses. DCA is almost invariably recommended to investors on the basis of this false comparison, so cognitive error appears to be a key factor explaining the strategy's continued popularity.

#### **IV. The Arithmetic of the Cognitive Error**

In this section we demonstrate more formally that it is only by making a misleading comparison that DCA appears to offer superior profits. We consider investing over a series of  $n$  discrete periods. The price of the asset in each period  $i$  is  $p_i$ . The alternative investment strategies differ in the quantity of shares  $q_i$  that are purchased in each period. We evaluate profits at a

subsequent point, after all investments have been made. If prices are then  $p_T$ , the profit made by any investment strategy is:

$$(1) \quad \Pi = p_T \sum_{i=1}^n q_i - \sum_{i=1}^n p_i q_i$$

We define DCA as a strategy which invests  $b$  dollars in each period ( $p_i q_i = b$ ). This gives us the profits that will result from following a DCA strategy:

$$(2) \quad \Pi_{dca} = p_T \sum_{i=1}^n \left( \frac{b}{p_i} \right) - nb$$

We assume that investors who use DCA do not believe that they can forecast market prices – in effect they assume that prices follow a random walk. As Brennan et al. (2005) show, mean reversion could under some limited circumstances lead DCA to outperform, but the case that is normally made for DCA makes no claim that it is exploiting market inefficiency – instead it is portrayed as a strategy which will outperform in any market. Furthermore, DCA commits investors to invest the same amount no matter what price movements they expect in the coming period. Those who (rightly or wrongly) believe that they can forecast short-term price movements are likely to reject DCA and follow other strategies instead.

We also assume that this random walk has zero drift.<sup>1</sup> Investors presumably believe that over the medium term their chosen securities will generate an attractive return, but they must also believe that the return over the short term (while they are using DCA to build up their position) is likely to be small. Investors who expect significant returns over the short term would prefer to invest immediately in one lump sum rather than delay their investments by following a DCA strategy. Given these assumptions, investors will assume ex ante that prices will remain flat, with  $E[p_T/p_i]=1$  for all  $i$ . Substituting this into Equation 2, we see that the ex ante expected profit from our DCA strategy is zero.

Our alternative investment strategy is to buy equal numbers of shares in each period ( $q_i=a$ ). Substituting this into Equation 1 gives us:

$$(3) \quad \Pi_{esal} = anp_r - a \sum_{i=1}^n p_i$$

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<sup>1</sup> In any case, the assumption of zero drift need not imply a loss of generality, since drift could be incorporated by defining prices not as absolute market prices, but as prices relative to a numeraire which appreciates at a rate which gives a fair return for the risks inherent in this asset ( $p_i^*=p_i/(1+r)^i$ , where  $r$  reflects the cost of capital and an appropriate risk premium). We could then assume that  $p_i^*$  has zero expected drift since investors who use DCA will not believe that they can forecast short-term relative returns for assets of equal risk (those who do would choose other strategies). The results derived above would continue to hold for  $p_i^*$ , with profits then defined as excess returns compared to the risk-adjusted cost of capital. Indeed, this assumes that funds not yet needed can be held in assets with the same expected return as the risky asset. This is clearly generous to DCA – if instead cash is held on deposit at a lower expected return, then DCA’s expected return will clearly be reduced by delaying investment.

The ex ante expected profit from this ESA strategy is also zero (this can be seen by substituting  $E[p_i]=E[p_T]$  for all  $i$ , as an equivalent expression of our driftless random walk). Thus DCA does not give superior expected returns.

This is an intuitive result. We can regard our total return as a weighted average of the returns made on the amounts invested in each period. ESA and DCA differ only in giving different relative weights to these individual period returns. But if prices are believed to follow a random walk with zero drift the expected return will be zero for each period and varying the relative weight given to different periods' returns cannot change the expected aggregate return.<sup>2</sup> By contrast, DCA's popular supporters suggest that even when investors have no belief that they can forecast market returns they can nevertheless expect to beat the market by using DCA.

As we saw in the previous section, the total amount invested under ESA1 ( $a\sum p_i$ ) is likely to differ from the amount ( $nb$ ) invested under DCA. But the comparison that is usually presented by proponents of DCA assumes that the two techniques invest equal total amounts. Thus in order to duplicate the conventional "proof" of the benefits of DCA, we need to rescale the number of shares bought under ESA1 by the fixed factor ( $nb/a\sum p_i$ ), so that an exactly equal amount is invested by the two strategies. This gives us the expected profits resulting from strategy ESA2:

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<sup>2</sup> The expected profit made by any investment strategy is shown below. Our assumption of a random walk implies that future price movements ( $p_T/p_i$ ) are always independent of the past values of  $p_i$  and  $q_i$ .

$$E[\pi] = \sum_{i=1}^n (E[p_T q_i]) - \sum_{i=1}^n (E[p_i q_i]) = \sum_{i=1}^n \left( E \left[ \frac{p_T}{p_i} \right] E[p_i q_i] \right) - \sum_{i=1}^n (E[p_i q_i])$$

$$(4) \quad \Pi_{esa2} = \Pi_{esa1} \left( \frac{nb}{a \sum_{i=1}^n p_i} \right)$$

The use of foresight can be seen in the fact that the scaling factor depends on the average share price throughout the investment period. Only if this is known at the outset would we be able to buy the correct number of shares so that we end up spending exactly the same amounts under ESA2 and DCA. Substituting from Equation 3:

$$(5) \quad \Pi_{esa2} = \left( anp_r - a \sum_{i=1}^n p_i \right) \left( \frac{nb}{a \sum_{i=1}^n p_i} \right)$$

$$(6) \quad = \frac{bn^2 p_r}{\sum_{i=1}^n p_i} - nb$$

Subtracting Equation (6) from Equation (2) we find:

$$(7) \quad \Pi_{dca} - \Pi_{esa2} = nbp_r \left( \frac{1}{n} \sum_{i=1}^n \left( \frac{1}{p_i} \right) - \frac{n}{\sum_{i=1}^n p_i} \right)$$

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But the random walk has zero drift, so  $E[p_t/p_i]=1$  for all  $i$  and expected profits are zero regardless of the amount  $p_i q_i$  which is invested in each period.

The term in brackets is non-negative for positive  $p_i$ , and strictly positive if they are not all equal. This follows directly from the arithmetic mean-harmonic mean inequality.<sup>3</sup>

This achieves our objective. We have shown that the expected profits from a DCA strategy are identical to those from our ESA1 strategy (both give zero expected profits). By contrast, DCA gives higher expected profits than our ESA2 strategy which scales the level of investment so as to spend exactly the same total amount as DCA. However, ESA2 is not a feasible strategy, since it uses perfect foresight to invest in a systematically loss-making fashion. It is only on this biased comparison (DCA vs ESA2) that DCA appears to make greater returns, yet it is exactly this comparison which is implicitly being made when we note that DCA buys at an average cost which is lower than the average price.

This biased comparison suggests that DCA makes profits of  $(P_T - \text{average purchase cost})$  per share whereas other strategies make  $(P_T - \text{average price})$ . Thus DCA appears to shift the whole distribution of possible profits upwards by the extent of the differential. The error involved in comparing DCA's average cost with the average price is a subtle one. With this not previously having been properly identified, it should not be surprising that – faced with such apparently obvious benefits - many investors choose DCA, disregarding contrary research showing that it is mean-variance inefficient.

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<sup>3</sup> The arithmetic-harmonic mean inequality is usually stated as:  $(x_1 + \dots + x_n) / n \geq n / (\frac{1}{x_1} + \dots + \frac{1}{x_n})$  for positive  $x_i$ , so we have substituted  $x_i = 1 / p_i$ . This inequality follows directly from Jensen's inequality that  $E[f(x)] \geq f(E[x])$  for any convex function  $f(\cdot)$ , using the function  $f(x) = 1/x$ .

## V. The Sub-Optimality of DCA

Previous research has shown that DCA is mean-variance inefficient. Other papers have investigated whether DCA outperforms on alternative measures of risk, but in this section we show that DCA is a sub-optimal strategy regardless of the investor's risk preferences. For this purpose we use the payoff distribution pricing model derived by Dybvig (1988).

To illustrate this, Figure 1 shows a binomial model of a DCA strategy over four periods. The equity element of the portfolio is assumed to double in a good outturn and halve in a bad outturn. At the start of the first period 16 is invested in equities, with 48 in cash. A further 16 of this cash is invested each period. All paths are assumed to be equally likely. The key to this technique is comparing the terminal wealths with their corresponding state price densities (the state price divided by the probability – in this case  $16(1/3)^u(2/3)^d$ , where  $u$  and  $d$  are the number of up and down states on the path concerned<sup>4</sup>). The better outturns generally correspond to the lower state price densities, but there are exceptions. DDUU results in a larger terminal wealth than UUUD despite seeing fewer lucky outturns. Similarly, DDDU beats UUDD and UDUD. This can be interpreted as DCA failing to make effective use of some relatively lucky paths.

[Figure 1 here]

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<sup>4</sup> More generally, the state price densities of one period up and down states are  $(1/(1+r\Delta t))(1 - ((\mu - r)\Delta t / \sigma\sqrt{\Delta t}))$  and  $(1/(1+r\Delta t))(1 + ((\mu - r)\Delta t / \sigma\sqrt{\Delta t}))$  respectively, where  $r$  is the continuously compounded annual risk-free interest rate and the risky asset has annual expected return  $\mu$  and standard deviation  $\sigma$ . The corresponding one period risky asset returns are  $(1 + \mu\Delta t + \sigma\sqrt{\Delta t})$  and  $(1 + \mu\Delta t - \sigma\sqrt{\Delta t})$ . See Dybvig (1988).

The inefficiency of DCA can be demonstrated by deriving an alternative strategy which generates exactly the same 16 outturns at lower cost. This is done by changing our strategy so that the best outturns occur in the paths with the lowest state price densities (and hence the largest number of up states), so we swap the outturns for DDUU and UUUD in Figure 1, and those for DDDU and UUDD. The state prices can then be used to determine the value of earlier nodes (this determines the proportion of the portfolio which must be held in cash at each point in order to duplicate the terminal wealth outturns of a DCA strategy), and this in turn determines the initial capital required to generate these outturns. This alternative strategy is shown in Figure 2 and requires only 62.2 initial capital (23.1 in equities and 39.1 in cash), compared to 64 above. This shows the degree to which DCA is inefficient. This improvement is achieved without additional borrowing, merely by making more use of existing capital in early periods and reducing exposure in later periods.

[Figure 2 here]

A key advantage of this method is that it establishes this result without needing to specify the investor's utility function. Generating identical terminal wealth outturns at lower initial cost can be considered better under any plausible utility function.

The amount of cash which must be held at each point is shown below the equity holdings. We can see that the optimized strategy holds considerably less cash during the first two periods than DCA. This supports the interpretation put forward by Rozeff (1994) that DCA is inefficient

because it offers poor time diversification, taking too little exposure in the early periods, leaving terminal wealth disproportionately sensitive to returns in later periods.

However, the doubling or halving of equity values at each step of this tree would – for volatility levels typical of developed equity markets – correspond to a gap of several years between successive investments. This extreme assumption allows us to illustrate dynamic inefficiencies in a very short tree, but it is unrealistic for most investors. For a more plausible strategy we consider a 12 step tree corresponding to a DCA strategy of equal monthly investments over a one year horizon. This tree contains 4096 outturns, and so is not shown here.<sup>5</sup>

[Table 3 here]

Table 3 shows the efficiency loss derived using binomial trees calibrated to a range of different assumptions for the market risk premium and volatility. Efficiency losses are roughly proportional to the assumed risk premium, which again supports the interpretation that these losses stem from the returns foregone by holding an excessively large proportion of the portfolio in cash during the early periods (with an inefficient concentration of relative risk in the later periods). The absolute level of volatility has very little impact on the level of inefficiency, which

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<sup>5</sup> As a robustness check, these calculations were replicated for an 18-step binomial tree, giving 262,144 outturns (the maximum which was computationally practical). The resulting inefficiency estimates were very similar to those in Table III, being larger by a maximum of 0.01%. This demonstrates that the granularity inherent in the 12-step tree does not affect our conclusions.

is a reassuring indication that the results generated here are not dependent on the assumed distribution of returns.

Furthermore, these estimates of the inefficiency resulting from using DCA can be regarded as lower bounds. The optimized strategy generates the same outturns as DCA at a lower cost, showing that DCA is inefficient regardless of the form taken by investor risk preferences. This is a powerful result, but in practice there is no strong reason why investors' preferred option would be to replicate DCA's outturns. Instead there are likely to be other strategies which – given investors' particular preferences – will be even more attractive alternatives to DCA. For example, Rozeff (1994) assumed mean-variance preferences and found that a lump-sum investment outperformed DCA over a twelve month horizon by around 1% (using S&P equity index data 1926-1990).

## **VI. Assessing the Wider Welfare Implications**

We have demonstrated that comparing DCA's lower average cost with the average market price is extremely misleading. This directly undermines the key argument put forward in favor of DCA, and may have rather more impact than previous research in convincing investors to abandon the strategy. In this section we investigate the possible wider effects of such a shift on investor welfare.

The previous section demonstrated that DCA is sub-optimal regardless of the form taken by investor risk preferences. The results shown in Table 3 are conservative estimates of the efficiency losses caused by DCA, but nevertheless they are economically significant – for

instance, investors would generally regard sustained return differentials of this scale as relevant in assessing the performance of competing fund managers. Moreover, these efficiency losses come in stark contrast to the common belief that DCA actually increases expected returns, so investors who use DCA are also likely to make sub-optimal portfolio choices as a result of basing their plans on a biased estimate of future expected returns. For both these reasons, demonstrating why DCA does not boost expected returns should be expected to lead to better decision-making.

It might be objected that the techniques used above to estimate the inefficiency of DCA assume that investors have an initial lump sum to invest, whereas this may not in fact be the case. If investors have completely regular monthly income and expenditure streams, then the optimal strategy may in any case be to save a fixed dollar amount each month, in which case the issue is moot, since the investor is pushed into a DCA strategy by default. However, even small variations in income or required expenditure would mean that a DCA strategy would be non-optimal, implying excessive scrimping in a tight month or holding excess cash balances in a relatively cash-rich month.

We also need to take into account the welfare impact of the behavioral finance effects identified by Statman (1995). We demonstrated above that prospect theory cannot justify the use of DCA. However, by committing investors to make regular investments, DCA may help overcome the myopia which is otherwise likely to lead to inadequate saving over the long run. Furthermore, framing the investment in an artificially flattering way (by making the misleading comparison between DCA's average cost and the average price) could itself boost investor

welfare. By giving investors less choice in the short-term, DCA can also reduce the pain of regret stemming from ill-timed investments.

Investors, or their advisors, may feel able to judge whether these benefits outweigh the disbenefits resulting from DCA's inefficiency in risk/return terms. Indeed, a few advisers and commentators explicitly recommend DCA as a mechanism for gradually encouraging nervous investors back into the market following significant losses. This is a valid argument, but it needs to be understood that any such benefits come at the cost of following a strategy which is sub-optimal in risk/return terms. A wider realization that DCA does not boost returns should improve welfare by helping investors and their advisors to assess whether the behavioral benefits of using DCA outweigh the costs.

Statman also notes that the rigid timetable imposed by DCA would prevent investors from trying to time the market, thus removing their tendency to buy at the top of upswings and sell at the bottom of downturns. It can be argued that this indirect benefit outweighs DCA's sub-optimality in risk/return terms, but advocating DCA for this reason locks investors into a second-best strategy. The best advice for investors would be to avoid trying to time the market and to aim to make regular savings, but without committing themselves to an unnecessarily rigid DCA strategy. We should also be extremely uncomfortable with the idea of boosting investors' welfare by encouraging them to believe (or at least failing to discourage them) that DCA boosts expected returns. This would deliberately leave investors believing a demonstrable falsehood. We should instead seek to publicize both errors - warning investors against trying to time the market, but also demonstrating that DCA generally does not raise expected returns.

Moreover, the belief that DCA raises expected returns is also widespread among investment professionals. This is clearly undesirable. DCA is again likely to be associated with over-estimation of returns and excess holdings of cash. Possible psychological benefits to fund managers cannot justify the use of a strategy which worsens investment performance for their clients.

## **VII. Conclusions**

Previous research has struggled to explain why DCA remains very popular despite being mean-variance inefficient. A number of proposed explanations assume specific non-variance forms of investor risk preferences, but this paper has demonstrated that these should be rejected, since DCA is a sub-optimal strategy regardless of the form taken by investor preferences over terminal wealth.

Furthermore, previous explanations have not properly addressed the point that is generally central to the case put forward for DCA: that it buys securities at an average cost which is below the average price. This paper shows that this in effect compares DCA with a strategy which uses perfect foresight to invest more when prices are about to fall and less when they are about to rise. It is only because of this false comparison that DCA appears to offer superior returns.

This brings two advantages. First, it gives us a better explanation of DCA's continued popularity by identifying the cognitive error in the key argument put forward by its proponents. Second, demonstrating this error is likely to increase investor welfare by helping them to make

better informed decisions about whether to use DCA. The current widespread use of DCA by investors suggests that the benefits could be significant.

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**Table 1**

**Illustrative Comparison Of Strategies As Share Prices Fall**

(a) The DCA strategy invests a fixed \$60 per period, and is compared to strategies which buy Equal Share Amounts (ESA) of (b) 20 shares, (c) 30 shares per period. Falling prices mean that ESA1 invests a lower dollar total than DCA. ESA2 is the only ESA strategy which invests the same amount as DCA, but choosing the right number of shares in period 1 requires knowledge of future share prices.

Period	Share price	(a) DCA		(b) ESA1		(c) ESA2	
		Shares purchased	Investment	Shares purchased	Investment	Shares purchased	Investment
1	\$3	20	\$60	20	\$60	30	\$90
2	\$1	60	\$60	20	\$20	30	\$30
3	\$2	30	\$60	20	\$40	30	\$60
Total		110	\$180	60	\$120	90	\$180

**Table 2**  
**Illustrative Comparison Of Strategies As Share Prices Rise**

(a) The DCA strategy invests a fixed \$60 per period, compared to buying Equal Share Amounts (ESA) of (b) 20 shares and (c) 15 shares per period. Rising prices mean that ESA1 invests a larger dollar total than DCA. ESA2 is the only ESA strategy which invests the same amount as DCA, but choosing the right number of shares in period 1 requires knowledge of future share prices.

Period	Share price	(a) DCA		(b) ESA1		(c) ESA2	
		Shares purchased	Investment	Shares purchased	Investment	Shares purchased	Investment
1	\$3	20	\$60	20	\$60	15	\$45
2	\$4	15	\$60	20	\$80	15	\$60
3	\$5	12	\$60	20	\$100	15	\$75
Total		47	\$180	60	\$240	45	\$180

**Table 3**

**Quantifying The Inefficiency Of DCA (% of initial capital)**

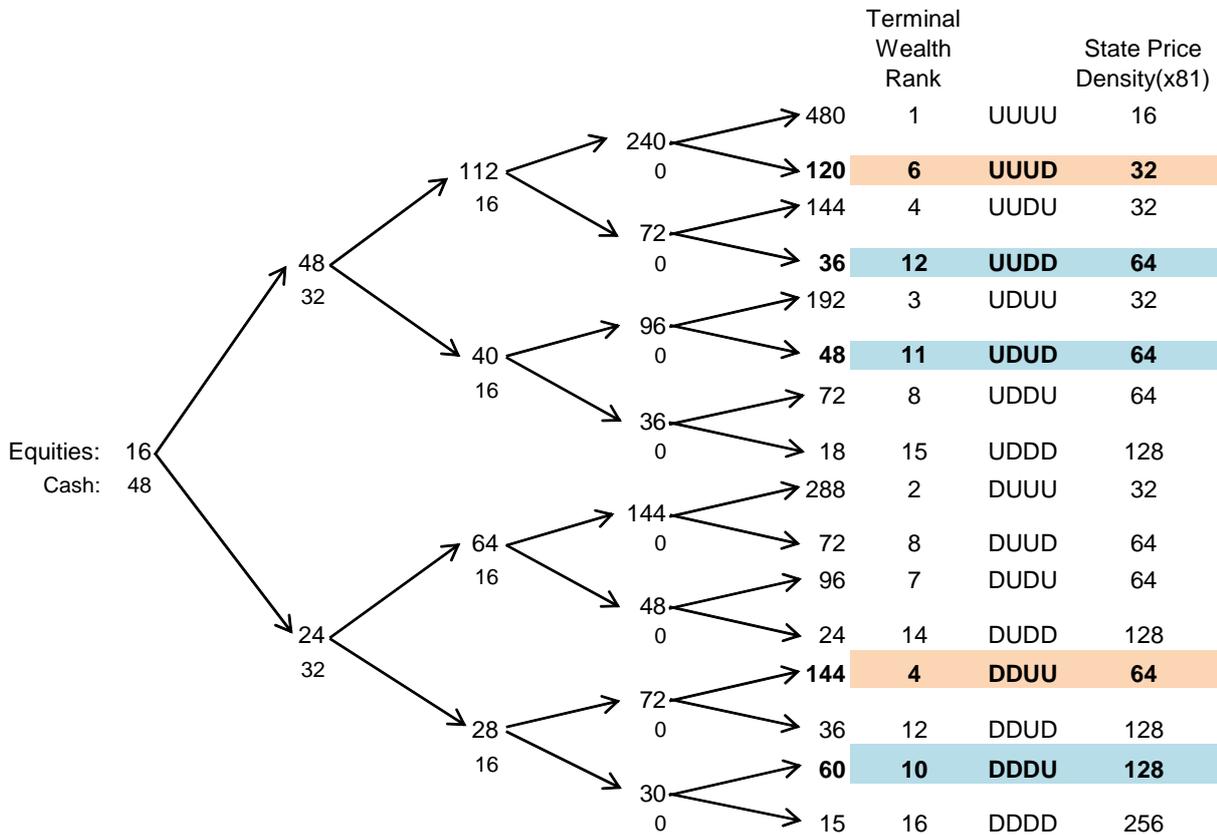
Dybvig's PDPM model is used to derive the cost of an optimized strategy which generates the same set of final portfolio values as those achieved by a DCA strategy which invests one twelfth of its initial capital at the start of each month. The table shows the percentage by which the capital required by the optimized strategy is lower than that required by the DCA strategy. These figures were derived using a 12 period binomial tree where returns are assumed IID with the binomial steps calibrated to give the risk premium and volatility shown. The risk-free rate is assumed to be 5%, but the results are not sensitive to this assumption (adjusting it to 0% or 10% alters the figures below by less than 0.005%).

Standard deviation of security (per annum)	Risk premium (per annum)			
	2%	4%	6%	8%
10%	0.12	0.24	0.35	0.46
20%	0.12	0.24	0.36	0.48
30%	0.12	0.24	0.36	0.48

**Figure 1**

**Simple Model of DCA Strategy**

This tree shows the value of the investor's equity and cash holdings at the start of each period. Equity values double in a good outturn and halve in a bad outturn. All paths are assumed to be equally likely. Investors start with 16 invested in equities (the upper figure at each node) and 48 in cash (the lower figure). They then invest a further 16 in each subsequent period, leaving zero at the start and end of the final period. The sub-optimality of this strategy stems from the fact that in some cases (highlighted) paths with a higher state price density achieve higher terminal wealth than luckier paths with a lower state price density.



## Figure 2 Optimized Strategy Giving Identical Outturns To DCA

This tree shows the amounts invested in equities and in cash at each point with amount of cash held at the start of each period set to replicate the outturns in Figure 1, but with these outturns optimized so that the largest outturns come in the states with the lowest state price density (compared with Figure 1, the outturns for UUUD and DDUU have been swapped, and the outturns for DDUU and DDDU). The lower total initial capital (62.2) required for this optimized strategy shows the degree to which the DCA strategy is inefficient.

