Exact Multivariate Structural Change Tests with Application to Energy Demand Models

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Introduction

- **Objective:** to propose finite sample tests for structural change in multivariate linear regressions [MLR] with applications to dynamic energy sources share equations.

- **Motivation:**

- Parameter constancy: an important issue in energy demand models [McAvinchey & Yannopoulos (2003), Arsenault, Bernard, Carr & Genest (1995)] $\oplus$ models are dynamic, data annual.

- Literature on multivariate break tests [e.g. Bai, Lumsdaine & Stock (1998)]: sparse compared to univariate.
  - A few available procedures [Cantrell, Burrows & Vuong (1991), Andrews (1993)] are sufficiently general to include MLR as special case.
  - Relevant results in:
    - statistics literature on multivariate outlier detection (typically no co-variates) [Hadi(1992), Caroni & Prescott (1992), Thode (2002), Qiu & Hawkins (2003)].
    - finance literature on event studies [Binder (1985a,b), Schipper& Thompson(1985)].

- Econometric, financial & statistics literature: disconnected. $\rightarrow$ In this paper, we adopt a unified testing approach.

- Exact multivariate break tests: available for a few special cases, which require normality (Hotelling-type) & not dynamic [Hooper & Zellner (1961), Dhrymes et al. (1972), Jayatissa & Farebrother (1977), Stewart (1997), Schipper & Thompson (1985).

- All other multivariate procedures are asymptotic, even in non-dynamic MLR.

- A number of studies [Dufour & Khalaf (2002) & ref. therein] have cast doubt on reliability of asymptotic MLR-based tests:

  - null distributions typically depend on error covariance parameters - whose number $\nearrow$ rapidly with dimension.
  - unknown break dates: dimensionality problems are compounded with multiple hypotheses concerns.

- Exact test solutions seem lacking even in univariate contexts [e.g. Dufour & Kiviet (1996. 1998)].

- Documented size control problems have motivated a number of bootstrap-based procedures [e.g. Christiano (1992), Diebold & Chen (1996), Banerjee, Lazarova & Urga (1998), Bekaert, Harvey & Lumsdaine (2002)].

Contribution

1. We extend tests for which finite-sample theory is available for Gaussian distributions [Schipper & Thompson (1985), Stewart (1997)] to non-Gaussian context.
2. We show that Bai, Lumsdaine, & Stock (1998)'s Sup-type test severely over-rejects and propose an exact variant of this test.
3. We consider predictive break test approaches which aim to generalize tests in Dufour (1980) and Dufour & Kiviet (1996) to MLR context.
4. We extend Wilks (1963)'s multivariate outlier test to the regression context and propose exact (non-Bonferonni based) versions of the test.
5. We apply our proposed procedures to the energy demand share equations estimated by Arsenault, Bernard, Carr & GenestLaplante (1995).

- Multivariate normality: not required

  → Error distribution specified up to:

    ▶ unknown scale matrix [Dufour & Khalaf (JE, 2002)]
    ▶ (possibly) a finite dimensional parameter.

- In Non-dynamic MLR:

  → Test statistics: ▶ nuisance parameters free null distributions

  → Monte Carlo test [Dufour (JE, 2006)] techniques ▶ yield exact tests

  with or without covariates..

- Given

    ▶ a Dynamic coefficient, or
    ▶ nuisance parameter in error distribution

  → Two-Stage

  consistent (or confidence set) set maximized Monte Carlo (CSMMC) procedure

  [Dufour (JE, 2006), Dufour & Kiviet (1996, 1998)]

Framework

$$Y = XB + U$$

- $Y \rightarrow T \times n$ matrix of observations on n dependent variables
- $X \rightarrow T \times k$ full-column rank matrix of fixed regressors
- $U \rightarrow T \times n$ matrix of error terms.
- Distributional assumption:▶ $U_t = JW_t, \quad t = 1, ..., T$ ◀

J unknown, **non-singular**
Distribution of $w = vec(W_1, \ldots, W_T)$ **known**
or **specified up to a finite dim. parameter** κ. Let

$$W = [W_1, \ldots, W_T]' = U\left(J^{-1}\right)'$$

$$\Sigma = JJ'.$$

- Special cases include:

$W_t \sim \mathcal{F}(\kappa_0), \; t = 1, \ldots, T$, [$\kappa_0$ specified]
$W_t \sim \mathcal{F}(\kappa), \; t = 1, \ldots, T$, [$\kappa$ unknown]
$\mathcal{F}(.)$: a known distribution

The energy demand model:

- Arsenault, Bernard, Carr & Genest (1995)'s Model: Three sectors: industrial, commercial and residential
- Energy sources are: fuel oil (O), natural gas (G), electricity (L) and coal (C).
- Market shares are modelled as:

$$\mathrm{MS}_{it} = \lambda\,\mathrm{MS}_{i,t-1} + X_t' B_i + U_{it}, \quad i = L, O, C, \quad t = 1, ..., T$$

$$\begin{aligned} B_L &= (b_L, b_{LL}, b_{LO}, b_{LC})' \\ B_O &= (b_O, b_{OL}, b_{OO}, b_{OC})' \\ B_C &= (b_C, b_{CL}, b_{CO}, b_{CC})' \end{aligned}$$

MS_{it} , and P_{it} : market share
& price of energy source $i = L$,O,C, for year t

$$X_t = \left(1, \ln\left[\frac{P_{Lt}}{P_{Gt}}\right], \ln\left[\frac{P_{Ot}}{P_{Gt}}\right], \ln\left[\frac{P_{Ct}}{P_{Gt}}\right]\right)'$$

U_{Lt}, U_{Ot}, U_{Ct}: Contemporaneously correlated error terms.

- λ: partial adjustment parameter $\rightarrow$ Same for all equations

- Case of **known** λ : ($\lambda = \lambda_0$):

  → Transform the model as:

  $$\mathrm{MS}_{it} - \lambda_0 \, \mathrm{MS}_{i,t-1} = X_t' B_i + U_{it}, \quad i = L, O, C, \quad t = 1, ..., T$$

  → Multivariate linear regression (MLR) model.

- Empirical results focus on:

$$\begin{aligned} W_t &\sim N[0,\, I_n] \Leftrightarrow U_t \sim N[0,\, \Sigma] \\ W_t &\sim \text{multivariate t}(\kappa_0): \quad [\kappa_0 \text{ specified}] \\ W_t &\sim \text{multivariate t}(\kappa): \quad [\kappa \text{ unknown}] \\ t &= 1, \ldots, T\,. \end{aligned}$$

  → Unknown λ, and/or Unknown κ : Maximized Monte Carlo [MMC] test techniques.

Exact version of BLS test

- Trimming period T_*
- Augment the model with multiplicative dummy variables:

$$Y = XB + D_s\Delta + U = Z_s\Theta + U$$

$$Z_s = \begin{bmatrix} X & D_s \end{bmatrix}, \quad \Theta = \begin{bmatrix} B \\ \Delta \end{bmatrix}$$

t-th row of D_s is $D_{ts}\overline{X}_t'$

$$\begin{aligned} D_{ts} &= 1, \;\; t > s, \\ &= 0, \;\; t \leq s. \end{aligned}$$

$$\overline{X} = XQ_X = [\overline{X}_1, \; ..., \overline{X}_T]'$$

$Q_X : K \times q_X$ regressor selection matrix

- The null hypothesis

$$H_{0s}^* : \qquad R^*\Theta = 0, \quad R^* = \begin{bmatrix} \mathbf{0}_{q_X \times K} & I_{q_X} \end{bmatrix}$$

$\longrightarrow$ The (Gaussian) Quasi Likelihood Ratio [QLR] criterion

$$-T\ln(\Lambda_s^*) \;=\; -T\ln\left(\left|\widehat{U}_s^{*\prime}\widehat{U}_s^*\right| \,/\, \left|\widehat{U}^{0\prime}\widehat{U}^0\right|\right)$$

$$\widehat{U}^0 \;:\; \text{OLS residual under } H_{0s}^*$$

$$\widehat{U}_s^* \;:\; \text{Unconstrained OLS residual}$$

$\longrightarrow$ Pivotality result

$$\boxed{-T\ln(\Lambda_s^*) = -T\ln\left(\frac{\left|W'\left(I-Z_s\left(Z_s'Z_s\right)^{-1}Z_s'\right)W\right|}{\left|W'\left(I-X(X'X)^{-1}X'\right)W\right|}\right)}$$

$\longrightarrow$ Joint null hypothesis: $\boxed{H_0^* \Longleftrightarrow \cap_{s\,\in\,[T_*+1,\;T-T_*-1]}\; H_{0s}^*}$

$\longrightarrow$ Sup-type statistic:

$$\boxed{\Lambda^* = \sup_{s\,\in\,[T_*+1,\;T-T_*-1]}\left\{-T\ln\left(\Lambda_s^*\right)\right\}}$$

Predictive test procedures

▶ Structural change after T_1

- Dummy variables augmented model

$$Y_{it} = X_t' B_i + \sum_{s=T_1+1}^{T} d_{ts}\gamma_{is} + U_{it}, \quad d_{ts} = 1, \quad t = s,$$

- Matrix notation:

$$Y = XB + \overline{D}\Gamma + U = Z\Pi + U$$

$$\overline{D} = \begin{bmatrix} 0_{T_1 \times T_2} \\ I_{T_2} \end{bmatrix}, \; Z = \begin{bmatrix} X & \overline{D} \end{bmatrix} = \begin{bmatrix} X_{(1)} & 0 \\ X_{(2)} & I_{T_2} \end{bmatrix}, \; \Pi = \begin{bmatrix} B \\ \Gamma \end{bmatrix}$$

Stability: $H_0 : R\Pi = 0, \quad R = \begin{bmatrix} 0_{T_2 \times K} & I_{T_2} \end{bmatrix}$

- QLR criterion

$$\blacklozenge \; -T\ln(\Lambda) = -T\ln\left(\left|\widehat{U}'\widehat{U}\right| / \left|\widehat{U}^{0\prime}\widehat{U}^{0}\right|\right) \; \blacklozenge$$

$\widehat{U}^0$& $\widehat{U}$: Cont. & unconst. OLS residual

- Our Pivotality result

$$\blacklozenge \; -T\ln(\Lambda) = -T\ln\left(\frac{\left|W'\left(I - Z\left(Z'Z\right)^{-1}Z'\right)W\right|}{\left|W'\left(I - X(X'X)^{-1}X'\right)W\right|}\right) \; \blacklozenge$$

▶ Structural change at individual dates after T_1 (dating break)

- $H_{0s} : R_s \Pi = 0, \; R_s : s$-th row of R, $\quad s = T_1 + 1, ..., T$
- QLR criterion:

$$\blacklozenge \; -T \ln(\Lambda_s) = -T \ln\left(\left| \hat{U}' \hat{U} \right| / \left| \hat{U}_s^{0\prime} \hat{U}_s^0 \right| \right) \; \blacklozenge$$

$\hat{U}_s^0 \& \widehat{U}$: Cont. & unconst. OLS residual

- Our Pivotality result

$$\blacklozenge \; -T \ln(\Lambda_s) = -T \ln\left(\frac{\left| W'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) W \right|}{\left| W' M_{(s)} W \right|} \right) \; \blacklozenge$$

$$\begin{aligned} M_{(s)} &= I - Z\left(Z'Z\right)^{-1} Z' \\ &\quad + Z\left(Z'Z\right)^{-1} R_s' \left[R_s \left(Z'Z\right)^{-1} R_s' \right]^{-1} R_s \left(Z'Z\right)^{-1} Z' \end{aligned}$$

- Sup-type statistic: $\mathcal{J}_{[T_1+1, \; T]}$: subset of $[T_1 + 1, \; T]$

$$\Lambda_{\max} = \sup_{s \, \in \, \mathcal{J}_{[T_1+1, \; T]}} \left\{ -T \ln(\Lambda_s) \right\}$$

An Adaptation of Wilks' Outlier Test

- Dummy variables augmented model

$$\boxed{Y_{it} = X_t' B_i + d_{ts}\gamma_{is} + U_{it},} \qquad s \in [1, ..., T]$$

- Matrix notation

$$\boxed{Y = XB + d_s\gamma_s + U = X_{(d_s)}\Pi_s + U}$$

$$X_{(d_s)} = \begin{bmatrix} X & d_s \end{bmatrix}, \ \Pi_s = \begin{bmatrix} B_L & ... & B_C \\ \gamma_{L,s} & ... & \gamma_{C,s} \end{bmatrix}$$

- Null hypothesis:

$$\boxed{H_{0s}^{**} : R_s^{**}\Pi_s = 0, \ \ R_s^{**} = (0, ..., 0, 1)' : 1 \times (K+1) \text{ vector}}$$

- QLR criterion

$$\blacklozenge \boxed{-T\ln(\Lambda_s^{**}) = -T\ln\left(\left|\widehat{U}_s^{**\prime}\widehat{U}_s^{**}\right| / \left|\widehat{U}^{0\prime}\widehat{U}^{0}\right|\right)} \blacklozenge$$

$\widehat{U}^0$ & $\widehat{U}_s^{**}$: Cont. & unconst. OLS residual

- Our Pivotality result

$$\blacklozenge \boxed{-T\ln(\Lambda_s^{**}) = -T\ln\left(\frac{\left|W'M_{(s)}^{**}W\right|}{\left|W'\left(I - X(X'X)^{-1}X'\right)W\right|}\right)} \blacklozenge$$

$$M_{(s)}^{**} = I - X_{(d_s)}\left(X_{(d_s)}'X_{(d_s)}\right)^{-1}X_{(d_s)}'$$

- Wilks' (min)-type statistic:

$$\Lambda_{\max}^{**} = \sup_{s \, \in \, \mathcal{J}_{[1,T]}} \left\{-T\ln\left(\Lambda_s^{**}\right)\right\} \qquad \underline{\underline{\text{pivotal}}}$$

Extensions to dynamic MLR

- Monte Carlo (MC) exact p-values [Dufour (2006)]:
  - ▶ Given the pivotality results, the test statistics

$$\text{Bai, Lumsdaine \& Stock type:} \;\; \Lambda^{*} \;\; -T\ln(\Lambda^{*}_{s})$$
$$\text{Predictive type:} \; -T\ln(\Lambda) \;\; -T\ln(\Lambda_{s}) \;\; \Lambda_{\max}$$
$$\text{Oulier type:} \;\; \Lambda^{**}_{\max} \;\; \Lambda^{**}_{s}$$

may easily be simulated to obtain exact MC p-values
if draws from the distribution
of $W_1, \ldots, W_T$ are available.

- ▶ Consider any pivotal test statistic $\mathcal{T}$ and

  let $\mathcal{T}_0$ be the observed value of $\mathcal{T}$

  (a) Generate N draws from the distribution of W
  (b) These yield N simulated values of the test statistic $\mathcal{T}$
  (c) The exact MC p-value is calculated from the rank of the observed $\mathcal{T}_0$ relative to the simulated ones:

$$\hat{p}_N(\mathcal{T}_0) = \frac{N\hat{G}_N(\mathcal{T}_0) + 1}{N+1}$$

$$N\hat{G}_N(\mathcal{T}_0) \ : \ \text{number of simulated criteria} \geq \mathcal{T}_0$$

  ▶ The test defined by the critical region:

$$\boxed{\hat{p}_N(\mathcal{T}_0) \leq \alpha}$$

  has size α exactly, if the distribution of $W_1, ..., W_T$ is specified.

  ▶ Share equations system:

  our tests are exact conditional on λ and (eventually) κ.

- Maximized Monte Carlo [MMC] test [Dufour (2006)]:
  - ▶ Let $\hat{p}_N(\mathcal{T}_0|\lambda,\kappa)$: MC p-value conditional on λ, κ.
  - ▶ Replacing λ and κ by consistent point estimates leads to:

    parametric bootstrap-type $p-$ value

    "local" MC (LMC) $p-$value

    ▶▶ Asymptotically valid test
  - ▶ Maximizing $\hat{p}_N(\mathcal{T}_0|\lambda,\kappa)$ over a relevant set for λ and κ :

    Exact MC test procedure: MMC
  - ▶ Maximization restricted over a set estimate $CS_{\lambda,\kappa}(\alpha_1)$ of level $1-\alpha_1$:

$$p_N^{\sup}(\mathcal{T}_0) = \sup_{\lambda\ ,\ \kappa \in CS_{\lambda,\kappa}(\alpha_1)} \hat{p}_N(\mathcal{T}_0|\lambda,\kappa)$$

  - ▶ Test defined by the critical region:

$$\boxed{p_N^{\sup}(\mathcal{T}_0) \leq \alpha - \alpha_1}$$

    has exact level α

    Confidence set MMC (CSMMC) test .

Size and power properties

- MC simulations:

  → 1000 replications, and $N = 99$ for the MC tests

  → U_t : drawn as $N[0,\ I_n]$

  → nominal test size is 5%.

■ Size simulation study

- **Size experiment 1:**

  → Normal MLR

  → 2 regressors (intercept, trend variate or N$(0, 1)$)

  → $T_* = I\,(.10T)$.

- **Size experiment 2:**

  → Normal MLR where variables in X are the observed regressors from our empirical data set.

  → $T_* = I\,(.15T)$, $T = 38$.

Empirical Size of Bai, Lumsdaine and Stock's test - Design 1

	Trend Regressor				Normal Regressor			
	$n = 5$		$n = 10$		$n = 5$		$n = 10$	
T	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*
40	35.8	4.0	83.3	4.7	30.7	5.2	76.1	4.7
50	29.0	4.5	68.2	4.7	22.8	4.2	58.6	4.2
60	22.6	5.4	51.9	4.5	17.3	4.8	44.9	5.0
80	19.5	4.3	36.4	4.2	14.3	4.6	31.4	6.2
100	16.4	5.1	30.4	5.4	9.8	4.2	25.6	6.1
140	13.3	5.8	19.5	4.2	8.3	5.3	16.0	5.4
180	11.3	4.9	17.8	5.5	7.4	4.5	13.3	5.7

Empirical Size of Bai, Lumsdaine and Stock's test - Design 2

	Paper & Allied	Petroleum & Coal	Residential	Other
Λ^*	4.03	5.59	4.48	5.05
Asy.	50.51	24.42	23.12	54.05

- **Size experiment 3:**

$\rightarrow$ 2 regressors: Intercept and a N$(0,1)$ variate.

$\rightarrow$ $T_* = I\,(.10T)$

$\rightarrow$ Predictive tests :

$$T_1 = I\,(\tau T) + 1, \text{ for } \tau = .5$$
$$T_1 = I\,(\tau T)\,, \text{ for } \tau = .85, .95.$$

Size of proposed structural change tests

		$-\mathbf{T}\ln(\Lambda)$		$\mathbf{\Lambda}_{\max}$		$\mathbf{\Lambda}^*$	$\mathbf{\Lambda}^{**}_{\max}$
n	**T**	$\boldsymbol{\tau} = .5$	$\boldsymbol{\tau} = .95$	$\boldsymbol{\tau} = .5$	$\boldsymbol{\tau} = .95$		
3	**25**	**.055**	**.058**	**.049**	**.058**	**.057**	**.049**
	40	**.057**	**.049**	**.045**	**.050**	**.060**	**.050**
	80	**.057**	**.040**	**.048**	**.046**	**.062**	**.051**
10	**25**	**.033**	**.051**	**.044**	**.051**	**.052**	**.056**
	40	**.051**	**.048**	**.064**	**.048**	**.048**	**.061**
	80	**.057**	**.056**	**.053**	**.050**	**.048**	**.060**

Bai, Lumsdaine & Stock type : Λ^*, $-T\ln(\Lambda_s^*)$

Predictive type : $-T\ln(\Lambda)$, $-T\ln(\Lambda_s)$, $\Lambda_{\max}$

Oulier type : $\Lambda^{**}_{\max}$, Λ^{**}_s

■ Power simulation study

- SAME model as in size experiment 3
- We assess power against several break structures and timings:

$\rightarrow$ A permanent break in intercepts only: (Design 4)

$$\begin{aligned} B &= \left[(0,1,1)',...,(0,1,1)'\right] \text{ for } t = 1,...,T_0 \\ B &= \left[(\xi_0,1,1)',...,(\xi_0,1,1)'\right] \text{ for } t = T_0+1,...,T. \end{aligned}$$

$\rightarrow$ A permanent break in all regression coefficients: (Design 5)

$$B = \left[(1,1,1)',...,(1,1,1)'\right] \text{ for } t = 1,...,T_0$$
$$B = \left[(1+\xi_0,1+\xi_0,1+\xi_0)',...,(1+\xi_0,1+\xi_0,1+\xi_0)'\right]$$
$$\text{for } t = T_0+1, T_0+2,...,T$$

- → A transitory break in all regression coefficients, at the beginning of the sample (Design 6)

$$B = \left[(1,1,1)', ..., (1,1,1)'\right] \text{ for } t \neq [\frac{1}{8}T]+1$$

$$B = \left[(1+\xi_0, 1+\xi_0, 1+\xi_0)', ..., (1+\xi_0, 1+\xi_0, 1+\xi_0)'\right]$$

$$\text{for } t = [\frac{1}{8}T]+1.$$

→ A transitory break in all regression coefficients, at the end of the sample (Design 7).

$$B = \left[(1,1,1)', ..., (1,1,1)'\right] \text{ for } t \neq [\frac{3}{4}T]+2$$

$$B = \left[(1+\xi_0, 1+\xi_0, 1+\xi_0)', ..., (1+\xi_0, 1+\xi_0, 1+\xi_0)'\right]$$

$$\text{for } t = [\frac{3}{4}T]+2.$$

→ A transitory break in intercepts, at the beginning of the sample (Design 8).

$$B = \left[(0,1,1)', ..., (0,1,1)'\right] \text{ for } t \neq [\frac{1}{8}T]+1$$

$$B = \left[(\xi_0,1,1)', ..., (\xi_0,1,1)'\right] \text{ for } t = [\frac{1}{8}T]+1.$$

→ A transitory break in intercepts, at the end of the sample (Design 9).

$$B = \left[(0,1,1)', ..., (0,1,1)'\right] \text{ for } t \neq [\frac{3}{4}T]+2$$

$$B = \left[(\xi_0,1,1)', ..., (\xi_0,1,1)'\right] \text{ for } t = [\frac{3}{4}T]+2.$$

Power of multivariate structural change tests under a permanent break $(\mathbf{n=10})$

$\boldsymbol{\xi}_0$	$\mathbf{T}$	$\mathbf{T}_0$	$-\mathbf{T}\ln(\Lambda)$ $\boldsymbol{\tau}=.5$	$-\mathbf{T}\ln(\Lambda)$ $\boldsymbol{\tau}=.85$	$\boldsymbol{\Lambda}_{\max}$ $\boldsymbol{\tau}=.5$	$\boldsymbol{\Lambda}_{\max}$ $\boldsymbol{\tau}=.85$	$\boldsymbol{\Lambda}^*$	$\boldsymbol{\Lambda}^{**}_{\max}$
5	**40**	**21**	**.873**	**.035**	**.999**	**.039**	**1.00**	**.057**
		34	**.897**	**1.00**	**.990**	**1.00**	**1.00**	**.061**
		38	**.742**	**1.00**	**.940**	**1.00**	**.965**	**.339**
	80	**41**	**.996**	**.036**	**1.00**	**.041**	**1.00**	**.051**
		68	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**.067**
		76	**.994**	**1.00**	**1.00**	**1.00**	**1.00**	**.363**
.5	**40**	**21**	**.997**	**.023**	**1.00**	**.039**	**1.00**	**.048**
		34	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**.041**
		38	**1.00**	**1.00**	**1.00**	**1.00**	**.923**	**.270**
	80	**41**	**1.00**	**.006**	**1.00**	**.029**	**1.00**	**.043**
		68	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**.039**
		76	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**.557**

Bai, Lumsdaine & Stock type : Λ^*, $-T\ln(\Lambda^*_s)$

Predictive type : $-T\ln(\Lambda)$, $-T\ln(\Lambda_s)$, $\Lambda_{\max}$

Oulier type : $\Lambda^{**}_{\max}$, Λ^{**}_s

Power of multivariate structural change tests under a transitory break

$\boldsymbol{\xi}_0$	n	T	$-\mathbf{T}\ln(\Lambda)$	$\boldsymbol{\Lambda}_{\max}$	$\boldsymbol{\Lambda}^*$	$\boldsymbol{\Lambda}^{**}_{\max}$
			Design 6			
1	**3**	**25**	**.004**	**.018**	**.932**	**.990**
		40	**.001**	**.020**	**.902**	**1.00**
		80	**.002**	**.019**	**.066**	**1.00**
	10	**25**	**.033**	**.034**	**.867**	**.997**
		40	**.015**	**.047**	**.501**	**1.00**
		80	**.005**	**.034**	**.054**	**1.00**
			Design 8			
7	**3**	**25**	**.003**	**.017**	**1.00**	**1.00**
		40	**.001**	**.020**	**.877**	**1.00**
		80	**.003**	**.021**	**.062**	**1.00**
	10	**25**	**.032**	**.036**	**.992**	**1.00**
		40	**.015**	**.047**	**.488**	**1.00**
		80	**.006**	**.037**	**.055**	**1.00**

Bai, Lumsdaine & Stock type : Λ^*, $-T\ln(\Lambda^*_s)$

Predictive type : $-T\ln(\Lambda)$, $-T\ln(\Lambda_s)$, $\Lambda_{\max}$

Oulier type : $\Lambda^{**}_{\max}$, Λ^{**}_s

Power study: **Summary of results**

- Predictive tests as well as our modified Bai, Lumsdaine and Stock test detect permanent changes.
- Predictive tests loose power if one "overshoots" the stable sub-sample; if we under-estimate T_1, power does not suffer much.
- The sup-type outlier test $\Lambda^{**}_{\max}$ performs best when the number of outliers is small relative to the sample size. Overall, this test is most suited to detect one outlier occurring at the beginning of the sample.
- All our outliers and predictive tests display good power to detect an outlier occurring at the end of the sample, while our modified Bai, Lumsdaine and Stock Λ^{*} cannot detect such a break.
- In general, each test displays good power for break structures compatible with its construction. While this is not unexpected, we see that size corrections are not achieved at the expense of power.

Empirical Results

- Energy consumption annual data sets for the Province of Québec, 1962-2000.
- We apply break tests discussed in this paper $\oplus$ exact procedures proposed in Beaulieu, Dufour and Khalaf (2006), Dufour, Khalaf and Beaulieu (2003, 2006) to test

  multivariate normality,
  serial correlation & GARCH
  λ: a nuisance parameter

- For all tests conducted, we report two MC p-values;
  - LMC, based on point estimate for λ
  - CSMMC; underlying set estimates, 97.5% level $\rightarrow$ CSMMC p-value is $\leq \alpha \Rightarrow$ test significant at level $\alpha + 2.5\%$.
  - Tables report max p-values over narrowest CS. For rejection cases, we further provide max. p-values over widest CS.
  - Predictive tests: based on 1962-85 sub-sample; (precedes oil and natural gas price deregulation in Québec).

Estimated market share equations

	Energy sources	
	Electricity	Oil
1. Paper & Allied		
Constant	0.056 (3.14)	-0.002 (-0.15)
Lagged dep.	0.936 (46.25)	0.936 (46.25)
Elect. price	-0.037 (-2.14)	0.045 (2.91)
Oil price	0.045 (2.91)	-0.109 (-5.04)
Coal price	-0.018 (-1.21)	0.023 (1.64)
2. Primary Metal		
Constant	0.037 (2.78)	-0.007 (-0.86)
Lagged dep.	0.953 (34.23)	0.953 (34.23)
Elect. price	-0.021 (-1.77)	0.020 (1.82)
Oil price	0.020 (1.82)	-0.041 (-2.10)
Coal price	-0.001 (-0.16)	0.011 (0.90)
3. Petroleum & Coal		
Constant	-0.031 (-0.64)	-0.003 (-0.41)
Lagged dep	0.894 (17.88)	0.894 (17.88)
Elect. price	0.073 (1.58)	0.015 (2.27)
Oil price	0.015 (2.27)	-0.010 (-1.19)
Coal price	-	-

Estimated market share equations (cont)

	Energy sources	
	Electricity	**Oil**
4. Other		
Constant	0.042 (3.23)	0.042 (3.09)
Lagged dep.	0.812 (32.00)	0.812 (32.00)
Elect. price	-0.017 (-1.98)	0.078 (7.12)
Oil price	0.078 (7.12)	-0.130 (-6.43)
Coal price	-0.008 (-1.80)	0.0004 (0.07)
5. Residential		
Constant	0.077 (7.12)	-0.022 (-4.25)
Lagged dep.	0.934 (73.74)	0.934 (73.74)
Elect. price	-0.040 (-5.11)	0.044 (6.24)
Oil price	0.044 (6.24)	-0.049 (-6.05)
6. Commercial		
Constant	0.085 (4.29)	-0.010 (-0.95)
Lagged dep.	0.906 (43.61)	0.906 (43.61)
Elect. price	-0.033 (-2.54)	0.024 (2.45)
Oil price	0.024 (2.45)	-0.066 (-5.00)

Estimated market share equations (cont)

Explanatory variables	Energy sources
	Coal
1. Paper & Allied	
Constant	0.009 (0.71)
Lagged dep.	0.936 (46.25)
Elect. price	-0.018 (-1.21)
Oil price	0.023 (1.64)
Coal price	0.013 (0.65)
2. Primary Metal	
Constant	0.005 (0.73)
Lagged dep.	0.953 (34.23)
Elect. price	-0.001 (-0.16)
Oil price	0.011 (1.90)
Coal price	-0.011 (-0.70)
4. Other	
Constant	0.012 (2.08)
Lagged dep.	0.812 (32.00)
Elect. price	-0.008 (-1.80)
Oil price	0.0004 (0.07)
Coal price	-0.005 (-1.04)

Normality and diagnostic tests
***p*-values**

Sector	**Normality tests**			
	MSK		**CSK**	
	LMC	**MMC**	**LMC**	**MMC**
Residential	**.511**	**.658**	**.468**	**.940**
Commercial	**.300**	**.419**	**.094**	**.181**
Paper and Allied	**.001**	**.001**	**.001**	**.001**
Primary Metal	**.557**	**.589**	**.577**	**.657**
Petroleum & Coal	**.002**	**.011**	**.001**	**.005**
Other	**.030**	**.136**	**.009**	**.061**

Sector	**Diagnostic tests**			
	GARCH		*Var. Ratio*	
	LMC	**MMC**	**LMC**	**MMC**
Residential	**.189**	**.254**	**.037**	**.758**
Commercial	**.049**	**.801**	**.008**	**.304**
Paper and Allied	**.627**	**.659**	**.223**	**.261**
Primary Metal	**.742**	**.825**	**.063**	**.356**
Petroleum & Coal	**.260**	**.684**	**.061**	**.217**
Other	**.648**	**.779**	**.045**	**.053**

Confidence sets (97.5%) for dynamic coefficient

Nominal level: 97.5%	Residential	Commercial	Paper & Allied
Based on entire sample			
$\mathbf{CS}^{*}_{\lambda}(\boldsymbol{\alpha}_1)$	[.765 .999]	[.615 .999]	[.521 .999]
$\mathbf{CS}^{**}_{\lambda}(\boldsymbol{\alpha}_1)$	[.874 .999]	[.766 .999]	[.871 .999]
$\overline{CS}_{\lambda}(\boldsymbol{\alpha}_1)$	[.907 .961]	[.861 .951]	[.891 .981]
Based on stable subsample			
$\mathbf{CS}^{*}_{\lambda}(\boldsymbol{\alpha}_1)$	[.346 .999]	[.308 .999]	[.451 .999]
$\mathbf{CS}^{**}_{\lambda}(\boldsymbol{\alpha}_1)$	[.506 .999]	[.428 .999]	[.652 .999]
$\overline{CS}_{\lambda}(\boldsymbol{\alpha}_1)$	[.685 .959]	[.600 .999]	[.841 .999]

Nominal level: 97.5%	Primary Metal	Petroleum & Coal	Other
Based on entire sample			
$\mathbf{CS}^{*}_{\lambda}(\boldsymbol{\alpha}_1)$	[.719 .999]	[.566 .999]	[.461 .999]
$\mathbf{CS}^{**}_{\lambda}(\boldsymbol{\alpha}_1)$	[.765 .999]	[.953 .999]	[.723 .981]
$\overline{CS}_{\lambda}(\boldsymbol{\alpha}_1)$	[.874 .995]	[.782 .999]	[.756 .868]
Based on stable subsample			
$\mathbf{CS}^{*}_{\lambda}(\boldsymbol{\alpha}_1)$	[.001 .999]	[.462 .999]	[.174 .999]
$\mathbf{CS}^{**}_{\lambda}(\boldsymbol{\alpha}_1)$	[.001 .999]	[.582 .999]	[.370 .999]
$\overline{CS}_{\lambda}(\boldsymbol{\alpha}_1)$	[.638 .996]	[.645 .999]	[.691 .887]

- $\overline{CS}_{\lambda}(\alpha_1)$: asy. SURE-based Wald-type;
- $CS^{*}_{\lambda}(\alpha_1)$ & $CS^{**}_{\lambda}(\alpha_1)$: exact, obtained via a union-intersection approach to combine exact univariate procedures from Dufour & Kiviet (1996).

Structural change tests in the share equations system

	LMC	**MMC**
	Residential	
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda^*)$	**.814**	**.864**
$\hat{\mathbf{p}}_N^{\text{sup}}(-T\ln(\Lambda))$	**.508**	**.552**
Dates Λ_s **significant at 5%**	$\varnothing$	$\varnothing$
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda_{\max})$	**.246**	**.300**
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda_{\max}^{**})$	**.560**	**.758**
Dates Λ_s^{**} **significant at 5%**	$\varnothing$	$\varnothing$
asymptotic Sup-W	**35.40 (27.13)**	
	Primary Metal	
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda^*)$	**.628**	**.906**
$\hat{\mathbf{p}}_N^{\text{sup}}(-T\ln(\Lambda))$	**.712**	**.782**
Dates Λ_s **significant at 5%**	$\varnothing$	$\varnothing$
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda_{\max})$	**.138**	**.228**
$\hat{\mathbf{p}}_N^{\text{sup}}(\Lambda_{\max}^{**})$	**.762**	**.848**
Dates Λ_s^{**} **significant at 5%**	$\varnothing$	$\varnothing$
asymptotic Sup-W	**67.93 (47.81)**	

$\underline{\textit{Bai, Lumsdaine \& Stock type}:}\ \Lambda^*,\ -T\ln(\Lambda_s^*)$

$\underline{\textit{Predictive type}:}\ -T\ln(\Lambda),\ -T\ln(\Lambda_s),\ \Lambda_{\max}$

$\underline{\textit{Oulier type}:}\ \Lambda_{\max}^{**},\ \Lambda_s^{**}$

	LMC	MMC
	Commercial	
$\hat{\mathbf{p}}_N^{\sup}(\Lambda^*)$	**.076**	**.210**
$\hat{\mathbf{p}}_N^{\sup}(-T\ln(\Lambda))$	**.776**	**.844**
Dates Λ_s **significant at 5%**	$\varnothing$	$\varnothing$
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max})$	**.354**	**.596**
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max}^{**})$	**.316**	**.502**
Dates Λ_s^{**} **significant at 5%**	$\varnothing$	$\varnothing$
asymptotic Sup-W	**48.21 (27.13)**	
	Petroleum and Coal	
$\hat{\mathbf{p}}_N^{\sup}(\Lambda^*)$	**.070**	**.152**
$\hat{\mathbf{p}}_N^{\sup}(-T\ln(\Lambda))$	**1.00**	**1.00**
Dates Λ_s **significant at 5%**	$\varnothing$	$\varnothing$
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max})$	**.874**	**.996**
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max}^{**})$	**.008**	**.012**
Dates Λ_s^{**} **significant at 5%**	**1965, 74**	**1965, 1974**
asymptotic Sup-W	**59.18 (27.13)**	

Bai, Lumsdaine & Stock type : $\Lambda^*, \; -T\ln(\Lambda_s^*)$

Predictive type : $-T\ln(\Lambda), \; -T\ln(\Lambda_s), \; \Lambda_{\max}$

Oulier type : $\Lambda_{\max}^{**}, \; \Lambda_s^{**}$

	LMC	**MMC**
	Paper and Allied	
$\hat{\mathbf{p}}_N^{\sup}(\Lambda^*)$	**.018**	**.050**
$\hat{\mathbf{p}}_N^{\sup}(-T\ln(\Lambda))$	**.004**	**.016**
Dates Λ_s significant at 5%	**1988-91**	**1988-90**
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max})$	**.014**	**.054**
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max}^{**})$	**.002**	**.004**
Dates Λ_s^{**} significant at 5%	**1966, 89**	**1966, 89**
asymptotic Sup-W	**133.31 (47.81)**	
	Other	
$\hat{\mathbf{p}}_N^{\sup}(\Lambda^*)$	**.202**	**.404**
$\hat{\mathbf{p}}_N^{\sup}(-T\ln(\Lambda))$	**.458**	**.490**
Dates Λ_s significant at 5%	$\varnothing$	$\varnothing$
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max})$	**.270**	**.528**
$\hat{\mathbf{p}}_N^{\sup}(\Lambda_{\max}^{**})$	**.112**	**.186**
Dates Λ_s^{**} significant at 5%	$\varnothing$	$\varnothing$
asymptotic Sup-W	**76.29 (47.81)**	

$$\underline{\textit{Bai, Lumsdaine \& Stock type}:}\ \Lambda^*,\ -T\ln(\Lambda_s^*)$$

$$\underline{\textit{Predictive type}:}\ -T\ln(\Lambda),\ -T\ln(\Lambda_s),\ \Lambda_{\max}$$

$$\underline{\textit{Oulier type}:}\ \Lambda_{\max}^{**},\ \Lambda_s^{**}$$

- Even a simple bootstrap-type correction reverses the decision of Bai, Lumsdaine & Stock asymp. test.
- For the petroleum and coal sector, beside detected outliers (in 1965 and 1974) $\rightarrow$ specification is upheld. Dates detected $\simeq$ adoption & abolition of Borden line policy.
- In contrast, break dummies have no effect on the normality test decision for the paper and allied industries.
  - Small values for coal use variable, occurring more frequently than predicted by a normal distribution.
  - Some industries in Québec also generate (generally, in a random fashion) their own electricity
  - Empirically, we find that fitting a multivariate student-t distribution leads to viable and stable specification for this industry.
  - We treat both the degrees-of-freedom κ and the lag coefficient λ as nuisance parameters. The 97.5% set estimate for κ is the interval $[2, 6]$ which reveals important kurtosis. The largest p-values for the statistics Λ^*, $-T\ln(\Lambda)$, $\Lambda_{\max}$ and $\Lambda^{**}_{\max}$ (respectively) are 0.402, 0.199, 0.498 and 0.919 over the confidence set $CS^*_{\lambda,\kappa}$ and 0.396, 0.220, 0.511 and 0.753, over the confidence set $CS^{**}_{\lambda,\kappa}$.

Conclusion

- Finite sample multivariate structural stability tests.
- We show how to correct the size of available tests, and introduce alternative predictive tests and tests for outlier observations.
- In dynamic models, we account for nuisance parameters through a two-stage confidence set maximized Monte Carlo (CSMMC) procedure
- These tests are applied to the energy demand model analyzed by Arsenault et al. (1995). In 2 out of 6 industrial sectors analyzed with yearly data which spans the 1962-2000 period, our tests allow to identify (and correct) specification problems arising from historical regulatory changes or (possibly random) industry-specific effects.
- The procedures we propose have potential useful applications in statistics, econometrics and finance.
- Our results suggest two promising research avenues:
  1. the development of further test strategies for the detection of multiple breaks;
  2. VAR extensions, dealing with (often many) more nuisance parameters.