Persistent deviations from market fundamentals or rational bubbles in stock market prices?

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Abstract

Tests for fractional integration in the log dividend yield of the S&P 500 are conducted in order to test the proposition that exogenous shocks have permanent effects. The presence of a unit root in the log dividend yield is consistent with "rational bubbles" in stock prices. Previous empirical studies employing univariate and bivariate tests for integer orders of integration or cointegration have had difficulty in rejecting the rational-bubbles hypothesis. Our findings, based on tests for fractional integration, yield robust rejections of the same hypothesis. The results strongly suggest that the log dividend yield is mean reverting.

Keywords: Asset pricing, Unit roots, Fractional integration, Rational bubbles

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1 Introduction

Recent developments in the US stock market have prompted Federal Reserve Chairman Alan Greenspan to coin the phrase "irrational exuberance" suggesting that asset prices were not grounded on market fundamentals. The dominant theory of financial asset pricing, the present-value model, purports that stock prices are equal to the sum of the expected discounted dividend sequence. Persistent deviations of stock prices from fundamental prices - price bubbles - do not necessarily reflect irrational behaviour on the part of market participants. Asset price bubbles are consistent with rational behaviour in cases where there is an expectation of a discounted capital gain from holding the stock into the indefinite future. Essentially, stock buyers will pay a price higher than that suggested by the fundamentals if they believe that someone else will subsequently pay an even higher price. Such price bubbles - known as "rational" - must be continually expanding in order to survive.

In the context of time-series analysis, rational bubbles are inconsistent with a cointegrating relationship between dividends and stock prices. Cointegration implies that two or more time series cannot drift apart indefinitely as they must satisfy a long-run equilibrium condition. Conversely, nonstationary deviations from the present-value model of stock prices are indicative of rational bubbles. Initial deviations from fundamental prices can be triggered by exogenous shocks or rumors and then perpetuated by self-fulfilling expectations. After the stock price reaches a high level the bubble bursts and can subsequently restart.

Campbell and Shiller (1987) test for cointegration in stock prices and dividends using annual data for the Standard and Poor's 500 composite stock price index from 1871 to 1986. The residual-based tests for cointegration used in their study yielded mixed results leading the authors to conclude that "deviations from the present-value model are quite persistent". The results of the same study were found to be sensitive to the choice of the discount rate. Such findings render models that are based on the assumption of a constant discount rate unattractive.

Campbell and Shiller (1988a,b) have extended the present-value model of stock prices under rational expectations to allow for a stochastic discount factor. The extended model implies that if dividend growth and the discount factor are stationary series, the log dividend yield should be stationary in the absence of rational bubbles; equivalently, stock prices and dividends should cointegrate. Froot and Obstfeld (1991) test for a unit root in the price-dividend ratio using Standard and Poor's data for the period 1900-1988. Based on Phillips-Perron (1988) unit-root tests, they report inability to reject nonstationarity in five of six cases. This amounts to lack of a cointegrating relationship between stock prices and dividends, suggesting that deviations from the present-value model are permanent - consistent with the presence of rational bubbles in stock prices. Craine (1993) applied the Augmented Dickey-Fuller (1981) unit-root test to the log dividend-price ratio (log dividend yield) using annual S&P 500 data from 1876 to 1988 and was also unable to reject the unit-root hypothesis.

More recent empirical work based on tests for integer orders of integration in the data has not been able to resolve the issue. Lamont (1998) uses quarterly data for the S&P 500 spanning the period 1947:1-1994:4. While application of Dickey-Fuller tests on the dividend yield cannot reject the unit-root hypothesis, bivariate tests proposed by Horvath and Watson (1995) support the existence of a cointegrating relationship between dividends and stock prices. I have been able to replicate Lamont's results using the Horvarth and Watson cointegration test on his data but was unable to find cointegration using the full available sample.

The inability of previous empirical work to obtain robust rejections of a unit root in the dividend yield is somewhat puzzling, as there are strong theoretical arguments that rule out rational bubbles. These arguments include the impossibility of negative bubbles on assets with limited liability. Further, a bubble cannot exist if there is an upper limit on the price of an asset. Stock price bubbles may be ruled out if firms impose an upper limit on stock prices by issuing stock in response to stock price increases.

In a general equilibrium context, Tirole (1982) has shown that bubbles cannot exist in a model with a finite number of infinite-lived rational agents. If a positive bubble existed in an asset, agents would sell the asset short, invest some of the proceeds to pay the dividend stream, and have positive wealth left over. Thus, arbitrage would rule out bubbles. Moreover, Tirole (1985) has shown that in the context of an overlapping-generations model with an infinite number of finite-lived agents, a bubble cannot arise when the interest rate exceeds the growth rate of the economy. This is because the bubble would eventually become infinitely large in relation to the wealth of the economy, thus violating some agent's budget constraint. Consequently, bubbles can only exist in dynamically inefficient overlapping generations models that have over-accumulated private capital and driven the interest rate below the growth rate of the economy.

This paper re-examines the empirical validity of permanent deviations from the present-value model of stock prices that are consistent with what could be described as "rational exuberance". I utilize a class of long-memory models known as Autoregressive Fractionally Integrated Moving Average (*ARFIMA*) processes. Fractionally integrated time series differ from both stationary and integrated processes in that they are persistent (they possess long memory) but are also mean reverting. The notion of fractional integration and cointegration allows more flexible modelling of the low frequency dynamics of stock dividends, prices, and their equilibrium relationship, while allowing significant deviations from equilibrium in the short run. Cheung and Lai (1993) found that allowing for a wider range of mean-reverting behaviour, through the notion of fractional cointegration, is important for a proper empirical assessment of Purchasing Power Parity.

In addition, the paper focuses on possible nonlinearities in the variance of the log dividend yield. Time-varying volatility in asset returns has attracted considerable attention in recent years. A rapidly expanding set of models has been developed to capture the serial correlation of volatility. The most well known of these models is perhaps the Autoregressive Conditionally Heteroscedastic ($ARCH$) model proposed by Engle (1982). Bollerslev (1986) generalized the $ARCH$ model by allowing modelling of the variance as an $ARMA(p, q)$ process hence the name $GARCH$.

Application of these recent techniques on a historical data set for the S&P 500 has led to robust rejections of the hypothesis of a unit root in the log dividend yield. Our results are inconsistent with the presence of rational bubbles in stock market prices but also reject temporary deviations of stock prices from fundamentals. The evidence presented in this paper points to long memory in the log dividend yield that is consistent with the type of "intrinsic" price bubbles suggested by Froot and Obstfeld (1991).

The following section provides a brief discussion of the present-value model under rational expectations and introduces the notion of rational bubbles in stock prices. Section 3 outlines the econometric methodology used in the paper. Data description and discussion of estimation results are included in Section 4. Simulation results are reported in Section 5, while Section 6 offers some conclusions.

2 Rational Bubbles

This section draws heavily from Campbell *et al.* (1997). Define the net simple return on a stock as:

$$R_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \tag{1}$$

where, R_{t+1} denotes the return on the stock held from time t to $t+1$ and D_{t+1} is the dividend in period $t+1$. The subscript $t+1$ denotes the fact that the return only becomes known in period $t+1$. Taking the mathematical expectation of (1), based on information available at time t, and rearranging we obtain:

$$P_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}} \right] \tag{2}$$

Solving (2) forward k periods yields the semi-reduced form:

$$P_t = E_t \left[\sum_{i=1}^{k} \left(\frac{1}{1 + R_{t+i}} \right)^i D_{t+i} \right] + E_t \left[\left(\frac{1}{1 + R_{t+k}} \right)^k P_{t+k} \right] \tag{3}$$

In order to obtain a unique solution to (3) we need to assume that the expected discounted value of the stock in the indefinite future converges to zero:

$$\lim_{k \to \infty} E_t \left[\left(\frac{1}{1 + R_{t+k}} \right)^k P_{t+k} \right] = 0 \tag{4}$$

The convergence assumption allows us to obtain the so-called fundamental value of the stock as the sum of the expected discounted dividend sequence:

$$F_t = E_t \left[\sum_{i=1}^{\infty} \left(\frac{1}{1 + R_{t+i}} \right)^i D_{t+i} \right] \tag{5}$$

Abandoning the convergence assumption - equation (4) - leads to an infinite number of solutions any one of which can be written in the form:

$$P_t = F_t + B_t \text{ where } B_t = E_t \left[\frac{B_{t+1}}{1 + R_{t+1}} \right] \tag{6}$$

The additional term in (6) appears in the price only because it is expected to be present in the next period with a value $B_t E_t(1 + R_{t+1})$. The term B_t is called a "rational bubble", in the sense that it is entirely consistent with rational expectations and the time path of expected returns.

The presence of time-varying expected stock returns has led to a nonlinear relation between prices and returns. Campbell and Shiller (1988a) suggest a loglinear approximation of equation (1) and write it as:

$$\begin{aligned} r_{t+1} &= \log(1 + R_{t+1}) = \log(P_{t+1} + D_{t+1}) - \log(P_t) \\ &= p_{t+1} - p_t + \log[1 + \exp(d_{t+1} - p_{t+1})] \end{aligned} \tag{7}$$

where, lower case letters represent the natural logarithm of a variable. Equation (7), a nonlinear function of the log dividend-price ratio, can be approximated around the mean by a first-order Taylor expansion:

$$r_{t+1} \approx \alpha + \lambda p_{t+1} + (1 - \lambda) d_{t+1} - p_t \tag{8}$$

where, α and λ are parameters.

Equation (8) is a linear difference equation for the log stock price. Solving forward and imposing the no rational bubble terminal condition:

$$\lim_{j \to \infty} \lambda^j p_{t+j} = 0 \tag{9}$$

we obtain:

$$p_t = \frac{\alpha}{1 - \lambda} + \sum_{j=0}^{\infty} \lambda^j \left[(1 - \lambda) d_{t+1+j} - r_{t+1+j}\right] \tag{10}$$

Finally, taking the mathematical expectation of (10) based on information available at time t and rearranging in terms of the log dividend-price ratio we obtain:

$$d_t - p_t = -\frac{\alpha}{1 - \lambda} + E_t \left[\sum_{j=0}^{\infty} \lambda^j \left[-\Delta d_{t+1+j} + r_{t+1+j}\right] \right] \tag{11}$$

Craine (1993), points out that if the dividend growth factor (Δd_t) and the log of stock returns (r_t) are stationary stochastic processes, then the log dividend-price ratio - log dividend yield - is a stationary stochastic process under the no rational bubble restriction. On the contrary, the presence of a unit root in the log dividend yield is consistent with rational bubbles in stock prices. Alternatively, one could test for cointegration between the log dividend and log price with the cointegrating vector restricted to (1, -1). The test is very robust as it does not rely on a constant discount factor and the evidence on dividend growth and stock returns seems to suggest that they are stationary stochastic processes.

3 Fractional Integration and Cointegration

A time series is defined as integrated of order d, denoted as $I(d)$, when applying the differencing operator $(1-L)^d$ renders it a stationary, invertible autoregressive moving average ($ARMA$) process. When d is not an integer, the series is said to be fractionally integrated. In the latter case the series is represented by an AutoRegressive Fractionally Integrated Moving Average ($ARFIMA$) model:

$$\Phi(L)(1-L)^d(y_t-\mu_t)=\Theta(L)\epsilon_t \tag{12}$$

where μ_t is the mean of y_t and L is the lag operator ($L^k y_t = y_{t-k}$). $\Phi(L) = 1-\sum_{i=1}^{p}\phi_i L^i$ and $\Theta(L)=1+\sum_{i=1}^{q}\theta_i L^i$ represent stationary autoregressive and moving average components respectively. Further, ϵ_t has an unconditional $N(0,\sigma^2)$ distribution, and d can take noninteger values. The fractional differencing operator is defined by

$$(1-L)^d = 1-dL-\frac{d(1-d)}{2!}L^2-\frac{d(1-d)(2-d)}{3!}L^3-\ldots \tag{13}$$

When in $d=0$ in equation (12), $z_t = y_t-\mu_t$ is a stationary $ARMA$ (p,q) process with constant mean and variance over time. Stationary series are mean reverting and have short memory since their autocorrelations decay geometrically. When $d=1$, z_t is a nonstationary process containing a unit root, $ARIMA(p,1,q)$. The effects of a shock persist undiminished in each period and accumulate over time, hence such series are known as integrated. Integrated processes have theoretically infinite variances, exhibit long stochastic swings, and are not mean reverting. On the other hand, fractionally integrated processes ($0 < d < 1$) are mean reverting and exhibit long but finite memory. The autocorrelations of an *ARFIMA* model die out at a slower rate than an *ARMA* model.

The *ARFIMA*(p, d, q) process is covariance stationary if $d < 0.5$. Assuming that $-0.5 < d < 0.5$ and $d \neq 0$ Hosking (1981) has shown that the autocorrelation function of an *ARFIMA* process is proportional to j^{2d-1} as $j \to \infty$. Consequently, the autocorrelations of an *ARFIMA* process decay hyperbolically to zero as $j \to \infty$. This is in contrast to the faster - geometric - decay of a stationary *ARMA* process. For $-0.5 < d < 0$ the process is called intermediate memory or "overdifferenced".

The ϵ_t term in equation (12) is the innovation in the process. There is considerable empirical evidence that the assumption of constant variance is inappropriate for many financial time series. In many cases, volatility has to be modelled as time varying. Engle (1982) defined as an Autoregressive Conditional Heteroscedastic (*ARCH*) process, all ϵ_t of the form:

$$\epsilon_t = \eta_t \sigma_t \tag{14}$$

where η_t is an independently and identically distributed (i.i.d.) process with $E(\eta_t) = 0$ and $Var(\eta_t) = 1$. Clearly, ϵ_t is serially uncorrelated with a mean equal to zero, but its conditional variance equals σ_t^2 and may change over time.

A variety of *ARCH* models exist in the literature the main difference among them being the functional form of σ_t^2. The conditional variance in the Engle (1982) formulation is a distributed lag of past squared innovations:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{15}$$

The *ARCH* model can describe volatility clustering. The conditional variance of ϵ_t is an increasing function of the shock that occurred in period $t-1$. A large absolute value in ϵ_{t-1} implies that σ_t^2 and ϵ_t (in absolute value) are expected to be large. As a way to model persistent movements in volatility without estimating a very large number of coefficients in a high order *ARCH* process, Bollerslev (1986) suggested the Generalized Autoregressive Conditionally Heteroskedastic, or *GARCH*, model:

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{16}$$

The exponential *GARCH* or *EGARCH* model of Nelson (1990) belongs to the family of absolute value *GARCH* models that make forecasts of future standard deviation linear in current and past standard deviations and have absolute values of ϵ_t driving revisions in the forecasts. Such models can accommodate asymmetric effects of negative and positive shocks. Negative innovations to stock returns have been found to increase volatility more than positive innovations of the same magnitude. The *EGARCH* model is written as

$$\log(\sigma_t) = \omega + \beta \log(\sigma_{t-1}) + \alpha[|\epsilon_t| - c\epsilon_t] \tag{17}$$

Testing for a fixed order of $I(d)$ in the context of *ARFIMA* models using a "full information" parametric analysis has some distinct advantages. Null-limit distributions are chi-squared and do not depend on the presence or absence of possibly breaking trends in μ_t, either in the data generating process or in the testing model. This is not the case in *AR* unit-root tests and stationarity tests such as the *KPSS*.

4 Estimation Results

The data set used in the estimation consists of the Standard and Poor's 500 composite stock market index and corresponding dividend yields. The sample spans the period 1871 to 2000. The annual data were obtained from Global Financial Data, Web site: http://www.globalfindata.com/. Primary sources for the data are the Cowles Commission (from 1871 through 1925) and the S&P composite (from 1926 through 2000). The monthly data set is posted at Robert Shiller's Web site: http://www.econ.yale.edu/.

Figure 1 plots the log of dividend and the log price of the S&P 500 annual dataset. The log of the dividend-price ratio for the same index is presented in the lower part of the same graph. There is no apparent trend in the log of the dividend yield during the sample period 1872-1988 used by Craine (1993). The sharp decline in the ratio that occurred during the last decade is due primarily to the rapid rise in stock prices.

The slow decay of the sample autocorrelation function presented in Figure 2 suggests high persistence in the log dividend-price ratio. The spectral density function, presented in the same Figure, has a peak at frequency zero suggesting that low frequency (long periodicity) cycles play a prominent role in explaining the variance of this series.

Most previous studies in this area have applied tests for integer orders of integration to the log dividend yield or have tested for integer cointegration between stock dividends and prices. As argued earlier, a nonstationary log dividend yield is consistent with the existence of rational bubbles, while stationarity implies that deviations from market fundamentals are short lived. The intermediate case of fractional integration, implying persistent deviations from market fundamentals but eventual return to them, is the main focus of this paper. However, in the interest of comparability with previous work I present some evidence on integer orders of integration in Table 1.

To investigate the order of integration of the variables, I test for unit roots using two alternative testing procedures to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root. Columns 3-5 in Table 1 report t-statistics for the augmented Dickey-Fuller (ADF) test — see Dickey and Fuller (1981).[1] The reported t-statistics are based on regressions with the following deterministic components: a constant and a linear trend $(\widehat{t^{\tau}})$, a constant only $(\widehat{t^{\mu}})$, and no deterministic components $(\widehat{t})$. The null hypothesis of a unit root in the series can be rejected if the reported t-statistic is less than its critical value displayed in the last two rows of Table 1.

The number of lagged differences of a series that is included in the ADF regressions to eliminate autocorrelation was chosen by the Akaike Information Criterion (AIC) and is reported in columns 6-8 of Table 1.

It is important to note that in the ADF test, the unit root is the null hypothesis to be tested and that the way in which classical hypothesis testing is carried out ensures that the null hypothesis is hard to reject. Kwiatkowski *et al.* (1992) argue that such unit root tests often fail to reject a unit root because they have low power against relevant alternatives, such as, fractionally integrated series. They propose tests, known as the $KPSS$ tests[2], of

[1] In particular, the null hypothesis of a unit root was tested using the following ADF unit root regression equation

$$\Delta y_t = \alpha_0 + \alpha y_{t-1} + \beta t + \sum_{j=1}^{k} c_j \Delta y_{t-j} + e_t,$$

where e_t is white noise, k the optimal lag length, and t a time trend. In terms of this equation, testing the unit root null involves testing the null that $\alpha = 0$.

[2] The null hypothesis of trend stationarity in y_t is tested by calculating the test statistic

$$\widehat{\eta}_{\tau} = (1/T^2) \sum_{t=1}^{T} S_t^2 / \widehat{\sigma}_k^2,$$

the hypothesis of stationarity against the alternative of a unit root. They argue that such tests should complement unit root tests. By testing both the unit-root and the stationarity hypotheses, one can distinguish between series that appear to be integrated, series that appear to be stationary, and series that are not very informative about whether or not they are stationary or have a unit root. Tests for the $I(0)$ hypothesis, such as the $KPSS$, have power against fractional alternatives, see Lee and Schmidt (1996). $KPSS$ tests for trend and level stationarity are presented in columns 9-10 of Table 1.

At the annual frequency, the ADF test can reject the unit-root null for the dividend growth and the log market return even at the 10 percent level of significance. The $KPSS$ cannot reject the null hypothesis of stationarity for the same series at the 5 percent level of significance. Under these conditions, the no-rational-bubble restriction implies stationarity of the log dividend yield. Evidence of a unit root in $d_t - p_t$ would be consistent with the presence of rational bubbles in stock prices.

The test results for the log dividend yield are not clear cut. For the 1871-1988 sample used by Craine (1993), the ADF test cannot reject the unit-root hypothesis at the 5 percent level of significance for detrended data, but it yields rejections for the remaining specifications of the deterministic components. Similarly, the $KPSS$ cannot reject stationarity in the log dividend-price ratio at the 5 percent level but it can at the 10 percent level of significance.

When the whole available sample (1871-2000) is used, the ADF test is unable to reject the unit-root null, even at the 10 percent level of significance. In addition, the $KPSS$ yields rejections of stationarity, even at the 5 percent level of significance. Clearly, the last 14 years of data have led to a substantial

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where $S_t = \sum e_i$, $t = 1, 2, \ldots, T$, e_t are the residuals from the regression of y_t on an intercept and a time trend, and $\widehat{\sigma}_k^2$ is a consistent estimate of the long-run variance of the errors of the above regression calculated using the Newey and West (1987) method as

$$\widehat{\sigma}_k^2 = \frac{1}{T}\sum_{t=1}^{T} e_t^2 + \frac{2}{T}\sum_{s=1}^{T} b(s,k) \sum_{t=s+1}^{T} e_t e_{t-s},$$

where T is the number of observations, $b(s,k) = 1 - s/(1+k)$ is a weighting function and k is the lag truncation parameter.

The null hypothesis of level stationarity is defined in exactly the same way, except that e are the residuals from a regression of y_t on a constant only. The corresponding test statistic is $\widehat{\eta}_\mu$.

increase in shock persistence.

Use of monthly data indicates an even further increase in persistence as documented in the lower part of Table 1. The unit-root null in the log dividend yield cannot be rejected for any choice of deterministic components even at the 10 percent level of significance. In addition the *KPSS* leads to strong rejections of the null of stationarity.

Horvath and Watson (1995) have generalized Vector-Error-Correction-Model-based tests for cointegration, proposed by Johansen (1988), to allow for known cointegrating vectors under both the null and alternative hypotheses. They report substantial power gains from these new methods. In a study of forward and spot exchange rates, they find cointegration in all currencies studied using tests that impose a cointegrating vector of (1 -1), while cointegration was found in only half the cases when the restriction was not imposed. Table 2 reports the results of Horvath and Watson cointegration tests between the log dividend (d) and the log price (p). This procedure tests the alternative of a known cointegrating vector - in this case (1 -1) - against the null of no cointegration.

The reported statistics are based on VARs with deterministic components and number of lags as indicated. The statistics are in all cases less than their critical values shown at the bottom of Table 2. Thus, there is no evidence of cointegration between the log dividend and the log price when the full available sample is used.

As the unit-root and cointegration tests presented so far allow for only integer orders of integration, the log dividend yield was checked for a fractional exponent in the differencing process using the Exact Maximum Likelihood (*EML*), Non-linear Least Squares (*NLS*), and Modified Profile Likelihood (*MPL*) estimators.[3] The *ARMA* part of these models was chosen using the *AIC* as was the case for integer unit-root tests. The results are reported in Tables 3-5. The unit-root hypothesis is tested by imposing the restriction $d = 1$ while the stationarity hypothesis is tested by imposing the restriction $d = 0$. The reported χ^2-statistics are followed by p-values in brackets. The p-values indicate the probability of a false rejection of the linear restriction. Thus, p-values that are less than a chosen level of significance indicate rejection of the relevant linear restriction. To be able to judge model adequacy we report residual tests for normality, *ARCH* effects, and autocorrelation.

The results for the sample period used in Craine (1993) are reported in Table 3. The χ^2-statistics reject both the null of stationarity and the unit-root null even at the 1 percent level of significance. The results are robust with respect to the choice of parametric estimator for the fractional differencing parameter. The residual tests cannot reject normality and the absence of *ARCH* effects at the 5 percent level of significance. In addition, the Portmanteau test does not detect autocorrelation. However, *ARCH* effects cannot be rejected at 10 percent level of significance for the models estimated by *MPL* and *NLS*.

Modelling the residuals as an *EGARCH* process preserves mean reversion in the log dividend yield (i.e. $d = 1$ is rejected). Thus, the evidence presented in Table 3 suggests that the log dividend yield is a fractionally integrated process. Unlike previous studies that have tested for integer orders of integration and cointegration we are able to obtain robust rejections of a unit root in the log dividend yield.

The results reported in Table 4, based on the whole available sample, suggest an increase in persistence signaled by higher point estimates of the fractional differencing parameter. Estimation by *MPL* failed when applied directly to the log dividend yield. The reported estimate of d was obtained, in this case, through the use of first-differenced data. The reported value

[3] For a detailed description of the different estimators of the fractional differencing parameter in the ARFIMA models see Doornik and Ooms (2001). In every case the residuals were checked for normality and ARCH effects and, when necessary, ARFIMA models incorporating ARCH effects were estimated using a package by Laurent and Peters (2002) that is based on the programming language Ox.

for d is $1 + \widehat{d_\Delta}$. Consequently, we can no longer identify a value for the mean of the log dividend yield: the constant drops out of the regressor set. It is evident that we can still obtain strong rejections of the unit-root and stationarity null hypotheses.

The residual tests are unable to reject the normality hypothesis and indicate lack of serial correlation. However, there is evidence of a strong $ARCH$ effect. To address this issue, we modelled the variance as an $EGARCH$ process. In spite of the rise in the point estimate of d we were still able to obtain strong rejections of the unit-root hypothesis. The $EGARCH$ residuals appear to be free of non-normality, $ARCH$ effects, and autocorrelation.

The results for the log dividend yield of the S&P 500 at monthly frequency are reported in Table 5. The $ARMA$ part of the $ARFIMA$ model is an $AR(4)$ $MA(0)$ process. The unit-root and the stationarity hypotheses are soundly rejected in favour of long memory in the series. However, the residuals exhibit strong non-normality and $ARCH$ effects as well as evidence of autocorrelation.

Modelling volatility as an $EGARCH$ process removes the autocorrelation and $ARCH$ effects from the residuals but evidence of non-normality remains. Again in spite of the rise in the point estimate of d we obtain strong rejections of the unit root and stationarity hypotheses in the log dividend yield.

5 Bootstrap Inference

Hypothesis tests and confidence intervals based on asymptotic theory can be misleading in finite samples. For example, the EML estimator of the $ARFIMA$ model can be severely biased, and empirical confidence levels for Wald tests may deviate substantially from the nominal levels - see Hauser (1999). In what follows, we use parametric bootstrap analysis to examine the relative performance of the estimators used in the previous section and check the robustness of inference in samples of size equal to the available samples.

Parametric bootstrap inference can be used to test for any value of d. In this case, we chose the parameter estimates of different estimation methods as the null hypothesis. This allows us to check the reliability of inference in small samples based on different estimators. Under each null hypothesis we generate 1000 pseudo-samples. The latter are exact drawings from completely specified $ARFIMA$ data generating processes with independent normal errors. For each pseudo-sample we compute parameter estimates and associated t-values for tests on the true (DGP) value.

Figures 5 and 6 summarize the bootstrap results for annual and monthly data respectively. The left-hand side of each graph presents the parametric bootstrap density of $\widehat{d}$ for each of the three estimators used in the paper. The right-hand side presents quantile-quantile (QQ) cross-plots of the sorted t-statistics against the quantiles of the Student-t distribution. The 45^o line is drawn for reference (the closer the cross plot to this line, the better the match).

The bootstrap results in Figure 5 clearly show the downward bias of the EML estimator for d. The bias in the estimators leads to a bias in the t-statistics which is reflected by the QQ plots. The MPL estimator is the least biased in the case of annual data followed by the NLS estimator. These observations are confirmed by the numerical results presented in Table 6. The ranking of the estimators is different in the case of monthly data as evidenced by Figure 6 and Table 7. NLS is, in this case, the least biased estimator of $\widehat{d}$ followed by EML and MPL.

MacKinnon (2002) argues that, in many cases, using simulated distributions to perform tests and construct confidence intervals yields inferences that are substantially more accurate than those based on asymptotic theory. I use parametric bootstrap inference to test for $d = 0.99$ for the log dividend yield in order to provide more evidence on the null of no mean reversion.

The simulated distributions are presented in Table 8 which provides critical values. It is clear that the probability of observing a realization of d, from a DGP where $d = 0.99$, equal to any of its estimated values is less than 0.01 in every case. Thus, bootstrap evidence strongly rejects the unit-root null. In addition, Table 8 provides critical values obtained by simulating the estimated models. It is clear that the probability of a realization of $d = 0$, or $d = 1$ from a DGP such as the estimated models is less than 0.01 in every case. We conclude that bootstrap inference strongly supports mean-reverting behaviour in the log dividend yield of the S&P 500 but also points to long memory.

The results reported in Table 9 confirm the above conclusions in the case of monthly S&P 500 data.

6 Concluding Remarks

This paper has employed fractional integration techniques combined with recent advances in volatility modelling to investigate the degree of persistence of the log dividend yield for the S&P 500. Contrary to previous literature that was based on tests for integer orders of integration or cointegration, we were able to obtain strong rejections of a unit-root restriction in the log dividend yield in favour of a fractional alternative.

The results reported in this paper are inconsistent with the presence of rational bubbles in stock market prices. The latter require nonstationary deviations from present-value prices. Our findings are robust to the choice of parametric estimator of the fractional differencing parameter and data frequency. Further, the results are invariant to modelling volatility as a time-varying process. There is evidence that the degree of shock persistence has increased in the 1990's. Further, the reported bootstrap inference fully supports the estimation results.

The empirical results reported in the paper are consistent with non-bubble hypotheses that can explain persistent but stationary deviations of asset prices from fundamentals. Stationary fads or noise trading can lead to departures from present value prices. Examples of models with fads or noise include Shiller (1984), Kyle (1985), Summers (1986), Campbell and Kyle (1993) and De Long et al. (1990).

Finally, the results are also consistent with the "intrinsic" rational bubble formulation of Froot and Obstfeld (1991). Unlike the traditional definition of rational bubbles employed in the present paper - where price increases are generated by extraneous events or rumors and perpetuated by self-fulfilling expectations - intrinsic bubbles are driven exclusively by the exogenous fundamentals. Intrinsic bubbles will remain constant over time for a given level of fundamentals as they are deterministic functions of fundamentals alone. This class of bubbles predicts that stable and highly persistent fundamentals lead to stable and highly persistent over- or undervaluations of asset prices.

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TABLE 1. TESTS FOR INTEGER ORDERS OF INTEGRATION

		ADF						*KPSS*	
Variable	Sample	$\widehat{t^{\tau}}$	$\widehat{t^{\mu}}$	$\widehat{t}$	*AIC* lags			$\widehat{\eta}^{\tau}$	$\widehat{\eta}^{\mu}$
		S&P 500 Annual Data							
Δd_t	1872-2000	−8.16	−7.99	−7.51	1	1	1	0.03	0.26
r_t	1872-2000	−9.85	−9.53	−1.06	1	1	2	0.03	0.36
$d_t - p_t$	1871-1988	−3.19	−3.08	−2.37	2	2	2	0.14	0.41
$d_t - p_t$	1871-2000	−1.43	−0.71	0.03	2	2	2	0.23	0.93
		S&P 500 Monthly Data							
Δd_t	1871:02-2000:12	−8.10	−7.92	−7.13	17	17	18	0.02	0.28
r_t	1871:02-2000:12	−7.53	−7.13	−2.19	18	18	17	0.22	0.84
$d_t - p_t$	1871:01-2000:12	−2.34	−1.37	−0.46	20	20	20	0.45	2.24
Crit. Val.									
5%		−3.43	−2.88	−1.95				0.146	0.463
10%		−3.13	−2.57	−1.62				0.119	0.347

NOTE: The null hypothesis in the augmented Dickey-Fuller (ADF) test is that the candidate series contains a unit root. The unit-root null can be rejected in favour of stationarity if the reported t-statistic is less than its critical value.

The null hypothesis in the $KPSS$ test is that of stationarity and can be rejected if the reported η- statistic is greater than its critical value. The reported $KPSS$ statistics are based on 4 lags for annual data and 24 lags for monthly data.

TABLE 2. HORVATH-WATSON TESTS FOR INTEGER COINTEGRATION

Variables	Sample	Lags	Constant and Trend	Constant
		S&P 500 Annual Data		
d and p	1871-1988	4	7.5	6.90
d and p	1871-2000	4	1.84	0.01
		S&P 500 Monthly Data		
d and p	1871-2000	24	2.00	0.69
Crit. Val.				
1%			13.73	13.73
5%			10.18	10.18
10%			8.30	8.30

NOTE: The null hypothesis is no cointegration and the alternative is a cointegrating vector (1 -1). The null can be rejected in favour of cointegration if the reported statistic is greater than its critical value.

TABLE 3. ESTIMATION RESULTS FOR THE LOG DIVIDEND YIELD

	S&P 500 Annual frequency 1876-1988			
	EML	*MPL*	*NLS*	*EML-EGARCH*
$\widehat{d}$	0.291 (0.089)	0.345 (0.105)	0.295 (0.093)	0.440 (0.093)
$\widehat{\vartheta_1}$	0.458 (0.121)	0.419 (0.137)	0.466 (0.121)	0.388 (0.089)
Constant	−0.150 (0.108)	−0.149 (0.150)	−0.189 (0.091)	−0.007 (0.183)
Cst(V)				−3.488 (0.164)
$GARCH(\widehat{\beta_1})$				0.327 (0.252)
$ARCH(\widehat{\alpha_1})$				−4.683 (6.721)
$EGARCH(\widehat{\theta_1})$				−0.132 (0.189)
$EGARCH(\widehat{\theta_2})$				−0.039 (0.081)
Tests for				
Normality	[0.277]	[0.241]	[0.172]	[0.451]
ARCH 1-1	[0.103]	[0.073]	[0.056]	[0.864]
Portmanteau	[0.487]	[0.474]	[0.298]	[0.211]
Linear Restrictions				
$d = 1$	[0.000] **	[0.000] **	[0.000] **	[0.000] **
$d = 0$	[0.001] **	[0.001] **	[0.001] **	[0.000] **

NOTE: The estimation sample is 1876-1988 ($T = 113$). All methods are applied directly to the logarithm of the dividend-price ratio.

The numbers in parentheses are standard errors of the estimated parameters. The numbers in brackets are p-values. A p-value less than 0.05 rejects the relevant null hypothesis at the 5 percent significance level and is followed by an asterisk. A p-value less than 0.01 rejects the null at the 1 percent significance level and is denoted by two asterisks.

The null hypotheses in the residual tests are respectively, normal errors, no *ARCH* effects, no autocorrelation.

TABLE 4. ESTIMATION RESULTS FOR THE LOG DIVIDEND YIELD

S&P 500 Annual frequency 1871-2000				
	EML	*MPL*	*NLS*	*EML-EGARCH*
$\widehat{d}$	0.467 (0.040)	0.631 (0.116)	0.624 (0.115)	0.709 (0.076)
$\widehat{\vartheta_1}$	0.407 (0.093)	0.242 (0.171)	0.252 (0.168)	0.319 (0.086)
Constant	−0.296 (0.522)	−	−0.058 (0.323)	−0.005 (0.168)
Cst(V)				−3.473 (0.161)
$ARCH(\widehat{\alpha_1})$				12.836 (30.636)
$EGARCH(\widehat{\theta_1})$				0.036 (0.086)
$EGARCH(\widehat{\theta_2})$				0.053 (0.133)
Tests for				
Normality	[0.389]	[0.167]	[0.188]	[0.722]
ARCH 1-1	[0.002] **	[0.003] **	[0.003] **	[0.916]
Portmanteau	[0.165]	[0.171]	[0.180]	[0.261]
Linear Restrictions				
$d = 1$	[0.000] **	[0.001] **	[0.001] **	[0.000] **
$d = 0$	[0.000] **	[0.000] **	[0.000] **	[0.000] **

NOTE: The estimation sample is 1871-2000 ($T = 130$). *EML*, *NLS* are applied directly to the logarithm of the dividend-price ratio, *MPL* is applied to Δ(logarithm of the dividend-price ratio). The numbers in parentheses are standard errors of the estimated parameters.

The numbers in brackets are p-values. A p-value less than 0.05 rejects the relevant null hypothesis at the 5 percent significance level and is followed by an asterisk. A p-value less than 0.01 rejects the null at the 1 percent significance level and is denoted by two asterisks.

The null hypotheses in the residual tests are respectively, normal errors, no $ARCH$ effects, no autocorrelation.

TABLE 5. ESTIMATION RESULTS FOR THE LOG DIVIDEND YIELD

	S&P 500 Monthly frequency 1871:01-2000:12			
	EML	*MPL*	*NLS*	*EML-EGARCH*
$\widehat{d}$	0.376 (0.093)	0.452 (0.127)	0.402 (0.118)	0.726 (0.100)
$\widehat{\phi_1}$	0.961 (0.095)	0.887 (0.127)	0.936 (0.119)	0.627 (0.104)
$\widehat{\phi_2}$	−0.156 (0.056)	−0.123 (0.060)	−0.144 (0.062)	−0.058 (0.033)
$\widehat{\phi_3}$	0.003 (0.035)	0.002 (0.033)	0.002 (0.034)	0.072 (0.032)
$\widehat{\phi_4}$	0.115 (0.028)	0.124 (0.027)	0.120 (0.029)	0.085 (0.025)
Constant	−0.355 (0.264)	−0.371 (0.480)	−0.340 (0.199)	0.449 (0.598)
Cst(V)				−6.497 (0.085)
$GARCH(\widehat{\beta_1})$				0.915 (0.014)
$EGARCH(\widehat{\theta_1})$				0.130 (0.024)
$EGARCH(\widehat{\theta_2})$				0.256 (0.027)
Tests for				
Normality	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗
ARCH 1-1	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗	[0.445]
Portmanteau	[0.005] ∗∗	[0.004] ∗∗	[0.004] ∗∗	[0.177]
Linear Restrictions				
$d = 1$	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗
$d = 0$	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗	[0.000] ∗∗

NOTE: The estimation sample is 1871:01-2000:12 ($T = 1560$). All methods are applied directly to the logarithm of the dividend-price ratio. The numbers in parentheses are standard errors of the estimated parameters.

The numbers in brackets are p-values. A p-value less than 0.05 rejects the relevant null hypothesis at the 5 percent significance level and is followed by an asterisk. A p-value less than 0.01 rejects the null at the 1 percent significance level and is denoted by two asterisks.

The null hypotheses in the residual tests are respectively, normal errors, no *ARCH* effects, no autocorrelation.

TABLE 6. PARAMETRIC BOOTSTRAP ESTIMATOR RESULTS
Annual Data

	EML		MPL		NLS	
	mean bias	RMSE	mean bias	RMSE	mean bias	RMSE
$\widehat{d}$	-0.0887	0.112	-0.0068	0.117	-0.0428	0.116
$\widehat{\theta}_1$	0.0512	0.112	0.0019	0.128	0.0280	0.122
Constant	-0.0010	0.502	-0.0003	0.005	-0.0003	0.005

TABLE 7. PARAMETRIC BOOTSTRAP ESTIMATOR RESULTS
Monthly Data

	EML		MPL		NLS	
	mean bias	RMSE	mean bias	RMSE	mean bias	RMSE
$\widehat{d}$	-0.0536	0.096	-0.0697	0.105	-0.0147	0.118
$\widehat{\phi}_1$	0.0507	0.096	0.0662	0.105	0.0126	0.118
$\widehat{\phi}_2$	-0.0300	0.064	-0.0321	0.062	-0.0140	0.065
$\widehat{\phi}_3$	0.0030	0.036	0.0032	0.036	0.0024	0.0355
$\widehat{\phi}_4$	-0.0124	0.112	-0.0124	0.032	-0.0084	0.032
Constant	-0.0213	0.502	-0.0085	0.464	-0.0253	0.3415

TABLE 8. PARAMETRIC BOOTSTRAP CRITICAL VALUES
Annual Data

	EML		MPL		NLS	
	$\widehat{d}$=0.99	$\widehat{d}$=0.47	$\widehat{d}$=0.99	$\widehat{d}$=0.63	$\widehat{d}$=0.99	$\widehat{d}$=0.62
0.99	1.28	0.48	1.26	0.90	1.17	0.82
0.95	1.12	0.47	1.17	0.83	1.12	0.76
0.90	1.07	0.46	1.13	0.77	1.07	0.72
0.50	0.95	0.39	0.98	0.63	0.94	0.58
0.10	0.82	0.28	0.84	0.48	0.80	0.44
0.05	0.78	0.25	0.80	0.44	0.75	0.40
0.01	0.71	0.17	0.72	0.36	0.67	0.32

NOTE: The first column shows the percent of simulated d values that are smaller than the corresponding critical values in the Table.

TABLE 9. PARAMETRIC BOOTSTRAP CRITICAL VALUES
Monthly Data

	EML		MPL		NLS	
	$\widehat{d}$=0.99	$\widehat{d}$=0.38	$\widehat{d}$=0.99	$\widehat{d}$=0.45	$\widehat{d}$=0.99	$\widehat{d}$=0.40
0.99	1.42	0.46	1.43	0.49	1.53	0.73
0.95	1.18	0.43	1.27	0.48	1.32	0.58
0.90	1.12	0.42	1.18	0.47	1.15	0.53
0.50	0.98	0.33	1.00	0.40	0.99	0.38
0.10	0.85	0.22	0.86	0.28	0.86	0.25
0.05	0.82	0.18	0.83	0.24	0.82	0.20
0.01	0.78	0.11	0.76	0.15	0.77	0.13

NOTE: The first column shows the percent of simulated d values that are smaller than the corresponding critical values in the Table.

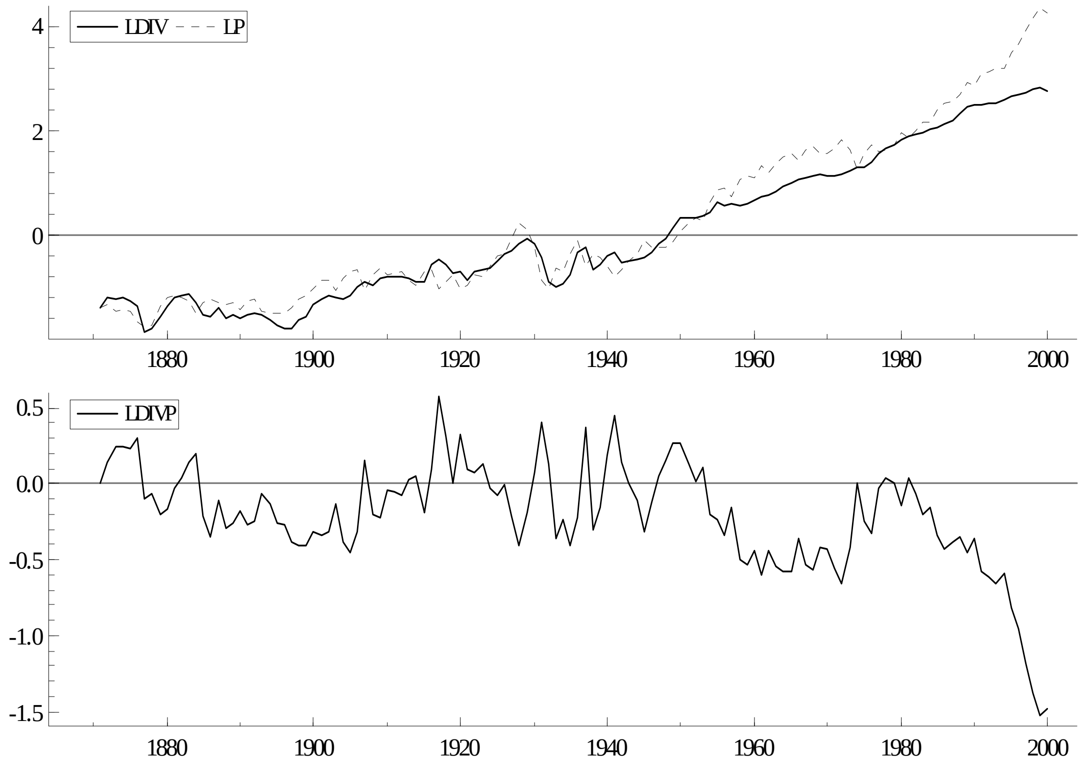

Figure 1: S&P500 Annual Data

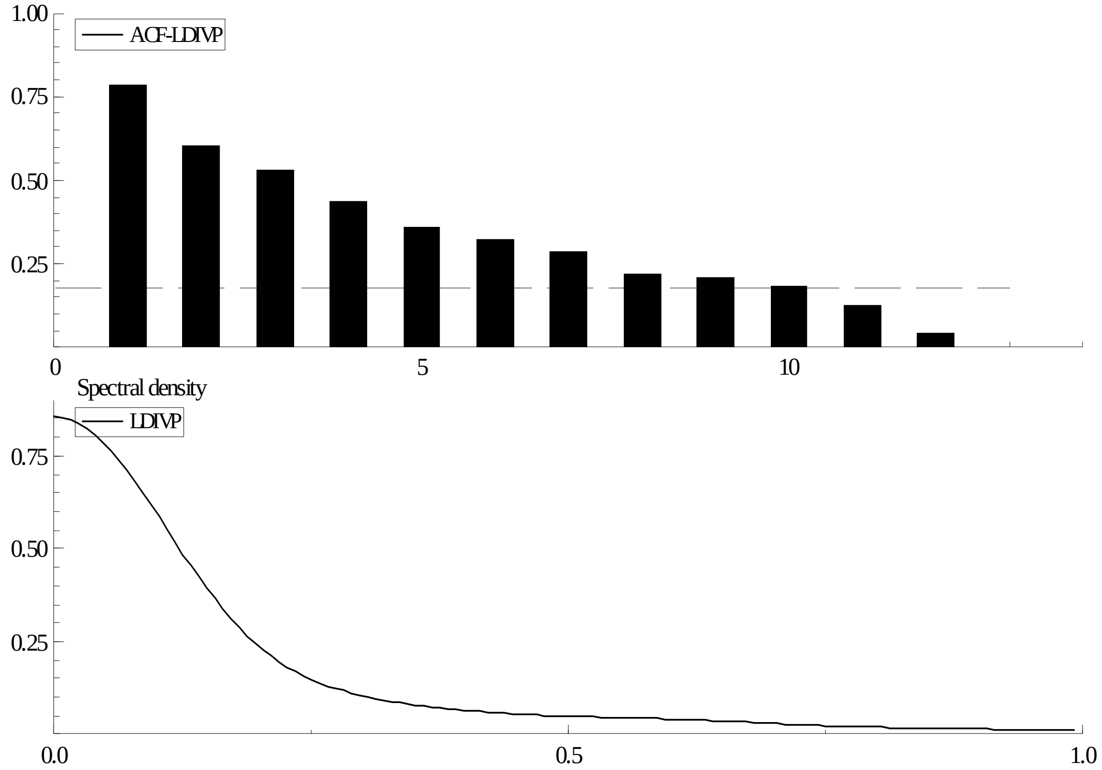

Figure 2: Sample Autocorrelation Functions and Spectral Densities of the S&P 500 Log Dividend-Price Ratio (Annual Data)

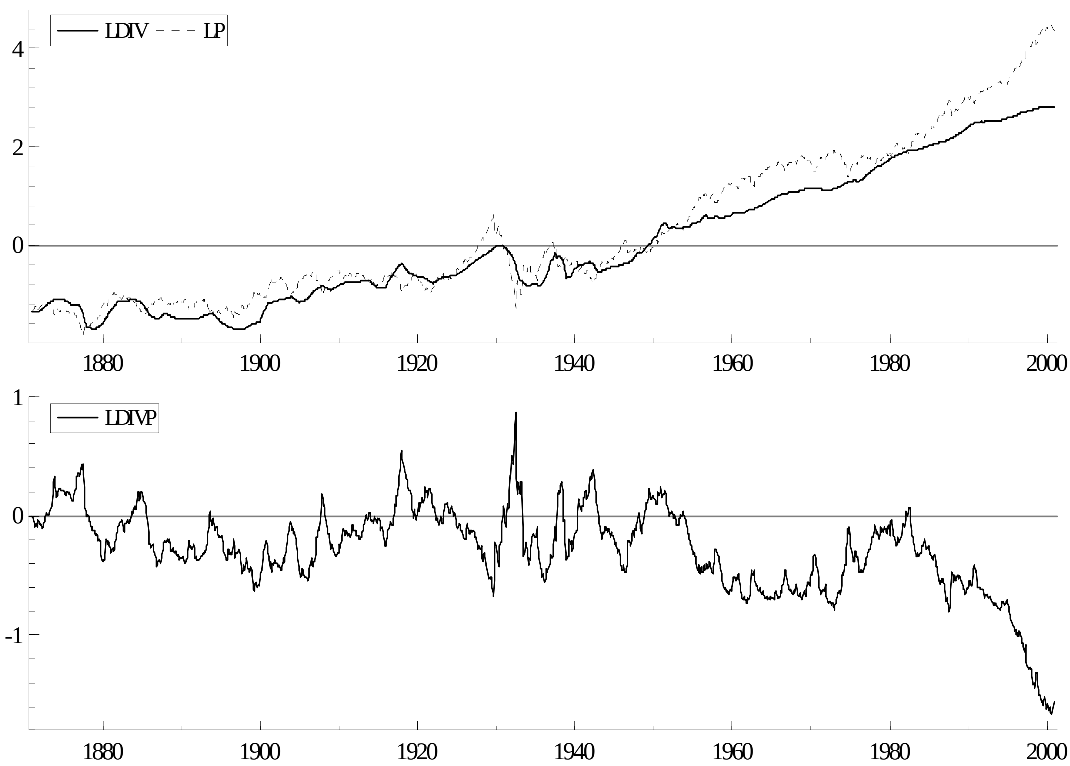

Figure 3: S&P 500 Monthly Data

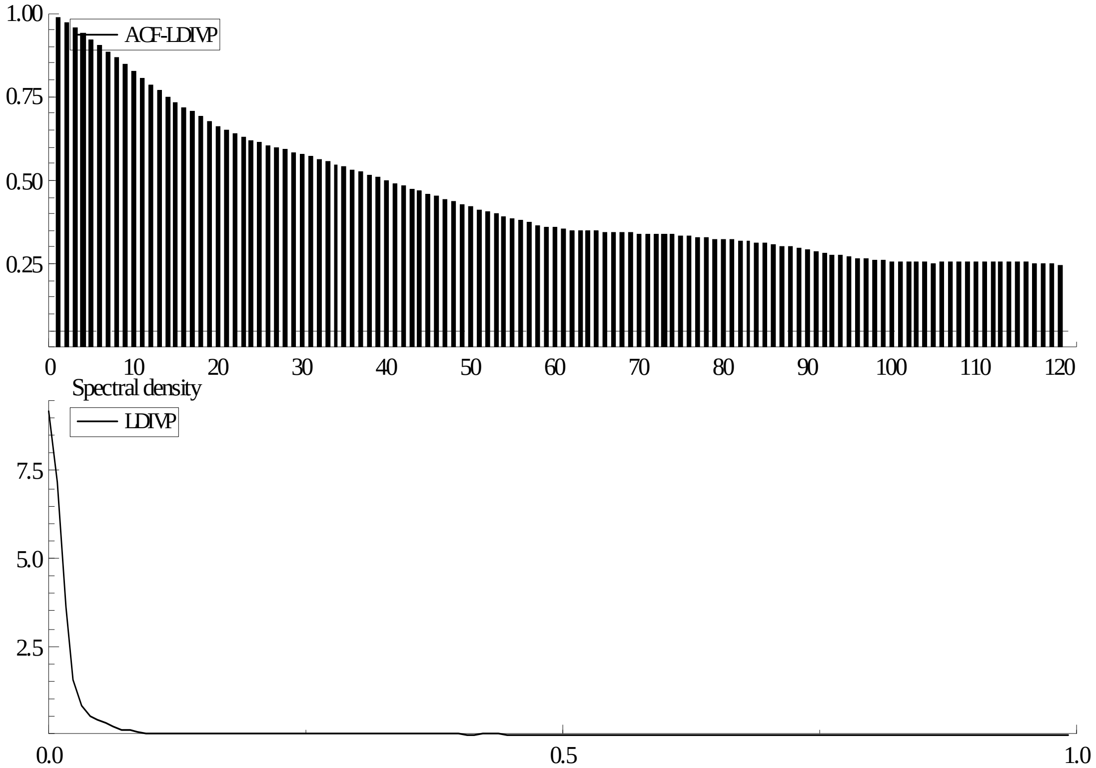

Figure 4: Sample Autocorrelation Functions and Spectral Densities of the S&P 500 Log Dividend-Price Ratio (Monthly Data)

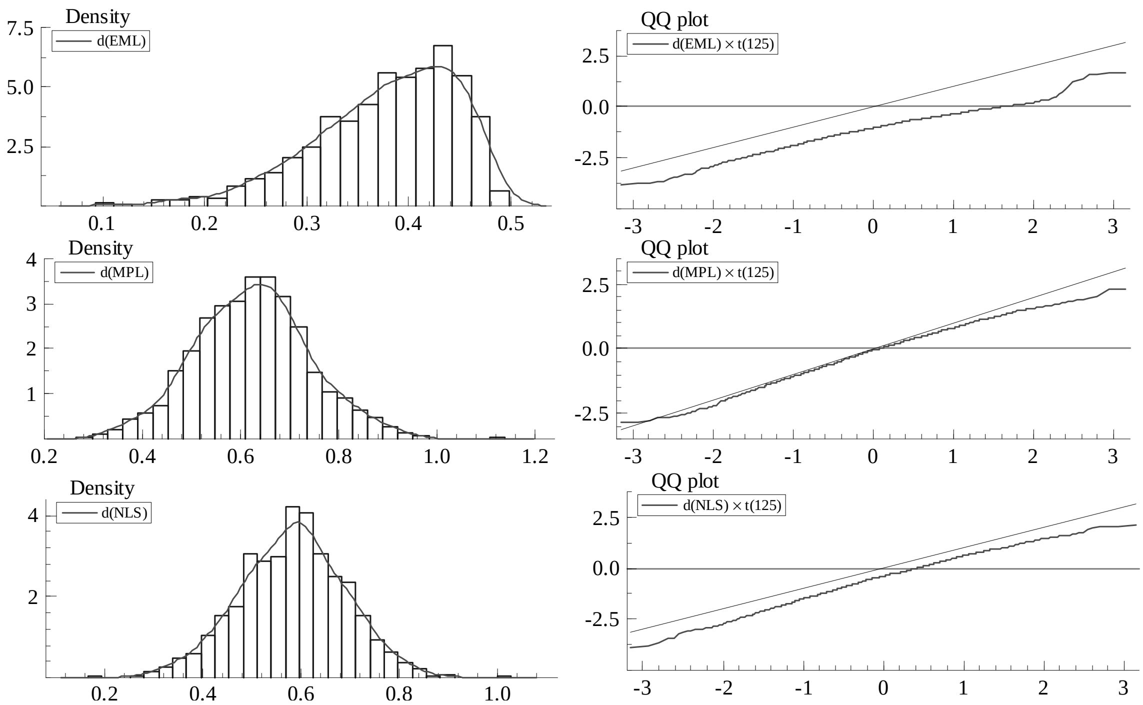

Figure 5: Estimates of Parametric Bootstrap Densities for $\widehat{d}$ and QQ-plots for the corresponding t-statistics (Annual Data).

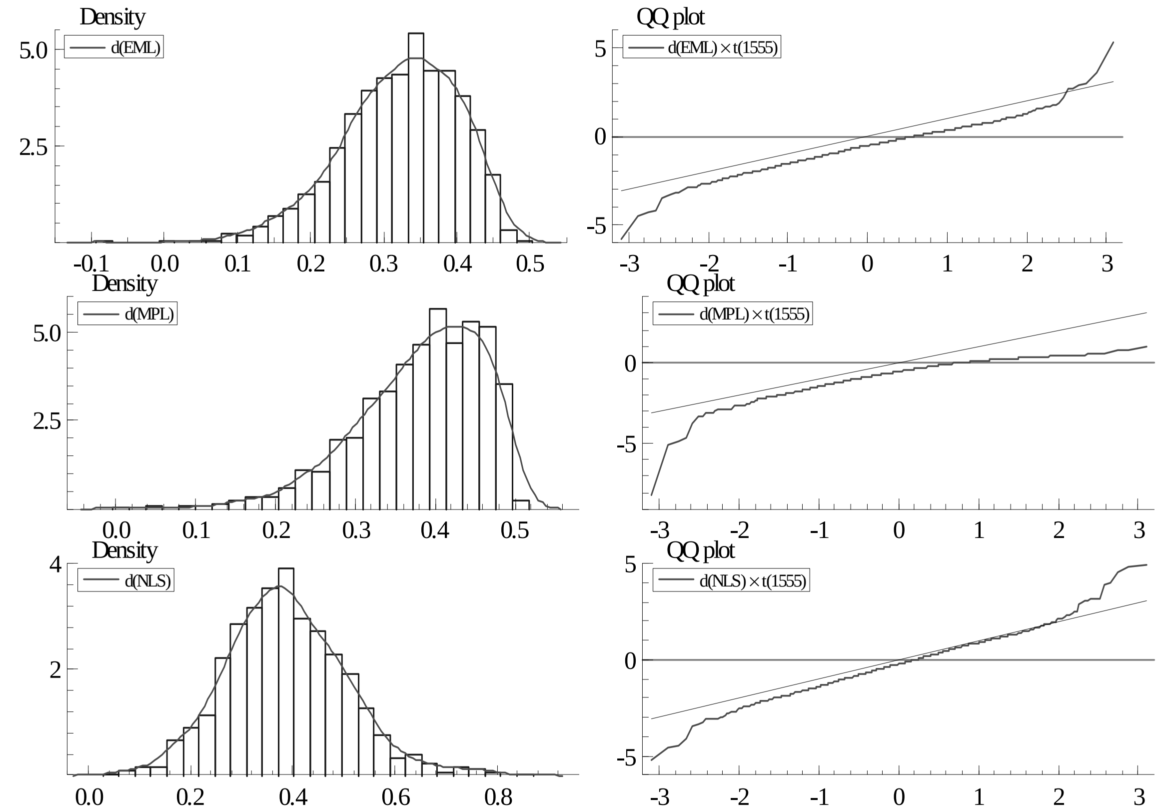

Figure 6: Estimates of Parametric Bootstrap Densities for $\widehat{d}$ and QQ-plots for the corresponding t-statistics (Monthly Data).